CERN-TH/99-206
UNIL-IPT/99-2
hep-th/9907194

arXiv:hep-th/9907194v2 27 Aug 1999

AN ALL-ORDER DISCONTINUITY AT THE ELECTROWEAK PHASE TRANSITION

M. Laine[a,b,1], M. Shaposhnikov[c,2]

[a]*Theory Division, CERN, CH-1211 Geneva 23, Switzerland*

[b]*Dept. of Physics, P.O.Box 9, FIN-00014 Univ. of Helsinki, Finland*

[c]*Institute for Theoretical Physics, University of Lausanne, BSP-Dorigny, CH-1015 Lausanne, Switzerland*

Abstract

We define a non-local gauge-invariant Green's function which can distinguish between the symmetric (confinement) and broken (Higgs) phases of the hot SU(2)×U(1) electroweak theory to all orders in the perturbative expansion. It is related to the coupling of the Chern-Simons number to a massless Abelian gauge field. The result implies either that there is a way to distinguish between the phases, even though the macroscopic thermodynamical properties of the system have been observed to be smoothly connected, or that the perturbative Coleman-Hill theorem on which the argument is based, is circumvented by non-perturbative effects. We point out that this question could in principle be studied with three-dimensional lattice simulations.

CERN-TH/99-206
UNIL-IPT/99-2
August 1999

[1]mikko.laine@cern.ch
[2]mikhail.shaposhnikov@ipt.unil.ch

Introduction. Non-Abelian gauge theories based on groups SU(N), $N \geq 2$, with scalar fields in the fundamental representation have no known gauge-invariant order parameters that can distinguish between the symmetric (confinement) and broken (Higgs) phases [1]. This leads to the conclusion that the confinement phase and the Higgs phase can be smoothly connected [2]. In other words, there may be no separate Higgs and confinement phases but just one Higgs-confinement phase. This consideration tells that finite temperature phase transitions in spontaneously broken gauge theories are not a generic phenomenon: depending on the parameters of the theory the system may go through a first order phase transition (or, in special cases, a second order one), but it may also be that there is no phase transition at all.

These expectations have been confirmed in lattice Monte Carlo simulations. In particular, no phase transition is observed for sufficiently large scalar self-coupling for three-dimensional (3d) theories like SU(2) with a doublet of Higgses [3]. As 3d theories can be considered as the effective theories of high temperature four-dimensional (4d) theories these results mean that there is no finite temperature phase transition in these systems in some region of the parameter space (for Higgs boson masses larger than the W boson mass).

The behaviour of an Abelian gauge theory is different. Although there is no local gauge-invariant order parameter that can distinguish between the Coulomb and the Higgs phases, there are non-local ones, such as the photon mass. In the Higgs phase the photon mass is non-zero, while in the Coulomb phase it is zero, clearly indicating a difference between the phases. This statement is correct in all orders of perturbation theory, and it has also been tested in lattice simulations [4]. There are also other non-perturbative order parameters in the Abelian case, such as the tension of an infinitely long vortex [5].

Here we are interested in theories that contain both Abelian and non-Abelian parts and contain scalar fields in the fundamental representation. We will deal essentially with the electroweak theory (or some typical extensions thereof such as the MSSM, with a similar pattern of electroweak symmetry breaking), although the results remain valid for many models of the type $G\times$U(1) where G is a simple group. As in both previous cases, there are no local gauge-invariant order parameters that can select the symmetric or the Higgs phase. A non-local order parameter associated with the existence of a massless vector excitation does not distinguish between the phases either, as the photon in the Higgs phase and the hypercharge vector boson in the symmetric phase remain massless in all orders of perturbation theory (this statement has also been checked with non-perturbative 3d lattice simulations [6]). The projection of the massless state to the gauge-invariant Abelian hypercharge field strength was found to be a smooth function of the temperature at sufficiently large Higgs masses [6], and all the other observables measured (expectation values of local gauge-invariant operators; other masses) also behaved smoothly. Thus, it seems that the confinement and Higgs phases can be smoothly connected also for the SU(2)$\times$U(1) theory and that there may be no phase transition.

In this paper we define a non-local Green's function which nevertheless jumps when one goes from the symmetric to the Higgs phase *independently* of the scalar self-coupling (or the Higgs mass) to all orders in perturbation theory. The existence of a massless vector excitation (or, in other words, the presence of an unbroken Abelian gauge symmetry) in both phases of the electroweak theory is essential for the argument. Therefore, one *can* separate the confinement phase from the Higgs phase in the SU(2)×U(1) gauge theory to all orders in the perturbative expansion. It remains to be seen if this statement is valid beyond perturbation theory.

Phase transition in 3d with a topological mass term. Let us consider the 3d SU(2)×U(1) theory with a single Higgs doublet. This theory is the high temperature limit for the standard 4d electroweak theory and many of its extensions, obtained by integrating out fermions, the non-zero Matsubara modes of bosons, and the zero Matsubara modes of the temporal components of the gauge fields [7]. Below we will see how the same results can be derived directly in 4d. The 3d action is

$$S = \int d^3x \Big\{ \frac{1}{4} B_{ij} B_{ij} + \frac{1}{2} \mathrm{Tr}\, F_{ij} F_{ij} + (D_i\phi)^\dagger (D_i\phi) + m_3^2 \phi^\dagger\phi + \lambda_3 (\phi^\dagger\phi)^2 \Big\}, \qquad (1)$$

where $F_{ij}^a = \partial_i A_j^a - \partial_j A_i^a + g\epsilon^{abc} A_i^b A_j^c$, $F_{ij} = T^a F_{ij}^a$, $D_i\phi = (\partial_i - igA_i + i(g'/2)B_i)\phi$, $A_i = T^a A_i^a$, $B_{ij} = \partial_i B_j - \partial_j B_i$ and $T^a = \tau^a/2$ (the τ^a are the Pauli matrices). The 3d gauge couplings g, g' have the dimension $\mathrm{GeV}^{1/2}$.

Let us first consider a somewhat more complicated theory by adding to Eq. (1) topological mass terms [8] for the U(1) and SU(2) fields: $S \to S + \Delta S$, where

$$\Delta S = i\Big(\mu_1 N_{\mathrm{CS}}^{(1)} - \mu_2 N_{\mathrm{CS}}^{(2)}\Big), \qquad (2)$$

$$N_{\mathrm{CS}}^{(1)} = \frac{g'^2}{16\pi} \int d^3x \epsilon_{ijk} B_{ij} B_k, \qquad (3)$$

$$N_{\mathrm{CS}}^{(2)} = \frac{g^2}{8\pi} \int d^3x \epsilon_{ijk} \mathrm{Tr}\, \left(F_{ij} A_k + \frac{2}{3} ig A_i A_j A_k \right). \qquad (4)$$

The theory with a topological mass term for the SU(2) field is mathematically consistent beyond perturbation theory only if μ_2 is quantized [8], $\mu_2 = 0, \pm 1, \pm 2, ...$, while the topological mass for the U(1) field may be arbitrary. In the following we will nevertheless take $\mu_1 = \mu_2 = \mu$: it is in this case that we find a discontinuity computable to all orders in perturbation theory, and this is also the limit which has an interpretation in the 4d finite temperature electroweak theory (see below).

Consider now the vacuum polarization tensor (the inverse propagator) for the U(1) field. In momentum representation, in a general covariant gauge with the gauge parameter ξ, it is given by

$$G_{ij}^{-1}(k,\mu) = (k^2\delta_{ij} - k_i k_j)\Pi_1(k^2,\mu) + i\epsilon_{ijl} k_l \Pi_2(k^2,\mu) + \xi^{-1} k_i k_j, \qquad (5)$$

where

$$G_{ij}(k,\mu) = \int d^3x\, e^{ikx} \langle B_i(x) B_j(0) \rangle. \tag{6}$$

The parity odd part $\Pi_2(k^2,\mu)$ is non-vanishing for $\mu \neq 0$, and gauge-independent. We will be interested in the limit $\Pi_2(k^2 \to 0, \mu)$.

Consider first $\Pi_2(0,\mu)$ in the symmetric phase. At the lowest order of perturbation theory, it is proportional to the topological mass,

$$\Pi_2(0,\mu) = -i\mu \frac{g'^2}{4\pi}. \tag{7}$$

However, the theory under consideration satisfies all the requirements of the Coleman-Hill (CH) theorem [9]: there is an unbroken Abelian gauge symmetry and no massless charged excitations (that is, as long as we stay away from the transition point $m_3^2 \approx 0$; the result however does not depend on m_3^2). Thus, all corrections to $\Pi_2(0,\mu)$ vanish beyond 1-loop level. Consequently, $\Pi_2(0,\mu)$ can be computed exactly at the 1-loop level. As is easy to understand, there are in fact no 1-loop corrections either, because on the 1-loop level the vacuum polarization tensor is given by scalar loops which contain no trace of the topological mass term. Thus, in the symmetric phase the relation in Eq. (7) is valid to all orders in perturbation theory.

Let us then compute $\Pi_2(k^2,\mu)$ (deep) in the Higgs phase. We are interested in the limit $k^2 \to 0$, so that only external lines associated with what would be massless particles in the absence of the topological mass term are important. We thus concentrate on the electromagnetic field $Q_i = -\sin\theta A_i^3 + \cos\theta B_i$ where $\tan\theta = g'/g$, and the idea is to apply again the CH theorem, but now for Q_i. One of the conditions of the theorem is actually not valid: there is a bilinear mixing between the massive field $Z_i = \cos\theta A_i^3 + \sin\theta B_i$ and the photon field Q_i of the form $\epsilon_{ijk} Z_i \partial_j Q_k$. However, this mixing can be formally removed (i.e., the quadratic part of the Lagrangian can be diagonalized) by a shift of Z_i: in momentum space,

$$\begin{aligned}
Z_i(k) &\to Z_i(k) + \Big[\Big(\delta_{ij} - \frac{k_i k_j}{k^2} \Big) A(k^2) + 2i\epsilon_{ijl} k_l B(k^2) \Big] Q_j(k), \\
A(k^2) &= \frac{8k^2}{k^2 + m_Z^2} \Big(\frac{g^2 + g'^2}{16\pi} \mu \Big)^2 \Delta^{-1}(k^2) \cos 2\theta \sin 2\theta, \\
B(k^2) &= i \Big(\frac{g^2 + g'^2}{16\pi} \mu \Big) \Delta^{-1}(k^2) \sin 2\theta, \\
\Delta(k^2) &= k^2 + m_Z^2 + \frac{k^2}{k^2 + m_Z^2} \Big(\frac{g^2 + g'^2}{4\pi} \mu \cos 2\theta \Big)^2.
\end{aligned} \tag{8}$$

This brings the effective Lagrangian in a form consistent with the CH conditions. This transformation is non-local, but it is infrared insensitive and does not break the U(1) gauge invariance of Q_i, and thus does not invalidate the CH-theorem.

As a result of writing the action in terms of Q_i, Z_i in the Higgs phase and making the transformation in Eq. (8), there is a topological mass term for the field Z_i, but none for Q_i [3]. Thus, in the vacuum polarization tensor for the massless field Q_i,

$$\Pi_2^{(Q)}(0,\mu) = 0 \tag{9}$$

at the tree-level. An explicit computation of 1-loop diagrams to order μ gives zero for $k^2 \to 0$ (we have carried out this computation before making the shift in Eq. (8)), and higher order corrections are absent due to the CH theorem. Thus, the values of $\Pi_2(0,\mu)$ associated with a massless field are different in the symmetric and Higgs phases, and this is valid in all orders of perturbation theory.

The fact that the IR-sensitive part of the effective action shows such behaviour, suggests that the partition function of the system (the minimum of the effective action) may also behave non-analytically at the transition point for $\mu \neq 0$. In other words, it is likely that this theory exhibits a genuine first or second order phase transition independent of the value of the scalar self-coupling: there are no massless excitations in the symmetric phase, but there is a massless particle in the Higgs phase.

Phase transition in 3d without topological mass term. The result obtained above can be applied to the original 3d theory in Eq. (1), without the topological mass term. Note that in perturbation theory, the quantization of μ is not essential and the statement about the absence of higher order corrections to $\Pi_2(0,\mu)$ is valid for real values of μ. Thus, one can consider the derivative with respect to μ of $G_{ij}^{-1}(k,\mu)$ for the theory with $\mu \neq 0$, and then take $\mu = 0$. The Green's function that results from this operation is given by

$$D_{ij}(k) = G_{il}^{-1}(k,0) S_{lm}(k) G_{mj}^{-1}(k,0), \tag{10}$$

where

$$S_{ij}(k) = \int d^3x e^{ikx} \Big\langle B_i(x) B_j(0) \Big(N_{\rm CS}^{(1)} - N_{\rm CS}^{(2)} \Big) \Big\rangle \tag{11}$$

is a Green's function computed in the theory with $\mu = 0$ [4]. The statement is that in the symmetric phase,

$$D \equiv \lim_{k^2 \to 0} D_{ij}(k) \frac{\epsilon_{ijl} k_l}{k^2} = 2 \frac{d}{d\mu} \Pi_2(0,\mu) \Big|_{\mu=0} = -i \frac{g'^2}{2\pi} \quad \text{(symmetric phase).} \tag{12}$$

This is valid in all orders of perturbation theory.

[3] In fact, one does generate a parity-odd term $\sim \epsilon_{ijk} Q_i \partial_j Q_k$ proportional to $\mu^3 k^2 / m_Z^4$, but this does not contribute in the limit $k^2 \to 0$ we are interested in.

[4] To avoid confusion let us note that this Green's function is not related to the rate of sphaleron transitions which was studied numerically in the crossover region in the context of the pure SU(2)+Higgs model in [10].

In the Higgs phase, in contrast, $D = 0$. As we have argued above, the massless field Q_i does not couple to $N_{\rm CS}^{(1)} - N_{\rm CS}^{(2)}$ in any order of perturbation theory for $k^2 \to 0$, so that $S_{ij}(k)$ in Eq. (11) does not have a term behaving as $\epsilon_{ijl} k_l/k^4$. At the same time, the $G_{ij}^{-1}(k,0)$ appearing in Eq. (10) and defined in Eq. (6), still behaves as $\propto k^2$ for $k^2 \to 0$. This leads to the vanishing of D in the Higgs phase.

A subtle point in this argument is that the quantity obtained from Eq. (12) is gauge-invariant only with respect to topologically trivial (small) SU(2) gauge transformations. To make the order parameter completely gauge-invariant one should replace $N_{\rm CS}^{(2)}$ by some operator which is completely gauge-invariant but coincides with $N_{\rm CS}^{(2)}$ in perturbation theory. Such an operator can be defined in the 4d theory with chiral fermions, see below. Here we recall a definition within the 3d theory [11, 12] which can be successfully discretized and implemented in practical lattice simulations [13].

The idea is to replace $N_{\rm CS}^{(2)}$ with the difference $N_{\rm CS}^{(2)} - N_{\rm CS,cl}^{(2)}$, where $N_{\rm CS,cl}^{(2)}$ is a particular integer times 2π depending on the initial field configuration, with the properties that $N_{\rm CS}^{(2)} - N_{\rm CS,cl}^{(2)}$ is gauge-invariant and that $N_{\rm CS,cl}^{(2)}$ vanishes in case the initial field configuration is "perturbative" (i.e., $gA^2 \ll \partial A$). Such a difference can be obtained with the following algorithm. Suppose we have some (sufficiently smooth) 3d gauge field configuration. Introduce a fictitious time variable τ and "cool" the configuration according to

$$\frac{\partial \Phi}{\partial \tau} = -\frac{\delta S}{\delta \Phi}, \tag{13}$$

where Φ is a generic notation for all real field components. The cooling, which is a gauge covariant procedure for static gauge transformations, is to be continued all the way from $\tau = 0$ to a classical vacuum configuration at $\tau \to \infty$. Define the non-Abelian and Abelian electric fields as $E_i^B = \frac{\partial B_i}{\partial \tau}, E_i^a = \frac{\partial A_i^a}{\partial \tau}$, compute the topological number densities $\epsilon_{ijk} E_i^B B_{jk}$ and $\epsilon_{ijk} E_i^a F_{jk}^a$, and integrate the difference of them over space and the fictitious time τ. This gives a gauge-invariant result $N_{\rm CS}^{(2)} - N_{\rm CS,cl}^{(2)}$, which can replace the SU(2) Chern-Simons number in Eq. (11): if the initial configuration is perturbative, the system should behave essentially as in the Abelian case, so that indeed $N_{\rm CS,cl}^{(2)} = 0$.

For a completely gauge-invariant measurement one should also replace B_i by the hypermagnetic field $\epsilon_{ijk} B_{jk}$ in Eqs. (6), (11)[5]. Note that for the measurement of $G_{ij}(k,0)$, $N_{\rm CS}^{(2)}$ is not needed.

Phase transition in 4d. The consideration above can be carried out also directly in the finite temperature 4d theory. Let us take as an example the standard electroweak theory, with its real fermionic content. Consider the finite temperature (one can also have a finite chemical potential for *conserved* charges) Green's function defined in

[5] In principle, one can then relate the Green's function in Eq. (11) to the expectation value $\langle (N_{\rm CS}^{(1)} - N_{\rm CS}^{(2)}) \rangle$, measured in an appropriate inhomogeneous external hypermagnetic field.

Eq. (10), but now in the 4d theory and with the replacement of S_{ij} by

$$S_{ij}(k) = \int d^3x e^{ikx} \langle B_i(x) B_j(0) (B+L) \rangle, \tag{14}$$

where $B+L = \int d^3x j_0^{B+L}$ and j_μ^{B+L} is the baryon + lepton number current, generated by the global transformation $\psi_i \to \exp(i\alpha)\psi_i$. We will be interested in the limit $k^0 = 0, \mathbf{k} \to 0$. The part of $S_{ij}(k)$ antisymmetric in i, j contains a term linear in $\mathbf{k}$, precisely as in Eq. (12). Now, the proof of the CH theorem (the absence of renormalization of the term linear in $\mathbf{k}$ beyond 1-loop level) in fact does not essentially depend on the number of dimensions (as long as there are no new IR-problems) and is directly applicable to our case. An explicit 1-loop computation gives a non-zero value in the symmetric phase since the Abelian hypercharge field contributes to the anomaly in the baryonic current. At the same time, an explicit computation in the Higgs phase gives zero, as there is no contribution to the baryonic current anomaly coming exclusively from the electromagnetic field. This statement does not depend on the mass spectrum of the theory, nor on whether there is a finite temperature T, or a finite chemical potential μ, with the associated Z_0, Q_0-condensate (this does not break the spatial U(1)-invariance). Thus, the object defined in Eq. (14) can serve as a distinction between the confinement and Higgs phases to all orders in perturbation theory.

In the 4d SU(2)×U(1) theory without fermions one could use again the Chern-Simons number as in Eq. (11), instead of the baryon + lepton number.

Conclusions. We proposed a gauge-invariant Green's function which can distinguish between the symmetric and Higgs phases of the electroweak theory. It is related to Chern-Simons number of the gauge fields and has a different value in the two phases, computable to all orders in perturbation theory. The most important ingredients of our consideration are the existence of an unbroken Abelian U(1) group in the symmetric and Higgs phases, the non-trivial photon mixing in the Higgs phase, the absence of massless charged particles, and the use of the Coleman-Hill theorem. It remains to be seen whether the CH theorem is valid beyond perturbation theory in the present context.

Obviously, this consideration does not allow us to fix the parameters of the theory at which the jump of the Green's function takes place, precisely as in the consideration of the photon mass in the case of the Abelian Higgs model. Nevertheless, if non-perturbative effects were absent, it would tell that there is a non-analytic behaviour in a certain Green's function when one goes from the symmetric to the Higgs phase. This does not as such necessarily mean that the vacuum energy of the 3d theory (or the partition function of the finite temperature 4d theory) has any singularity at the point of the transition, since the Chern-Simons number does not appear as a term in the Lagrangian for $\mu = 0$ and serves only as an external probe.

There is a physical consequence from this result. Suppose that one has a high temperature system with a non-zero chemical potential μ_{B+L} for the baryon + lepton number. This system is unstable as $B+L$ is not conserved due to the anomaly, but can nevertheless be described on the perturbative level by an effective action of the type in Eqs. (1), (2), with the replacement $\mu \to -i\mu_{B+L}$ [14]. Now, the resulting system is unstable even perturbatively in the symmetric phase due to the coupling of the Chern-Simons number to the massless hypercharge vector field, which leads to a spontaneous generation of a hypermagnetic field [15]. However, in the Higgs phase no magnetic field is generated perturbatively, due to the absence of a coupling of $N_{\rm CS}$ to a massless mode, as we have discussed above.

We thank K. Kajantie, S. Khlebnikov and K. Rummukainen for many helpful discussions. The work of M.L. was partly supported by the TMR network *Finite Temperature Phase Transitions in Particle Physics*, EU contract no. FMRX-CT97-0122. The work of M.S. was partly supported by the Swiss Science Foundation, contract no. 21-55560.98.

References

[1] S. Elitzur, Phys. Rev. D 12 (1975) 3978.

[2] E. Fradkin and S.H. Shenker, Phys. Rev. D 19 (1979) 3682; T. Banks and E. Rabinovici, Nucl. Phys. B 160 (1979) 349.

[3] K. Kajantie, M. Laine, K. Rummukainen and M. Shaposhnikov, Phys. Rev. Lett. 77 (1996) 2887 [hep-ph/9605288]; F. Karsch, T. Neuhaus, A. Patkós and J. Rank, Nucl. Phys. B (Proc. Suppl.) 53 (1997) 623 [hep-lat/9608087].

[4] K. Kajantie, M. Karjalainen, M. Laine and J. Peisa, Phys. Rev. B 57 (1998) 3011 [cond-mat/9704056].

[5] K. Kajantie, M. Laine, T. Neuhaus, J. Peisa, A. Rajantie and K. Rummukainen, Nucl. Phys. B 546 (1999) 351 [hep-ph/9809334].

[6] K. Kajantie, M. Laine, K. Rummukainen and M. Shaposhnikov, Nucl. Phys. B 493 (1997) 413 [hep-lat/9612006].

[7] K. Kajantie, M. Laine, K. Rummukainen and M. Shaposhnikov, Nucl. Phys. B 458 (1996) 90 [hep-ph/9508379]; Phys. Lett. B 423 (1998) 137 [hep-ph/9710538].

[8] S. Deser, R. Jackiw and S. Templeton, Phys. Rev. Lett. 48 (1982) 975.

[9] S. Coleman and B. Hill, Phys. Lett. B 159 (1985) 184.

[10] H.P. Shanahan and A.C. Davis, Phys. Lett. B 431 (1998) 135 [hep-ph/9804203].

[11] M.E. Shaposhnikov, Nucl. Phys. B 299 (1988) 797.

[12] J. Ambjørn, M.L. Laursen and M.E. Shaposhnikov, Nucl. Phys. B 316 (1989) 483.

[13] J. Ambjørn and A. Krasnitz, Nucl. Phys. B 506 (1997) 387 [hep-ph/9705380]; G.D. Moore, Phys. Rev. D 59 (1999) 014503 [hep-ph/9805264]; G.D. Moore and K. Rummukainen, hep-ph/9906259.

[14] A.N. Redlich and L.C. Wijewardhana, Phys. Rev. Lett. 54 (1985) 970; A.J. Niemi and G.W. Semenoff, Phys. Rev. Lett. 54 (1985) 2166.

[15] M. Joyce and M. Shaposhnikov, Phys. Rev. Lett. 79 (1997) 1193 [astro-ph/9703005].