Preprint typeset in JHEP style. - HYPER VERSION

SNS-PH/00-08, Bicocca-FT/00/03

The Fate of the Radion in Models with Metastable Graviton

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Abstract: We clarify some general issues in models where gravity is localized at intermediate distances. We introduce the radion mode, which is usually neglected, and we point out that its role in the model is crucial. We show that the brane bending effects discussed in the literature can be obtained in a formalism where the physical origin is manifest. The model violates positivity of energy due to a negative tension brane, which induces a negative kinetic term for the radion. The very same effect that violates positivity is responsible for the recovery of conventional Einstein gravity at intermediate distances.

Keywords: brane world, graviton, radion.

Contents

1. Introduction

After the work of Randall and Sundrum (RS) [1] it is well understood that theories with non-compact extra-dimensions can localize gravity on three-branes [1]. A very interesting modification of this setup was recently proposed by Gregory, Rubakov and Sibiryakov (GRS) in [3] (see also [4]). The theory is effectively five-dimensional both at small and at large scales while it localizes gravity at intermediate scales. Since five-dimensional effects are visible only at distances larger than the universe radius, the model can have an acceptable phenomenology and many attractive features. It was also noticed in [5, 6] that the GRS model offers a completely new viewpoint on the cosmological constant problem. Since the five-dimensional cosmological constant is zero, our brane-world will be naturally flat. From a different perspective, 4-d gravity disappears in our world at very large distances possibly changing the impact of a non-zero 4-d vacuum energy.

It was unfortunately pointed out in [5] that the model has a serious drawback which makes it probably inconsistent. The background indeed requires a negative-tension brane in the bulk. Alternatively said, positivity of energy is violated. Versions of the same setup, where the negative tension brane is smoothened in a kink solution of the five-dimensional theory, suffer from a similar problem [5]. Such smooth solutions have a stress-energy tensor that violates the weak-dominance energy condition. Many other papers [7, 6, 8, 9, 10, 11, 12] have addressed the question of consistency in the GRS model. These papers opened a debate on the capability of reproducing the correct Einstein theory at intermediate distances. The claim that the GRS

[1] For an earlier suggestion on graviton trapping see [2].

model has serious phenomenological difficulties[6] has been criticized through an actual computation [8, 9]. It was also pointed out that the very same effect that is responsible for the agreement with the Einstein theory at intermediate distances, shows up as a four-dimensional anti-gravity force at very large distances [9]. Based on an RG-inspired computation, in [11] it was claimed that anti-gravity disappears at large distances so that the GRS model is well behaved at all scales. The anti-gravity effect could be interpreted as an effect of the internal inconsistence of the model [10], therefore it is extremely important to understand if it really exists.

The modest purpose of this letter is to reintroduce a mode which was mostly ignored in this debate and which we believe should deserve much more credit. Indeed the position of the negative-tension brane is a modulus of the GRS solution corresponding to a localized four-dimensional scalar. With some abuse of language we will indicate this modulus as the radion. It was claimed by many authors that the radion can be frozen or stabilized and therefore it is not crucial for the physics and internal consistency of the model. However, as one can expect, and as we will prove explicitly, the kinetic coefficient of the radion is proportional to the brane tension, which is negative. This fact makes the issue of radion stabilization very unclear and probably not even well posed. We stress that the presence of a negative tension brane free to fluctuate in the bulk seems essential to the existence of quasi-localized gravity. The negative radion kinetic term very likely makes the theory inconsistent at some level, but we think it is worth understanding more about this model.

We show that an accurate analysis of the radion dynamics sheds light on the various effects discussed in [6, 8, 9, 10, 11]. The radion is responsible both for the recovery of Einstein gravity at intermediate distances and for the negative energy problem. The violation of energy positivity does not happen at experimentally accessible scales, but at very large distances and in the form of anti-gravity, therefore the GRS model could be phenomenologically acceptable. We stress that all these effects have already been discussed in various papers [7, 6, 8, 9, 10, 11], but in a formalism that, in our opinion, somehow obscures their simple and intuitive physical meaning. We prefer to choose a gauge for the gravity perturbations where there is no brane bending. In this gauge, the propagating degrees of freedom have a direct interpretation as 4-dimensional physical particles.

The role of the radion versus the bending effect discussed in references [8, 9] will be clarified. Our results can be useful for studying other models as well. For example, we discuss the non-compact model in [16]. We also comment on the difference between our results and reference [11] which uses RG-inspired arguments.

2. The Effective Action for Graviton and Radion

We consider the five-dimensional background given by

$$ds^2 = e^{2\phi(z)}\,\eta_{\mu\nu}\,dx^\mu dx^\nu \,+\, dz^2\,; \tag{2.1}$$

the flat 4-d metric $\eta_{\mu\nu}$ has the signature $(-1, 1, 1, 1)$, which we use throughout the paper. The metric (2.1) is the most general five-dimensional metric with four-dimensional Poincaré invariance and we mostly assume that it is a solution of a five-dimensional gravity theory with three-brane sources,

$$\int dx^5 \sqrt{-\hat{g}}\,(2M^3\,R \,-\, \Lambda_a) \,-\, \sum_i \delta(z - z_i)\,\tau_i \int dx^4 \sqrt{-g_{_{in}}} \tag{2.2}$$

where $g_{_{in}}$ is the induced metric on each brane. The 5-d cosmological constant Λ_a may vary in the different domains of space-time, labelled by a, delimited by three-branes. We shall consider the case $\Lambda_a \leq 0$. The derivative of the warp factor jumps at the three-brane positions by an amount related to the tensions of the branes $\Delta\phi'(z_c) = -\frac{\tau_i}{12M^3}$. Tensions and cosmological constants in the bulk must satisfy these jump conditions. As a consequence there is a fine-tuning in order to get flat four-dimensional space.

The non-compact RS model [1] has $\phi(z) = -k|z|$. The bulk metric is AdS$_5$ with cosmological constant $\Lambda = -24M^3k^2$ and there is a brane of positive tension $\tau_1 = 24M^3k$ at the origin. An orbifold $\mathbb{Z}_2$ symmetry $z \to -z$ is also imposed. The compact RS model [17] is obtained by compactifying the z direction on a circle and introducing a second brane at the other orbifold point $z = r$ [17]. The tension of the second brane is negative $\tau_2 = -\tau_1$. Since the negative tension brane is sitting at an orbifold fixed point, the negative energy mode associated with its fluctuations in the transverse direction is projected out.

A modified non-compact RS model is obtained by introducing a second brane in the bulk at the position $z = r$ [16]. From now on, this will be called the Lykken-Randall (LR) model[2]. The five-dimensional cosmological constant is now different in the various regions of space. The warp factor for the LR model is

$$\phi(z) \,=\, \left[\begin{array}{cc} -k_{_L}z & 0 \leq z \leq r \\ -k_{_R}z \,+\, (k_{_R} - k_{_L})r & z \geq r \end{array}\right. \quad . \tag{2.3}$$

So the cosmological constant equals respectively $\Lambda_1 = -24Mk_{_L}^2$ and $\Lambda_2 = -24Mk_{_R}^2$ to the left and to the right of the second brane. The matching conditions at the branes fix $\tau_1 = 24M^3k_{_L}$, $\tau_2 = 24M^3(k_{_R} - k_{_L})$. The right brane has positive tension only in the case $k_{_R} > k_{_L}$. Provided $k_{_R} > 0$, the model localizes gravity and a radion

[2]In [16], the second brane is just a probe, with negligible effects on the background. In this paper we consider the general case of a brane with arbitrary tension.

field. As shown below, in the LR model with negative tension brane the radion field has a negative kinetic term and a problem with positivity of energy. For models with positive tension branes, the radion field can be stabilized with a mechanism like that in [13]. In the compact RS and in the LR model (for observers living on the right brane), phenomenological agreement with observations requires that the radion is stabilized. Since we will mostly use the LR background as a toy model, we will not stabilize the radion in this paper.

The GRS model can be formally obtained from the LR by taking $k_R = 0$. The space is flat outside the brane region and the theory is really five-dimensional. However, gravity is localized at intermediate scales [3]. The four-dimensional graviton exists as a metastable state in the Kaluza-Klein (KK) spectrum [3, 7, 6]. The model can be conveniently thought of as a regularized version of the RS model, with the position r of the right brane as a regulator. When $r \to +\infty$ we recover the RS model with a four-dimensional graviton at all finite scales. For finite r, the graviton is converted to a metastable state decaying at large distances. Notice that $k_R = 0$ requires that the brane at $x = r$ has negative tension. This point is not invariant under the orbifold symmetry and therefore the translational degrees of freedom of this brane are not projected out. The presence of a negative tension object free to fluctuate is expected to cause troubles at some point.

The Lagrangian (2.2) can be generalized to include scalar fields in the bulk. These extra scalar fields, with a suitable potential, can be used to stabilize the radion. They can also be used to mimic the singular three-brane background with a smooth solution of a five-dimensional Einstein theory coupled to scalars. In this language, a negative tension brane will appear as a kink solution of an ill-defined bulk theory. Positivity of energy shows up in the smooth solution as the requirement that $\phi'' \leq 0$. This follows from the weak dominance energy condition or, even more simply, from an equation of motion that involves the kinetic term for the five-dimensional scalar fields λ_a,

$$\phi'' = -\sum_{ab} G^{ab}\partial\lambda_a\partial\lambda_b \quad . \tag{2.4}$$

Our purpose now is to study fluctuations of the background (2.1) corresponding to the four-dimensional graviton and radion. Many results on the RS and GRS models have been obtained using a particular coordinate system where the fluctuations $h_{\mu\nu}$ are transverse and traceless. This simplifies the equations, but in this system the branes are bent. In order to read the physical potential on a brane, the fluctuation is transformed to Gaussian Normal (GN) coordinates with respect to the brane. The bending must be carefully taken into account in any computation of gravitational potentials, since it is crucially responsible for the correct reproduction of the four-dimensional Einstein gravity [18, 19]. With two branes there are further complications, simply because there is in general no coordinate system that is GN with respect to both. In other words, in coordinates where $\hat{g}_{zz} = 1$, $\hat{g}_{\mu z} = 0$ and the brane at $z = 0$ is flat, the brane at $z = r$ is in general bent. To avoid these kind of problems, we choose to work in a gauge where both branes are flat and where the physical interpretation of each mode is manifest. Our basic coordinate frame is a slight generalization of GN to a two brane system. This frame can be easily constructed as follows. We start from GN coordinates with the respect to the brane in $z = 0$; the second brane will be defined by $F(x, z) = 0$. For a small fluctuation $f(x)$, $F = z - r - f(x)$, so that the bending of the second brane is fully encoded in a 4-d scalar function. One can easily show (see the appendix) that by an infinitesimal coordinate change $(x,\, z) \to (x',\, z')$ the metric can be put in the form

$$ds^2 \;=\; a^2(z')\,[\eta_{\mu\nu} \,+\, \gamma_{\mu\nu}(x', z')]\,dx'^{\mu}dx'^{\nu} \,+\, (1 \,+ \chi(z')f(x'))\,dz'^2 \quad ; \tag{2.5}$$

with the two branes sitting at $z' = 0$ and $z' = r$. As a result, relaxing the condition $\hat{g}_{zz} = 1$ we can have "parallel" branes, still keeping $\hat{g}_{\mu z} = 0$. We shall omit the primes in what follows. Following ref. [20], in order to distinguish the fluctuations of spin 0 and spin 2, we further parametrize the linear perturbations as

$$ds^2 \;=\; a(z,x)^2 \left[\eta_{\mu\nu} \,+\, \tilde{h}_{\mu\nu}(x,z) \,+\, 2\epsilon(z)\,\partial_\mu\partial_\nu f(x)\right] dx^\mu dx^\nu \,+\, b(z,x)^2\,dz^2 \quad ; \tag{2.6}$$

where

$$\begin{aligned} a^2 \;&=\; e^{-2kz}\,[1 + B(z)f(x)] \\ b^2 \;&=\; \frac{(\partial_z \log a)^2}{k^2} \simeq \left(1 - \frac{B'}{k} f\right) \;. \end{aligned} \tag{2.7}$$

The fields $f(x)$ and $\tilde{h}_{\mu\nu}(x,z)$ will end up giving respectively the radion and the tower of spin 2 gravitons. The case $\tilde{h}_{\mu\nu}(x,z) = h_{\mu\nu}(x)$ corresponds to the four-dimensional fluctuations of the graviton. As usual, the wave-function of the graviton is proportional to the unperturbed warp factor e^{-2kz} [1]. Notice that the above profile for $f(x)$ is the appropriate one for a modulus. When f is constant, the metric (2.6) can be reduced to the unperturbed AdS$_5$ with the trivial change of coordinates $\tilde{z} = z - \frac{B(z)f}{2k}$. With the parametrization (2.7) and the additional relation

$$B(z) \;=\; 2\left[e^{2kz} \,+\, k\,e^{-2kz}\,\partial_z\epsilon(z)\right] \;, \tag{2.8}$$

the equations of motion for $\tilde{h}_{\mu\nu}$ and f decouple. This can be explicitly checked using the Einstein equations: the $\mu\nu$ equation does not depend on f so that $\tilde{h}_{\mu\nu}$ solves the pure spin 2 equation. The other equations imply $\Box f = 0$. [3]. From a 4-dimensional

[3] To find the effective lagrangian for the zero modes we could also use the direct method of refs. [14, 15]. Even though this parametrization cannot even solve the linearized Einstein equations [20] for non-trivial fields, it is adequate to describe the zero modes at the two derivative level. We explicitly checked this in the compact RS model and in the LR model with $k_R > 0$. We preferred to use the method of ref. [20], since it is completely well defined also for $k_R = 0$ and because it nicely shows the physical properties of the modes.

viewpoint the choice in eq. 2.8 corresponds to a Weyl frame where the spin 2 fields are not kinetically mixed to the spin 0 radion.

Notice that the function $\epsilon(z)$ can be arbitrarily modified by coordinate changes that preserve $\hat{g}_{\mu z} = 0$. However in general these coordinate changes also shift and bend the branes. This is why we kept ϵ free in the above. As it will become clear below, the physically meaningful quantities are the values of $\partial_z \epsilon$ at the brane positions. Fortunately these quantities are not modified by reparametrizations that keep the branes straight and are, moreover, fixed by the Israel junction conditions. A particular metric of form (2.7) with $\epsilon \equiv 0$ was obtained in ref. [20] for the radion of the compact RS model. Notice that if B is of the form

$$B(z) \sim 2\left[k\, e^{-2kz}\, \partial_z \epsilon(z)\right] , \tag{2.9}$$

the equations of motions are satisfied for any f, since the f dependent part of the metric can be gauged away. We stress that the great simplification obtained in [20] is that the equations of motion for $\tilde{h}_{\mu\nu}$ and f are decoupled. Notice that in our gauge the calculation of the spin-two KK excitations goes through as usual. The massive modes have an invertible 4-dimensional kinetic term and satisfy the condition $\tilde{h}^{(n)\mu}_{\mu} = 0$, $\partial^\mu \tilde{h}^{(n)}_{\mu\nu} = 0$ just by Einstein's equations. For the zero mode $\tilde{h}^{0}_{\mu\nu}(x,z) \equiv h_{\mu\nu}(x)$ we can use the remaining 4-d reparametrization $x^\mu \to x^\mu + \xi^\mu(x)$, $z \to z$ to go to a standard gauge, for instance the harmonic one, without affecting the position of the branes. Then the propagator of our graviton zero mode (when it is normalizable) has already the correct 4-dimensional tensor structure.

Our branes are located at $z = 0$ and $z = r$, with a $\mathbb{Z}_2$ symmetry around the origin. As discussed in [20] the radion corresponds to a non-trivial metric excitation which is however pure gauge in the region $|z| > r$. So to find the radion, we need to patch together two different copies of metric (2.7). In the region $|z| < r$, B will be given by eq. (2.8) with $\epsilon = \epsilon_{_L}$, $k = k_{_L}$. For $|x| > r$, B is given by (see eq. 2.3)

$$B_{_R}(z) = 2\left[k\, e^{-2k_R z + 2(k_R - k_L) r}\, \partial_z \epsilon_{_R}(z)\right] \quad . \tag{2.10}$$

The matching conditions between different regions in space-time with different values of k requires continuity for the functions $a, b, \epsilon, \partial_z\epsilon$. The conditions on the first derivative of a is automatically satisfied with the ansatz for a and b in eq. (2.7). For instance, at $z = r$ the jump of the extrinsic curvature is given by

$$[K_{\mu\nu}] = q a^2 \left[\frac{1}{2} + \left(B + \frac{B'}{2k}\right)\frac{1}{2}\right]\eta_{\mu\nu} + \left[a^2 q \epsilon + \left(\epsilon'_{_R} - \epsilon'_{_L}\right) a^2\right]\partial_\mu \partial_\nu f , \tag{2.11}$$

with $q = \lim_{z\to r}\left[\partial_z \log a^2_{_R} - \partial_z \log a^2_{_L}\right]$. As a result the Israel junction conditions,

$$[K_{\mu\nu}] = -\left(2M^2\right)^{-1}\left(T_{\mu\nu} - \frac{1}{3} T_{\alpha\beta}\, g^{\alpha\beta}_{in}\, g_{in\,\mu\nu}\right) \tag{2.12}$$

involving the brane energy-momentum tensor $T_{\mu\nu} = -\frac{1}{2}(\hat{g}_{zz})^{-1/2}\,\tau_i\, g_{in\,\mu\nu}$, simply require that ϵ' is continuous through the brane.

The radion f is a localized 4-dimensional field, whose kinetic term we want to calculate. Let us consider the LR model with generic $k_{_L}$ and $k_{_R}$. Our general formulae can be applied to the GRS model as well, provided the limit $k_{_R} \to 0$ exists. Set $\tilde{h}_{\mu\nu} = 0$ first. The matching conditions at the origin, imposing the $\mathbb{Z}_2$ symmetry, give $\partial_z \epsilon_{_L}(0) = 0$, $B_{_L}(0) = 2$. The matching conditions at $z = r$, $B_{_L}(r) = B_{_R}(r)$, $\partial_z \epsilon_{_L}(r) = \partial_z \epsilon_{_R}(r)$ fix

$$B_{_L}(r) \;=\; 2\, e^{2k_L r} \frac{k_{_R}}{k_{_R} - k_{_L}} \quad . \tag{2.13}$$

Since we considered a pure gauge solution in the region outside the brane, the contribution of $|z| > r$ to the action is just a constant, which is irrelevant for our computation of the kinetic term. The only contribution to the effective action comes from the region between the two branes. Expanding the Lagrangian (2.2) and considering only derivative terms, we obtain

$$\begin{aligned} \mathcal{L} &= -M^3 \int_{-r}^{r} dz \sqrt{-\hat{g}}\, \delta\hat{g}^{MN}(E_{MN}) = 2M^3 \int_0^r dz \frac{3B'(z)}{k_{_L}} (f\Box f) \\ &= 6M^3\, \frac{B_L(r) - B_L(0)}{k_{_L}} (f\Box f) \;=\; \frac{24M^3}{k_{_L}} \left(e^{2k_L r} \frac{k_{_R}}{k_{_R} - k_{_L}} - 1 \right) \left(\frac{f\Box f}{2}\right) \quad , \end{aligned} \tag{2.14}$$

where E_{MN} are the five-dimensional equations of motion. We always work on the full real axis for z with a $\mathbb{Z}_2$ identification. The previous computation is greatly simplified by the fact that the four-dimensional components $E_{\mu\nu}$ are identically zero[4]. Notice that the final result, before integration in z, is a total derivative. The kinetic term only depends on the value of the function $B(z)$ at the branes positions. This is a welcome fact since, while the B function itself is gauge dependent, its value on the branes is fixed by the matching conditions.

Our 4-dimensional action truncated to the zero modes will be normalized as follows,

$$\mathcal{L} \;=\; 2\,\hat{M}_r^2 \int dx^4 \sqrt{-g}\, R_4 \;+\; \frac{C_r}{2} \int \sqrt{-g} dx^4\, f\Box f \quad , \tag{2.15}$$

with the kinetic term given by

$$C_r \;=\; \frac{24M^3}{k_{_L}} \left(e^{2k_L r} \frac{k_{_R}}{k_{_R} - k_{_L}} - 1 \right) \quad . \tag{2.16}$$

[4]Since we considered a flat four-dimensional metric, this is just the statement (or a check) that the equations of motion for g and f are decoupled.

The four-dimensional Planck mass is easily computed by integrating the graviton wave-function [17, 1],

$$2\hat{M}_r^2 \,=\, 4M^3 \left(\int_0^r e^{-2k_L z} + \int_r^\infty e^{-2k_R z+2(k_R-k_L)r} \right) \,=\, \frac{2M^3}{k_L} \left(1 + e^{-2k_L r} \frac{k_L - k_R}{k_R} \right) . \tag{2.17}$$

Notice that r explicitly appears in the above formulae. r is related to the VEV of f by a non-linear transformation.

3. The Role of the Radion in the GRS Model

We can now extract information about the GRS model. In the limit $k_R \to 0$ the radion has a finite, and negative, kinetic term $C_r = -24M^3/k$, independent of r. Notice that the entire contribution to C_r comes from $B(0)$, since $B(r) = 0$. As noticed in [20], in the GRS model the radion profile is zero on the second brane. This is simply due to the fact that the space is flat to the right. The four-dimensional Planck mass, on the other hand, diverges in the limit $k_R \to 0$. This is a signal that the graviton zero mode is not normalizable in the GRS model. However, conventional four-dimensional gravity is mimicked at intermediate scales by metastable states from the KK tower. The integration over arbitrarily light KK modes produces a propagator for an effective four-dimensional graviton. To correctly reproduce the Einstein theory, this propagator must have the form

$$< h_{\mu\nu}\, h_{\rho\sigma} > \sim \left(\frac{\eta_{\mu\rho}\,\eta_{\nu\sigma} \,+\, \eta_{\nu\rho}\,\eta_{\mu\sigma}}{2} - \frac{\eta_{\mu\nu}\,\eta_{\rho\sigma}}{2} \right) \frac{1}{q^2} \,=\, \left(\frac{P_2}{2} - \frac{P_0}{2} \right) \frac{1}{q^2} \quad , \tag{3.1}$$

where we have neglected terms involving q_μ in the tensor structure. It was noticed in [6] that the KK modes unfortunately always give a contribution to the effective propagator with a behavior appropriate to a 4-dimensional massive graviton

$$\frac{\Delta}{4}(\frac{P_2}{2} - \frac{P_0}{3})\frac{1}{q^2} \quad , \tag{3.2}$$

where $\Delta = k_L/M^3$ was explicitly computed in ref. [3]. Based on this discrepancy in the tensorial structure of the propagator, the authors in [6] concluded that GRS-like models are in contradiction with experimental results on gravitational forces. However physics is slightly more subtle. By five-dimensional general covariance, also the radion couples to the trace of the energy momentum so it contributes a factor P_0/C_r to the effective propagator. Numerical factors felicitously combine in such a way that the full (KK + radion) propagator will behave as prescribed by the four-dimensional Einstein theory. Notice that the fact that the radion has a negative kinetic term is crucial for this argument. It is certainly true that, as stressed in [10], whenever gravity is not localized but only obtained trough KK modes, one needs a state with negative norm to change the $-1/3$ in the propagator into $-1/2$[5]. In the GRS model, it not difficult to identify this state with the radion.

This result deserves some comments. That the discrepancy between massive and massless propagators can be cured was already pointed out in [8, 9], in analogy with a similar phenomenon in the RS model. A computation including the brane bending effect [8, 9] shows that the full graviton propagator at intermediate distances has the right form form (3.1). We just showed that the bending effect can be equivalently explained by the existence of a localized and physical radion mode. At this point two remarks are in order. First, notice that the equivalence is consistent because the bending computation in [8, 9] is correct only for a radion that has not been stabilized[18, 21]. It has been shown (for the compact RS model) [21] that the stabilization mechanism induces an extra correction to the bending, as required by physical intuition. Second, the analogy used by ref. [8] with the RS model is somehow misleading. In the RS model, the bending effect is an artifact of the gauge choice. By using our gauge where the graviton is not transverse traceless, the four-dimensional graviton automatically shows up with the right propagator. In the GRS model, on the other hand, the bending has a physical meaning. This is due to the fact that, in the GRS model, the graviton is not really a zero-mode, but a metastable state made up with massive KK modes.

In conclusion, the behavior of gravity at intermediate scales in the GRS model is correct, as already pointed out by many authors [8, 9]. At very large scales, the metastable graviton disappears and we are left with a radion with negative kinetic term. Notice that on this point we disagree with ref. [11]. They used a RG-inspired reasoning to argue that, even at very large distances, the sick negative energy problem disappears and the model is well behaved at all scales. The argument in [11] assumes implicitly that the radion is stabilized and explicitly that there is no problem with the positivity of energy. Under these conditions, the physics at large distances is apparently well behaved. The system of two branes would look at very large scales like a single object without any trapped massless modes[6]. However, the crucial issue in the GRS model is whether these conditions can be physically realized. From the viewpoint of this paper the results of ref. [11] would for instance be obtained if the radion had a mass of the order of $k_L e^{-3k_L r}$, the graviton width. This way the graviton and radion potentials would turn off together at large distances. However, giving a mass to a field with negative kinetic term clearly does not remove the associated instability. In any event, the negative energy problem will show up at some energy

[5]The authors of ref.[10] call generically this state a *ghost*, without trying to identify it in the model at hand.

[6]This composite system has effectively zero tension. This, by itself, is not necessarily a problem, since the composite brane sits at an orbifold fixed point. Indeed one can consider a new model by flipping all the signs of the brane tensions in GRS. At large distance we have a tensionless brane, but with a radion of positive kinetic term trapped on it

scale.

To further clarify the role of the radion and to check the consistency of our method, we can use our results for models with $k_R > 0$ where we do have a graviton zero mode. The four-dimensional Lagrangian for the zero modes contains the graviton and the radion. By construction, we do not have problems with gauge fixing and brane bending. The propagator for the graviton is the correct one, as in equation (3.1). Let us assume that the radion is not stabilized. Let us compare the bending computation with our method based on just the physical zero modes. Without stabilization, the bending effect only depends on the warp factor derivative at the origin $z = 0$, which is model independent. In particular, it does not depend on k_R. The four-dimensional Planck mass $\hat{M}_r$, however, depends on k_R. The propagator on the $z = 0$ brane truncated to zero modes is

$$\frac{1}{8\hat{M}_r^2}\left(P_2 - \frac{2}{3}P_0\right)\frac{1}{q^2} - \frac{1}{24M_L^2}\frac{P_0}{q^2} \quad , \tag{3.3}$$

where $M_L = M^3/k_L$ is the Planck mass in the RS model. Notice that Einstein gravity is not reproduced. We have instead a tensor-scalar gravity, as expected from the existence of a radion field. From the viewpoint of our "effective" Lagrangian for zero modes, we have two contributions, $\frac{1}{8\hat{M}_r^2}(P_2 - P_0)$ from the graviton, and $\frac{P_0}{C_r}$ from the radion. Agreement with the bending method requires

$$\frac{1}{\hat{M}_r^2} - \frac{24}{C_r} = \frac{1}{M_L^2} = \frac{k_L}{M^3} \quad . \tag{3.4}$$

Using eqs. (2.16) and (2.17), we can easily check that this equation is identically satisfied. This is another indication that our method is equivalent to brane bending calculations.

One final comment concerning the role of the radion in the recovery of locality is in order. Consider once more a LR model characterized by k_L, k_R and brane position r. We want to study the limit $r \to \infty$ with k_L fixed, as perceived by an observer doing experiments at a fixed distance on the Planck brane. This observer should recover the physics of the non compact RS model with warp factor k_L. The full propagator on this brane has the form

$$\frac{1}{\hat{M}_r^2}(P_2 - P_0)\frac{1}{q^2} + \frac{1}{C_r}\frac{P_0}{q^2} + \left(P_2 - \frac{2}{3}P_0\right) G_{KK}(q^2) \quad ; \tag{3.5}$$

G_{KK} indicates the contribution of the massive KK modes. Now, if we keep k_R and k_L fixed and send $r \to \infty$ we have that $C_r \to \infty$, so that the radion disappears, while $\hat{M}_r^2$ goes to the value $\to M^3/k_L$ proper for the non-compact RS model. Then the above formula gives the right leading $1/r$ potential of RS without any adjustment from the KK tower. On the other hand, consider a limit in which $r \to \infty$ with k_L fixed but with a scaling k_R such that C_r stays fixed. By eq. (2.16) this situation requires $k_R \sim e^{-2k_L r} k_L \ll k_L$, so that the bulk brane must have negative tension. It is important to remark that with this limiting procedure $\hat{M}_r^2$ goes to a value different than M^3/k_L. At first sight this seems to violate locality, as the second brane should be going out of the sight of the Planck observer. To reestablish locality it is necessary that the G_{KK} term from the KK tower develops an effective pole at $q^2 = 0$. This can fix the coefficient of the P_2 term, but it certainly upsets the P_0 term. The latter can only be adjusted by the propagation of an extra scalar, and the finite $1/C_r$ radion term is there precisely for this. Notice that the particular choice $k_R = 0$ in this discussion is just the GRS model. We stress that the need for a non-decoupling massless and ghost-like radion is *always* associated with the presence of a brane of negative tension.

This ghost, as suggested by ref. [10], serves the purpose of giving a smooth limit as the KK resonance mass (width) goes to zero. This smoothness is precisely the requirement of locality. Something vaguely similar can be done for the simple Abelian Higgs model. In the unitary gauge the lagrangian is

$$L = \frac{1}{4e^2} F_{\mu\nu} F^{\mu\nu} - m^2 A_\mu A^\mu + A_\mu J^\mu \tag{3.6}$$

where J^μ is the matter field contribution to the current, the analogue of the brane energy momentum in our case. In the limit $m \to 0$ the photon keeps having 3 helicity states and moreover its propagator becomes singular. A smooth limit can be taken by adding an object with negative kinetic term and with lagrangian

$$L = \frac{1}{2}(\partial\Phi)^2 + \frac{\Phi}{m} \partial_\mu J^\mu. \tag{3.7}$$

Diagrammatically, the propagation of Φ eliminates the singular $k_\mu k_\nu / m^2$ terms in the photon exchange and eliminates the third unwanted helicity state. Formally this can be seen by adding the above two lagrangians and redefining $A_\mu \to A_\mu + \partial_\mu \Phi / m$. Now, Φ only appears as a Lagrange multiplier term $\Phi \partial_\mu A^\mu$: integrating over Φ we get a gauge fixing condition $\partial_\mu A^\mu = 0$. So we get for $m \to 0$ a massless photon (2 helicities) with an invertible kinetic term (in the Landau gauge). For finite m, the above simply corresponds to Stueckelberg's Lagrangian (or B-field formalism) in Landau gauge.

In the course of this discussion we only focussed on what sees an observer on the Planck brane, whatever k_R is. Notice than from this point of view, as soon as $k_L r$ is large enough there is no real need to stabilize the radion in order to recover a good approximation to Einstein gravity. This happens both for positive and negative bulk brane tension, and in particular for the GRS case of $k_R = 0$. On the other hand, if we had been concerned with what sees an observer on the bulk brane, then the recovery of Einstein gravity would have required the stabilization of the radion. This is because the radion couples a lot more strongly to this brane (at least for $k_R \neq 0$).

4. Conclusions

In this letter, we have clarified the role of the radion in the GRS model. We want to stress that, in our opinion, even before any actual computation of how gravity behaves in the GRS model, one must face the basic objection made in [5]. Positivity of energy is a basic requirement. In a model where it is violated, we expect to see dangerous effects at some point. Similar objections have been made also in [10]. The interplay between violation of energy-positivity and recovery of Einstein gravity is however intriguing. As already stressed in [6, 10], a negative energy state seems to be necessary for recovering the correct Einstein gravity, and our analysis confirms this fact. However, it is true that the negative energy effect only appears at very large distances, bigger than the universe radius, with a proper choice of parameters. It is a four-dimensional negative energy effect in a five-dimensional flat universe. It is not clear how such an effect can be consistently incorporated in a sensible theory. A possible question now is whether consistent modifications of the GRS model or mechanisms for solving the radion problem exist. We believe that, contrary to the claim in [11], this problem has not yet been solved in the GRS model. Since the negative norm state played a crucial role in our analysis, at first look it seems a difficult task to find a positive-energy model that, at the same time, reproduces the correct gravity behavior. However, the intrinsic attractiveness of the model and the fact the Einstein gravity is correctly reproduced at intermediate scales strongly calls for some way out.

A. Linear perturbations

Let us start from the GN metric

$$\hat{g}_{AB} = \begin{pmatrix} g_{\mu\nu}(x,\, z) & 0 \\ 0 & 1 \end{pmatrix} \quad . \tag{4.1}$$

with the first brane sitting at $z = 0$ and the second brane defined by the equation

$$z = r + f(x) \quad . \tag{4.2}$$

Let us consider the coordinate change

$$z \,=\, z' \,+ f(x)\,\chi(z')\,, \qquad \chi(0) \,=\, 0\,, \qquad \chi(r) \,=\, 1 \quad . \tag{4.3}$$

In the coordinates $(x,\, z')$ the branes are located at $z' = 0$ and $z' = r$ and the metric has the form

$$\hat{g}'_{AB} = \begin{pmatrix} g'_{\mu\nu}(x,\, z') & \chi(z')\,\partial_\mu f(x) \\ \chi(z')\,\partial_\mu f(x) & [1 \,+\, f(x)\,\partial_{z'}\chi(z')]^2 \end{pmatrix} \quad . \tag{4.4}$$

This parametrization shows, as it is obvious, that in order to keep the branes parallel it suffices to introduce a scalar field $f(x)$. With a further coordinate transformation $x^\mu = x'^\mu + \xi^\mu(x', z')$ we can eliminate the off-diagonal terms. For our purposes it suffices to consider an infinitesimal bending f and work in the linearized approximation. Then in eq.(4.4) we have

$$g'_{\mu\nu} \;=\; a^2(z)\;[\eta_{\mu\nu} \;+\; \gamma_{\mu\nu}(x,z)] \tag{4.5}$$

where $\gamma_{\mu\nu}$ represents a small perturbation. The required transformation is

$$\xi^\mu \;=\; \psi(z')\,\eta^{\mu\nu}\partial_\nu\phi\,, \qquad \text{with} \quad \chi(z') \;+\; a^2(z')\partial_{z'}\psi(z') \;=\; 0\;. \tag{4.6}$$

As a result, we get a metric (2.5)

$$ds^2 \;=\; a^2(z')\left[\eta_{\mu\nu} \;+\; \gamma'_{\mu\nu}(x',z')\right]dx'^\mu dx'^\nu \;+\; [1\;+2\partial_{z'}\chi(z')f(x')]\,dz'^2 \quad . \tag{4.7}$$

Acknowledgments

We thank M. Mintchev and A. Strumia for helpful discussions.
A. Z. is partially supported by the European Commission TMR program ERBFMRX-CT96-0045, wherein he is associated to the University of Torino. L. P. and R. R. are partially supported by the EC under TMR contract ERBFMRX-CT96-0090.

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