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A REMARK ON SPHALERON ERASURE OF BARYON ASYMMETRY

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Abstract

We complete an existing result for how the baryon asymmetry left over after a period of full thermal equilibrium depends on different lepton asymmetries.

Introduction. Baryon plus lepton number $(B + L)$ is violated by anomalous electroweak processes, and at the high temperatures appearing in the Early Universe these processes are fast enough to be in thermal equilibrium [1]. This leaves only three conserved global charges,

$$X_i = \frac{1}{n_F} B - L_i, \tag{1}$$

where $i = 1, ..., n_F$, and n_F denotes the number of generations[3].

For cosmological applications it is important to know what is the value of the baryon number B in an equilibrium system with given X_i (see, e.g., a recent work [2]). This problem has been addressed in a number of papers [3]–[10], and different answers to the same questions have been given. The confusing points were related to accounting for the scalar degrees of freedom and to fixing the boundary conditions for the gauge charges (electric charge versus hypercharge in the Higgs phase of the theory).

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[3] In extensions of the Minimal Standard Model even these charges may be violated, for instance due to flavour non-diagonal slepton masses or Majorana-type right-handed neutrino masses, but we assume here that the X_i are strictly conserved.

In the limit of small Yukawa couplings the baryon number can be written in the form

$$B = f_0 \sum_i X_i + \sum_i X_i \Big(f_1 \frac{m_i^2}{T^2} + f_2 h_i^2 \Big), \tag{2}$$

where $m_i = h_i \phi/\sqrt{2}$ are the lepton masses, h_i are the lepton Yukawa couplings, ϕ is the expectation value of the Higgs field, T is the temperature, and $\sum_i X_i = B - L$. The f_i are some functions of T, ϕ, the number of fermionic flavours n_F, and the number of Higgs scalars n_S [4]. The function f_0 was computed correctly in the symmetric phase of the EW theory in [4], and in the Higgs phase in [10]; the limit $\phi \gg T$ of f_0 coincides with the results of refs. [5, 6, 7, 8].

The situation with f_1 and f_2 is more obscure. In ref. [4] it was claimed that $f_1 = (4/13\pi^2)A$, $f_2 = (1/13\pi^2)A$ with $A \simeq 1$ for both symmetric and the Higgs phases provided the temperature is much larger than the vector boson masses (however, no explicit expression for A was presented). This result was stated to be wrong in [8] based on a reasoning related to the boundary conditions, and another form for f_1 was derived. In [8] it was said, moreover, that in the symmetric phase $B = 0$ if $B - L = 0$, meaning effectively that $f_2 = 0$. In [9], on the contrary, a specific non-vanishing expression was derived for f_2 in the limit $\phi \ll T$.

The question of how to really carry out the computation for generic ϕ, T was finally clarified in [10], but no estimates were presented for f_1 and f_2, as only f_0 was computed. We feel that the correct results for f_1 and f_2, valid both in the symmetric and the Higgs phases, should appear in the literature at last, and this is the aim of the present note.

Method. Let us briefly recall the method of computing the baryon number [10]. We have n_F conserved global charges X_i, with which we can associate chemical potentials. However, it is more convenient to introduce first the n_F+1 chemical potentials μ_B, μ_{L_i}, and impose the constraint following from the sphaleron processes,

$$n_F \mu_B + \sum_{i=1}^{n_F} \mu_{L_i} = 0 \tag{3}$$

only later on. We now have to compute the effective potential $V(\phi, A_0^a, B_0; \mu_B, \mu_{L_i}, T)$, where ϕ is the Higgs expectation value, and A_0^a, B_0 are the temporal components of the SU(2) and U(1) gauge fields. The fields A_0^a, B_0 have to be included, since the chemical potentials break Lorentz invariance and induce expectation values for them. The minimization $\partial V/\partial A_0^a = \partial V/\partial B_0 = 0$ corresponds to the neutrality of the system with respect to gauge charges. From the gauge-invariant value of the effective potential at the minimum, we get $B = -\partial V/\partial \mu_B, L_i = -\partial V/\partial \mu_{L_i}$ (these are really the baryon

[4] The definitions of f_1, f_2 are actually not completely unique, since a term proportional to ϕ^2 in f_2 can equivalently be presented as a contribution to f_1. We make here a certain division based on the way in which the two terms arise in our computation.

and lepton numbers per unit volume). Utilising Eq. (3), we can finally eliminate μ_B, μ_{L_i} to obtain an expression of the desired form $B = f(X_i)$.

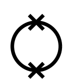
We will work at high temperatures, assuming $\mu_B, \mu_{L_i} \ll T$ and $\phi \lesssim$(a few)$\times T$. The expectation values of A_0^a, B_0 are proportional to the μ's, so it is sufficient to keep terms up to quadratic order in $A_0^a, B_0, \mu_B, \mu_{L_i}$. It is sufficient to choose only A_0^3 non-zero. The bosonic degrees of freedom (Higgses, gauge fields) only contribute to terms quadratic in A_0^3, B_0, while μ_B, μ_{L_i} come from the fermionic contributions. We denote the fermionic terms involving $A_0^a, B_0, \mu_B, \mu_{L_i}$ by

$$= \quad \bar{\psi}\gamma_0\Big[-\mu \pm \frac{i}{2}\tilde{A}a_L + \frac{i}{2}\tilde{B}(Y_L a_L + Y_R a_R)\Big]\psi, \tag{4}$$

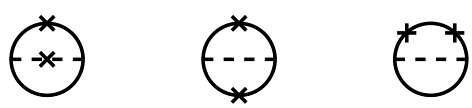
where μ is either $\mu_B/3$ (for quarks) or μ_{L_i} (for leptons), a_L, a_R are the left and right projectors, $Y_{L,R}$ are the corresponding hypercharges, and $\tilde{A} \equiv gA_0^3, \tilde{B} \equiv g'B_0$.

Baryon asymmetry. In order to account for flavour dependent contributions to the baryon asymmetry, we have to supplement the effective potential computed in [10] with the dominant terms differentiating between the μ_{L_i}'s. Such terms must involve leptonic Yukawa couplings, and can arise either as mass corrections in the 1-loop effective potential, or as 2-loop corrections directly proportional to the Yukawa couplings [5]. For $\phi \sim T$ both types of terms ($\sim m_i^2/T^2, h_i^2$) are of the same order of magnitude and must be included simultaneously.

The dominant fermionic 1-loop mass corrections come from the graph



On the other hand, denoting the Higgs with a dashed line, the 2-loop graphs potentially contributing are (we need only terms with at least one power of μ_{L_i})



However, it is easy to see that the first 2-loop graph does in fact not contribute. The reason is that viewed as a self-energy correction for the Higgs field, the fermionic loop does not have a term linear in μ, because the insertions of $\gamma_0\mu$ to the two different fermion lines cancel each other; thus the result is $\propto \tilde{A}^2, \tilde{B}^2$.

Furthermore, it turns out the second 2-loop graph does not contribute either. It is proportional to the divergent integral

$$I = \sum\!\!\!\!\!\!\int_{P,Q} \frac{p_f(p_f + q_f)}{(P+Q)^2 Q^2 (P^2)^2}, \tag{5}$$

[5] At the order we are working, quark Yukawa couplings are always associated with terms involving μ_B, and thus do not make a distinction between the different generations.

where $P = (p_f, \mathbf{p})$ and p_f are the fermionic Matsubara frequencies. It turns out that the coefficient of this integral gets contributions also from the last graph, and altogether the result is again just a higher order correction of the form $\tilde{A}^2, \tilde{B}^2$.

The remaining finite contribution from the last 2-loop graph, together with the 1-loop contribution as well as all the bosonic contributions, give the effective potential

$$
\begin{aligned}
V &= \frac{1}{2}\tilde{A}^2\Big[\frac{1}{4}\phi^2 + \Big(\frac{2}{3} + \frac{n_S}{6} + \frac{n_F}{3}\Big)T^2\Big] + \frac{1}{2}\tilde{B}^2\Big[\frac{1}{4}\phi^2 + \Big(\frac{n_S}{6} + \frac{5n_F}{9}\Big)T^2\Big] + \frac{1}{4}\tilde{A}\tilde{B}\phi^2 \\
&+ \frac{i}{3}\tilde{B}T^2\Big(\frac{n_F}{3}\mu_B - \sum_i \mu_{L_i}\Big) - \frac{n_F}{9}\mu_B^2 T^2 - \frac{1}{4}\sum_i \mu_{L_i}^2 T^2 \\
&- \frac{i}{8\pi^2}\tilde{A}\sum_i \mu_{L_i} m_i^2 + \frac{T^2}{8\pi^2}\sum_i (3i\tilde{B}\mu_{L_i} + 2\mu_{L_i}^2)\Big(\frac{m_i^2}{T^2} + \frac{h_i^2}{4}\Big). \qquad (6)
\end{aligned}
$$

Working as outlined above, we obtain for the physical case ($n_F = 3, n_S = 1$)

$$
\begin{aligned}
B &= 4\frac{77T^2 + 27\phi^2}{869T^2 + 333\phi^2}\sum_i X_i \\
&+ \frac{11}{2\pi^2}\frac{47T^2 + 18\phi^2}{869T^2 + 333\phi^2}\sum_i X_i \frac{m_i^2}{T^2} + \frac{1}{16\pi^2}\frac{1034T^2 + 405\phi^2}{869T^2 + 333\phi^2}\sum_i X_i h_i^2. \qquad (7)
\end{aligned}
$$

The first term coincides with the one found in [10]. The latter line is the dominant one if $\sum_i X_i = 0$. Using the notation of Eq. (2) we have $f_1 = (4/13\pi^2)(143/148) \approx (4/13\pi^2) \cdot 0.9662$ and $f_2 = (1/13\pi^2)(585/592) \approx (1/13\pi^2) \cdot 0.9882$ for $\phi \gg T$, and $f_1 = (4/13\pi^2)(611/632) \approx (4/13\pi^2) \cdot 0.9668$ and $f_2 = f_1/4$ for $\phi \ll T$, in numerical agreement with the estimates in [4]. In the symmetric phase $\phi \ll T$, the analytic result for f_2 agrees with that given in [9].

When higher order corrections are taken into account, one expects $f_i \to f_i[1 + \mathcal{O}(\alpha_W/\pi, \alpha_S/\pi, h^2/\pi^2, m^2/(\pi T)^2)]$, where h is a general Yukawa coupling and m a general mass.

In summary, we have derived the leading order expressions for the flavour dependent contributions to the baryon asymmetry remaining after a period of full thermal equilibrium, Eq. (7). The result should be evaluated at $\phi \sim T$ if the sphaleron processes fall out of thermal equilibrium after the system has smoothly passed from the symmetric to the Higgs phase. If, on the contrary, there is a strong first order electroweak phase transition such that the sphaleron processes are always switched off in the Higgs phase (but at the same time no new $B + L$ asymmetry is generated during the transition due to, say, too little CP-violation), it should be evaluated at $\phi = 0$.

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