OPTIMAL EXPERIMENTS WITH SEISMIC SENSORS

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ABSTRACT

In this paper, we consider the problem of detecting and locating buried land mines and subsurface objects by using seismic waves. We demonstrate an adaptive seismic system that maneuvers an array of receivers, according to an optimal positioning algorithm based on the theory of optimal experiments, to minimize the number of distinct measurements to localize the mine. The adaptive localization algorithm is tested using numerical model data as well as laboratory measurements performed in a facility at Georgia Tech. It is envisioned that the future systems should be able to incorporate this new method into portable mobile mine-location systems.

1. INTRODUCTION

Buried land mines and similar subsurface structures pose a huge threat to resettling civilians. It takes significant time and resources to clear out regions contaminated by mines, so it is important to develop efficient detection and localization systems to create a safer environment. Georgia Tech has built a laboratory to collect the real data needed to investigate buried land mine and subsurface target detection problem [1]. In laboratory data, the detection schemes using seismic waves have been extensively tested and shown to have satisfactory mine detection probabilities [1, 2].

Seismic waves, scattered from man-made targets, induce resonances that result in a stronger sustained reflection from mines than from clutter objects. Hence, it is possible to use seismic imaging to discriminate land mines from common types of clutter such as rocks, wood, etc. To detect a mine, a seismic wave is launched from a source at a known location. The seismic wave then travels through the soil and interacts with objects under the ground. The resulting propagating waves in an elastic medium are of two main types: surface waves and body waves. The existing research concentrates on the reflected surface waves (Rayleigh waves) for detection, because the Rayleigh waves carry most of the returned energy.

Typical imaging methods are time consuming and expensive if measurements are taken over 2D grids with large apertures in order to have sufficient image resolution over the space of interest. Once a complete image is formed from a large data set, it is then searched to find targets [2]. However, to image any single target, only a small subset of the measurements is actually required, but this subset is not known ahead of time. Therefore, if we want to reduce the time or the resources needed to localize a target, we can use maneuvering receiver(s) to take the minimum number of measurements needed, if we can develop an adaptive algorithm to find the best receiver positions. With each new measurement we want to maximize the information gained about the target. In our case we use a maneuvering 3x3 array to create an efficient system to detect and locate mines. In the method proposed here, any one image, created at successive measurements, has low resolution. However, as the array maneuvers, the cumulative imaging operation improves the resolution around the true mine location.

The array movement is based on the theory of optimal experiments [3]. We employ a 2D sensor array with known relative receiver positions. Starting at an arbitrary array position, we calculate an initial estimate of the target location. Then, the variance of the location estimate is calculated, by using the Fisher information matrix (FIM). Based on the expected value of the FIM, the next optimal array position is determined by using the theory of optimal experiments [3, 4]. The search for the optimal array position maximizes the determinant of the Fisher information matrix. The two steps involved in the maneuver strategy for a mobile array of sensors are shown in Figs. 1(a) and (b).

The following sections will describe the data model, which leads to the target position estimate and its performance bounds. Then the algorithm for determining the next optimal array position is shown. Performance of the algorithm is demonstrated by using the experimental data collected in a laboratory setting [1].

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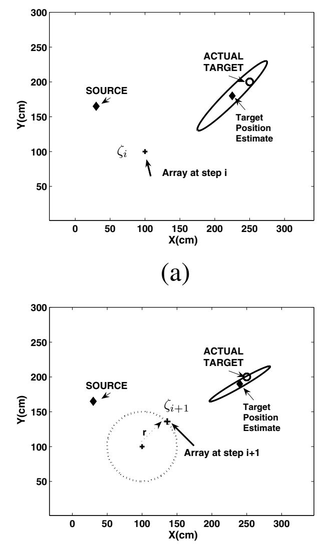


(a)

(b)

Fig. 1. Algorithm mechanics illustrated: (a) Source generates a probing pulse, and the waves are reflected from the target and collected by the array. At step i, z_i denotes the target position and ζ_i the array center. (b) Estimate of the next array position ζ_{i+1} by using the cumulative Fisher Information Matrix measure and a circular constraint on the movement.

2. DATA MODEL, TARGET LOCATION ESTIMATION AND PERFORMANCE BOUNDS

Consider a single seismic source and an array that consists of P seismic receivers, where the source and receivers are co-planar. We model the soil as a highly dispersive medium with frequency dependent velocity. Hence, the signal processing is done in the frequency domain, even though the measurements are taken in the time domain. The source generates a probing pulse which is reflected from buried targets and is collected by the seismic sensors. The probing pulse and reflected waves at each sensor are separated by using the algorithm in [6]. The reflected signal from the target can be represented as [5]:

$$\mathbf{y}(\omega) = \mathbf{A}(\zeta, \mathbf{z}, \omega)\mathbf{s}(\omega) + \mathbf{n}(\omega), \tag{1}$$

where $\mathbf{y}(\omega) \in \mathcal{C}^{P\times 1}$ is the noisy array output vector, $\mathbf{n}(\omega) \in \mathcal{C}^{P\times 1}$ is a complex additive noise, and $\mathbf{s}(\omega) \in \mathcal{C}^{K\times 1}$ is the signal vector. The array manifold $\mathbf{A}(\zeta, \mathbf{z}, \omega)$ has elements given by the Green's function:

$$g(r, r', \omega) = \frac{i}{4} H_0^{(1)}\left(\frac{\omega}{v(\omega)}|r - r'|\right) \tag{2}$$

where $H_0^{(1)}$ is the zero-order Hankel function of the first kind, and $v(\omega)$ is the frequency-dependent velocity. Spectrum analysis of the surface waves is used to determine the velocity vs. frequency [6].

Let $\mathbf{Y}_t = \left[\, \mathbf{y}_t^T(\omega_1)\,,\ \ldots\ ,\ \mathbf{y}_t^T(\omega_N)\,\right]^T$, $\mathbf{Y}_t \in \mathcal{C}^{PN\times 1}$, be the data vector, formed by aggregating the Fourier transform at frequencies ω_i of the received data $\mathbf{y}_t$ at each seismic sensor during the batch period t, where $i = 1, 2, \ldots, N$. The choice of the Fourier transform frequencies is discussed later. Under the *i.i.d.* Gaussian noise assumption, the probability density function for the current received data is given by [7]:

$$p(\mathbf{Y}_t) = \prod_{l=1}^{N} \frac{1}{\pi^P \sigma_t^{2P}} \exp\left\{-\frac{1}{\sigma_t^2}||\mathbf{y}_t(\omega_l) - \mathbf{A}_t(\omega_l)\mathbf{s}_t(\omega_l)||^2\right\} \tag{3}$$

Using (3), one can calculate the negative log-likelihood function of the data:

$$L^- = NP\log(\pi\sigma_t^2) + \frac{1}{\sigma_t^2}\sum_{l=1}^{N}||\mathbf{y}_t(\omega_l) - \mathbf{A}_t(\omega_l)\mathbf{s}_t(\omega_l)||^2. \tag{4}$$

The ML estimate, maximizing the log-likelihood, can be determined by minimizing L^-. In (4), both the target signal and the noise variance are unknown. Therefore, we first estimate the noise variance by fixing the target position in $\mathbf{A}_t(\omega)$ and the source signal $\mathbf{s}(\omega)$. The ML estimate of the noise variance σ_t^2 is given by:

$$\hat{\sigma}_t^2 = \frac{1}{NP}\sum_{l=1}^{N}||\mathbf{y}_t(\omega_l) - \mathbf{A}_t(\omega_l)\mathbf{s}_t(\omega_l)||^2. \tag{5}$$

When the estimated noise variance is used in conjunction with (4), the ML target signal estimate can be calculated:

$$\hat{s}_t(\omega_l) = \left(\mathbf{A}_t^H(\omega_l)\mathbf{A}_t(\omega_l)\right)^{-1}\mathbf{A}_t^H\mathbf{y}_t(\omega_l). \tag{6}$$

Substituting (5) and (6) into (4), one can determine the ML cost function to minimize as a function of $\mathbf{z}$:

$$J_t(\mathbf{z}) = \sum_{l=1}^{N}\left|\left|\left\{I - \mathbf{A}_t(\omega_l)\left(\mathbf{A}_t^H(\omega_l)\mathbf{A}_t(\omega_l)\right)^{-1}\mathbf{A}_t^H(\omega_l)\right\}\mathbf{y}_t(\omega_l)\right|\right|^2 \tag{7}$$

The target location estimate is then given by the minimum of the cost function (7):

$$\mathbf{z} = \arg\min_{\mathbf{z}} J_t(\mathbf{z}). \tag{8}$$

The Cramér-Rao lower bound (CRLB) is an information theoretic inequality, which provides a lower bound for the variances of the unbiased estimators. The Cramér-Rao lower bound is the inverse of the Fisher information matrix (FIM). Assuming that the variance of the additive noise in (1) is known, the log-likelihood function for a single target can be written as:

$$L(\zeta_t, \mathbf{z}) \doteq -\frac{1}{\sigma_t^2}\sum_{l=1}^{N}||\mathbf{y}_t(\omega_l) - \mathbf{a}_t(\zeta_t, \mathbf{z}, \omega_l)s_t(\omega_l)||^2 \tag{9}$$

where $\mathbf{a}_t(\zeta_t, \mathbf{z}, \omega)$ is the propagation (steering) vector from the array center to the target position. The $(i,j)^{\text{th}}$ element of the FIM is given by the partial derivative of (9) with respect to the i^{th} and j^{th} parameters of the vector $\mathbf{z}$ [7]:

$$\begin{aligned} \mathbf{F}_{i,j}(\mathbf{z}, \zeta_t) &= E_y \left\{ \frac{\partial^2 L(\mathbf{z}, \zeta_t)}{\partial z_i \partial z_j} \right\} \\ &= -\frac{2}{\sigma_t^2} \sum_{l=1}^{N} \Re \left\{ \left(\frac{\partial \mathbf{a}_t(\mathbf{z}, \zeta_t, \omega_l)}{\partial z_i} \right)^H \frac{\partial \mathbf{a}_t(\mathbf{z}, \zeta_t, \omega_l)}{\partial z_j} \right\} \end{aligned} \tag{10}$$

where $E_y\{.\}$ denotes the expected value, and $\mathbf{F}$ is the Fisher information matrix as a function of the target position z and the array center ζ. The elements of the steering vector are given in terms of 2-D Green's function in (2). The partial derivative of the steering vector is calculated with respect to the target coordinates for a fixed array center.

3. MOVEMENT OF THE SEISMIC ARRAY VIA OPTIMAL EXPERIMENTS

In the previous section, we described how to determine the target position and its FIM which represents the uncertainty about the estimates as a function of the array center position. Recall that the sensors in the 2D array have known locations with respect to the array center ζ. Suppose that we have estimated the target location at batch t, and now we are interested in determining the next optimal array center position candidate for the batch $t+1$. Our approach in selecting the new sensor position to reduce the expected uncertainty in the estimated target coordinates is to minimize the determinant of the CRLB, or equivalently, maximize the determinant of the FIM as a function of the array center. In the literature of optimal experiments, this technique is called D-optimal design [3], and has been applied to metal detectors in [4]. Other approaches might minimize the trace of the CRLB or minimize its maximum eigenvalue.

Let q represent the determinant of the FIM. The cumulative effect of the measurements up to batch t can be written as:

$$q\left(\{\zeta_1, \ldots, \zeta_t\}\right) = |F(\zeta_1, \ldots, \zeta_t)| = \left| \sum_{j=1}^{t} F(\zeta_j) \right| \tag{11}$$

where $|\cdot|$ stands for determinant and F_t represents the FIM at batch t. The logarithmic increase due to the additional measurements at batch $t+1$ is given by:

$$\begin{aligned} \delta_q(\boldsymbol{\zeta}_{t+1}) &= \ln q\left(\{\zeta_1, \ldots, \zeta_{t+1}\}\right) - \ln q\left(\{\zeta_1, \ldots, \zeta_t\}\right) \\ &= \ln \left| I + F(\zeta_{t+1}) B_t^{-1} \right| \end{aligned} \tag{12}$$

where I is an identity matrix, and $B_t = \sum_{j=1}^{t} F(\zeta_j)$. To achieve the maximum expected information gain, the next optimal array center can be determined by

$$\zeta_{t+1} = \arg\max_{\zeta} \ln \left| I + F(\zeta_{t+1}) B_t^{-1} \right|. \tag{13}$$

In this optimization problem, there are additional constraints that come from the configuration of the seismic system. First of all, the target reflections do not behave as an omni-directional active source. Hence, we need to make sure that the receiving array is between the source and the targets all the time to receive the reflected waves. One way to impose this condition is to use a movement step size of radius r from the previous array center position as shown in Fig. 1. As a result, the maximum of (13) is calculated on a circle of radius r, where the center of the circle is at the previous optimum array center position.

4. PROCESSING OF EXPERIMENTAL DATA

An experiment has been conducted in our laboratory setting, where buried mines in a sandbox are used as targets [1]. A shaker is used as a seismic source, where the input signal is a differentiated Gaussian pulse centered at 450 Hz. In the experiment, the seismic sensors are ground contact accelerometers. The target is a TS-50 (anti-personnel) landmine buried at a depth of 1 cm. We estimate the wavenumber for the reflected signals at different frequencies by using the algorithm presented in [6]. To separate the forward and reflected waves, a linear array of fifteen sensors is used, however only three are kept for use in the maneuver algorithms. With three linear arrays, a total of nine sensors (3×3) are used in actual imaging.

Once the data is collected and the waves are separated the next step is to estimate the target position. The initial estimate is shown as a surface plot (Fig. 2). The surface plot is obtained by using (7), and this cost function is calculated at each point in a 2D grid. The minimum of $J_t(\mathbf{z})$ gives the target position estimate. However, the inverse of this function is plotted in the surface plots. Based on an initial estimate, the next optimal array position is determined by using (13). This function is calculated at each grid point as a function of array center position, using the estimated target position from the previous step. The surface plot is shown in Fig. 3, along with the circle constraint, at a radius of 30cm.

Once the next optimum array position is determined and the array is moved to a new position, a new batch of data is collected. We then append the new data set to the existing data. The new target position estimate and the next optimum movement are determined by using the cumulative data. Further steps are shown in Figs. 4. With each successive step the target position estimate is improved, along with

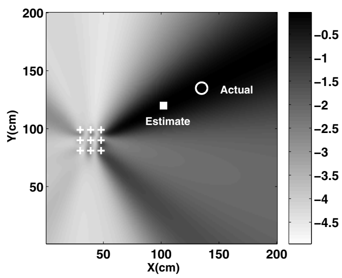
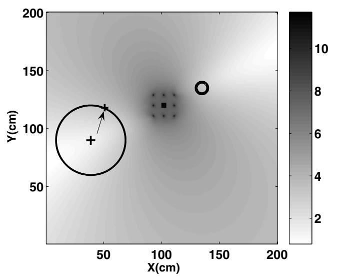
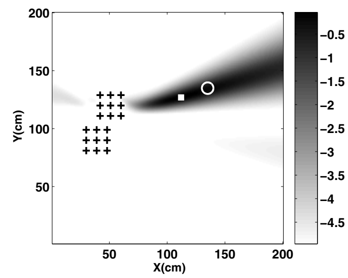


Fig. 2. Initial target location estimate is done using the ML cost function (7), shown on a dB scale.

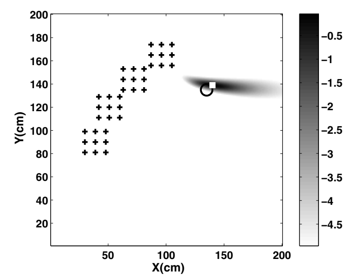


(a)

Fig. 3. Next Optimal array position: Surface plot obtained by using (13), shown on a linear scale, with a circle of radius 30 cm.



(a)



(b)

Fig. 4. Target position estimate (db scale) (a) after two optimal moves. (b) after four optimal moves.

a decrease in the uncertainty ellipse, because the cumulative estimation is effectively increasing the aperture.

5. CONCLUSIONS

The algorithm presented in this paper shows that it is possible to control a maneuvering seismic array to find buried targets from reflected surface waves. A complete mine finding system would require one more step to distinguish a land mine from clutter. Since the maneuver algorithm can obtain very accurate estimate of the target location, the array would be positioned to exploit the "resonance property" of buried land mines to make this final confirmation. The example in the paper uses a total of 180 seismic measurements to locate the target. This can be compared with a conventional method that would scan an entire grid of size (100×100) [1, 2] in order to find the same target within an area of (2×2) meters. Furthermore, the whole scan would take a few hours to isolate a resonating target.

6. REFERENCES

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