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Is There a Hot Electroweak Phase Transition at $m_H \gtrsim m_W$?

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Abstract

We provide non-perturbative evidence for the fact that there is no hot electroweak phase transition at large Higgs masses, $m_H = 95$, 120 and 180 GeV. This means that the line of first order phase transitions separating the symmetric and broken phases at small m_H has an end point $m_{H,c}$. In the minimal standard electroweak theory 70 GeV $< m_{H,c} <$ 95 GeV and most likely $m_{H,c} \approx 80$ GeV. If the electroweak theory is weakly coupled and the Higgs boson is found to be heavier than the critical value (which depends on the theory in question), cosmological remnants from the electroweak epoch are improbable.

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The transition between the high temperature symmetric (or confinement) phase and the low T broken (or Higgs) phase in the standard electroweak theory or its extensions is known to be of first order for small values of the Higgs mass m_H. This follows from perturbative studies of the effective potential [1] and non-perturbative lattice Monte Carlo simulations [2, 3, 4]. In the region of applicability of the perturbative expansion the strength of the electroweak phase transition decreases when m_H increases. However, the nature of the electroweak phase transition at "large" Higgs masses, $m_H \gtrsim m_W$ remains unclear, since the perturbative expansion for the description of the phase transition is useless there. This letter contains the results of the first non-perturbative MC analysis of the problem for "large" Higgs masses, $m_H = 95, 120, 180$ GeV. We shall show that the system behaves very regularly there, much like water above the critical point. As there is no distinction between liquid water and vapor, there is no distinction between the symmetric and broken phases; there is no long-range order.

In ref. [5] it has been shown that in a weakly coupled electroweak theory and in most of its extensions (supersymmetric or not) the hot EW phase transition can be described by an SU(2)×U(1) gauge-Higgs model in three Euclidean dimensions. (We stress that our study is not applicable for a strongly coupled Higgs sector, where the perturbative scheme of dimensional reduction is not valid.) Since the effects of the U(1) group are perturbative deep in the Higgs phase and high in the symmetric phase, the presence of the U(1) factor cannot change the qualitative features of the phase diagram of this theory. Thus we shall neglect the U(1) factor and work in the limit $\sin\theta_W = 0$. The effective Lagrangian is

$$L = \frac{1}{4}G^a_{ij}G^a_{ij} + (D_i\phi)^\dagger(D_i\phi) + m_3^2\phi^\dagger\phi + \lambda_3(\phi^\dagger\phi)^2, \qquad (1)$$

where G^a_{ij} is the SU(2) field strength, ϕ is the scalar doublet and D_i is the covariant derivative. The three parameters of the 3d theory (gauge coupling g_3^2, scalar self-coupling λ_3 and the scalar mass m_3^2) depend on temperature and on underlying 4d parameters and can be computed perturbatively; the explicit relations for the MSM are worked out in [5] and for MSSM in [6]. The phase structure of the theory (1) depends on one dimensionless ratio, $x = \lambda_3/g_3^2$, because the dimensionful coupling g_3^2 can be chosen to fix the scale, while the change of the second dimensionless ratio $y = m_3^2(g_3^2)/g_3^4$ corresponds to temperature variation. For $y \gg 1$ (large T) the system is in the strongly coupled symmetric phase, while at $y \ll -1$ (low T) the system is in the weakly coupled Higgs phase. In presenting our results we will use a more physical set of variables m_H^* and T^* instead of x and y. The parameter m_H^* is the tree-level Higgs mass in the 4d SU(2)+Higgs theory and T^* is the temperature there. The exact relationship between (x, y) and (m_H^*, T^*) is given in eqs. (2.9–10) of [3].

An essential point in understanding the phase structure of the theory is the fact that the 3d gauge-Higgs system (as well as the underlying electroweak theory) does

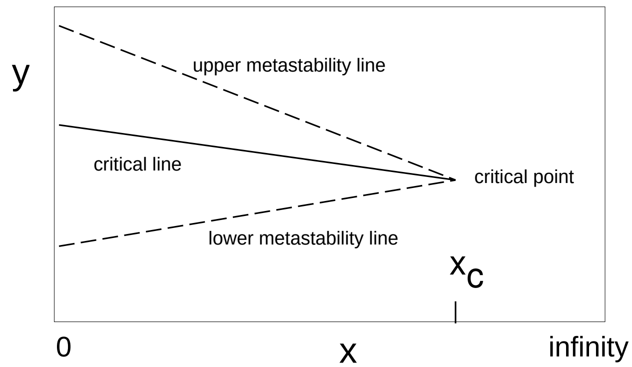


Figure 1: The schematical phase diagram for the SU(2) gauge-Higgs theory. Solid line is the phase transition and dashed lines indicate the metastability region.

not have a true gauge-invariant order parameter which can distinguish the symmetric high temperature phase and low temperature Higgs phase [7, 8]. There is no breaking or restoration of the gauge symmetry across the phase transition, just because physical observables are always gauge invariant. The physical spectrum of the corresponding Minkowskian $(2+1)$ theory in the Higgs phase consists of three massive vector bosons and one scalar excitation, perfectly mapping to the spectrum of low lying resonances (three vector bound states of scalar constituent "quarks" and one scalar bound state) in the symmetric phase. The corresponding scalar (π) and vector (V) gauge-invariant operators are given by $\pi = \phi^\dagger\phi$, $V_j^0 = i\phi^\dagger \overleftrightarrow{D}_j \phi$, $V_j^+ = (V_j^-)^* = 2i\phi^\dagger D_j\tilde{\phi}$, where $\tilde{\phi} = i\sigma_2\phi^*$.

In lattice non-abelian gauge-Higgs systems with matter in the fundamental representation and fixed length of the scalar field, the Higgs (weakly coupled) and symmetric (strongly coupled) phases are continuously connected [8]. This suggests the phase diagram on the (x, y) (Higgs mass-temperature) plane shown in Fig. 1. The knowledge of the phase diagram and the value of x_c is essential for cosmological applications. If $x_c = \infty$, the electroweak phase transition did occur in the early Universe at the electroweak scale independent of the parameters of the electroweak theory. This means that substantial deviations from thermal equilibrium took place at this scale, which might leave some observable remnants such as the baryon asymmetry of the universe (for a review see [9] and references therein). In the opposite situation of finite x_c the EW phase transition never took place for a region of parameters of the underlying theory; in this case it is extremely unlikely that there are any remnants from the electroweak epoch.

There were up to now no solid results on the phase structure of the continuum 3d (and, therefore, high temperature) gauge-Higgs theory. Various arguments in favour and against finite x_c are listed below.

1. $x_c = \infty$? The limit $x \to \infty$ corresponds formally to $g_3^2 = 0$, i.e. to the pure scalar model with SU(2) global symmetry. The latter is known to have a second order phase transition, suggesting that $x_c = \infty$ in the SU(2) gauge-Higgs system. The weakness of this argument is revealed when the particle spectra of the two theories are compared: the pure scalar theory below the critical point contains massless scalar particles – Goldstone bosons – but the spectrum of the gauge theory contains only massive modes.

The ϵ-expansion predicts a first order phase transition for any finite value of x, suggesting again that $x_c = \infty$ [10]. However, it relies on the hope that $\epsilon = 1$ is small and, therefore, is not conclusive.

2. $x_c = finite$? The absence of a true order parameter for the gauge-Higgs system is certainly consistent with finite x_c. Moreover, because there is no symmetry breaking, the existence of a line of second order phase transitions starting at x_c is very unlikely. However, the proof of the fact that the Higgs and symmetric phases are continuously connected [8] refers to a lattice system with a finite cutoff and is not applicable to a continuum system we are interested in.

A study of one-loop Schwinger-Dyson equations for this system argues in favour of a finite value of x_c [11]. However, this analysis relies heavily on the applicability of perturbation theory near the phase transition point. This is known to break down at $m_H \sim m_W$.

In this letter we present strong non-perturbative evidence for the fact that the line of first order phase transitions has a critical end-point at a finite value of x, $0.09 < x_c < 0.17$, and most likely $x_c \approx \frac{1}{8}$. In terms of the physical Higgs mass in the MSM this means that the phase transition ends between $m_H = 70$ and 95 GeV, probably near $m_H = 80$ GeV.

The lattice action corresponding to (1) is, in standard notation,

$$\begin{aligned} S &= \beta_G \sum_x \sum_{i<j} (1 - \frac{1}{2}\mathrm{Tr}\, P_{ij}) + \\ &- \beta_H \sum_x \sum_i \frac{1}{2}\mathrm{Tr}\, \Phi^\dagger(\mathbf{x}) U_i(\mathbf{x}) \Phi(\mathbf{x}+i) + \\ &+ \sum_x \frac{1}{2}\mathrm{Tr}\, \Phi^\dagger(\mathbf{x})\Phi(\mathbf{x}) + \beta_R \sum_x [\frac{1}{2}\mathrm{Tr}\, \Phi^\dagger(\mathbf{x})\Phi(\mathbf{x}) - 1]^2. \end{aligned} \tag{2}$$

Here $g_3^2 a = 4/\beta_G, x = \beta_R \beta_G/\beta_H^2$; y is given in terms of β_H, x, β_G in [3]. The continuum limit $a \to 0$ corresponds to $\beta_G \to \infty, \beta_H \to 1/3, \beta_R \to 0$.

Among the many tests of the order of the transition we shall use here (I) the finite size scaling analysis of the $\phi^\dagger\phi$ susceptibility and (II) the analysis of the correlation lengths. We define the dimensionless $\phi^\dagger\phi$ susceptibility

$$\chi = g_3^2 V \left\langle (\phi^\dagger\phi - \langle\phi^\dagger\phi\rangle)^2 \right\rangle \tag{3}$$

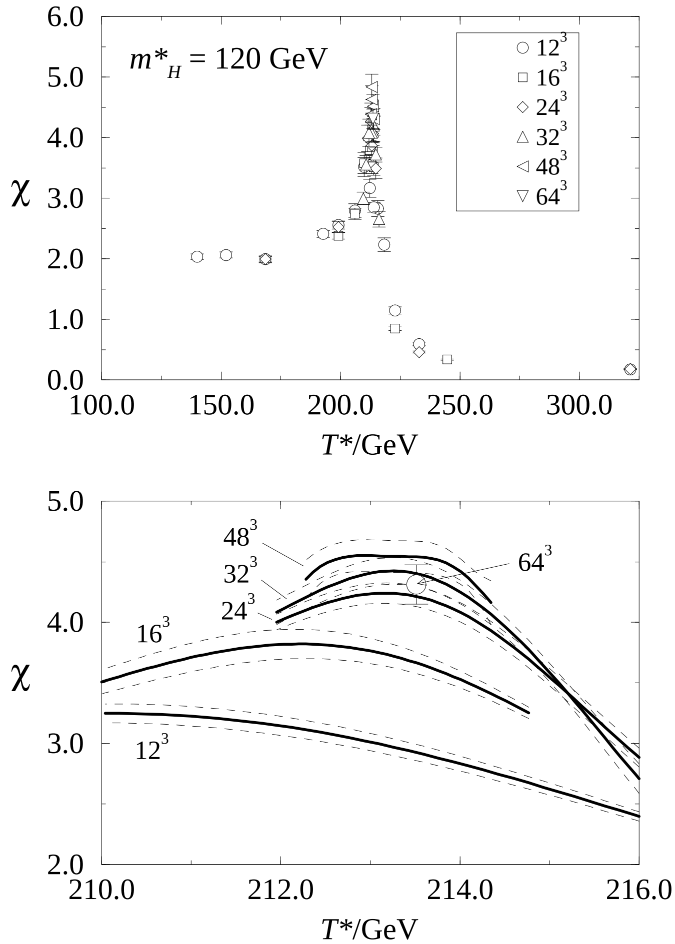


Figure 2: $\phi^\dagger\phi$ susceptibility at $m_H^* = 120$ GeV plotted as a function of T^* for lattices of various sizes. The lower panel shows in more detail the region near the maximum; the continuous lines with error bands result from multihistogram reweighting. The maximum values are plotted in Fig. 3.

and measure it as a function of T^*. For each volume we find the provisional 'transition temperature' $T^*_{t,V}$ where χ attains its maximum value $\chi_{\max}$. There are now 3 distinct possibilities: a) In a first order phase transition $\langle\phi^\dagger\phi\rangle$ has a discontinuous jump Δ_ϕ, and $\chi_{\max} \propto V \times \Delta_\phi^2$. b) In a second order transition χ displays critical behaviour, and $\chi_{\max} \propto V^\gamma$, where γ is a critical exponent [12]. c) If there is no transition, χ is regular and remains finite when $V \to \infty$ (on a system with periodic boundary conditions).

Fig. 2 shows $\chi(T^*)$ measured from lattices of sizes 12^3–64^3 for $m_H^* = 120\,\mathrm{GeV}$ and $\beta_G \equiv 4/(g_3^2 a) = 8$. The data exhibits a strong peak at $T^* \sim 213\,\mathrm{GeV}$, suggesting

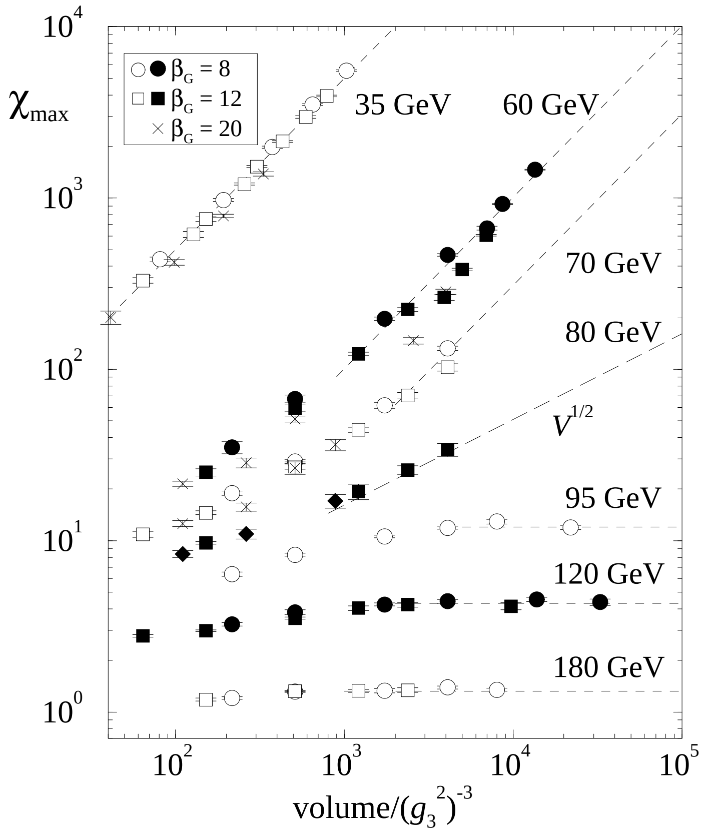


Figure 3: The maximum values $\chi_{\rm max}$ for different m_H^* plotted as a function of V. The dashed lines are lines $\sim V, V^{1/2}, V^0$.

a possibility of a phase transition. However, on closer inspection (bottom panel of Fig. 2), one sees that $\chi_{\rm max}$ remains finite (within the statistical accuracy), and the provisional transition is only a sharp – but regular – cross-over. The maximum values $\chi_{\rm max}$ for different m_H^* are shown as a function of V in Fig. 3. Note that the natural unit g_3^2 is used in writing

$$V(g_3^2)^3 = (V/a^3)(4/\beta_G)^3 = (4N/\beta_G)^3. \tag{4}$$

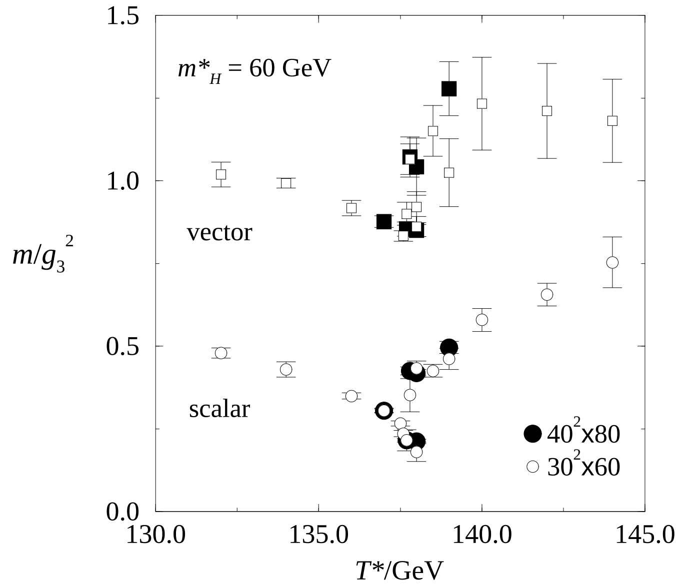
In Fig. 3 we use 3 different lattice spacings (β_G = 8, 12, 20); no significant finite lattice spacing effect can be observed (the scatter in $m_H^* = 60\,{\rm GeV}$ is due to the large variation in lattice geometries: some volumes are long cylinders, some cubes). The pattern of Fig. 3 very clearly suggests that the behaviour of the system changes around $m_H^* = 80\,{\rm GeV}$ from a 1st order transition to no transition. The line $\sim V^{1/2}$ corresponds to mean field critical behaviour. The data in Fig. 3 cannot yet distinguish the true critical exponent, nor whether $m_H^* = 80\,{\rm GeV}$ is actually above or below the critical m_H^*: near the critical point increasingly large volumes are needed in order to see the asymptotic behaviour.

Another evidence of the absence of the phase transition comes from the study of the correlation lengths of the system. If $x_c = \infty$ then the phase transition becomes weaker when the Higgs mass is increased. The jump of the order parameter $\phi^\dagger\phi$ gets smaller together with the mass of the scalar excitation. At the same time, the vector correlation length may remain finite at the transition point, making the resolution of the nature of the phase transition to be a very difficult problem to solve numerically because of the increasing hierarchy of the scalar and vector masses. A typical signature of this situation is a drastic increase of the scalar correlation length for all x at some value of $y(x)$.

If, on the contrary, x_c is finite, then for $x > x_c$ all correlation lengths of the system are finite, and expectation values of different gauge-invariant operators are continuous functions of y. After some minimum size, finite volume effects become negligible. In this case a reliable lattice MC analysis, which is hardly possible to carry out near x_c, becomes comparatively quite simple at large Higgs masses.

On Figs. 4 and 5 we present the behaviour of the scalar and vector masses (the inverse π and V_j^0 correlation lengths) for $m_H^* = 60$, 80, 120 and 180 GeV near the transition/cross-over temperature. Fig. 4a clearly demonstrates the jump of the correlation lengths typical of 1st order transitions. Fig. 4b shows the power-like decrease of the mass of the scalar excitation with no change of the vector mass across the critical region. In contrast, the behaviour of scalar and vector masses is smooth for $m_H^* = 120$ and 180 GeV (Fig. 5), signalling the absence of the phase transition. Within the statistical accuracy, the masses and the susceptibility χ are independent of the lattice spacing, showing that the observed behaviour is not a lattice artefact and persists in the continuum limit.

To summarize, we demonstrated that the Higgs and confinement phases of 3d SU(2) gauge-Higgs model can be continuously connected. This means that the electroweak phase transition in weakly coupled electroweak theories is absent in a part of their parameter space. For the minimal standard model the critical value of the Higgs mass is near 80 GeV.



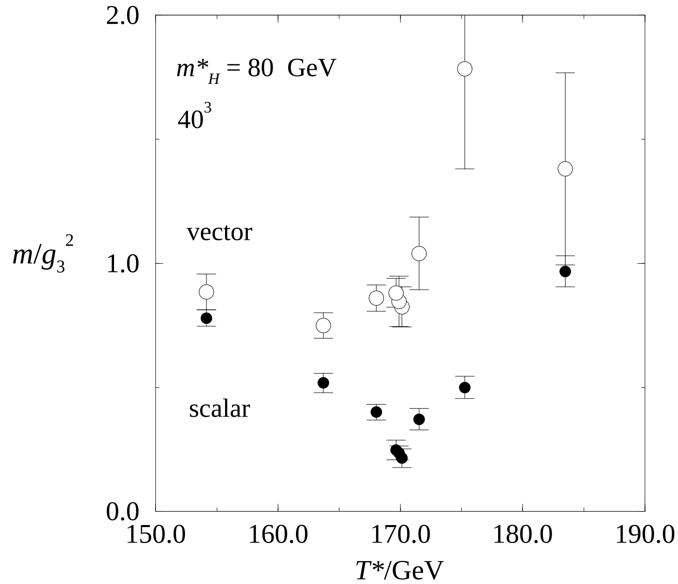


Figure 4: The scalar and vector mass dependence on the temperature for "small" Higgs masses, $m_H^* = 60$ and 80 GeV

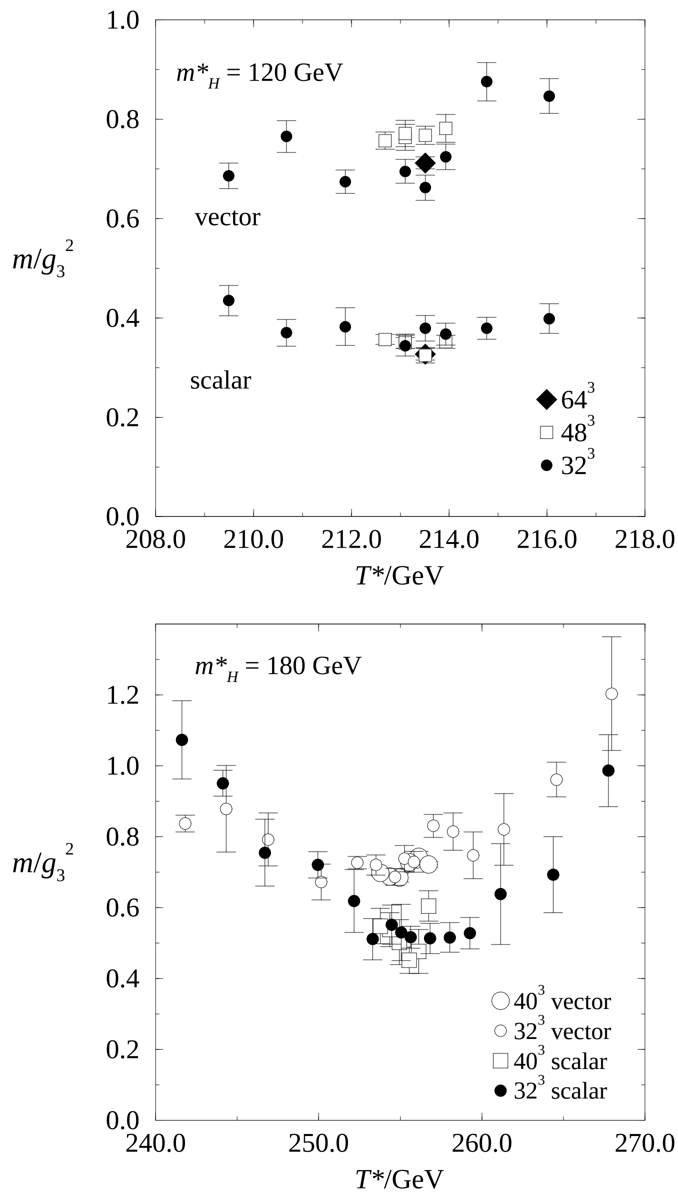


Figure 5: The scalar and vector mass dependence on the temperature for "large" Higgs masses, $m_H^* = 120$ and 180 GeV.

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