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The Ising model universality of the electroweak theory*

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Lattice simulations have shown that the first order electroweak phase transition turns into a regular cross-over at a critical Higgs mass $m_{H,c}$. We have developed a method which enables us to make a detailed investigation of the critical properties of the electroweak theory at $m_{H,c}$. We find that the transition falls into the 3d Ising universality class. The continuum limit extrapolation of the critical Higgs mass is $m_{H,c} = 72(2)\,\mathrm{GeV}$, which implies that there is no electroweak phase transition in the Standard Model.

The Standard Model (SM) finite temperature phase transition has been studied in great detail with lattice Monte Carlo simulations. The transition is of the first order at small Higgs masses, but it has been found to turn into a regular cross-over when $m_H \gtrsim 75\,\mathrm{GeV}$ [1–4]. A second order transition appears at the endpoint of the first order transition line, and the macroscopic behaviour of the system is determined by the *universal* properties of the endpoint. While the location of the endpoint and the mass spectrum near it have been studied before, the critical properties of the endpoint itself have not been resolved so far. In this talk we show that the universality class of the SM endpoint is of the 3d Ising type. A full description of this work can be found in ref. [5].

At high temperatures, the static properties of the SM and many of its extensions, can be accurately described with an effective 3d SU(2) gauge + Higgs theory [6]:

$$\mathcal{L} = \frac{1}{4}F^a_{ij}F^a_{ij} + |D_i\phi|^2 + m_3^2|\phi|^2 + \lambda_3|\phi|^4. \qquad (1)$$

The theory is fixed by the dimensionful gauge coupling g_3^2 and by the ratios

$$x = \lambda_3/g_3^2, \qquad y = m_3^2(\mu)/g_3^4\,, \qquad (2)$$

where $m_3^2(\mu)$ is the renormalized mass parameter.

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We have omitted the U(1) sector of the SM; this is justified, since the U(1) gauge boson remains massless at any temperature and does not affect the transition qualitatively.

The lattice action in standard formalism is

$$\begin{aligned} S &= \beta_G \sum_{x;i,j}(1 - \tfrac{1}{2}\mathrm{Tr}\, P_{ij}) \\ &- \beta_H \sum_{x;i} \tfrac{1}{2}\mathrm{Tr}\, \Phi^\dagger(x)U_i(x)\Phi(x+i) \\ &+ \sum_x \left[\tfrac{1}{2}\mathrm{Tr}\,\Phi^\dagger\Phi + \beta_R\big(\tfrac{1}{2}\mathrm{Tr}\,\Phi^\dagger\Phi - 1\big)^2\right] \qquad (3) \\ &\equiv S_G + S_{\mathrm{hopping}} + S_{\phi^2} + S_{(\phi^2-1)^2}. \end{aligned}$$

The universal behaviour does not depend on the lattice spacing, which we keep fixed at $a \equiv 4/(g_3^2\beta_G) = 4/(5g_3^2)$. For the full lattice $\leftrightarrow$ continuum relations, see [5] and references therein.

The phase diagram of the SU(2)+Higgs theory is shown in Fig. 1. What kind of critical behaviour can one expect? Formally, the Higgs field has $\mathrm{SU(2)_{gauge}}\times\mathrm{SU(2)_{custodial}}$ symmetry, but this remains unbroken at all temperatures. Indeed, the mass spectrum of the system has been investigated in detail both above and below the critical point, and only one scalar excitation (which couples to $\phi^\dagger\phi$) becomes light in its neighbourhood. Thus, one would expect Ising-type universality, but also mean field-type or multicritical behaviour is, in principle, possible.

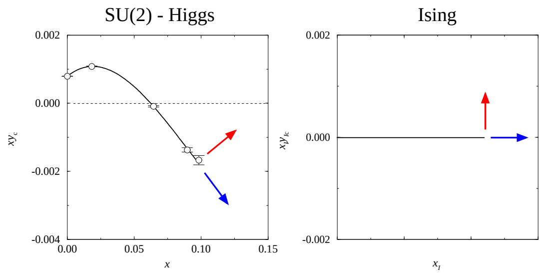

Figure 1. The phase diagrams of the SU(2)+Higgs (left) and the Ising (right) models.

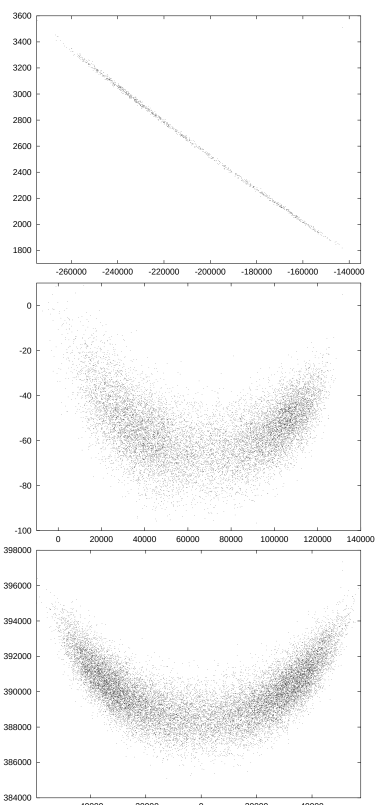

Figure 2. *Top:* A density plot of the 3d SU(2)+Higgs model at the critical point, shown on the $S_{(\phi^2-1)^2}$ vs. $S_{\rm hopping}$ plane. *Middle:* The same as above, after a shift and a rotation. *Bottom:* A density plot of the 3d Ising model on the energy vs. magnetization plane.

Adopting now Ising-model terminology, let us call the two critical directions in Fig. 1 the magnetization like (M; perpendicular to the transition line) and the energy like (E; along the transition line). Due to the lack of an exact order parameter, the mapping of the M-like and E-like directions of the SU(2)+Higgs model to the Ising model is non-trivial. This is illustrated in Fig. 2, where the probability density at the critical point is plotted on the $(S_{\rm hopping}, S_{(\phi^2-1)^2})$-plane. Only after a suitable rotation of the axes is the striking similarity with the Ising model revealed. This rotation closely corresponds to the rotation of the directions shown in the phase diagrams in Fig. 1.

However, there is no reason to restrict ourselves only to the two observables $S_{\rm hopping}$ and $S_{(\phi^2-1)^2}$. Any number of operators can contribute to the true M-like and E-like directions. In order to improve on the projection, it is important to consider a large number of operators. Our method works as follows:

(a) Locate the infinite volume critical point (for details, see ref. [5]), where all of the subsequent analysis is performed.

(b) Using several volumes, measure the fluctuation matrix $M_{ij} = \langle s_i s_j \rangle$, $s_i \equiv S_i - \langle S_i \rangle$. We used up to 6 operators: those in eq. (3), together with the operators ($V = \Phi/|\Phi|$)

$$S_R = \sum_x |\Phi| \, , \quad S_L = \sum_{x,i} \tfrac{1}{2} {\rm Tr}\, V^\dagger(x) U_i(x) V(x+i).$$

(c) Calculate the eigenvalues λ_α and -vectors V_α of M_{ij}. Some of the eigenvectors correspond to “critical” observables like M or E, and the rest are “trivial.” They can be classified either by inspecting the probability distributions $p(V_\alpha)$ and $p(V_\alpha, V_\beta)$, or by looking at the finite volume behaviour of the eigenvalues. For example, the M- and E-like eigenvalues diverge with the critical exponents as (L is the length of the lattice)

$$\lambda_M \propto L^{3+\gamma/\nu} \, , \qquad \lambda_E \propto L^{3+\alpha/\nu} \, . \tag{4}$$

The “trivial” eigenvalues diverge as L^3. A somewhat related method has been used to study the critical behaviour of the 4d U(1)+Higgs model [7].

The M-like and E-like eigenvalues are shown in Fig. 3 ($\chi_a = \lambda_a/L^3$); the other eigenvalues

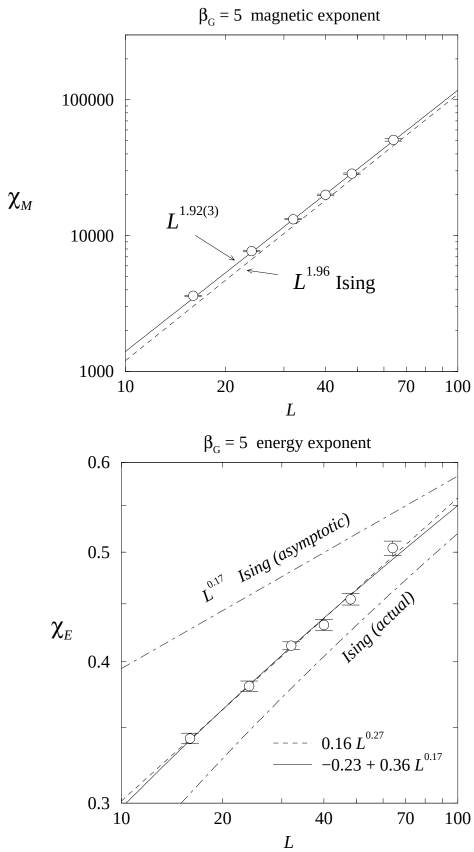

Figure 3. The divergence of χ_M (*top*) and χ_E (*bottom*) as a function of the lattice size. The slope of χ_E does not agree with the asymptotic Ising value, but it is consistent with the *measured* [8] Ising model behaviour at these lattice sizes.

do not show any critical behaviour. The results are consistent with the Ising model ones. The positive value of α clearly excludes O(N) models with $N \geq 2$ ($\alpha < 0$) and the mean field behaviour ($\alpha = 0$).

We have performed the analysis also with 4 instead of 6 operators. The results remain stable, although in some cases small deviations from the Ising behaviour begin to appear. This shows both the robustness of the method and the importance of including a large enough number of operators in the analysis.

The Ising-type universality seen in the SM is quite compatible with the lack of a true order parameter. The effective $\pm M$-symmetry is not a symmetry of the action, but it is dynamically generated at the critical point. In this respect the system is completely analogous to the critical point in liquid-vapour transitions.

In the simulations above the lattice spacing was fixed through $\beta_G = 4/g_3^2 a = 5$. We have also located the critical point at $\beta_G = 8$, and Gürtler *et al* [3] have published results at $\beta_G = 12$ and 16. This allows us to calculate the continuum limit extrapolation of the critical point, with the result $x_c = 0.0983(15)$. In the Standard Model this corresponds to $m_H = 72(2)\,\mathrm{GeV}$. Since the experimental lower limit is $\sim 88\,\mathrm{GeV}$, this excludes the existence of the SM phase transition. Nevertheless, a first order phase transition is still allowed in several extensions of the SM; most notably, it can occur in the Minimal Supersymmetric SM.

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