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# Unity of Supersymmetry Breaking Models

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## ABSTRACT

We examine the models with gauge group  $U(1)^{k-1} \times \prod_{i=1}^k SU(n_i)$ , which are obtained from decomposing the supersymmetry breaking model of Affleck, Dine and Seiberg containing an antisymmetric tensor field. We note that all of these models are distinct vacua of a single  $SU(N)$  gauge theory with an adjoint superfield. The dynamics of this model may be analyzed using the duality of Kutasov and Schwimmer and the deconfinement trick of Berkooz. This analysis leads to a simple picture for supersymmetry breaking for  $k = 2$ , complementing that of previous work. We examine the flat directions of these models, and give straightforward criteria for lifting them, explaining the requisite peculiar form of the superpotential. For all cases with  $k > 2$ , the duality argument fails to give supersymmetry breaking dynamics, and we identify a class of problematic flat directions, which we term  $2m$ -baryons. We study in some detail the requirements for lifting these directions, and uncover some surprising facts regarding the relationship between  $R$ -symmetry and supersymmetry breaking in models with several gauge groups.

## 1. Introduction

In previous papers[1] it was shown that a large class of models could break supersymmetry. These models could be obtained from the Affleck–Dine–Seiberg model [2,3,4] with an antisymmetric tensor and antifundamentals simply by removing group generators[5] and decomposing the representations under a subgroup of the initial gauge symmetry. The models considered had gauge group  $SU(n_1) \times SU(n_2) \times U(1)$ ; in what follows, we refer to these as the  $n_1-n_2-1$  models. An interesting aspect is that the gauge group factors of the resulting models are in different phases. Nonetheless, models with dynamical superpotentials, quantum confinement, smooth confinement, and a more weakly coupled dual phase could all be shown to exhibit supersymmetry breaking. Related work appears in Refs. [6,7,8,9] and references therein.

Although the gauge dynamics of the specific examples was different, there was a unifying picture for the mechanism of dynamical supersymmetry breaking which applied to all models. In all cases in which there was not initially a dynamical superpotential for at least one of the gauge groups (that is a superpotential generated by instantons or gaugino condensation which would drive fields to large values), the dynamics of one of the gauge groups was such that most flavors of the other gauge group were given mass. In the low energy theory there was always a dynamical superpotential for this second gauge group with the few remaining light flavors. In all cases, the low energy theory contains a field  $\phi$  which is set to zero by the equation of motion of a composite field. This is inconsistent with another equation of motion, as the dynamical superpotential removes the origin  $\phi = 0$ . This alone would not suffice to break supersymmetry if there were to exist runaway directions. Another common feature of all models considered in Refs. [1] was that all flat directions could be lifted by a renormalizable superpotential (though sometimes with a rather mysterious flavor structure for the couplings, to be explained later). The common features of the supersymmetry breaking mechanism suggests the existence of a more unified picture of all models. In this paper we show that these models can be analyzed simultaneously by the introduction of a massive adjoint field. The individual models can be understood as particular vacua of the theory with the adjoint.

It might seem surprising that much can be learned from the theory with an adjoint, which is notoriously difficult to understand. At present, models with an adjoint are understood only in the presence of other fields in the fundamental representation and in the presence of a superpotential.[10] Our theories on the other hand contain an antisymmetric

tensor of each gauge group present. However, we can use the deconfinement trick of Ref. [11] to interpret the theory as the low energy limit of a confined  $Sp(m)$  theory. The other remarkable aspect is that it is precisely the theory with a superpotential considered by Kutasov that is required for our analysis. In fact, the superpotential  $\Sigma^{k+1}$  is necessary in order to generate the vacua of the form  $U(1)^{k-1} \times \prod_{j=1}^k SU(n_j)$ . Note that the models considered previously[1] all correspond to  $k = 2$ .

The very interesting result is that for the  $k = 2$  theories, there exists a general and very similar mechanism to that of Refs. [1] through which supersymmetry breaking can be understood simultaneously for all the examples. In particular, there is a field whose equation of motion from the superpotential is set to zero which is inconsistent with low energy dynamics of a dynamical superpotential. However, here the dynamical superpotential is generated by the  $Sp$  dynamics, which is the only possible way all theories could have been understood simultaneously! This is after many flat directions are lifted through the combination of the superpotential and  $SU$  duality.

This remarkably compact picture of all models suggests probing further to the applicability of this picture to other models, the most obvious generalization being the  $k > 2$  models. We find the dual picture is quite different. Unlike the  $k = 2$  model, the dual theory is actually more strongly coupled. Furthermore there are many more massless fields charged under  $Sp$  which remain in the low energy theory, so that the analysis based on the  $Sp$  dynamics is inconclusive.

This leads to the suspicion that the theories based on  $U(1)^{k-1} \times \prod_{j=1}^k SU(n_j), k > 2$ , do not in fact break supersymmetry. We argue based on explicit examples and a general analysis of  $R$ -symmetry that these theories are unlikely to break supersymmetry. One key element is the emergence of new types of flat direction, which we term  $2m$ -baryons (dibaryons in the  $m = 1$  case), which cannot be lifted by the renormalizable superpotential, or indeed, any superpotential which preserves an  $R$ -symmetry.

In this analysis, we learn some new facts about the role of  $R$ -symmetry. In particular, many examples can be shown to break supersymmetry with only an effective  $R$ -symmetry applying to a low-energy version of the theory. Furthermore it is not always necessary that the  $R$ -symmetry be anomaly-free under all gauge group factors.

In the next section we define our model and analyze the nonperturbative dynamics of the  $k = 2$  theory. In section 3, we examine what can be learned from the strongly coupled  $k = 3$  theory. In section 4, we show that all flat directions are lifted for the  $k = 2$  theories.

Particular attention is paid to the dibaryon and  $2m$ -baryonic flat directions. In section 5, we consider in more detail some examples. We argue based on these models that we do not expect the  $k > 2$  vacua to give a supersymmetry breaking vacuum. We also clarify the role of  $R$ -symmetry in these models. We conclude in the final section. An appendix completes the proof of the lifting of flat directions in the  $n_1-n_2-1$  models. A second appendix shows that there are no dangerous flat directions in the deconfined theory which would restore supersymmetry.

## 2. Supersymmetry Breaking and Duality for $k = 2$

We begin by considering an  $SU(N)$  gauge theory, for odd  $N$ . We take matter in the following representations of the gauge group:

$$\begin{aligned}\Sigma &= \mathbf{Adj} \\ A &= \square \\ \bar{F}_I &= \overline{\square}, \quad I = 1, \dots, (N-4).\end{aligned}\tag{2.1}$$

This is just the matter content of Ref. [4], with the addition of an adjoint superfield. In general, we will have a superpotential for  $\Sigma$  of the form

$$W_\Sigma = \sum_{j=2}^{k+1} \frac{s_{k+1-j}}{j} \text{tr} \Sigma^j.\tag{2.2}$$

For generic  $s_j$ , there are several discrete classical vacua, in which  $\Sigma$  is massive. First, there is a vacuum at  $\langle \Sigma \rangle = 0$ , which has the spectrum of the ADS model.[4] In addition, there are vacua with gauge group  $U(1)^{k-1} \times \prod_{j=1}^k SU(n_j)$  where  $\sum n_j = N$ . For  $k = 2$ , these models have been studied in Refs. [1].

If supersymmetry is to be broken, it is clear that couplings must be added to (2.2). For  $k = 2$ , we consider a superpotential of the form

$$W = \frac{1}{2} m \text{tr} \Sigma^2 + \frac{1}{3} s_0 \text{tr} \Sigma^3 + \lambda_1^{IJ} \bar{F}_I A \bar{F}_J + \lambda_2^{IJ} \bar{F}_I A \Sigma \bar{F}_J + \lambda_3^{IJ} \bar{F}_I \Sigma A \Sigma \bar{F}_J\tag{2.3}$$

For each of the vacua mentioned above, this superpotential reduces, for arbitrary  $\lambda_i$ , to the form necessary for supersymmetry breaking [4,1].

In this paper, we will show that supersymmetry is broken dynamically in the full theory (2.3), for all  $m$ . This may at first sight seem rather strange; at  $m = 0$  it would

appear that there is a supersymmetry-preserving ground state at the origin, and there are effectively too many flavors to lead to any non-perturbative effects. At least in the absence of the superpotential, there are effectively<sup>4</sup>  $N_{F,eff} = 2N - 3$ ; one might expect then that the theory is in a non-Abelian Coulomb phase. However, as we will show, and is clear from the supersymmetry breaking analyses, the superpotential is quite relevant<sup>5</sup>. We find that there is a useful dual description in which the Yukawa terms are mass terms; the physics of these is then clear. A full analysis of the dual theory leads to supersymmetry breaking (in a way apparently independent of the existence of the adjoint field) given certain minimum requirements on the Yukawa couplings. It is interesting that there is a link between the classical lifting of flat directions and a simple dual non-perturbative phenomena (gaugino condensation). As mentioned in the introduction, the phenomena found here is very similar to that of Refs. [1]. In the present case however, it is the duality of Refs. [10] that is used for the analysis.

### 2.1. Deconfinement and Duality

As we have indicated above, there is some indication that the idea that the model (2.3) is in the non-Abelian Coulomb phase may be fallacious. The picture that we would like to advocate is that the Yukawa couplings are, at the would-be infrared fixed point, relevant operators which cause flow away to some new point (perhaps strong coupling, or another fixed point). However, the description at hand is unable to make clear the appropriate physics, as the Yukawa couplings are cubic, quartic and quintic terms. This may not be the case in a dual description however.

A dual description of the theory (2.1),(2.3) is not available. However, it is easy to derive one by combining the results of Refs. [11,10]. The technique is to ‘deconfine’ the antisymmetric tensor by introducing a new gauge group  $Sp(m)$ , for  $2m = N - 3$ , and matter (classified under  $SU(N) \times Sp(m)$ ):

$$\begin{aligned} Y &= (\square, \square) \\ Z &= (\mathbf{1}, \square) \\ \bar{P} &= (\overline{\square}, \mathbf{1}) \end{aligned} \tag{2.4}$$

and a superpotential  $W = c(YZ)\bar{P}$ . The  $Sp(m)$  group is confining; the gauge invariant fields are  $A = (YY)/\mu'$  and  $P = (YZ)/\mu'$ . A superpotential  $Y^N Z \sim A^{m+1} P$  is generated

<sup>4</sup> This is the quantity that appears in the  $\beta$ -function,  $N_{F,eff} = \sum_i T(R_i)$ .

<sup>5</sup> to our discussion, as well as in the RG sense.

and the tree-level superpotential leads to a mass term for  $P$  and  $\bar{P}$ . Thus the low energy theory is equivalent to the theory of an antisymmetric tensor without a superpotential.

In the case at hand, we construct an  $SU(N) \times Sp(m)$  theory with matter

$$\begin{aligned}\Sigma &= (\mathbf{Adj}, \mathbf{1}) \\ Y &= (\square, \square) \\ Z &= (\mathbf{1}, \square) \\ \bar{F}_I &= (\overline{\square}, \mathbf{1}) \quad I = 1, \dots, (N-4) \\ \bar{P} &= (\overline{\square}, \mathbf{1})\end{aligned}\tag{2.5}$$

and the superpotential

$$\begin{aligned}W = &\frac{1}{2}m \operatorname{tr} \Sigma^2 + \frac{1}{3}s_0 \operatorname{tr} \Sigma^3 + c Y \bar{P} Z \\ &+ \frac{1}{\mu'} \left\{ \lambda_1^{IJ} Y \bar{F}_I Y \bar{F}_J + \lambda_2^{IJ} Y \bar{F}_I Y \Sigma \bar{F}_J + \lambda_3^{IJ} Y \Sigma \bar{F}_I Y \Sigma \bar{F}_J \right\}\end{aligned}\tag{2.6}$$

We are now in possession of a theory that we know how to dualize:  $SU(N)$  with an adjoint and  $N_F = N - 3$  flavors, and  $W \supset \operatorname{tr} \Sigma^3$ . Treating  $Sp(m)$  as a spectator, the dual theory [10] will have gauge group  $SU(\tilde{N} = N - 6) \times Sp(m)$ . The matter includes  $SU(\tilde{N})$ -singlet fields  $M_I^a \sim Y^a \bar{F}_I$ ,  $M_0^a \sim Y^a \bar{P}$ ,  $N_I^a \sim Y^a \Sigma \bar{F}_I$  and  $N_0^a \sim Y^a \Sigma \bar{P}$ , as well as dual quarks  $y^a$ ,  $\bar{f}^I$ ,  $\bar{p}$ . There is also the adjoint  $\tilde{\Sigma}$  and the spectator  $Z$ . The superpotential is[12]

$$\begin{aligned}\tilde{W} = &-\frac{1}{3}s_0 \operatorname{tr} \tilde{\Sigma}^3 - \frac{1}{2}\tilde{m} \operatorname{tr} \tilde{\Sigma}^2 + \frac{1}{\mu'} \tilde{\lambda}_1^{IJ} (M_I M_J) + \frac{1}{\mu'} \tilde{\lambda}_2^{IJ} (M_I N_J) + \frac{1}{\mu'} \tilde{\lambda}_3^{IJ} (N_I N_J) \\ &+ c(M_0 Z) + \frac{s_0}{\mu'^2} \left( M_I \bar{f}^I \tilde{\Sigma} y + M_0 \bar{p} \tilde{\Sigma} y + (\tilde{b} M_I + N_I) \bar{f}^I y + (\tilde{b} M_0 + N_0) \bar{p} y \right)\end{aligned}\tag{2.7}$$

where  $SU(\tilde{N})$  contractions are understood, and  $Sp(m)$  contraactions appear in parentheses. Also, we have  $\tilde{\lambda}_1 = \lambda_1 - b\lambda_2 + b^2\lambda_3$ ,  $\tilde{\lambda}_2 = \lambda_2 - 2b\lambda_3$  and  $b = m/2s_0$ ,  $\tilde{b} = -bN_c/\tilde{N}_c$ . There is a subtlety here, as the field  $M$  has dimension less than one at the fixed point[10], and thus should decouple in the infrared. However, we also have a mass term for these fields, and it is appropriate to integrate them out of the theory. The number of massive modes clearly is dictated by the rank of the Yukawa coupling matrices.

To study this, collect the mesons together as  $\Phi_I = (M_I, N_I)$ , and then the mass term can be written as  $\Phi \cdot \mathcal{M} \cdot \Phi$ , where

$$\mathcal{M} \equiv \begin{pmatrix} \tilde{\lambda}_1 & \tilde{\lambda}_2 \\ -\tilde{\lambda}_2^T & \lambda_3 \end{pmatrix}\tag{2.8}$$

The mesons also couple linearly to dual mesons  $\Phi \cdot R$ , where  $R = (\tilde{b}\bar{f}y + \bar{f}\tilde{\Sigma}y, \bar{f}y)$ . If  $\mathcal{M}$  has maximal rank, we can integrate out all of the components of  $\Phi$  and then only  $m - 1$  flavors of  $Sp(m)$  are left; this theory will generate a dynamical superpotential. We find

$$\tilde{W} = \tilde{W}_{\tilde{\Sigma}}(\tilde{\Sigma}) + W_{irr}(y, \bar{f}, \tilde{\Sigma}) + \frac{s_0}{\mu^2} \bar{p}(N_0 y) + \frac{\Lambda_m^{m+2}}{(y^{\tilde{N}} N_0)^{1/2}} \quad (2.9)$$

where  $\tilde{W}_{\tilde{\Sigma}}$  is as in (2.7), and  $W_{irr}$  is a low-energy tree-level superpotential of the form  $R \cdot \mathcal{M}^{-1} \cdot R$ .

It is convenient to define  $\tilde{A} = yy/\tilde{\mu}$ ,  $p = yN_0/\tilde{\mu}$ , to rewrite the superpotential as

$$\tilde{W} = \tilde{W}_{\tilde{\Sigma}}(\tilde{\Sigma}) + W_{irr}(A, \bar{f}, \tilde{\Sigma}) + \frac{s_0 \tilde{\mu}}{\mu^2} p \cdot \bar{p} + \frac{\Lambda_m^{m+2}}{\tilde{\mu}^{(m-1)/2} (\tilde{A}^{m-2} p)^{1/2}} \quad (2.10)$$

The superpotential is thus very similar to that found in Refs. [1] for  $SU(n_1) \times SU(n_2) \times U(1)$ . Therefore, the F-terms for  $p$  and  $\bar{p}$  are inconsistent.

We note however that we have not yet shown that flat directions are absent. It is important to verify that such flat directions do not exist so that there is no runaway direction along which the potential slopes to zero. To show that this does not occur, in a later section, we will analyze the moduli space of the individual vacua and argue that they are compact. In all cases, the low energy theory contains a field which is set to zero by the equation of motion of a composite field. This is inconsistent with another equation of motion, as the dynamical superpotential removes the origin. It is encouraging that duality has given us the type of superpotential that we would expect, similar to those of Refs. [1]; apparently the supersymmetry breaking mechanisms found in those vacua extend to the full theory.

We have also obtained some insight into the necessary form of the Yukawa couplings: they must have sufficient rank as to induce non-perturbative dynamics for the  $Sp(m)$  group, and thus induce supersymmetry breaking. This is a necessary condition, but it is not sufficient. There are further restrictions, which are related to the comments above. To see this, let us explicitly invert the Yukawa matrix

$$\mathcal{M}^{-1} = \begin{pmatrix} \tilde{\lambda}_2^{-T} \lambda_3 X^{-1} & -X^{-T} \\ X^{-1} & \tilde{\lambda}_2^{-1} \tilde{\lambda}_1 X^{-T} \end{pmatrix} \quad (2.11)$$

where  $X = \tilde{\lambda}_2 + \tilde{\lambda}_1 \tilde{\lambda}_2^{-T} \lambda_3$ . An illuminating example is the case  $\lambda_3 = \tilde{\lambda}_1 = 0$ ; then, for non-singular  $\lambda_2$ ,  $\mathcal{M}$  is invertible. However, in this case, many of the dual meson couplings

are missing, and so we will expect flat directions to exist. Thus in such a case, the  $Sp$  group will indeed induce a non-zero energy state, but there will presumably be runaways along these classically flat directions. Thus, the full condition on the  $\lambda_i$ -matrices involves a study of flat directions. In the following sections, we will study this problem in more detail.

The form of the dual superpotential found here is somewhat intriguing. Note that  $W_{\tilde{\Sigma}}$  and  $W_{irr}$  apparently play no direct role in the supersymmetry breaking (although they are the offspring of terms necessary to lift flat directions). Both of these terms are of an identical form to those of the electric theory. But also, the dual Yukawa coupling scales, apart from shifts (due to a shift in the adjoint field under duality) like the *inverse* of the electric Yukawa coupling  $\mathcal{M}$ . This hints at some underlying duality in Yukawa couplings.

## 2.2. $N = 7$

Strictly speaking, the analysis above is valid for  $N \geq 9$ , as it is only for these values for which there is a dual gauge group. For  $N = 7$ , the infrared physics is confining.[12] The physics of the confining phase may be understood, as sketched in Ref. [12], through duality. In fact, the  $N = 7$  case (which would have a dual gauge group “ $SU(1)$ ”) may be read off from (2.7), with the replacement of  $\tilde{\Sigma} = 0$ .<sup>6</sup> We thus obtain a confining superpotential

$$\begin{aligned}\tilde{W} = & \frac{1}{\mu'} \tilde{\lambda}_1^{IJ} (M_I M_J) + \frac{1}{\mu'} \tilde{\lambda}_2^{IJ} (M_I N_J) + \frac{1}{\mu'} \tilde{\lambda}_3^{IJ} (N_I N_J) + c(M_0 Z) \\ & + \frac{s_0}{\mu^2} \left( (\tilde{b} M_I + N_I) \bar{f}^I y + (\tilde{b} M_0 + N_0) \bar{p} y \right)\end{aligned}\tag{2.12}$$

In addition to this, there may be a non-perturbative contribution, presumably of the form  $\det \sum M$ . The exact form of this superpotential is not known; fortunately, we do not need to know its detailed form for the present discussion. We need only that it does not depend on the field  $\bar{p}$ .

Again, we must integrate out massive meson singlet fields, and we then obtain a result similar in form (up to the effect of the above-mentioned non-perturbative contribution) to (2.9). We see that the physics relevant to supersymmetry breaking of the  $SU(7)$  model is essentially identical to that for  $N \geq 9$ .

<sup>6</sup> In the following, we have fixed what we believe to be minor typos in Ref. [12](sec 6.2), related to setting  $\tilde{\Sigma}_s = 0$ , instead of  $\tilde{\Sigma} = 0$ .

### 2.3. Duality Without $\Sigma$

The analysis of the flat directions which we will present applies to the particular vacua of interest. We will not, apart from a few comments in Section 4.4, analyze the flat directions including the  $\Sigma$  field which is integrated out of the theory. This is sufficient since it is only the vacua which reproduce the supersymmetry breaking theories of interest. Nonetheless, it would be useful to verify that the  $\Sigma$  field serves only as a device to generate the desired vacua, and does not play an essential role in supersymmetry breaking dynamics. For this reason, we show that an analysis very similar to that of the previous section can be done once the vacuum is chosen. That is for a particular  $n_1-n_2-1$  model, one can deconfine the antisymmetric tensor via  $Sp$  dynamics, eliminating the  $\Sigma$  field altogether.

Consider the  $n_1-n_2-1$  model which is parented by the  $SU(N)$  ADS model with  $N = n_1 + n_2$ . The antisymmetric tensor  $A$  decomposes into  $A_1$ ,  $A_2$ ,  $T$ . Notice that the deconfinement trick we used before for  $A$  can be used to decompose its fragments as well. Doing so we have an  $SU(n_1) \times SU(n_2) \times Sp(m)$ , with  $2m = N - 3$  and matter content

$$\begin{aligned} Y_1 &= (\square, \mathbf{1}, \square) \\ Y_2 &= (\mathbf{1}, \square, \square) \\ Z &= (\mathbf{1}, \square) \\ \bar{F}_{1I} &= (\overline{\square}, \mathbf{1}, \mathbf{1}) \\ \bar{F}_{2I} &= (\mathbf{1}, \overline{\square}, \mathbf{1}) \quad I = 1, \dots, (N-4) \\ \bar{P}_1 &= (\overline{\square}, \mathbf{1}, \mathbf{1}) \\ \bar{P}_2 &= (\mathbf{1}, \overline{\square}, \mathbf{1}) \end{aligned} \tag{2.13}$$

The superpotential is

$$W = \frac{1}{\mu'} g_1^{IJ} Y_1 \bar{F}_{1I} Y_1 \bar{F}_{1J} + \frac{1}{\mu'} g_2^{IJ} Y_2 \bar{F}_{2I} Y_2 \bar{F}_{2J} + \frac{1}{\mu'} g_3^{IJ} Y_1 \bar{F}_{1I} Y_2 \bar{F}_{2J} + (c_1 Y_1 \bar{P}_1 + c_2 Y_2 \bar{P}_2) Z \tag{2.14}$$

where the matrices  $g_i$  can be written as linear combinations of the  $\lambda_i$  matrices of eq. (2.3). The discussion parallels the one of the previous section. The confined  $Sp(m)$  theory is the  $n_1-n_2-1$  model with the relevant low energy degrees of freedom  $A_1 \sim Y_1^2$ ,  $A_2 \sim Y_2^2$ ,  $T \sim Y_1 Y_2$  and  $P_{1,2} \sim Y_{1,2} Z$ . A superpotential  $\sim A^{m+1} P$  (with obvious notation) is also generated and the tree level superpotential leads to mass terms for  $P_{1,2}$  and  $\bar{P}_{1,2}$ . Let us choose  $n_1 > n_2$ . Then for  $5 \leq n_2 \leq (N-1)/2$ , both  $SU(n_1)$  and  $SU(n_2)$  admit an equivalent dual description [13]. The dual gauge group is  $SU(\tilde{n}_1) \times SU(\tilde{n}_2) \times U(1)$ ,

with  $\tilde{n}_{1,2} = N - 3 - n_{1,2}$ . The matter includes  $SU(\tilde{n}_1) \times SU(\tilde{n}_2) \times U(1)$  singlet fields  $M_{1I}^a \sim Y_1^a \bar{F}_{1I}$ ,  $M_{2I}^a \sim Y_2^a \bar{F}_{2I}$ ,  $M_{10}^a \sim Y_1^a \bar{P}_1$ ,  $M_{20}^a \sim Y_2^a \bar{P}_2$  as well as dual quarks  $y_{1,2}^a$ ,  $\bar{f}_{1,2}$ ,  $\bar{p}_{1,2}$ . The field  $Z$  is a spectator. The superpotential is now

$$\begin{aligned}\tilde{W} = & \frac{1}{\mu'} g_1^{IJ} M_{1I} M_{1J} + \frac{1}{\mu'} g_2^{IJ} M_{2I} M_{2J} + \frac{1}{\mu'} g_3^{IJ} M_{1I} M_{2J} + (c_1 M_{10} + c_2 M_{20}) Z \\ & + M_{1I} y_1 \bar{f}_{1I} + M_{10} y_1 \bar{p}_1 + M_{2I} y_2 \bar{f}_{2I} + M_{20} y_2 \bar{p}_2\end{aligned}\quad (2.15)$$

Now if both  $g_3$  and  $g_3 + g_1^T g_3^{-1} g_2$  are non-singular<sup>7</sup> we can integrate out all  $M_{1I}$  and  $M_{2J}$ ; moreover also  $Z$  and the combination  $c_1 M_{10} + c_2 M_{20}$  pair up and get a mass. Calling  $N_0 = c_2 M_{10} - c_1 M_{20}$ , we are left with the coupling  $N_0(y_1 \bar{p}_1 + y_2 \bar{p}_2)$ . Then when  $Sp(m)$  dynamics generates a superpotential, we are lead to supersymmetry breaking, as was the case in the theory with adjoint. In this case however we have a proof that all flat directions are lifted and thus we can rigorously conclude that there is a stable vacuum. A discussion of that appears in Section 4 and in Appendix A. Notice that to reach our conclusion, we have taken the limit where the dual gauge group  $SU(\tilde{n}_1) \times SU(\tilde{n}_2) \times U(1)$  gauge factor is weakly coupled and acts just as a spectator: its only role is to lift flat directions at the classical level.

### 3. Supersymmetry Breaking and Duality for $k > 2$

We now wish to attempt a construction similar to that of previous sections for the cases  $k > 2$ . We begin again with the matter (2.1) and a superpotential of the general form

$$W = \sum_{j=2}^{k+1} \frac{s_{k+1-j}}{j} \text{tr} \Sigma^j + \sum_{m,n=1}^k \lambda_{mn}^{IJ} \bar{F}_I \Sigma^{m-1} A \Sigma^{n-1} \bar{F}_J + \dots \quad (3.1)$$

Generically, the model has several classical vacua on the Coulomb branch. To find these vacua, we consider solutions to the  $\Sigma$  equation of motion. This gives

$$\sum_{j=1}^k s_{k-j} x^j + \Delta = 0 \quad (3.2)$$

with the Lagrange multiplier  $\Delta$  being determined by the tracelessness of  $\Sigma$ ,  $\Delta = -\frac{1}{N_c} \sum_{j=1}^k s_{k-j} \text{tr} \Sigma^j$ . A given vacuum will be determined by the number of each solution to (3.2) appearing in  $\langle \Sigma \rangle$ , that is, the number of distinct eigenvalues.

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<sup>7</sup> There is a clear analogy here with the matrix  $\mathcal{M}$  of Section 2.1.

We follow the construction of the  $k = 2$  case, deconfining the antisymmetric tensor field in terms of an  $Sp(m = N - 3)$  gauge group. The resulting superpotential is

$$W = \sum_{j=2}^{k+1} \frac{s_{k+1-j}}{j} \text{tr} \Sigma^j + \sum_{m,n=1}^k \lambda_{mn}^{IJ} (Y \Sigma^{m-1} \bar{F}_I) (Y \Sigma^{n-1} \bar{F}_J) + c Y \bar{P} Z + \dots \quad (3.3)$$

The dual theory[10] will have gauge group  $SU(\tilde{N} = (k-1)N - 3k) \times Sp(m)$ . The matter includes  $SU(\tilde{N})$ -singlet fields  $(M_{(j)})_I^a \sim Y^a \Sigma^{j-1} \bar{F}_I$ ,  $(M_{(j)})_0^a \sim Y^a \Sigma^{j-1} \bar{P}$ , as well as dual quarks  $y^a$ ,  $\bar{f}^I$ ,  $\bar{p}$ . There is also the adjoint  $\tilde{\Sigma}$  and the spectator  $Z$ . The superpotential is

$$\begin{aligned} \tilde{W} = & \sum_{j=2}^{k+1} \frac{\tilde{s}_{k+1-j}}{j} \text{tr} \tilde{\Sigma}^j + \sum_{j=1}^k \sum_{\ell} c_{\ell,j} (M_{(j)})_0^a \bar{p} \tilde{\Sigma}^\ell y^b \mathcal{J}_{ab} + \sum_{j=1}^k \sum_{\ell} c_{\ell,j} (M_{(j)})_I^a \bar{f}^I \tilde{\Sigma}^\ell y^b \mathcal{J}_{ab} \\ & + \sum_{m,n=1}^k \tilde{\lambda}_{mn}^{IJ} (M_{(m)})_I^a (M_{(n)})_J^b \mathcal{J}_{ab} + c Z^a (M_{(1)})_0^b \mathcal{J}_{ab} \end{aligned} \quad (3.4)$$

where  $SU(\tilde{N})$  contractions are understood, and  $\mathcal{J}_{ab}$  is the invariant tensor of  $Sp(m)$ . The  $\tilde{\lambda}$  matrices are linear combinations of the original Yukawa matrices, while  $c_{\ell,j}$  are functions of the couplings appearing in the  $\Sigma$ -dependent part of the superpotential.

The number of massive modes is again dictated by the rank of the Yukawa couplings  $\lambda_{mn}$ . However, even if the number of massive modes is maximal, there are still  $\frac{\tilde{N}+k-1}{2} = (2m+1)k/2 - m - 2$  flavors of  $Sp(m)$  left massless (if  $k$  is even).<sup>8</sup> This is due essentially to the presence of  $y$ : since  $\tilde{N}$  grows with  $k$ , there are always many flavors left; only in the case  $k = 2$  are there just  $m - 1$  flavors. The  $Sp(m)$  theory will not then generate a superpotential, and it is not clear that supersymmetry is (or is not) broken. The most conservative view is that it is not.

The analysis is clearly inconclusive. It is clear that for the  $k > 2$  models,  $Sp$  dynamics in the dual theory will not in and of itself suffice to prove supersymmetry breaking. In principle, the  $SU$  dynamics can be relevant, and lead to a supersymmetry breaking vacuum. Indeed this is precisely what happens in the vacua that are dual to those special electric vacua where  $SU(N)$  is only broken to  $SU(n_1) \times SU(n_2) \times U(1)$ . In other words, the  $k > 2$  case contains  $k = 2$  particular vacua. Consider for instance  $k = 3$  and focus on the vacua of the magnetic theory where  $SU(\tilde{N} = 2N - 6) \rightarrow SU(N - 3) \times SU(\tilde{n}_1) \times SU(\tilde{n}_2) \times$

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<sup>8</sup> For odd  $k$ , there is one massless mode in the  $M$  mass matrix, and we get at least  $(2m+1)(k+1)/2 - m - 3$  flavors instead.

$U(1)^2$ . The first  $SU$  factor is confining with a quantum modified moduli space. When the confining dynamics is accounted for, one reproduces the results of the  $k = 2$  case considered previously. This analysis is analogous to that of Ref. [10].

In general however, it is not clear what role the  $SU$  dynamics will play. These models therefore merit further investigation, which we do in a later section. We will find that an essential distinction of the  $k > 2$  models is that there exist flat directions involving only the fragments from the decomposition of the ADS adjoint field which are not lifted by the cubic superpotential. It appears to be impossible to lift these directions without introducing a supersymmetric minimum.

#### 4. $k = 2$ Flat Directions

Let us now return to the case  $k = 2$ , and show that there are no flat directions. We begin with a discussion of the individual vacua with  $SU(n_1) \times SU(n_2) \times U(1)$  gauge group. Following this, we return to the full theory with  $\Sigma$  in Section 4.4.

##### 4.1. Cubic Invariants

We study the  $n_1-n_2-1$  model with a generic cubic superpotential. We will derive a set of necessary requirements as well as a set of sufficient ones that the Yukawa matrices must satisfy in order to classically lift all flat directions. Rank maximality will turn out to be a necessary requirement, without which there are unlifted flat directions. Other requirements on the orientation in flavour space and on the eigenvalues of Yukawa matrices will turn out to be sufficient to give a simple proof that all flat directions are lifted. The interesting result is that the Yukawa matrices which satisfy all our requirements clearly represent a set of “non-zero measure” in the space of couplings, *i.e.*, they represent a generic choice of couplings. In other words, flat directions are not lifted only at special points in the space of Yukawa couplings. However specifying what all these points are, *i.e.*, giving a set of necessary *and* sufficient requirements for lifting flat directions, seems to require considerably more effort, for which there is no apparent motivation. On the contrary, our analysis makes clear that the somewhat mysterious choice of Yukawa matrices which seemed to be needed in Refs. [1] corresponds to just one particular point in the vast set of matrices which satisfy our sufficient requirements. In the following discussion, we will make clear which points in parameter space that we avoid. In this section, and in Appendix

A, we will prove that the cubic flat directions are all lifted. In the following subsection, we will do the same for those corresponding to the higher order invariants.

We denote by  $A_{1,2}$  and  $T$  the antisymmetric and mixed tensors respectively, while  $\bar{F}_1 = (\bar{n}_1, 1)$  and  $\bar{F}_2 = (1, \bar{n}_2)$ . As a convention, we will take  $n_1 > n_2$ . The cubic invariants are

$$X_1^{ij} = \bar{F}_1^i A_1 \bar{F}_1^j, \quad X_2^{IJ} = \bar{F}_2^I A_2 \bar{F}_2^J, \quad M^{Ij} = \bar{F}_2^I T \bar{F}_1^j. \quad (4.1)$$

(where  $I, i = 1, \dots, n$ , for  $n = n_1 + n_2 - 4$ .) The cubic superpotential may be written<sup>9</sup>

$$W = g_{ij} X_1^{ij} + f_{IJ} X_2^{IJ} + \delta_{Ij} M^{Ij} \quad (4.2)$$

where  $g_{ij}, f_{IJ}$  are antisymmetric matrices. We will show below that a necessary condition for the classical lifting of all flat directions is that  $g, f, \delta$  be of maximal rank. In particular  $\delta$  must be invertible, so that by rotations and rescaling it can be taken equal to the identity matrix  $\delta = \mathbf{1}$ .

Let us study the F-term constraints given by the above superpotential. By contracting the equations of motion to form invariants, we get two classes of constraints, respectively linear and quadratic. The linear ones are the following.

$$\begin{aligned} \bar{F}_1^k \partial_{\bar{F}_1^i} W &= 2g_{ij} X_1^{kj} + \delta_{Ii} M^{Ik} \\ &\rightarrow -2g X_1 + M = 0 \end{aligned} \quad (4.3)$$

and

$$\begin{aligned} \bar{F}_2^K \partial_{\bar{F}_2^i} W &= 2f_{IJ} X_2^{KJ} + \delta_{Ii} M^{Ki} \\ &\rightarrow -2f X_2 + M^T = 0 \end{aligned} \quad (4.4)$$

while from  $A_1 \partial_{A_1}$ ,  $A_2 \partial_{A_2}$  and  $T \partial_T$  we get

$$\text{tr}(g X_1) = \text{tr}(f X_2) = \text{tr}(M) = 0. \quad (4.5)$$

In the second lines of eqs. (4.3) and (4.4), we have written the expressions in matrix form.

There are many quadratic constraints; examples are

$$(T \bar{F}_1^k)(A_1 \bar{F}_1^i) \partial_T W = \delta_{Ij} X_1^{ij} M^{Ik} \rightarrow X_1 M = 0 \quad (4.6)$$

$$(T \bar{F}_2^K)(A_2 \bar{F}_2^L) \partial_T W = \delta_{Ij} X_2^{LI} M^{Kj} \rightarrow -MX_2 = 0 \quad (4.7)$$

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<sup>9</sup> We have redefined the couplings  $g_i$ ; compare to the confined form of eq. (2.14).

One can easily show that, when  $\delta$  is nonsingular, given eqs. (4.3)–(4.7), the remaining quadratic constraints are redundant.

It is easy to show that if the rank maximality is not satisfied the full set of equations admits non zero solutions. For instance if  $\delta_{11} = 0$  in the diagonal basis, we have that  $M_{11}$  is totally unconstrained. And similarly for  $X_{1,2}$  entries when  $g, f$  are not of maximal rank. So we will from now on assume maximal rank for these matrices.

Since  $n$  is odd, the generic antisymmetric matrices  $g$  and  $f$  will have rank  $n - 1$ . By a change of basis which leaves  $\delta$  invariant we can always put one of them, say  $g$ , in the form

$$g = \begin{pmatrix} g' & 0 \\ 0 & 0 \end{pmatrix} \quad (4.8)$$

where  $g'$  is an invertible  $(n - 1) \times (n - 1)$  matrix. The matrix  $f$  will be of the form

$$f = \begin{pmatrix} f' & \rho \\ -\rho^T & 0 \end{pmatrix} \quad (4.9)$$

where  $\rho$  is an  $n - 1$  vector, and  $f'$  is  $(n - 1) \times (n - 1)$ . Now,  $n - 1$  is even so that it is clear that  $f'$  will be singular only at special points in parameter space. Then since we do not lose much of parameter space and since it makes the discussion simpler we will assume that  $f'$  is invertible. In Appendix A we show that it is sufficient to impose some additional mild requirements on  $f'$  and  $g'$  in order to easily conclude that all cubic flat directions are lifted. We stress once more that the resulting space of matrices is still of non-zero measure, consistent with what one may call a principle of “genericity”. Among the special points which are removed from this space will be the point  $f' \propto (g')^{-1}$ . The removal of this point is indeed necessary, as it was shown in [1] that there are unlifted flat directions. Moreover the superpotentials of Refs. [1], in particular, are simple examples which satisfy these requirements.

The genericity requirements derived in Appendix A place no restriction on the vector  $\rho$  in eq. (4.9). Notice that for a generic  $\rho \neq 0$ , the superpotential preserves no non-anomalous R-symmetry. An anomaly-free  $R$  symmetry for these models is flavor dependent; a generic superpotential breaks the  $R$  symmetries.

#### 4.2. Higher order invariants

We now consider invariants with more than three fields. These involve  $\epsilon$  and  $\bar{\epsilon}$  tensors for each group. By  $U(1)$  invariance and use of Fierz identities there are two classes of such

invariants. There are antibaryons, involving just  $\bar{F}_{1,2}$  contracted to  $\bar{\epsilon}_1\bar{\epsilon}_2$ . These objects only exist for  $n_2 \geq 4$ , so that the smallest model which has these D-flat directions is 5–4–1. The other class of invariants are baryonic, which involve fields contracted to  $(\epsilon_1\epsilon_2)^p$ . This class may be further divided into two subclasses, one involving matter fields  $\bar{F}_{1,2}$  and one whose elements are made purely of  $A_{1,2}$  and  $T$ . It turns out that elements in this second subclass, which we will refer to as  $2m$ -baryons (or in the simplest case, dibaryons) vanish identically. Since their vanishing is a peculiarity of  $k = 2$  models, we will discuss them in more detail in the next section. In this section, we study the antibaryons and the baryons which involve  $\bar{F}_{1,2}$ .

The lifting of antibaryons is a consequence of  $\partial_{A_{1,2}}W = 0$ , together with  $f$  and  $g$  being of maximal rank. Indeed let us assume that there remains a flat direction which has non-zero overlap with an antibaryon. By combined flavor and gauge rotations we can go to a basis where the vacuum expectation values are

$$\begin{aligned} (\bar{F}_1)_i^\alpha &= v_1^\alpha \delta_i^\alpha \quad 1 \leq i \leq n_1 & (\bar{F}_1)_i^\alpha &= 0 \quad i > n_1 \\ (\bar{F}_2)_I^A &= v_2^A \delta_I^A \quad 1 \leq I \leq n_2 & (\bar{F}_2)_I^A &= 0 \quad I > n_2 \end{aligned} \tag{4.10}$$

where  $\alpha$  and  $A$  are gauge indices. The e.o.m. of  $A_{1,2}$  are then

$$\begin{aligned} \partial_{A_{1\alpha\beta}} W &= (\bar{F}_1)_i^\alpha g^{ij} (\bar{F}_1)_j^\beta = v_1^\alpha v_1^\beta g^{\alpha\beta} = 0 \quad 1 \leq \alpha, \beta \leq n_1 \\ \partial_{A_{2AB}} W &= (\bar{F}_2)_I^A f^{IJ} (\bar{F}_2)_J^B = v_2^A v_2^B f^{AB} = 0 \quad 1 \leq A, B \leq n_2 \end{aligned} \tag{4.11}$$

with no sum over  $\alpha, \beta$  and  $A, B$ . The necessary condition for the F-flatness of the antibaryon is the existence of a null  $n_1 \times n_1$  submatrix in  $g$  and of a null  $n_2 \times n_2$  submatrix in  $f$ . This however conflicts with  $g$  being of maximal rank (recall  $n_1 > n_2$  and  $g$  is  $(n_2 + n_1 - 4) \times (n_1 + n_2 - 4)$ ). Indeed it can be easily shown that the existence of a  $n_1 \times n_1$  null submatrix bounds the rank of  $g$  to be  $\leq (n_1 + n_2 - 5) - (n_1 - n_2 + 3)$ , which is strictly smaller than the maximal rank  $n_1 + n_2 - 5$ . We conclude that all antibaryons have to vanish on the equations of motion.

Now consider the baryons which involve at least one power of  $\bar{F}_{1,2}$ . We will show that they may be reduced, by the F-term constraints, to products of operators which either vanish trivially, or are proportional to  $2m$ -baryon operators. The crucial remark here comes by contemplating the F-term constraints of matter fields suitably contracted to give such invariants. They are

$$(P_1)\partial_{\bar{F}_1}W = (P_1)(T\bar{F}_2) + (P_1)(A_1\bar{F}_1) = 0 \tag{4.12}$$

$$(P_2)\partial_{\bar{F}_2}W = (P_2)(T\bar{F}_1) + (P_2)(A_2\bar{F}_2) = 0 \quad (4.13)$$

where  $P_1$  and  $P_2$  are combinations of fields and  $\epsilon$ 's with, respectively, the same quantum numbers as  $\bar{F}_1$  and  $\bar{F}_2$ . Now, since the Yukawa matrix for  $\bar{F}_1 T \bar{F}_2$  is invertible, the above equations tell us that each operator  $O_1 = P_1 T \bar{F}_2$ ,  $O_2 = P_2 T \bar{F}_1$  is equal by the equations of motion to a combination of objects with one less power of  $T$ . Heuristically, we can write this result as

$$O_1 = O_1 \times \left( \frac{A_1 \bar{F}_1}{T \bar{F}_2} \right) = O_1 \times r_1 \quad O_2 = O_2 \times \left( \frac{A_2 \bar{F}_2}{T \bar{F}_1} \right) = O_2 \times r_2 \quad (4.14)$$

where, for example,  $\times r_1$  refers to the substitution in an operator of  $T\bar{F}_2$  with  $A_1\bar{F}_1$ . This result may now be iterated, until one gets on the right hand side an operator which does not have *enough* powers of  $T$ , and thus vanishes trivially. It then follows that the original  $O_{1,2}$ , as well as all the intermediate operators in the chain, must vanish by the equations of motion.

Thus all operators vanish by equations of motion, provided they can be written in the form  $(\dots)T\bar{F}_2$  (or  $(\dots)T\bar{F}_1$ ). It can be shown by fierzing that any operator can be brought to a form where only  $T$ 's interpolate between  $\epsilon$ 's, while each  $A$  has either both indices contracted to the same  $\epsilon$  or one contracted to  $\bar{F}$ . Consider then a situation in which one  $\epsilon$  is contracted to  $T$ 's,  $A_1$ 's and to  $(A_1\bar{F}_1)$ 's

$$(T^p)_K (A_1^q)_L [(A_1\bar{F}_1)^r]_M \epsilon^{KLM} \quad (4.15)$$

where  $K, L, M$  are appropriate groups of indices. By fierzing the gauge index of one  $\bar{F}_1$  into the  $\epsilon$ , we get a combination of the same operator plus one in which an  $\bar{F}_1$  is contracted to a  $T$ . The original object is then of the form  $(P)(T\bar{F}_1)$ . The same conclusion is obtained by considering  $A_2$ 's and  $A_2\bar{F}_2$ 's instead. The only other possibility is that no  $T$  is contracted to an  $\epsilon$  together with  $(A_1\bar{F}_1)$  or  $(A_2\bar{F}_2)$ . Then either the operator has a factor which is proportional to a  $2m$ -baryon, or there is an  $\epsilon$  for  $SU(n_1)$  which is only contracted to  $A_1$ 's, where we assume odd  $n_1$  for the sake of the argument. The latter case vanishes trivially, while the former will be shown to vanish in the next section. We have thus succeeded in eliminating flat directions involving  $\bar{F}$ 's.

Before moving on to  $2m$ -baryons, let us see how the above discussion works for the

simplest chain of invariants, that with only one power of  $\bar{F}$ . These are given by (odd  $n_1$ )

$$\begin{aligned} I_k &= A_2^{\frac{n_2}{2}-k} A_1^{\frac{n_1+1}{2}-k} T^{2k} \bar{F}_1 = I_0 \times \left(\frac{T^2}{A_1 A_2}\right)^k = I_0 (r_1 r_2)^{-k} \quad k = 0, \dots, k_{\max} \\ J_m &= A_2^{\frac{n_2}{2}-m} T^{2m+1} A_1^{\frac{n_1-1}{2}-m} \bar{F}_2 = J_0 \times \left(\frac{T^2}{A_1 A_2}\right)^m = J_0 (r_1 r_2)^{-m} \quad m = 0, \dots, m_{\max} \end{aligned} \quad (4.16)$$

where for  $n_1 > n_2$  we have  $(k_{\max}, m_{\max}) = (n_2/2, n_2/2 - 1)$ , while for  $n_1 < n_2$  we have  $(k_{\max}, m_{\max}) = (n_1/2 - 1/2, n_1/2 + 1/2)$ .

Notice that  $I_0 \equiv 0$ , and the above operators form the chain

$$I_0 \xleftarrow{r_1} J_0 \xleftarrow{r_2} I_1 \cdots J_{\max} \xleftarrow{r_2} I_{\max} \quad (4.17)$$

where by multiplying repeatedly by  $r_{1,2}$  we move on the chain from  $I_{\max}$  down to  $I_0$  (we consider here  $n_1 > n_2$ ). Now the equations of motion (4.14) simply become

$$0 \equiv I_0 = J_0 = I_1 = J_1 = \cdots = J_{\max} = I_{\max} \quad (4.18)$$

so that all the operators are zero and the corresponding flat directions are lifted.

#### 4.3. 2m-baryons

In this section we discuss the baryonic invariants that are made only of pieces of the antisymmetric tensor of the original ADS model and not the antifundamentals. The main result is that these objects vanish for the  $n_1-n_2-1$  models ( $k = 2$ ) while they exist, and are important, for the models with  $k > 2$ . For this reason, this discussion applies to general  $k$ . The gauge group is

$$U(1)^{k-1} \times \prod_{i=1}^k SU(n_i)$$

where  $\sum_i n_i = N$  and the antisymmetric tensor breaks down into:

$$\begin{aligned} A_i &= (\mathbf{1}, \dots, \mathbf{1}, \square_i, \mathbf{1}, \dots, \mathbf{1}) \quad i = 1, \dots, k \\ T_{ij} &= (\mathbf{1}, \dots, \mathbf{1}, \square_i, \mathbf{1}, \dots, \mathbf{1}, \square_j, \mathbf{1}, \dots, \mathbf{1}) \quad i, j = 1, \dots, k \quad i \neq j \end{aligned} \quad (4.19)$$

It is simple to see algebraically, since all gaugeable  $U(1)$ 's are gauged, that the most general baryonic operator can be written in the form

$$B_{\{m_i, p_{ij}\}} = \prod_i A_i^{m_i} T_{ij}^{p_{ij}} \quad (4.20)$$

with

$$2m_i + \sum_j p_{ij} = nn_i \quad (4.21)$$

for each  $i$ , where  $n$  is the number of  $\epsilon$ 's (for each group). There is the possibility that there is more than one contraction for each case, so there should be an additional label. Moreover, by summing eq. (4.21) over  $i$ , since  $N = \sum_i n_i$  is odd, we find that  $n$  is even. We then conclude that any baryon has an even number  $n = 2m$  of  $\epsilon$  tensors for each gauge group factor, so it is indeed a  $2m$ -baryon (i.e., the  $U(1)$  quantum numbers of the operator are all  $2m$ .).

Let us study the case  $k = 2$  first. To see that no  $2m$ -baryonic invariants exist, it is simplest to explore directly the required  $D$ -term constraints. We require

$$2A_1^\dagger A_1 + {}^T T^\dagger T = c_1 1. \quad (4.22)$$

$$2A_2^\dagger A_2 + T^\dagger T = c_2 1. \quad (4.23)$$

$$2n_2 \text{tr}(A_1^\dagger A_1) - 2n_1 \text{tr}(A_2^\dagger A_2) + (n_2 - n_1) \text{tr}(T^\dagger T) = 0 \quad (4.24)$$

Now, by diagonalizing  $A_1$  and  $A_2$ , eq. (4.22) tells us that  $\text{rank}(T) = \text{rank}({}^T T^\dagger T)$  is either odd or zero, while eq. (4.23) tells us that  $\text{rank}(T) = \text{rank}(T^\dagger T)$  is even (zero included). This, for consistency, requires  $T$  to be of zero rank, *i.e.*  $T = 0$ . Now eq. (4.22) can only be solved for  $A_1 = 0$ , so that by eq. (4.24),  $A_2$  has to also be zero.

However, once we consider  $k > 2$ , one can show that  $2m$ -baryonic flat directions exist. Indeed consider the  $k = 3$  case with gauge group  $SU(n_1) \times SU(n_2) \times SU(n_3) \times U(1)^2$ . The antisymmetric tensor of  $SU(N)$  decomposes as

$$A = \begin{pmatrix} A_1 & T_{12} & T_{13} \\ * & A_2 & T_{23} \\ * & * & A_3 \end{pmatrix} \quad (4.25)$$

with notation as in eqs. (4.19). Let us assume  $n_1$  and  $n_2$  are even. Then for example, the following direction is flat:

$$T_{12} = \begin{pmatrix} 0 & \cdots & 0 \\ \cdot & \cdots & \cdot \\ 0 & \cdots & x/\sqrt{2} \\ 0 & \cdots & 0 \end{pmatrix} \quad T_{23} = \begin{pmatrix} 0 & \cdots & 0 \\ \cdot & \cdots & \cdot \\ 0 & \cdots & x/\sqrt{2} \\ 0 & \cdots & 0 \end{pmatrix} \quad (4.26)$$

$$T_{13} = \begin{pmatrix} 0 & \cdots & 0 \\ \cdot & \cdots & \cdot \\ 0 & \cdots & x/\sqrt{2} \end{pmatrix} \quad (4.27)$$

with

$$\begin{aligned} A_1 &= x \operatorname{diag}(\sigma, \dots, \sigma, \sigma/\sqrt{2}) \\ A_2 &= x \operatorname{diag}(\sigma, \dots, \sigma, \sigma/\sqrt{2}) \\ A_3 &= x \operatorname{diag}(\sigma, \dots, \sigma, 0) \end{aligned} \quad (4.28)$$

The  $U(1)$  D-terms are important here. Indeed there exist  $SU(n_1) \times SU(n_2)$  flat directions involving only  $A_1$ ,  $A_2$  and  $T_{12}$  which are lifted by the  $U(1)$ 's. Similar flat directions can also be found in the case in which all three  $n_i$  are odd. Obviously the above is enough to assess the existence of these objects for the case  $k \geq 3$ , since it can be obtained for higher  $k$  by group reduction. We will consider these flat directions more fully in Section 5.

#### 4.4. Flat directions in the full model with $\Sigma$

Finally we would like to make a comment on the associated flat directions in the original theory with the adjoint  $\Sigma$ , *i.e.* those involving just  $A$  and  $\Sigma$ . Since the  $\bar{F}$  fields are not excited,  $F$ -flatness is just given by

$$\begin{aligned} \frac{\partial W}{\partial \Sigma} &= c_k \Sigma^k + \dots + c_1 \Sigma + b1 = 0 \\ b &= -\frac{1}{N} \sum_j c_j \operatorname{tr}(\Sigma^j) \end{aligned} \quad (4.29)$$

On the other hand, D-flatness corresponds to

$$A^\dagger A + \Sigma^\dagger \Sigma - \Sigma \Sigma^\dagger = c1. \quad (4.30)$$

Now it can be easily shown that for  $k = 2$  eq. (4.29) constrains  $\Sigma$  to be such that  $D_\Sigma = \Sigma^\dagger \Sigma - \Sigma \Sigma^\dagger$  has even rank, and thus eq. (4.30) has only the trivial solution  $A = 0$ ,  $D_\Sigma = 0$ . This does not happen at  $k > 2$ . The peculiarity of  $k = 2$  is more easily seen by considering the simple potential  $W = \operatorname{tr} \Sigma^{k+1}$ . Now at  $k = 2$  the equation of motion is just  $\Sigma^2 = 0$ , which implies that  $\Sigma^\dagger \Sigma$  and  $\Sigma \Sigma^\dagger$  are orthogonal to each other. Thus we have  $\operatorname{rank}(D_\Sigma) = 2\operatorname{rank}(\Sigma \Sigma^\dagger)$ . Now already at  $k = 3$  we have  $\Sigma^3 = 0$  and we cannot conclude much on the rank of  $D_\Sigma$ , which can now in fact be odd. Take for instance  $SU(3)$  and

$$\Sigma = u \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & \sqrt{2} \\ 0 & 0 & 0 \end{pmatrix} \quad (4.31)$$

Now we have

$$[\Sigma^\dagger, \Sigma] = |u|^2 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix} \quad (4.32)$$

Indeed the similar ways in which  $k = 2$  stands out as a special case in the above discussion and in duality may be worth further investigation.

## 5. Examples and the Role of $R$ -Symmetry

In the previous sections, we have found various pieces of evidence consistent with supersymmetry breaking for the  $k = 2$  models. On the other hand, the  $k > 2$  models behave quite differently; the duality construction failed to show supersymmetry breaking, and furthermore, we have been able to identify unlifted flat directions. In this section, we consider these models further, paying particularly close attention to the flat directions and  $R$ -symmetry, both in the general case and in specific examples. We will consider adding additional operators to the superpotential, in an attempt to classically lift the  $2m$ -baryons. In the examples, it is not possible to lift all  $2m$ -baryonic flat directions (when they exist) and preserve an  $R$ -symmetry. When they are not lifted, there is a gauge symmetry breaking minimum (a runaway direction) in which the dynamics responsible for breaking supersymmetry does not occur. On the other hand, when the  $2m$ -baryons are lifted and no  $R$ -symmetry is preserved, there exists a supersymmetric solution to the equations of motion, which would be expected on the basis of the argument of Nelson and Seiberg.[14]

However, the role of  $R$ -symmetry is not always clear. In examples where the superpotential is nongeneric, an  $R$ -symmetry is not essential[14]. A trivial example of this is given by the superpotential  $W = \phi_1 + \phi_2^2 + \phi_2^3$  (or, more generally, by a superpotential which decomposes as the sum of two terms  $W_{1,2}$  which depend on different fields and such that  $W_1$  is  $R$ -symmetric while  $W_2$  is not). In some cases, indeed, the lack of genericity of the microscopic superpotential may lead to an *effective*  $R$ -symmetry of some low-energy version of the theory. In the above mentioned case this is what happens after  $\phi_2$  is integrated out. If on the other hand, the superpotential is generic to a sufficient degree, *i.e.* it involves enough independent operators, one would not expect the symmetries of the low-energy theory to include an  $R$ -symmetry not present in the original theory. In a subsection below, we reanalyze the  $4 - 3 - 1$  model with a general cubic superpotential in a particular limit in which there exists a hierarchy of strong mass scales. With such a general dimension three superpotential there is no quantum  $R$ -symmetry. Nonetheless the superpotential of the low energy theory has an  $R$ -symmetry and supersymmetry is broken, consistent with the analysis of Ref. [14]. It seems that this low energy symmetry originates because of the simple, non generic, *just* cubic form of the microscopic  $W$ . The remaining examples we consider, corresponding to  $k = 3$  models, will not have this luxury; higher dimensional operators are required to lift dangerous flat directions. No  $R$ -symmetry of these theories can be identified when the flat directions are lifted and the models will not

break supersymmetry.

Before proceeding to specific examples, we give a general discussion. It is useful for us to first focus on a particular subset of  $D$ -flat directions. First, there are the dimension-three flat directions which, as in the analogue eq. (4.1), we call  $X$  and  $M$ . The second set of flat directions we distinguish are the  $2m$ -baryons,  $Y$ , discussed in detail above. In particular we focus on the dibaryons. These have dimension  $N$ , the size of the original ADS  $SU(N)$  gauge group from which the model was derived. It is important to notice that the dibaryon operators, unlike the other higher dimension operators, cannot be lifted by the renormalizable superpotential terms, since each term involves at least two  $\bar{F}$ -type fields. Therefore, additional operators must be present in the superpotential in order to lift these flat directions classically. The dibaryons can be lifted either by including the dibaryon in the superpotential, or by including a higher order invariant with a single  $\bar{F}$  factor.

We now give an argument that it is impossible to put in all  $X$ ,  $M$ , and  $Y$  operators while maintaining an anomaly-free  $R$ -symmetry. It is readily checked that if  $M$  and  $X$  operators are present in the superpotential, the dibaryon operator carries  $R$ -charge

$$R_Y = 2N - 2 \sum_i n_i \bar{F}_i \quad (5.1)$$

where we have used the field name to represent its  $R$ -charge. On the other hand, the anomaly cancellation condition for  $SU(n_i)$  requires that

$$2(N - n_i - 2) = \sum_j n_j \bar{F}_j. \quad (5.2)$$

Clearly one cannot satisfy this constraint for all  $n_i$ . Furthermore, the  $R$ -charge of  $Y$  will not be two, and so the dibaryon will break this  $R$ -symmetry. If one looks for flavor-dependent R-symmetries, one finds that the only flavor symmetries allowed by the maximal rank condition on the Yukawas that could mix with  $R$  are non-anomalous. The same conclusions are reached if we try to lift the dibaryon by including a higher order invariant with only one  $\bar{F}$ .

The above argument shows that one cannot consistently include all dimension-three gauge invariant operators in the superpotential, maintain an anomaly-free  $R$  symmetry, and lift the dibaryon classically. This might suggest that one cannot construct a supersymmetry breaking model based on this gauge and field content. This argument is suggestive

but inconclusive. In fact it is unclear that any of the  $R$  symmetry requirements above are essential for each gauge group. First of all, it is not clear that all  $M$  operators need be included. Omitting these operators introduces new flat directions which break one of the  $SU(n_i)$  symmetries. However, if it is assumed that another  $SU(n_j)$  gauge group is strong first, the resulting flat direction can be lifted quantum mechanically. So although it is clear that at least one type of  $M$  operator must be included (where type refers to which type of  $A$  and  $\bar{F}$  operator it involves), it is not clear that *all* are necessary.

Second, it is not really clear that the dibaryonic operators need to be included. Although it is obvious that the dibaryonic operators are not lifted by the dimension three superpotential operators, they can conceivably be lifted quantum mechanically.[6,15]

The third point is perhaps the most subtle. It is not necessary to incorporate all anomaly constraints, even for non-Abelian groups. This goes against conventional wisdom, but we give two arguments why this can be the case. There are two arguments in the literature concerning the special role of  $R$ -symmetry in supersymmetry breaking. The first argument[4] is that if there is a spontaneously broken global symmetry and no flat directions the theory is likely to break supersymmetry since the massless pseudoscalar has no massless scalar partner. The second argument[14] shows that if there is a generic superpotential and an  $R$ -symmetry, there are more equations which must be satisfied for a supersymmetric minimum than unknowns, and therefore the theory will not have a supersymmetric minimum.

Now let us consider both of these arguments in turn. Suppose we have a classical symmetry which is anomalous with respect to a particular gauge group factor, but that factor does not contribute a superpotential in the electric phase. A  $U(1)$  gauge symmetry is an example of such a factor, but a gauge group in the non-Abelian Coulomb phase or free magnetic phase might also have this property, depending on how it is perturbed. In this case, the Kähler potential (and higher derivatives terms) alone would violate the global symmetry, typically through higher dimensional operators which depend on the dynamical scale  $\Lambda$  of the theory. The axion would only get a mass from the Kähler potential *after* supersymmetry is broken. So if one is asking whether there can be a supersymmetric minimum, the fact that the global symmetry is anomalous is irrelevant.

Now let us consider the Nelson-Seiberg argument. Since this argument only depends on the equations of motion, one can look directly at the superpotential to see how the anomaly constraint enters. It enters precisely as we have considered above; that is, if

there exists an operator generated by strong dynamics present in the superpotential, it is one of the terms considered when analyzing the equations of motion. Clearly if there is an instanton-generated term, for example, it should be consistent with the  $R$ -symmetry. This is guaranteed if the  $R$ -symmetry is not anomalous with respect to the gauge group being considered. However, if the  $R$ -symmetry is anomalous with respect to a factor which does not generate an operator in the superpotential, it is clearly irrelevant to the Nelson-Seiberg argument. The conclusion is that even for non-Abelian gauge groups, one does not necessarily need to require an anomaly free  $R$ -symmetry.

For the above three reasons, it is very difficult to make completely generic statements about all models, since it might be that there exists a particularly clever choice of operators such that some flat directions remain or the  $R$ -symmetry is anomalous, but nonetheless there exists a supersymmetry breaking minimum. For example, in the  $n_1 - n_2 - 1$  models, it is not essential to preserve an exact  $R$ -symmetry. The analysis of the  $4 - 3 - 1$  model in the next section demonstrates that there is an effective  $R$ -symmetry in some low-energy version of the theory which is sufficient to guarantee supersymmetry breaking. The  $4 - 3 - 1$  model represents an example where the  $SU$  dynamics cannot be neglected, *i.e.* the scales of both gauge groups appear in the effective superpotential. On the other hand, the proof of dynamical supersymmetry breaking in the deconfined version of the  $n_1 - n_2 - 1$  models without an adjoint relied solely on the  $Sp$  dynamics of the dual phase. Therefore, the  $SU(\tilde{n}_1) \times SU(\tilde{n}_2)$  can be taken very weak without spoiling the susy breaking dynamics, which is all determined by  $Sp(m)$ . It is easy to verify in this case that there is an  $R$ -symmetry which is anomaly-free with respect to the  $Sp$  gauge group (but anomalous with respect to  $SU$ ).

The  $n_1 - n_2 - n_3$  models on the other hand will probably not break supersymmetry. In these models, there are generally two possible formulations of the superpotential. In one, the dibaryon direction is not lifted. An  $R$ -symmetry is preserved, but the flat direction leads to a supersymmetric minimum at infinity. In the second formulation, the dibaryon is included, but there is no remaining  $R$ -symmetry and there is a supersymmetric minimum at finite field value. This will be demonstrated in the examples which follow. As for the general  $n_1 - n_2 - n_3$  models in the dual phase we may argue in the following way. The  $Sp$  dynamics alone in the dual theory is not enough to generate a superpotential. Therefore, the dynamics of at least one of the  $SU$  factors must be relevant if supersymmetry is to be broken. This suggests that if supersymmetry is to be broken, for the purpose of classifying

$R$ -symmetries one must also impose some  $SU$  anomaly constraint, and not only the  $Sp$  anomaly constraint. By imposing the constraint for both  $Sp$  and  $SU(\tilde{n}_1)$  we find, again, that in the presence of the most general cubic superpotential all dibaryons have charge  $R_Y = 2(k - 1)N - 6k - 4 - 4\tilde{n}_1$ . This expression can equal 2 only for special groups.

### 5.1. Example1: The $4 - 3 - 1$ Model

In this section we reanalyze this model in order to illustrate the role of an  $R$ -symmetry, whether exact or accidental. It is known that this model does break supersymmetry, unlike the models considered later in this section. However, unlike Ref. [1], we will consider the model with a generic cubic superpotential, which will not in general preserve an  $R$ -symmetry. Nonetheless the model can be shown to break supersymmetry with an  $R$  symmetry of a low-energy effective superpotential as we now discuss.

Since this model is completely confining, its behavior is distinctive when compared to the larger  $n_1-n_2-1$  cases where at least one of the group factors is, at least naively, in a non-Abelian Coulomb phase. In this case the confining superpotential, derived in Ref. [1], involves both strong scales. Thus we expect that the anomaly constraints of both groups are relevant to  $R$ -symmetry considerations. As we will describe below the situation is however more subtle. Let us analyze it in detail. The field content under  $SU(4) \times SU(3) \times U(1)$  is just given by

$$A(6,1)_6, \quad \bar{Q}(1,\bar{3})_{-8}, \quad T(4,3)_{-1}, \quad \bar{F}_I(\bar{4},1)_{-3}, \quad \bar{Q}_i(1,\bar{3})_4 \quad (5.3)$$

where  $i, I = 1, 2, 3$  are the flavor indices. The most general cubic tree level superpotential of the model can be written as

$$W = g^{12}\bar{Q}\bar{Q}_1\bar{Q}_2 + f^{12}A\bar{F}_1\bar{F}_2 + \lambda^{iJ}T\bar{Q}_i\bar{F}_J \quad (5.4)$$

where  $\lambda$  is a rank 3 matrix, which is in general non-diagonal. We have already proven that the above  $W$  lifts all flat directions. For diagonal  $\lambda$  there is an anomaly-free  $R$ -symmetry [1] under which the fields have charges  $A(0), \bar{F}_3(0), \bar{F}_{1,2}(1), \bar{Q}_{1,2}(5/3), \bar{Q}_3(8/3), \bar{Q}(-4/3), T(-2/3)$ . This symmetry is indeed anomalous under the  $U(1)$  gauge group, but this anomaly will not play a role as there is no strong dynamics associated with the  $U(1)$ . However for general  $\lambda$  there is no quantum  $R$ -symmetry. More generally we notice that whenever one of  $g^{12}, f^{12}, \Lambda_4, \Lambda_3$  vanishes or when  $\lambda$  is diagonal there exists a non-anomalous  $R$ -symmetry. Incidentally we notice also that in the case of generic Yukawa

matrices the only non-anomalous  $U(1)$  is indeed the gauged one. Nonetheless the model with general couplings, and no  $R$ -symmetry, does break supersymmetry. For this model the full confining superpotential is known.[1] Nonetheless we find it instructive to consider the limit  $\Lambda_3 \gg \Lambda_4$  and study the effective theory below the scale of  $SU(3)$  confinement. We want to show that, while the original microscopic theory does not preserve an  $R$ -symmetry, the effective low-energy one does indeed have an accidental  $R$  in the superpotential. The original breaking appears only in the Kähler potential and in higher derivatives terms. This effective  $R$ -symmetry plays the usual role[14] in supersymmetry breaking.

The  $SU(4)$  gauge theory below the scale of  $SU(3)$  confinement contains 4 flavors ( $F_i$ ,  $\bar{F}_I$ ), one antisymmetric tensor  $A$ , and 4 singlets  $\bar{b}^i$ . In terms of the original fields we have  $F_i \sim T\bar{Q}_i$ ,  $\bar{b}^i = \epsilon^{ijk}\bar{Q}\bar{Q}_j\bar{Q}_k$  for  $i = 1, 2, 3$  and  $\bar{F}_4 \sim T^3$ ,  $F_4 \sim T\bar{Q}$ ,  $\bar{b}^4 \sim \bar{Q}_1\bar{Q}_2\bar{Q}_3$ . The superpotential of the confined theory can be shown to be just the tree level plus the confining terms

$$W = g^{12}\bar{b}_3 + f^{12}A\bar{F}_1\bar{F}_2 + \lambda^{Ij}\bar{F}_I F_j + \frac{1}{\Lambda_3^5}(\bar{F}_4 F_i \bar{b}^i - \det F_i) \quad (5.5)$$

where the indices of  $\lambda$ , the tree level Yukawa, run from 1 to 3. The original Yukawa couplings  $\lambda$  are now mass terms for three of the four  $SU(4)$  flavors, so that it is appropriate to integrate them out. Indeed the remaining light fields are just spectators of this decoupling and the low energy  $W$  is just obtained from (5.5) by setting the massive fields to zero

$$W_{eff} = g^{12}\bar{b}_3 + \bar{F}_4 f_4 \bar{b}^4 \quad (5.6)$$

This result is easily derived by redefining  $\bar{F}_3$  in such a way that  $\lambda$  has the form

$$\lambda^{Ij} = \begin{pmatrix} \lambda^{11} & \lambda^{12} & \lambda^{13} \\ \lambda^{21} & \lambda^{22} & \lambda^{23} \\ 0 & 0 & \lambda^{33} \end{pmatrix}. \quad (5.7)$$

In this basis, the e.o.m. of  $\bar{F}_3$  and  $F_{1,2}$  imply  $F_3 = 0$  and  $\bar{F}_{1,2} \propto \bar{F}_4$ . Then both  $A\bar{F}_1\bar{F}_2$  and  $\det F$  vanish by the e.o.m., while the linearity of  $W$  in  $F_{1,2}$  leads to the vanishing of the other terms involving the heavy fields and to the simple result (5.6). Moreover the scale of the low energy theory is just given by<sup>10</sup>  $\tilde{\Lambda}_4^{10} = \det \lambda \Lambda_4^8 \Lambda_3^5$ . The low energy  $SU(4)$  dynamics generates a superpotential from gaugino condensation and the full low energy  $W_{eff}$  will be

$$W_{eff} = g^{12}\bar{b}^3 + \frac{1}{\Lambda_3^5}M_{44}\bar{b}^4 + \left( \frac{\tilde{\Lambda}_4^{10}}{PfA M_{44}} \right)^{\frac{1}{2}} \quad (5.8)$$

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<sup>10</sup> The dimensions do not match here since we have not canonically normalized composites.

where  $M_{44} = \bar{F}_4 F_4$ . This expression preserves an  $R$ -symmetry. The reason for this is that a number of sources of explicit  $R$ -breaking have decoupled. In particular,  $W_{eff}$  does not depend at all on  $f$  while the dependence on  $\Lambda_4$  and  $\lambda$  is all coming via  $\det(\lambda)\Lambda_4^8$ . Notice that, for any  $\lambda$ , the latter expression is neutral under the  $R$  symmetry defined below eq. (5.4). Eq. (5.8) is exact; it can be derived from the full effective superpotential of Ref. [1] by integrating out heavy mesons along generic  $PfA \neq 0$ . Alternatively one could derive it by considering the most general low-energy effective  $W$  under the simple assumption that  $SU(3)$  confinement gives a mass to  $\bar{F}_{1,2,3}$  and  $F_{1,2,3}$ . One can then use the constraints from the  $SU(3) \times SU(3)$  flavor symmetry of the original theory, under which the fields transform as  $\bar{Q}_i(3,1)$ ,  $\bar{F}_I(1,3)$ ,  $g(3,1)$ ,  $f(1,3)$  and  $\lambda(\bar{3},\bar{3})$ . Flavor symmetry and holomorphy then constrain the Yukawa couplings to appear in the low energy theory only via the three expressions  $I_1 = g^{ij}\bar{b}^k\epsilon_{ijk}$ ,  $I_2 = \det(\lambda)$  and  $I_3 = g^{ij}f^{IJ}\lambda^{kL}\epsilon_{ijk}\epsilon_{IJL}$ . Notice the field independent  $I_{2,3}$  are indeed  $R$  preserving: so holomorphy and flavor symmetry alone are already enough to infer  $R$  invariance of the low energy  $W$ ! (The fact that the last expression above does not appear in  $W$  is not even necessary for our purpose.)

The occurrence of such an accidental  $R$ -symmetry is indeed analogous to what happens in the  $SU(2)$  model of ref. [16]. We can trace the origin of  $R$  to the specific form of the microscopic superpotential. Had we added additional quintic or higher order invariants to the original  $W$ , the equations of motions would not have lead to the vanishing of all the terms involving heavy fields, and we would not be left with an  $R$  symmetry.

Notice that there should be no exactly massless  $R$ -axion associated to the breaking of  $R$ , as the Kähler potential and the terms with higher covariant derivatives do break  $R$ . It is also clear, that, since an exact  $R$  symmetry is recovered in the limit  $\Lambda_4 \rightarrow 0$ , the mass of a possible  $R$ -axion scales like  $\Lambda_3(\Lambda_4/\Lambda_3)^p$  with  $p > 0$ . For comparable scales  $\Lambda_3 \sim \Lambda_4$  there should be no approximately massless Goldstone boson. This should be compared to other models where supersymmetry is broken without an  $R$ -symmetry. In Ref. [14], the 3-2-1 model with the addition of one flavor  $s$ ,  $\bar{s}$  of  $SU(3)$  is considered. This model, with the most general renormalizable superpotential, supports no  $R$ -symmetry. Nonetheless the model breaks supersymmetry for any finite value of the mass term  $m\bar{s}s$ . At  $m = 0$  supersymmetry is restored. Refs. [16][9] also give models which do break supersymmetry without an  $R$  symmetry. However in both these cases, there are non-renormalizable operators which stabilize some flat direction. The resulting axion mass, though suppressed by some power of  $1/m_{Pl}$ , may still be large enough to suppress axion production in stars. Models with

supersymmetry breaking induced by higher dimensional operators need however to have a strong dynamics much above the “minimal”  $10^{4-5}$  GeV.

The common feature of the previous supersymmetry breaking models without an  $R$  symmetry is that there is a dimensionful parameter in the tree level Lagrangian,  $m$  in Ref. [14] and  $1/m_{\text{Pl}}$  in Ref. [16][9]. The 4-3-1 model which we have just described, on the other hand, has all its scales generated by dimensional transmutation, and still the  $R$  axion gets a mass. This seems to be the first example of this type. Indeed this is somehow similar to  $R$  breaking via the addition of an  $R$ -color gauge group, which was discussed, though without explicit examples, in Ref. [14].  $R$ -color is assumed weaker than the dynamics responsible for supersymmetry breaking, and its only purpose is to make  $R$  anomalous. In the limit in which  $\Lambda_3 \gg \Lambda_4$ , the role of  $SU(4)$  is similar to that of an  $R$ -color factor.

To conclude we comment on this result. The main point is that an effective  $R$ , limited to the low energy superpotential, can result from a microscopic  $W$  which is general enough to lift all flat directions and break  $R$ , though not *completely* generic. A cubic  $W$  at  $k = 2$  seems to have this remarkable property. Notice that had we studied 4-3-1 with comparable scales for the two groups there would not have been an  $R$  symmetric low energy theory. In that case we would have concluded that supersymmetry is broken, even without  $R$ , due to the non-genericity of the full superpotential. It is interesting that by moving the scales we can go from a picture where  $W$  is non-generic and there is no  $R$  to one in which a low energy  $W$  is generic but also  $R$ -symmetric. We reiterate that the addition of enough higher dimensional operators would of course eventually restore supersymmetry. The nice thing about  $k = 2$  models, which can make them appealing in applications, is that this “non-genericity” just results from renormalizability. In this sense it is natural. Supersymmetry is broken at  $k = 2$  just because we can lift all flat directions with a very limited set of operators. The  $k > 2$  models do not have this critical behaviour, since the lifting of flat directions requires too many operators. On the other hand at  $k = 1$ , *i.e.* the original ADS models, the cubic superpotential does not break  $R$  and there is an axion.

### 5.2. Example 2: The 3 – 1 – 1 Model

The field content of this model is obtained by decomposing the ADS model based on gauge group  $SU(5)$  into its components under an  $SU(3) \times U(1) \times U(1)$  subgroup. The antisymmetric tensor decomposes as

$$A \rightarrow \bar{Q}(-4/3, 0) \oplus Q_1(1/3, 1) \oplus Q_2(1/3, -1) \oplus S_3(2, 0) \quad (5.9)$$

where there is only one  $A$  type field,  $\bar{Q}$ , and there are three  $T$  type fields. The antifundamental decomposes as

$$\bar{F} \rightarrow \bar{Q}_1(2/3, 0) \oplus S_1(-1, -1) \oplus S_2(-1, 1) \quad (5.10)$$

We define the flat directions

$$X_1 = S_1(\bar{Q}_1 Q_2), \quad X_2 = S_2(\bar{Q}_1 Q_1), \quad X_3 = S_1 S_2 S_3 \quad (5.11)$$

$$Y = (\bar{Q} Q_1)(\bar{Q} Q_2) S_3 \quad (5.12)$$

$$Z_1 = (\bar{Q} Q_1)(\bar{Q}_1 Q_2), \quad Z_2 = (\bar{Q} Q_2)(\bar{Q}_1 Q_1) \quad (5.13)$$

And the superpotential is of the form

$$W = X_1 + X_2 + X_3 + Y + \frac{\Lambda_3^7}{Z_1 - Z_2} \quad (5.14)$$

It is readily checked that this superpotential does not support an  $R$ -symmetry and that there are consistent supersymmetric solutions to the equations of motion.

We now ask whether it is possible to preserve an  $R$ -symmetry by omitting superpotential terms (in this example, the instanton generated term requires that an  $R$ -symmetry be nonanomalous with respect to  $SU(3)$ ). One can consider removing one or more of the  $X$  operators and/or the  $Y$  operator from the superpotential. First consider removing  $X_1$  or  $X_2$ . We see that in this case, we will not lift the operators  $Z_1$  or  $Z_2$  which would then be a runaway direction. It can readily be seen that without the  $Y$  operator, the equations of motion require that  $X_1$  and  $X_2$  vanish, which would require  $Z_1$  and  $Z_2$  to diverge. So we conclude we cannot omit any of the above superpotential operators (without adding something else) if we are to get a minimum at finite expectation value. This is not surprising as it is expected that the  $X$  operators should have maximal rank in order to avoid dangerous flat directions. This minimum preserves supersymmetry.

There exist other possible superpotentials to lift the flat directions of the theory. It can be checked that the other possible models work similarly. One might note the similarity of this model to models with an antisymmetric tensor for even  $N$ . These models break supersymmetry at any finite field value if the operator  $A^{N/2}$  is omitted from the superpotential. However, without the operator, there is a runaway direction and a supersymmetric minimum exists at infinity. With the inclusion of the operator, the  $R$ -symmetry is destroyed and there is a supersymmetric minimum at finite field value. In the  $3 - 1 - 1$

model, the theory with the  $Y$  operator has a supersymmetric minimum, while the theory without it has a supersymmetric minimum at infinity. The presence of the new flat direction  $Y$ , not present in the  $n_1 - n_2$  models, is critical to the analysis of supersymmetry breaking.

### 5.3. Example 3: The $2 - 2 - 1$ Model

We next consider an example with two non-Abelian gauge groups in the decomposition of  $SU(5)$ , namely  $SU(2)_L \times SU(2)_R \times U(1)$ , where we have labelled the two  $SU(2)$ 's for convenience of notation. The field content for this decomposition is

$$A \rightarrow Q_L(-3, 1) \oplus Q_R(-3, -1) \oplus V(2, 0) \oplus S_L(2, 2) \oplus S_R(2, -2) \quad (5.15)$$

and

$$\bar{F} \rightarrow S(4, 0) \oplus F_L(-1, -1) \oplus F_R(-1, 1) \quad (5.16)$$

There are many possible flat directions in this model. The dimension-three flat directions are  $X = F_L V F_R$ ,  $X_L = S F_L Q_L$ ,  $X_R = S F_R Q_R$ , the dimension four flat directions are  $Z_L = \det V \cdot (F_L Q_L)$ ,  $Z_R = \det V \cdot (F_R Q_R)$ ,  $T = S \cdot Q_L V Q_R$ ,  $R_L = S_L S_R F_L Q_L$ ,  $R_R = S_L \cdot S_R F_R Q_R$ ,  $W_L = S_L \cdot F_L V Q_R$ ,  $W_R = S_R F_R V Q_L$ , and the dimension-five invariants are  $Y_1 = \det V \cdot Q_L V Q_R$  and  $Y_2 = S_L S_R Q_L V Q_R$ . There are constraints among these directions but they do not affect the following analysis. The flat directions can be lifted by the superpotential

$$W = X + X_L + X_R + Y_1 + Y_2. \quad (5.17)$$

This superpotential does not preserve an anomaly-free  $R$ -symmetry however. In fact this theory does not break supersymmetry. It is interesting to see this explicitly. The strong dynamics associated with the product of  $SU(2)$  groups with this field content was worked out in Ref. [9]. The superpotential which results is

$$\begin{aligned} W_{eff} = & A (B_L B_R u - B_L \Lambda_R^4 - B_R \Lambda_L^4 - M_{11} M_{22} M_{12} M_{21}) \\ & + M_{11} + S B_L + S B_R + u M_{22} + S_L S_R M_{22} \end{aligned} \quad (5.18)$$

where the bound states of  $SU(2)$  are  $B_L = F_L Q_L$ ,  $B_R = F_R Q_R$ ,  $u = \det V$ ,  $M_{11} = F_L V F_R (= X)$ ,  $M_{12} = F_L V Q_R$ ,  $M_{21} = Q_L V F_R$ ,  $M_{22} = Q_L V Q_R$ . One can check explicitly that the equations of motion can be solved. The situation is very similar to the previous

example. Without the  $Y$  operators, there would have been runaway directions at which supersymmetry is restored. With the inclusion of the  $Y$  operators, there is a supersymmetric minimum at finite field value.

Again, we are left with the question of whether or not one can lift dangerous flat directions while preserving an  $R$ -symmetry. Let us first assume that we include the dimension-three superpotential as above. We also impose the anomaly constraints since both  $SU(2)$ 's are confining and their associated  $\Lambda$ 's appear explicitly in the superpotential. One can then check that there is a two parameter family of  $R$ -symmetries, under which the charges are  $F_R = -2F_L + 4 + Q_R$ ,  $V = -2 + F_L - Q_R$ ,  $S = -2 + 2F_L - 2Q_R$ ,  $Q_L = -3F_L + 4 + 2Q_R$  (the  $R$ -charges of  $S_L$  and  $S_R$  are also free). One can then derive the charges of the flat directions. Most flat directions are already lifted. The directions  $T$  and  $Y_1$  are not. However they have  $R$ -charge 0 and  $-2$ , respectively. In order to preserve an  $R$ -symmetry in the superpotential,  $T$  and  $Y_1$  can only appear multiplying a flat direction which has been lifted. Therefore one cannot lift the  $Y_1$  operator consistent with an  $R$ -symmetry and the presence of the remaining terms in the superpotential. We conclude that there is no superpotential which will preserve an  $R$ -symmetry and have a supersymmetry breaking minimum, at least with this choice of tree-level superpotential.

There are however other combinations of tree level superpotential which might be tried. If either  $X$ , or both  $X_L$  and  $X_R$  are removed, there would be flat directions along which both  $SU(2)$ 's are Higgsed, and there would be no dynamical supersymmetry breaking. If only  $X_L$  is removed,  $Z_L$  must be lifted or else once again both  $SU(2)$ 's would be Higgsed. So the operator  $Z_L$  should be included in the superpotential. In this case,  $X_L$  is still not lifted and can obtain a nonzero expectation value. If this were the case, there is an  $SU(2)_R$  theory with one flavor due to the nonzero vev's of the fields in  $X_L$ . One can then check that without  $R_L$  or  $Y_L$  there would be a supersymmetry breaking vacuum at finite field value, but of course there are runaway directions. Once one of these operators is included, there is a supersymmetric vacuum.

We conclude there is no model with this field content which breaks supersymmetry.

#### 5.4. Example 4: The $3 - 2 - 2 - 1 - 1$ Model

The last model we consider explicitly is based on decomposing  $SU(7)$  to  $SU(3) \times$

$SU(2)_L \times SU(2)_R \times U(1) \times U(1)'$ . The fields decompose as

$$\begin{aligned} A \rightarrow & \bar{Q}(\bar{3}, 1, 1)_{(0, 8/3)} \oplus S_L(1, 1, 1)_{(2, -2)} \oplus S_R(1, 1, 1)_{(-2, -2)} \\ & \oplus T_L(3, 2, 1)_{(1, 1/3)} \oplus T_R(3, 1, 2)_{(-1, 1/3)} \oplus V(1, 2, 2)_{(0, -2)} \end{aligned} \quad (5.19)$$

and

$$\bar{F}_7^I \rightarrow \bar{F}^I(\bar{3}, 1, 1)_{(0, -4/3)} \oplus f_L^I(1, 2, 1)_{(-1, 1)} \oplus f_R^I(1, 1, 2)_{(1, 1)} \quad (5.20)$$

We can once again ask whether it is possible to lift the dibaryonic flat directions as well as all other potentially dangerous flat directions.

Again there are numerous possibilities. For example, we can include all dimension-three operators  $T_L \bar{F} f_L$ ,  $T_R \bar{F} f_R$ ,  $f_L V f_R$  since, as we have seen, in addition to lifting some of the dimension-three flat directions, these operators permit most higher-dimension operators to be lifted as well. Since the  $SU(3)$  group is confining, and below this scale (after integrating out massive flavors), the  $SU(2)$  groups are confining (and it is their dynamics which is relevant), we need to impose the  $R$ -anomaly constraints associated with these three gauge groups in order to consider an  $R$ -symmetry which can serve as a useful guide to supersymmetry breaking.

We now need to lift the dibaryon flat directions. If the  $\bar{Q} \bar{F} \bar{F}$  operators are not present in the superpotential, there are flat directions along which the  $SU(3)$  symmetry is Higgsed. The theory then reduces to the  $2 - 2 - 1 - 1$  model (with some additional singlets) and will not break supersymmetry. It is readily seen that one cannot include the dibaryon  $V^2 T_L^2 T_R^2 \bar{Q}$  in the superpotential while preserving an  $R$ -symmetry. The only possibility is to lift the dibaryon operators with lower dimension invariants. However, to include such an operator,  $\bar{F} T_R^2 T_L^2$  would prevent including an operator  $\bar{Q} \bar{F} \bar{F}$  in the superpotential, if an  $R$ -symmetry is to be preserved. It is interesting to note that if we do not preserve an  $R$ -symmetry, and write a generic potential to lift all flat directions,  $SU(3)$  confines and again one is reduced to the  $2 - 2 - 1 - 1$  model so supersymmetry is not broken.

### 5.5. $n_1-n_2-n_3$ Models in Dual Phase

The models considered above all had a gauge group in the confining phase. It is interesting to ask why the mechanism for breaking supersymmetry which was common to the  $m - n$  models no longer applies. Although in models with confinement, there can be other sources of supersymmetry breaking, all the  $m - n$  models that we studied could be considered in the limit where first one the the groups got strong. This resulted in a theory

in which Yukawa couplings became mass terms, so that there were sufficiently few flavors for the other gauge group that a dynamical term ensued. Furthermore, as a remnant of the dynamics of the first group, there was a mass term between some field which only coupled in this term and another field which occurred in the dynamical superpotential. Because this field was set to zero by the equations of motion, there was no consistent solution to the equations of motion in a theory without flat directions.

In the  $n_1 - n_2 - n_3$  theories, several of the pieces are missing. First of all, the field whose equation of motion set a field to zero now multiplies the sum of two fields, so neither are necessarily vanishing. Second, even after integrating out massive flavors, there can still be so many flavors that neither group generates a dynamical potential. Although it could be that the groups are in the non-Abelian Coulomb phase and no definite conclusion can be made, there is no evidence that such a theory will break supersymmetry. Finally, it might be that one or the other group has sufficiently few flavors to generate dynamically a superpotential. But as in the explicit examples we have studied, we expect that there will be a supersymmetric solution to the equations of motion in a theory in which the dibaryons are lifted.

## 6. Conclusions

To summarize, we have found that the addition of an adjoint superfield gives a compact way to investigate a large class of product group models. For the  $k = 2$  models, supersymmetry breaking was understood in the dual picture through the (deconfining)  $Sp$  dynamics. For the  $k > 2$  models, the dual description uncovered no dynamics which would lead to supersymmetry breaking.

An interesting aspect of the analysis of the dual phase was that maximal rank Yukawa matrices were required in order to reduce the number of flavors sufficiently (for  $k = 2$ ) that there was a gaugino condensation contribution to the superpotential. An explicit analysis of flat directions yielded further insight into this requirement. We found that most maximal rank superpotentials can lift all flat directions for  $k = 2$ .

We have also argued that  $k > 2$  models, that is models with more than three non-Abelian factors and/or more than one Abelian factor, do not break supersymmetry. This can be attributed to flat directions which could not be lifted while maintaining a sufficiently nongeneric superpotential, or while maintaining an  $R$ -symmetry.

Searching for an anomaly-free  $R$ -symmetry can be an unreliable guide to determining whether supersymmetry is broken. One often requires only an effective  $R$  symmetry, and not even that when the superpotential is non-generic (the latter in agreement with Ref. [14]). We have found in particular that similarly to examples presented there in which massive fields can be integrated out to produce an effective  $R$ -symmetry, dynamically massive fields can be integrated out to do the same. This behaviour is found in the 4–3–1 model with generic superpotential for example, where the anomaly-free  $R$ -symmetry of the superpotential of Ref. [1] was not in fact necessary. Furthermore, we argued that anomaly-free  $R$ -symmetries are required only when the dynamics of the associated gauge group is somehow reflected in the superpotential. Otherwise, an anomalous  $R$ -symmetry is permitted, as it is in the case of Abelian gauge factors.

It is clear that the tools we have developed should be useful in exploring other models with higher rank tensors. For example, a model with symmetric tensors can be treated the same way.

However, it is also clear that there is a lot about dynamical supersymmetry breaking that we have yet to understand. Analyzing flat directions is almost always difficult and subtle. Furthermore it is not clear when these flat directions are dangerous, as they might be lifted by strong dynamics. Finally we have found  $R$ -symmetries useful, but often inconclusive. Essentially the same 4–3–1 model with and without an  $R$ -symmetry breaks supersymmetry. On the other hand, clearly the fact that we cannot include dibaryonic operators is related to  $R$ -symmetry breaking. It would be worthwhile to have even stronger tools for analyzing potential dynamical supersymmetry breaking theories.

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## Appendix A.

In this appendix, we complete the proof of Section 4.1 that all cubic invariants in the

$n_1 - n_2 - 1$  models are lifted. Recall the form of the F-term constraints

$$\begin{aligned} M &= 2gX_1 = 2X_2f \\ MX_2 &= X_1M = 0 \\ \text{tr}(M) &= 0 \end{aligned} \tag{A.1}$$

As noted in the text, since  $n$  is odd, the generic antisymmetric matrices  $g$  and  $f$  will have rank  $n - 1$ . By a change of basis which leaves  $\delta$  invariant we can always put one of them, say  $g$ , in the form

$$g = \begin{pmatrix} g' & 0 \\ 0 & 0 \end{pmatrix} \tag{A.2}$$

where  $g'$  is an invertible  $(n - 1) \times (n - 1)$  matrix. The matrix  $f$  will be of the form

$$f = \begin{pmatrix} f' & \rho \\ -\rho^T & 0 \end{pmatrix} \tag{A.3}$$

where  $\rho$  is an  $n - 1$  vector, and  $f'$  is  $(n - 1) \times (n - 1)$ . As explained in section 4, by genericity, we will assume  $f'$  to be invertible as well. To study eqs. (A.1) in this basis it is useful to decompose also  $X_{1,2}$  as

$$X_{1,2} = \begin{pmatrix} X'_{1,2} & v_{1,2} \\ -v_{1,2}^T & 0 \end{pmatrix} \tag{A.4}$$

The first of eqs. (A.1) then gives

$$gX_1 = \begin{pmatrix} g'X'_1 & g'v_1 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} X'_2f' - v_2\rho^T & X'_2\rho \\ -v_2^Tf' & -v_2^T\rho \end{pmatrix} = X_2f \tag{A.5}$$

which implies

$$v_2 = 0 \tag{A.6}$$

$$v_1 = (g')^{-1}X'_2\rho \tag{A.7}$$

$$X'_1 = (g')^{-1}X'_2f'. \tag{A.8}$$

Another useful set of constraints are

$$X_1gX_1 = \begin{pmatrix} X'_1g'X'_1 & X'_1g'v_1 \\ -v_1^Tg'X'_1 & -v_1^Tv_1 \end{pmatrix} = 0 \tag{A.9}$$

$$X_2fX_2 = \begin{pmatrix} X'_2f'X'_2 & 0 \\ 0 & 0 \end{pmatrix} = 0 \tag{A.10}$$

which, together with eqs. (A.6)–(A.8), give

$$X'_1 g' X'_1 = 0 \quad (\text{A.11})$$

$$X'_2 f' X'_2 = 0 \quad (\text{A.12})$$

In fact, eq. (A.11) is redundant: it may be deduced from eqs. (A.8) and (A.12). It also important to keep in mind that  $X'_1$  is antisymmetric, so for example,

$$(g')^{-1} X'_2 f' = f' X'_2 (g')^{-1}. \quad (\text{A.13})$$

The constraint given by this equation is crucial. Notice that for the particular point  $f' \propto (g')^{-1}$  it would be trivially satisfied for any  $X'_2$ , leading to unlifted flat directions [1]. For example, taking both matrices equal to the identity matrix, which might have seemed the most obvious choice, will not work. It is also useful to rearrange the above equations to deduce

$$X'_2 (g')^{-1} X'_2 = 0. \quad (\text{A.14})$$

We will now focus on the above two equations and show that there exists a set of non zero measure of  $f'$ ,  $g'$  for which they imply  $X'_1 = X'_2 = 0$ . Then eqs. (A.6)–(A.8) and (A.1) allow us to conclude that the complete set of cubic invariants  $M$ ,  $X_1$  and  $X_2$  has to vanish by the equations of motion.

It is useful to notice that, while keeping the form  $M_{Ij} \sim \delta_{Ij}$ , it is possible to make further rotations and rescaling such that  $g'$  and  $f'$  in  $[(n-1)/2] \times [(n-1)/2]$  blocks have the form

$$g' = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \quad (\text{A.15})$$

$$f' = \begin{pmatrix} f_1 & H \\ -H^T & f_2 \end{pmatrix} \quad (\text{A.16})$$

where  $f_{1,2} = -f_{1,2}^T$ . A crucial assumption that we are going to make here is that  $H$  have  $\ell = (n-1)/2$  distinct eigenvalues, so that it can be diagonalized by a similarity transformation

$$H = U^{-1} d \ U \quad d = \text{diag}(d_1, \dots, d_k) \quad d_i \neq d_j \quad i \neq j \quad (\text{A.17})$$

Finally, we make an additional change of basis which simplifies the discussion; we define

$$V = \begin{pmatrix} U^{-1} & 0 \\ 0 & U^T \end{pmatrix} \quad (\text{A.18})$$

and make the redefinitions  $X'_2 \rightarrow (V^{-1})^T X'_2 V^{-1}$ ,  $f' \rightarrow V f' V^T$ ,  $(g')^{-1} \rightarrow V(g')^{-1} V^T$ , which replace  $H$  with  $d$  while leaving  $g'$  invariant. By decomposing  $X'_2$  in  $\ell \times \ell$  blocks

$$X'_2 = \begin{pmatrix} b_1 & \Delta \\ -\Delta^T & b_2 \end{pmatrix} \quad (\text{A.19})$$

eq. (A.13) reduces to

$$[d, b_2] = \Delta^T f_1 - f_1 \Delta \quad (\text{A.20})$$

$$[d, b_1] = \Delta f_2 - f_2 \Delta^T \quad (\text{A.21})$$

$$[\Delta^T, d] = b_2 f_2 - f_1 b_1 \quad (\text{A.22})$$

while eq. (A.14) becomes

$$b_1 b_2 - \Delta^2 = 0 \quad (\text{A.23})$$

$$b_1 \Delta^T + \Delta b_1 = 0 \quad (\text{A.24})$$

$$b_2 \Delta + \Delta^T b_2 = 0. \quad (\text{A.25})$$

The models studied in Refs. [1] are a particular example of the case  $f_{1,2} = 0$ . In this case, eqs. (A.20)–(A.21) imply  $b_{1,2} = 0$ , since the eigenvalues of  $d$  are all non-degenerate. Moreover, eq. (A.22) constrains  $\Delta$  to be diagonal, so that by eq. (A.23) we must have  $\Delta = 0$  and all cubic invariants must then vanish.

In Ref. [1], the superpotential that was chosen corresponds to

$$H_{ij} = \delta_{\text{mod}_k(i), \text{mod}_k(j+1)} \quad (\text{A.26})$$

whose  $\ell$  eigenvalues are all non degenerate and are given by the  $\ell$ -th roots of the identity:  $d_j = e^{i2\pi j/\ell}$ . We stress that the non-degeneracy of the  $d$ 's (eq. (A.17)) is the main reason why those particular examples succeed in lifting all flat directions. For non-degenerate  $d$ 's, eqs. (A.20)–(A.22) are maximally constraining. Indeed in the limiting situation of  $H = 1$  and  $f_{1,2} = 0$  there would be flat directions.

Notice that one easily gets the same result also in the case in which just one of  $f_{1,2}$ , say  $f_1$ , vanishes while  $f_2 \neq 0$ . In order to make our case that flat directions are lifted at generic points we should now consider the case where both  $f$ 's are non zero. This is however more involved, and we have not found a simple argument showing that  $X_2 = 0$ . It is however possible to reach this conclusion for small enough, but otherwise general,  $f_1$  and  $f_2$ . For our purpose of showing removal of flat directions over a set of non-zero measure

of the space of parameters this suffices. On a case by case basis it is not difficult to study the above equations for any size of the  $f$ 's. It is straightforward to do that for the case  $\ell = 2$ , which corresponds to models generated by the  $SU(9)$  ADS, and the result is that  $X_2 = 0$ . The proof for the case  $f_{1,2} \ll d_j$  is just done by “perturbing” the one we gave at  $f_{1,2} = 0$ . The off-diagonal entries in eqs. (A.20)–(A.22) allow one to solve for  $b_{1,2}$  and for  $\Delta_{ij}$ ,  $i \neq j$ , in terms of  $\{\Delta_{ll}\}$ , which we will now simply indicate with  $\Delta_l$ . In particular we have  $b_{1,2} = \mathcal{O}(f_{2,1})\Delta$ , while  $\Delta_{ij} = \mathcal{O}(f_1 f_2)\Delta$ , for  $i \neq j$ . It is then easy to see that the diagonal entries of eq. (A.23) have the form

$$\Delta_l^2 + B_{lmn}\Delta_m\Delta_n = 0, \quad l = 1, \dots, k \quad (\text{A.27})$$

where  $B_{lmn}$  is of order  $f_1 f_2$ . For  $B$  small enough, the above set of  $\ell$  equations has only the solution  $\Delta_l = 0$ . Indeed, if to the contrary, we assume a non-zero solution exists, then consider the equation for the largest  $\Delta_{\max}$  in  $\{\Delta_l\}$

$$|\Delta_{\max}|^2 = |B_{l_0 mn}\Delta_m\Delta_n| < \mathcal{O}(f_1 f_2)|\Delta_{\max}|^2. \quad (\text{A.28})$$

This, for  $f_1 f_2$  small enough, has only the solution  $\Delta_{\max} = 0$ , i.e.  $\Delta_l = 0$ . To conclude let us list the requirements on  $f'$  for which we have been able to prove complete lifting of flat directions:

- i)  $d_i \neq d_j$  for  $i \neq j$  for the eigenvalues of  $H$ .
- ii)  $|f_1 f_2| \ll |d_i - d_j|$  for any  $i \neq j$  and where on the left-hand side we mean the product of any entry of  $f_1$  with any entry of  $f_2$ . While this is not the full space of matrices it is clearly a set of non-zero measure, proving that a generic cubic superpotential lifts all the cubic flat directions. Notice that no  $R$ -symmetry is preserved with this superpotential when  $\rho$  in eq. (A.3) is non zero.

## Appendix B.

In this Appendix we briefly discuss the lifting of flat directions in the deconfined theory. We will just consider the theory without adjoint with tree level superpotential given by eq. (2.14). By construction, this theory reduces to the original  $n_1-n_2-1$  model when  $Sp(m)$  confines. This means that at any finite point on the  $Sp$  moduli space we can integrate out the massive  $P$ ,  $\bar{P}$  and get a theory which has no flat directions. The question remains whether there are flat directions along which  $Sp$  is higgsed on which the

potential slopes to infinity. To answer this question we need to understand the space of classically flat directions of the deconfined theory. The invariants are easily obtained by forming  $Sp$  mesons first. This procedure gives the  $A_{1,2}$  and  $T$  of the  $n_1-n_2-1$  theory, plus  $P_{1,2} \sim ZY_{1,2}$ . The latter are however set to be zero by the equations of motion of  $\bar{P}_{1,2}$ . So the invariants that are not obviously vanishing are formed from the fields of the  $n_1-n_2-1$  theory plus  $\bar{P}_{1,2}$ . The analysis goes through similar to those of Section 4 and Appendix A. The trilinear and baryonic invariants are shown to vanish by the equations of motion for maximal rank *generic* Yukawa matrices  $g, f, \delta$ . Indeed one is reduced to eqs. (4.3)–(4.7) for the cubic invariants, provided the condition  $\text{Det}(\delta + 4g^T \delta^{-1} f) \neq 0$  holds as well. Notice that this is precisely the condition of maximal rank for the meson mass matrix of the dual theory. The difference with respect to the confined case is however that antibaryons are not lifted by the classical equations of motion. This is because  $Y_{1,2}$  couple quadratically (rather than linearly) to matter and their equations of motion certainly do not constrain the antibaryons at  $Y_{1,2} = 0$ . However, the only directions which can remain flat will not involve the  $Y_1, Y_2$ , and  $Z$  fields. Therefore, any remaining flat directions do not Higgs the  $Sp$  gauge group. In other words, the classical moduli space of the deconfined theory consists just of all the antibaryons that are formed from  $\bar{F}_1^i, \bar{P}_1$  and  $\bar{F}_2^I, \bar{P}_2$ . Along any such flat direction  $SU(n_1) \times SU(n_2) \times U(1)$  is completely higgsed, while  $Sp(m)$  remains strongly coupled. Then we still expect that the  $Sp$  can be treated as confining and that the antibaryons are indeed lifted by quantum effects. To see this explicitly we analyze the theory far away along one such antibaryon  $\bar{B} \sim \bar{\phi}^{n_1+n_2}$ , where  $\bar{\phi}$  is the elementary field that makes up  $\bar{B}$ . For the sake of the argument we may consider an antibaryon which does not involve  $\bar{P}_{1,2}$  so that by a flavor rotation the field vev has the form

$$\begin{aligned} (\bar{F}_1)_i^\alpha &= \bar{\phi} \delta_i^\alpha \quad 1 \leq i \leq n_1 & (\bar{F}_1)_i^\alpha &= 0 \quad i > n_1 \\ (\bar{F}_2)_I^A &= \bar{\phi} \delta_I^A \quad 1 \leq I \leq n_2 & (\bar{F}_2)_I^A &= 0 \quad I > n_2 \end{aligned} \tag{B.1}$$

where  $U(1)$  invariance fixes the vev to be the same for  $\bar{F}_1$  and  $\bar{F}_2$ . For large enough  $\bar{\phi}$  the Kähler metric is flat in  $\bar{\phi}$ . The original  $Sp(m)$  theory has  $m+2$  flavors. For maximal rank Yukawa couplings, along an antibaryon, between  $(n_1 - n_2 + 3)/2$  and  $m+2$  flavors get massive. Remember that  $n_1 \geq n_2 + 1$ , so that at least two  $Sp$  flavors get massive. Let us denote the massive  $Sp$  fundamentals by  $Q_A$  for  $A = 1, \dots, 2p$  and the massless ones by  $q_a$  for  $a = 1, \dots, 2m+2-2p$ . Moreover there are massless  $Sp$  singlets  $\bar{\psi}_k$  coming from the antifundamentals of the broken  $SU(n_1) \times SU(n_2)$ . The  $\psi_k$  parameterize indeed the other unexcited antibaryons, thus we take by definition  $\psi_k = 0$  on the antibaryon  $\bar{B}$ . The

classical superpotential in terms of these fields has the form

$$W = \bar{\phi}^2 \Gamma_0^{AB} Q_A Q_B + \bar{\phi} \psi_k \Gamma_1^{kab} q_a q_b + \psi_k \Gamma_2^{kab} q_a q_b + \dots \quad (\text{B.2})$$

where  $\det \Gamma_0 \neq 0$  gives mass to  $p$  flavors of  $Sp(m)$  and the first two terms originate from the quartic terms in the original  $W$ , while the third originates from  $\bar{P}_{1,2} Z Y_{1,2}$ . The dots represent terms quadratic in  $\psi$ , terms of the type  $\bar{\phi} \psi Q_A Q_B$ , and terms like  $\bar{\phi} \psi Q q$ , which are not relevant for the following discussion where we focus on  $\psi \sim 0$ . Notice that the property that  $Y_{1,2}$  and  $Z$  be zero by the classical equations of motion have to be reproduced by the above superpotential at the point  $\psi = 0$ . This property is trivially satisfied for the massive flavors  $Q$  as  $\Gamma_0$  is non singular. On the other hand the light mesons  $m_{ab} = q_a q_b$  have to be set to zero by the  $\psi$  e.o.m.. In order for this to hold true the fields  $\Psi^{ab} = \psi_k (\Gamma_1^{kab} + \Gamma_2^{kab} / \bar{\phi})$ , for  $a, b = 1, \dots, 2m + 2 - 2p$  should be linearly independent so that  $\partial_{\Psi^{ab}} W = m_{ab} = 0$ . Now, by integrating the massive  $Q$ 's out we get a low-energy  $Sp$  dynamics which generates a superpotential, described by the following  $W_{eff}$

$$W_{eff} \sim \left( \frac{\bar{\phi}^{2p} \Lambda_{Sp}^{m+1}}{Pf(m_{ab})} \right)^{\frac{1}{p-1}} + \bar{\phi} \Psi^{ab} m_{ab} \quad (\text{B.3})$$

Notice that the  $Sp$  dynamics lifts the origin while the  $\Psi$  e.o.m. conflict with that. In other words, along the antibaryon  $Sp$  becomes stronger, the origin in  $m_{ab}$  is lifted, but this conflicts with the e.o.m. constraint that cubic invariants like  $A_1 \bar{F}_1^j \bar{F}_1^i$  be zero. In the above equation we have neglected multiplicative corrections of order  $\psi/\bar{\phi} \ll 1$ , this is because we can always rotate our fields in such a way that only one antibaryon is nonvanishing. A similar discussion holds for antibaryons which overlap with  $\bar{P}_{1,2}$ . Notice indeed that the Kähler metric for  $\bar{\phi}$  is asymptotically flat. Then it is manifest that the above superpotential does not give asymptotically vanishing vacuum energy for  $\bar{\phi} \rightarrow \infty$ . For instance, by imposing  $F$ -flatness for the mesons we get the following superpotential for the antibaryons

$$W \sim \bar{\phi}^{1+p/(m+1)} (Pf\Psi)^{1/(m+1)}. \quad (\text{B.4})$$

According to this equation, at large  $\bar{\phi}$ , the fields  $\Psi$  are driven away from the origin. This is because  $Pf\Psi$  is the product of  $m + 1 - p < m + 1$  fields whose Kähler metric is non singular at  $\Psi = 0$ . However as  $\Psi$  is driven away from the origin, the antibaryon  $\bar{\phi}$  will be pushed towards smaller values by  $|\partial_{\bar{\phi}} W|^2$ . Then we expect that  $\bar{\phi}$  and  $\Psi$  will end up being comparable, *i.e.* an antibaryon giving mass to all flavors ends up being excited. In this

situation the above analysis has to be repeated at  $p = m + 1$ , for which there are no light  $Sp$  flavors. Then eq. (B.3) is just replaced by  $W \sim \bar{\phi}^{2+1/(m+1)}$  which pushes  $\bar{\phi}$  back to the origin. We conclude that the antibaryonic flat directions are lifted by the  $Sp$  quantum effects.

## References

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