Standard Model Higgs boson mass from inflation

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Abstract

We analyse one-loop radiative corrections to the inflationary potential in the theory, where inflation is driven by the Standard Model Higgs field. We show that inflation is possible provided the Higgs mass m_H lies in the interval $m_{\min} < m_H < m_{\max}$, where $m_{\min} = [136.7 + (m_t - 171.2) \times 1.95]\,\mathrm{GeV}$, $m_{\max} = [184.5 + (m_t - 171.2) \times 0.5]\,\mathrm{GeV}$ and m_t is the mass of the top quark. In the renormalization scheme associated with the Einstein frame the predictions of the spectral index of scalar fluctuations and of the tensor-to-scalar ratio practically do not depend on the Higgs mass within the admitted region and are equal to $n_s = 0.97$ and $r = 0.0034$ correspondingly.

Key words: Inflation, Higgs boson, Standard Model, Variable Planck mass, Non-minimal coupling
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1. Introduction

During the last decades a growing number of connections between cosmology and particle physics were established. However, finding a relation of cosmological inflation to low energy particle theory is a difficult task. In many models inflation is driven by some new physics at large energies which is not connected to the scale of the Standard Model (SM). In [1] it was suggested that the SM Higgs boson can play the role of the inflaton. At first sight, the properties of the electroweak Higgs boson (with the quartic coupling $\lambda \sim 0.1$ and the mass $m_H \sim 100\,\mathrm{GeV}$) are very far from those required for the inflaton field [2] (in the simplest $m^2\phi^2 + \lambda\phi^4$ model the typical choice of parameters, leading to successful inflation, is $\lambda \sim 10^{-13}$, $m \sim 10^{13}\,\mathrm{GeV}$). Nevertheless, addition of the non-minimal coupling of the Higgs fields to the Ricci scalar changes the situation. As was shown in [1], the SM action with gravity included

$$S_J = S_{\mathrm{SM}} + \int d^4x\sqrt{-g}\left(-\frac{M^2}{2}R - \xi\Phi^\dagger\Phi R\right) \qquad (1)$$

naturally leads to inflation. Here S_{SM} stands for the SM action, M is a mass parameter, nearly equal to the Planck mass in our case, R is the scalar curvature, Φ is the Higgs doublet, and ξ is the new coupling constant.

The fact that non-minimal coupling of the scalar field relaxes the requirement for the smallness of the quartic coupling, and also suppresses the generation of the tensor modes during inflation, was known for quite a long time [3, 4, 5, 6, 7, 8, 9]. Basically, if non-minimal coupling is present, the parameter that fixes the normalization of the CMB fluctuations is not the scalar self-coupling, but the combination λ/ξ^2. It is this point which allows the SM Higgs boson to be the inflaton at the same time.

The study of [1] was based on the classical scalar potential in the theory (1). It was argued there that the radiative corrections do not spoil the flatness of the potential, necessary for inflation. In Refs. [10, 11] it was conjectured that all the results of the tree analysis remain true if the Higgs mass lies in the interval[1] $m_H \in [129, 189]\,\mathrm{GeV}$, corresponding to the situation when the Standard Model remains a consistent quantum field theory up to the inflation scale M_P/ξ, or, to be on a safer side, all the way up to the Planck scale M_P.

The aim of this Letter is the analysis of the renormalization group improved effective potential for Higgs-inflaton. We will show that inflation is possible in the SM if the Higgs mass lies in the interval $m_{\min} < m_H < m_{\max}$, somewhat exceeding the range in which the SM can be valid up to the Planck scale, in accordance with our previous expectations.

The Letter is organized as follows. In Sec. 2 we review the Higgs-inflaton scenario and introduce the notations. In Sec. 3 we construct the renormalization group improved effective potential and discuss possible renormalization prescriptions for its computation. We also identify there an error made in a previous attempt [13] to include radiative corrections to Higgs-inflation. In Sect. 4 we present the numerical results. Sect. 5 is conclusions.

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[1] These specific numbers should be taken with a grain of salt, as they were quoted in [12] on the basis of compilation of previous computations and do not take into account the progress made in experimental determination of electroweak parameters.

2. Inflation in tree approximation

The simplest way to analyse the inflation in the model (1) is to make the conformal transformation to the "Einstein frame", where the gravitational term takes its usual form. This is achieved by rescaling the metric by the conformal factor Ω

$$g_{\mu\nu} \to \hat{g}_{\mu\nu} = \Omega^2 g_{\mu\nu}\,, \quad \Omega^2 = \frac{M^2 + \xi h^2}{M_P^2}\,, \tag{2}$$

where $M_P \equiv 1/\sqrt{8\pi G_N} = 2.44 \times 10^{18}\,\mathrm{GeV}$ is the reduced Planck mass, and h is the unitary gauge Higgs $\Phi(x) = \frac{1}{\sqrt{2}}\binom{0}{v+h(x)}$. Then, with redefinition of the field $h \to \chi$

$$\frac{d\chi}{dh} = \sqrt{\frac{\Omega^2 + 6\xi^2 h^2/M_P^2}{\Omega^4}}\,, \tag{3}$$

we get the action with usual gravity and canonically normalised scalar field χ with potential

$$U(\chi) = \frac{1}{\Omega^4\,[h(\chi)]}\frac{\lambda}{4}\left[h^2(\chi) - v^2\right]^2\,. \tag{4}$$

For large ξ the approximate solution for (3) is

$$\chi \simeq \begin{cases} h & \text{for} \quad h < \frac{M_P}{\xi}\,, \\ \sqrt{\frac{3}{2}} M_P \log \Omega^2(h) & \text{for} \quad \frac{M_P}{\xi} < h\,. \end{cases} \tag{5}$$

Therefore, the potential coincides with the standard one for small fields. At the same time, for large fields it becomes exponentially flat

$$U(\chi) \simeq \frac{\lambda M_P^4}{4\xi^2}\left(1 - \mathrm{e}^{-\frac{2\chi}{\sqrt{6}M_P}}\right)^2\,. \tag{6}$$

The inflation in the Einstein frame[2] can be analysed by the usual means [1, 16, 17]. One has a slow roll inflation ending at $h_{\rm end} \simeq (4/3)^{1/4} M_P/\sqrt{\xi}$, with the WMAP scale perturbations exiting the horizon $N \simeq 59$ e-foldings earlier at $h_{\rm WMAP} \simeq 9.14 M_P/\sqrt{\xi}$. The normalization of the CMB perturbations leads to the requirement

$$\xi \simeq \sqrt{\frac{\lambda}{3}}\frac{N_{\rm WMAP}}{0.0276^2} \simeq 44700\sqrt{\lambda}\,. \tag{7}$$

The spectral index and the tensor-to-scalar ratio are $n_s \simeq 0.97$ and $r \simeq 0.0034$, which lies well within the WMAP5 limits [18].

3. Renormalization group and effective potential

Following [1], we adopt the following procedure for computation of quantum corrections to the effective potential. First, we rewrite the theory in the Einstein frame with the use of the equations given in the previous Section. Second, we determine the particle masses (W, Z, Higgs, and t-quark) as a function of the background field χ:

$$m_W^2 = \frac{g^2 h^2}{4\Omega^2}\,,\; m_Z^2 = \frac{(g^2 + g'^2)h^2}{4\Omega^2}\,, \tag{8}$$
$$m_H^2 = \frac{d^2 U}{d\chi^2}\,,\; m_t^2 = \frac{y_t^2 h^2}{2\Omega^2}\,.$$

Here g, g', g are the electroweak SU(2), U(1) and strong SU(3) coupling constants, y_t is the top quark Yukawa coupling. Note that the only difference from the flat space case is the presence of the conformal factor Ω in the denominators of the masses. This is just the result of the transformation to the Einstein frame. Finally, we compute the radiative corrections with the use of the standard formula of [19] (cf. [20, 21]):

$$\delta U = \frac{6 m_W^4}{64\pi^2}\log\frac{m_W^2}{\mu^2} + \frac{3 m_Z^4}{64\pi^2}\log\frac{m_Z^2}{\mu^2} - \frac{3 m_t^4}{16\pi^2}\log\frac{m_t^2}{\mu^2}\,, \tag{9}$$

where μ is the normalization scale[3]. Note that we omitted the contribution from the Higgs field itself, since it is exponentially suppressed at large field values and thus can be safely neglected for analysis of inflation.

This procedure may be contrasted with that of [13]. These authors suggested to use the Jordan rather than Einstein frame for the computation of radiative corrections and made a transition to the Einstein frame of the 1-loop effective potential. This leads to the same structure of radiative corrections as in Eq. (9) but with replacement $\mu \to \mu/\Omega$.

Let us elaborate more on the difference between two prescriptions. For this end we replace the normalization point μ by an effective ultraviolet "cut off". Then two choices are possible with the cut off proportional to:

	I [1]	II [13]
Jordan frame	$M_P^2 + \xi h^2$	M_P^2
Einstein frame	M_P^2	$\dfrac{M_P^4}{M_P^2 + \xi h^2}$

Both choices correspond to imposing a *field-dependent* cut off in one or another frame. It is hard to say, which prescription should be used without knowledge of the behaviour of the quantum theory at the Planck scales. In fact, the prescription of [1] can be justified by the ideas of exact quantum scale invariance, discussed in [22, 23, 24]. In these papers it was proposed that all dimensional parameters, including the Planck mass, are generated by spontaneous breaking of exact scale invariance by an additional dilaton field. The SI-GR prescription of [23] exactly corresponds to the suggestion which was made in [1]. Though the choices of [1] and [13] are not physically equivalent, we will show that after the renormalization group improvement the predictions for the Higgs mass are *nearly the same.*

The one loop modification of the potential (9) is a good approximation, if the logarithms remain small. However, if one

[2] The same results can be obtained in the Jordan frame [7, 14, 15].

[3] In the $\overline{\rm MS}$ subtraction scheme $\log(x)$ should be replaced by $\log(x) - 3/2$ for t-quark and $\log(x) - 5/6$ for gauge bosons.

uses the physical values of coupling constants, Higgs, W, Z and top masses, which are defined at the electroweak scale, the logarithms are large in the inflationary region. Therefore, to connect the potential at inflation with the low energy parameters, one should apply the renormalization group procedure. This was not done in [13], which resulted in erroneous conclusions.

The one-loop renormalization group equations in the curved space are (no graviton loops are included) [25, 21, 20, 26]:

$$16\pi^2 \frac{dg}{dt} = -\frac{19}{6} g^3 , \tag{10}$$

$$16\pi^2 \frac{dg'}{dt} = \frac{41}{6} g'^3 , \tag{11}$$

$$16\pi^2 \frac{dg_3}{dt} = -7 g_3^3 , \tag{12}$$

$$16\pi^2 \frac{dy_t}{dt} = \frac{9}{2} y_t^3 - 8 g_3^2 y_t - \frac{9}{4} g^2 y_t - \frac{17}{12} g'^2 y_t , \tag{13}$$

$$\begin{aligned} 16\pi^2 \frac{d\lambda}{dt} &= 24\lambda^2 + 12\lambda y_t^2 - 9\lambda(g^2 + \frac{1}{3} g'^2) \\ &\quad - 6 y_t^4 + \frac{9}{8} g^4 + \frac{3}{8} g'^4 + \frac{3}{4} g^2 g'^2 , \end{aligned} \tag{14}$$

$$16\pi^2 \frac{d\xi}{dt} = \left(\xi + \frac{1}{6}\right)\left(12\lambda + 6 y_t^2 - \frac{9}{2} g^2 - \frac{3}{2} g'^2\right) , \tag{15}$$

where $t \equiv \log \mu / M_Z$.

The solution of these equations can be plugged in the expression for the effective potential (as usual, the μ-dependent constants should be substituted only in the tree level part)

$$\begin{aligned} U_{\mathrm{eff}}(\chi, \mu) &= U + \delta U \\ &= \frac{\lambda(\mu)}{4\xi^2(\mu)} f(\chi) + s(g, g', g_3, y_t) f(\chi) \log\left(\frac{m_t^2}{\mu^2}\right) \\ &\quad + \mu\text{-independent terms} . \end{aligned} \tag{16}$$

Note that the function

$$f(\chi) = M_P^4 \left(1 - \mathrm{e}^{-\frac{2\chi}{\sqrt{6} M_P}}\right)^2 \tag{17}$$

is in fact the same in the tree level term and one loop contributions (compare (6) with (9), (8)), and function $s(g, g', g_3, y_t)$ can be read of (9), (8).

The dependence of the effective potential on μ is artificial. To be more precise, $U_{\mathrm{eff}}(\chi, \mu)$ does not depend on μ at its extrema (in other points contributions the field renormalization must be taken into account). In our case, the potential becomes constant at $\chi \to \infty$, and, therefore, it should not depend on μ in this region (in other words, the energy density during inflation is a physical quantity and thus is μ-independent). With the use of Eqns. (14,15) one can easily check that this is indeed the case for both prescriptions discussed above,

$$\frac{d}{d\mu}\left[\frac{\lambda(\mu) M_P^4}{4\xi^2(\mu)} + \delta U\right] = 0 . \tag{18}$$

The running of ξ is essential for this result.

As far as the potential is μ-independent, we can choose the most convenient value of μ. The obvious choice is to take μ to make the logarithms vanish [27]

$$\mu^2 = \kappa^2 m_t^2(\chi) = \kappa^2 \frac{y_t(\mu)^2}{2} \frac{M_P^2}{\xi(\mu)} \left(1 - \mathrm{e}^{-\frac{2\chi}{\sqrt{6} M_P}}\right) . \tag{19}$$

Here κ is some constant of order one, introduced to imitate difference between m_t, m_W, m_Z, and also account for μ-independent terms that were dropped in (9). Then the final improved potential is given by the formula (6), where λ and ξ are taken at the scale μ, determined by (19). The parameter μ varies in a finite interval, $0 < \mu < \mu_{\max}$, corresponding to the χ change from 0 to ∞.

Making the analysis for the prescription of [13] is also simple, and boils down to just taking another value for μ:

$$\mu^2 = m_t^2(\chi) \Omega(\chi)^2 = \frac{y_t(\mu)^2}{2} \frac{M_P^2}{\xi(\mu)} \left(\mathrm{e}^{\frac{2\chi}{\sqrt{6} M_P}} - 1\right) . \tag{20}$$

Once the potential is determined, one can carry out the usual analysis of the slow-roll inflation, fixing ξ from COBE or WMAP normalization, and calculating spectral index n_s and tensor to scalar ration r, in complete analogy with [1, 16]. The only technical point here is that it is easier to use μ as an independent variable instead of χ. The advantage is that no inversion of Eqns. (19, 20) is required.

4. Numerical results

We solve the equations (10)-(15) with the initial conditions

$$\frac{g^2}{4\pi} = 0.034 , \ \frac{g'^2}{4\pi} = 0.010 , \ \frac{g_3^2}{4\pi} = 0.13 ,$$
$$y_t \frac{v}{\sqrt{2}} = 171.2\,\mathrm{GeV} , \ \sqrt{2\lambda} v = m_H , \ \xi = \xi_0$$

at $\mu = M_Z$. Here $v = 246.22\,\mathrm{GeV}$ and the central value of the mass of t-quark is specified for concreteness. With this solution we obtain the RG improved potential, which is then used for computation of the parameters of inflation. We take $\kappa = 1$.

We find that inflation can take place provided the Higgs mass lies in the interval

$$\begin{aligned} &m_{\min} < m_H < m_{\max} , \\ m_{\min} &= [136.7 + (m_t - 171.2) \times 1.95]\,\mathrm{GeV} , \\ m_{\max} &= [184.5 + (m_t - 171.2) \times 0.5]\,\mathrm{GeV} . \end{aligned} \tag{21}$$

If the mass is smaller than $m_{\min}$, the slope of the effective potential for large field values becomes negative, making inflation impossible. If the mass is larger than $m_{\max}$, the value of $\mu_{\max}$, corresponding to inflationary stage is close to the Landau pole in $\lambda(\mu)$, making the theory strongly coupled. The specific numbers in (21) correspond to $\mu_{\max}$ coinciding with the Landau pole for λ. More elaborate definitions of the applicability of the perturbative theory may be introduced (like $\lambda(\mu_{\max}) \sim 1$), and lead to slightly smaller $m_{\max}$.

The value of ξ (at the M_Z scale), leading to proper CMB normalization, is presented in Fig. 1. As expected, smaller ξ correspond to smaller Higgs masses, cf. (7). The small rise in

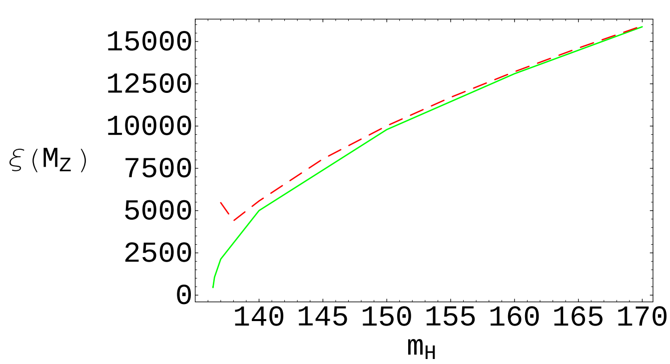

Figure 1: Non-minimal coupling parameter ξ as a function of the Higgs mass m_H. Solid line is for the choice I, dashed—for the choice II.

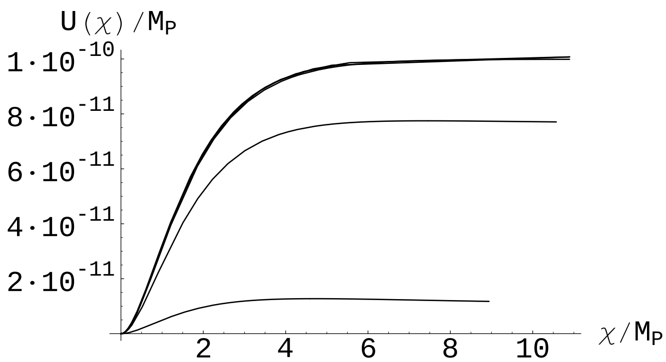

Figure 2: RG improved effective potential. The four nearly coinciding upper lines correspond to the choice I and $m_H = 137, 140, 170$ GeV and choice II with $m_H = 170$ GeV. The middle and lower lines correspond to the choice II with $m_H = 140$ GeV and 137 GeV, respectively.

ξ at small m_H for the prescription of [13] corresponds to the potential which starts to decrease at high field values. Note, that for the λ-ξ relation the approximate formula (7) can still be used, (except for the Higgs masses very close to the boundaries of the allowed region). Of course, ξ and λ in (7) should be calculated then at the scale μ, corresponding to inflation.

Figure 2 shows the resulting RG improved potential for several values of the Higgs mass. It is seen, that for the choice I the shape of the potential is nearly universal, while the overall normalization is always the same, due to the proper choice of ξ. The form of the potential (related to the $\lambda(\mu)/\xi^2(\mu)$ ratio) start to change only very close to the boundaries of the allowed mass for the Higgs field, when the zero or the pole of λ are close to the inflationary value of μ. For the choice II the change of μ during the inflationary epoch is larger, so the deviation of the potential from the tree level form is more pronounced, especially for small Higgs masses.

For the choice I the spectral index stays nearly constant over the whole admissible range of the Higgs masses, except for an extremely small region near the boundaries (see Fig. 3). The tensor-to scalar ratio r also stays constantly small. However,

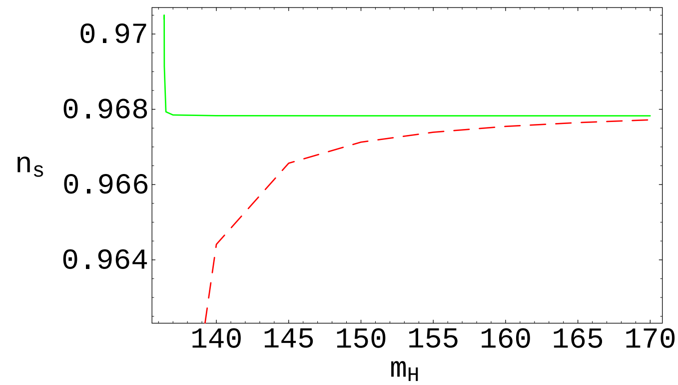

Figure 3: Spectral index n_s depending on the Higgs mass m_H. Solid line is for the choice I, dashed—for the choice II.

for the choice II the potential is different for the lower Higgs masses, and the spectral index becomes smaller. At masses $m_H < 137$ GeV the spectral index goes out of the WMAP allowed region $n_s > 0.93$ [18]. Thus, the choice made in [13] leads, after renormalization group improvement, to a window just slightly smaller, than the one, following from the choice I. Moreover, contrary to what was claimed in [13], the predictions of n_s and r depend on the Higgs mass in the allowed interval only weakly.

Several comments are now in order. The analysis we carried out in this Letter can be improved in several respects.

1. For the solution of the RG equations the initial conditions were specified with the use of the tree relations for the masses of the Higgs boson and t-quark. This should be modified accounting for the physical pole masses.
2. For the RG improvement of the potential we chose a unique scale μ related to the top mass (see Eq. (19)) and dropped the one-loop contribution completely. This can be accounted for.
3. The one-loop running of the couplings can be further replaced by the two-loop one.

However, these effects cannot change the main pattern of the Higgs-inflation and will only result in some modification of the window for the Higgs mass.

5. Conclusions

To summarize, the inflation in the Standard Model is possible for the Higgs masses in the window $136.7\,\text{GeV} < m_H < 184.5\,\text{GeV}$ (for $m_t = 171.2$ GeV). This roughly coincides with the domain of m_H, in which the SM can be considered as a consistent quantum field theory all the way up to the Planck scale. For the scale invariant normalization choice I the spectral index in the whole region is constant and satisfies the WMAP constraints, while in the normalization choice II from [13] the spectral index is m_H-dependent and leads to a slightly stronger limit $m_H > 137$ GeV (no change in the upper limit).

If one extends the SM by three relatively light singlet fermions (νMSM of [28, 29]), then the model (1) is able to address all *experimentally confirmed* indications for existence of physics beyond the SM, including neutrino oscillations, dark matter, baryon asymmetry of the Universe, and inflation. Further extending the model, by making it scale invariant via introduction of one more scalar field (the dilaton) [22] and adding unimodular constraint on gravity, allows to explain also the late time accelerating expansion of the Universe (Dark Energy). The scale-invariant quantum renormalization procedure of [23], applied to this model, allows to construct a theory were all mass parameters come from one and the same source, cosmological constant is absent due to the symmetry requirement, and no quadratically divergent corrections to the Higgs mass are generated. Various cosmological and experimental consequences of the model were studied in [28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 12, 38, 39, 16, 40, 41, 42, 43].

These considerations indicate that no intermediate energy scale between the Z mass and the Planck scale is required to deal with the observational and a number of fine-tuning problems of the SM. A crucial test of this conjecture and of the Higgs-inflation will be provided by LHC, if it finds nothing but the Higgs boson in a specific mass range, found in this Letter.

A closely related paper [44] appeared in the arXiv simultaneously with the current work, providing a different approach to the same problem. The conclusions of this paper about the possibility of Higgs-inflation are similar, but not exactly identical to ours.

The upgrade of the computation of this Letter to the two-loop level is performed in [45], where a more detailed comparison with [44] is also carried out.

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