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The νMSM, Inflation, and Dark Matter

Mikhail Shaposhnikov[1,2] and Igor Tkachev[2]

[1] *Institut de Théorie des Phénomènes Physiques, Ecole Polytechnique Fédérale de Lausanne, CH-1015 Lausanne, Switzerland*

[2] *Theory Division, CERN, CH-1211 Geneve 23, Switzerland*

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Abstract

We show how to enlarge the νMSM (the minimal extension of the standard model by three right-handed neutrinos) to incorporate inflation and provide a common source for electroweak symmetry breaking and for right-handed neutrino masses. In addition to inflation, the resulting theory can explain simultaneously dark matter and the baryon asymmetry of the Universe; it is consistent with experiments on neutrino oscillations and with all astrophysical and cosmological constraints on sterile neutrino as a dark matter candidate. The mass of inflaton can be much smaller than the electroweak scale.

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In refs. [1, 2] it has been shown that a simple renormalizable extension of the Minimal Standard Model (MSM), containing three right-handed neutrinos N_I of masses smaller than the electroweak scale, called νMSM, can explain simultaneously dark matter and the baryon asymmetry of the Universe, being consistent with neutrino masses and mixings observed experimentally[1]. The Lagrangian of the νMSM is

$$\mathcal{L}_{\nu\mathrm{MSM}} = \mathcal{L}_{\mathrm{MSM}} + \bar{N}_I i\partial_\mu \gamma^\mu N_I - F_{\alpha I}\, \bar{L}_\alpha N_I \Phi - \frac{M_I}{2}\, \bar{N}_I^c N_I + \text{h.c.}\,, \tag{1}$$

where $\mathcal{L}_{\mathrm{MSM}}$ is the Lagrangian of the MSM, Φ and L_α ($\alpha = e, \mu, \tau$) are respectively the Higgs and lepton doublets, F is a matrix of Yukawa coupling constants, and M_I are the Majorana masses. The Majorana mass matrix is taken to be real and diagonal.

The νMSM has two essential drawbacks. On the cosmological side, it does not explain the main features of the Universe, such as its homogeneity and isotropy on large scales and the existence of structures on smaller scales; these are believed to be coming from inflation [5]–[9][2]. On the theoretical side, it remains unclear why the potentially different energy scales, related to the electroweak symmetry breaking and to the Majorana neutrino masses, could be of the same order of magnitude. The aim of the present note is to show that a simple extension of the νMSM by a real scalar field (inflaton) with scale-invariant couplings (on the classical level) provides a possibility to have inflation and to fix the scales of the νMSM[3]. Moreover, we will show that the coupling of the inflaton to sterile neutrino gives rise to a novel mechanism of dark matter production. In order not to let the number of different names proliferate, we will be using in what follows the same name (νMSM) for the theory with the inflaton.

We propose the following Lagrangian to describe the physics at energies below the Planck scale:

$$\mathcal{L}_{\nu\mathrm{MSM}} \to \mathcal{L}_{\nu\mathrm{MSM}[\mathrm{M}\to 0]} + \frac{1}{2}(\partial_\mu \chi)^2 - \frac{f_I}{2}\, \bar{N_I}^c N_I \chi + \text{h.c.} - \mathrm{V}(\Phi, \chi)\,, \tag{2}$$

where the first term is the νMSM Lagrangian with all dimensionful parameters (Higgs and Majorana masses) put to zero, χ is a real scalar field (inflaton), and f_I are Majorana-type

[1] We do not include here the LSND anomaly [3], which will be tested in the near future [4].
[2] Note that the present day accelerated expansion of the Universe can be incorporated in MSM or νMSM by adding a cosmological constant.
[3] Though our motivation is similar to that of [10], the model we propose and its physics are entirely different.

Yukawa couplings. We parametrize the most general scale-invariant potential $V_s(\Phi,\chi)$ as follows

$$V_s(\Phi,\chi) = \lambda\left(\Phi^\dagger\Phi - \frac{\alpha}{\lambda}\chi^2\right)^2 + \frac{\beta}{4}\chi^4\ , \tag{3}$$

where λ, α and β are the scalar coupling constants, which we take to be positive. We will assume that the scale invariance is explicitly broken on the classical level in the inflaton sector only, so that

$$V(\Phi,\chi) = -\frac{1}{2}m_\chi^2\chi^2 + V_s(\Phi,\chi)\ . \tag{4}$$

This potential has a symmetry $\chi \to -\chi$ which could lead to a cosmological domain wall problem [11]. However, there are many different ways to solve this problem in inflationary cosmology, the simplest one is just to add to (4) a cubic term $\sim \chi^3$.

The requirement of scale invariance looks (and is) rather ad-hoc since this symmetry is broken by quantum corrections. We cannot provide any further motivation for this choice besides the one given already in ref. [12] (see also references therein). From a phenomenological side, it is this requirement that ensures the same source for the scale of electroweak symmetry breaking and for Majorana masses of sterile neutrinos. An explicit breaking of this symmetry by the inflaton mass term is also essential: if $m_\chi^2 = 0$ the electroweak symmetry breaking by radiative corrections only is impossible, because of the large mass of t-quark [12]. From (3,4) one gets the relations between the vev of the inflaton field, its mass m_I and the Higgs mass m_H: $\langle\chi\rangle \simeq m_H/2\sqrt{\alpha}$, $m_I \simeq m_H\sqrt{\beta/2\alpha}$. In what follows we will choose $\alpha > \beta/2$. In this case the inflaton is lighter than the Higgs boson, $m_I < m_H$.

Let us discuss different constraints on the parameters of this model coming from successful (chaotic) inflation scenario [13]. The inflaton potential must be sufficiently flat so as not to produce too large density fluctuations. In our case the flat direction is given by $\Phi^\dagger\Phi = \frac{\alpha}{\lambda}\chi^2$, since the constant $\lambda \sim 1$ must be large enough to have a Higgs boson of mass $\mathcal{O}(100)$ GeV. Along this direction the potential is simply $V \propto \frac{\beta}{4}\chi^4$. The constant β can now be fixed from the requirement to give correctly the amplitude of adiabatic scalar perturbations[4] observed by WMAP [14]. With the use of general expressions given, for instance, in [17], one gets $\beta \simeq 2.6\times 10^{-13}$. The flatness of the potential must not be spoiled by radiative corrections

[4] The pure χ^4 inflaton potential with minimal coupling to gravity is disfavoured by the WMAP3 data [14], producing too large tensor fluctuations. However, allowing for non-minimal couplings (see also the comment at the end of the paper) can bring this potential in agreement with the data [15, 16].

from the loops of the particles of the Standard Model and sterile neutrinos. This requirement gives $\alpha \lesssim 3\times10^{-7}$, $\quad f_I \lesssim 2\times10^{-3}$.

Another constraint could come from the requirement to have successful baryogenesis. After the end of inflation the Universe is reheated up to a certain temperature T_r, which must be considerably larger than the freezing temperature of anomalous electroweak fermion number non-conservation $T \simeq 130$ - 190 GeV [18] to allow sphaleron processes to convert the lepton asymmetry created in sterile–active neutrino transitions to baryon asymmetry [2, 19, 20]. In our model, the transfer of inflaton energy to the fields of the Standard Model goes through the inflaton–Higgs coupling, proportional to the parameter α. The energy in the Higgs field is then quickly distributed among all other fields of the MSM, since the typical coupling constants are quite large. The evolution of the energy $\rho = \frac{\pi^2 g^*}{30}T^4$ of the MSM particles can be found from equation

$$\frac{\partial\rho}{\partial t} + 4H\rho = R \ , \tag{5}$$

where H is the Hubble constant and R is the energy transfer rate from inflaton oscillations to the Higgs field, which can be found using the approach of Ref. [21, 22]

$$R \sim \frac{1}{\lambda}\alpha^2\omega\chi^4 \ . \tag{6}$$

Here $\omega^2 \sim \beta\chi^2$ is the typical frequency of inflaton oscillations. Taking into account that the Universe expands as radiation dominated after inflation (since we assumed that $m_\chi \ll \sqrt{\beta}M_{\rm Pl}$) one gets

$$T_r \sim M_{\rm Pl}\left(\frac{\alpha^2}{g^*\lambda}\right)^{1/4} \ . \tag{7}$$

For $\alpha > \beta \simeq 10^{-13}$ this temperature exceeds greatly the electroweak scale, as required.

Now, we are coming to the question of dark matter abundance in this model. The lightest sterile neutrino N_1, being sufficiently stable, plays the role of dark matter in νMSM. It can be created in active–sterile neutrino oscillations [23] or through the coupling to the inflaton [24, 25]. We will assume that the Yukawa constants $F_{\alpha1}$ are too small to make the first mechanism operative, $F_{\alpha1} \lesssim 10^{-12}$ [1, 2] and estimate the second effect only.

Owing to the Higgs–inflaton mixing, the inflaton with a mass 300 MeV $\lesssim m_I \lesssim m_H$ is in thermal equilibrium down to rather small temperatures $T \ll m_I$, thanks to reactions

$\chi \leftrightarrow e^\dagger e$, $\chi \leftrightarrow \mu^\dagger \mu$, etc[5]. This range of masses corresponds to $1 \times 10^{-13} \lesssim \alpha \lesssim 5 \times 10^{-8}$ and to the inflaton vev in the interval 4×10^5 GeV–3×10^8 GeV (we took $m_H = 200$ GeV for numerical estimates). Sterile neutrinos are produced in inflaton decays mainly at $T \simeq m_I$, and their distribution function $n(p,t)$ (p is the momentum of the sterile neutrino and t is time) can be found from the solution of the kinetic equation

$$\frac{\partial n}{\partial t} - Hp\frac{\partial n}{\partial p} = \frac{2m_I\Gamma}{p^2}\int_{p+m_I^2/4p}^{\infty} n_I(E)dE \ , \tag{8}$$

where the inverse decays $\chi \leftarrow N_1 N_1$ are neglected, H is the Hubble constant, E is the inflaton energy, $n_I(E)$ is the inflaton distribution, $\Gamma = f_1^2 m_I/16\pi$ is the partial width of the inflaton for $\chi \to N_1 N_1$ decay. For the case when the effective number of degrees of freedom is time-independent, an asymptotic ($t \to \infty$) analytic solution to (8) can be easily found:

$$n(x) = \frac{16\Gamma M_0}{3m_I^2} x^2 \int_1^\infty \frac{(y-1)^{3/2}dy}{e^{xy}-1} \ , \tag{9}$$

where $x = p/T$, and $M_0 \approx M_{\rm Pl}/1.66\sqrt{g^*}$, leading to the number density

$$N_0 = \int \frac{d^3p}{(2\pi)^3} n(p) = \frac{3\Gamma M_0 \zeta(5)}{2\pi m_I^2} T^3 \tag{10}$$

and to an average momentum of created sterile neutrinos $\langle p \rangle = \pi^6/(378\zeta(5))T = 2.45T$, which is about 20% smaller than that for the equilibrium thermal distribution, $p_T = 3.15T$.

For the inflaton with a mass $m_I < \mathcal{O}(500)$ MeV, taking $g^* =$ const is a bad approximation, since exactly in this region g^* changes from $g^* \sim 60$ at $T \sim 1$ GeV to $g^* \sim 10$ at $T \sim 1$ MeV, because of the disappearance of quark and gluon degrees of freedom. In this case a numerical solution of eq. (8) is necessary with the input of the hadronic equation of state, which is not known exactly. To make an estimate we took a phenomenological equation of state constructed in [28] on the basis of the hadron gas model at low temperatures, and on available information on lattice simulations and perturbative computations; we found that $N = f(m_I)N_0$, where the function $f(m_I)$ changes from 0.9 at $m_I = 70$ MeV to 0.4 at $m_I = 500$ MeV. An average momentum stays almost unchanged in this interval of m_I. At higher inflaton masses a good approximation to $f(m_I)$ is $f(m_I) \simeq (10.75/g^*(m_I/3))^{3/2}$, and to an average momentum is $\langle p \rangle \simeq 2.45T(10.75/g^*(m_I/3))^{1/3}$.

[5] If inflaton mass is larger than 500 GeV one can show that the inflaton does not equilibrate. In this case the computation of sterile dark matter abundance requires a detailed study of fermionic preheating, similar to [26, 27].

The abundance of dark matter sterile neutrinos can be further diluted by a factor S, which accounts for the entropy production in decays of heavier sterile neutrinos [25]. Collecting all factors together we get for the contribution of sterile neutrinos to the dark matter abundance:

$$\Omega_s \simeq \frac{0.26 f(m_I)}{S} \frac{\Gamma M_0 m_s}{m_I^2 \times 12 \text{ eV}} \frac{2\pi\zeta(5)}{\zeta(3)} \ , \tag{11}$$

where $m_s = f_1 \langle \chi \rangle$ is the dark matter sterile neutrino mass, and M_0 is taken with $g^* = 10.75$.

Let us proceed to numerical estimates. For the case under consideration, the Yukawa couplings $F_{\alpha 1}$ are very small and do not contribute to active neutrino masses [2]. So, the couplings $F_{\alpha 2,3}$ cannot be smaller than $\mathcal{O}(\sqrt{m_{\text{sol}} M}/v) \simeq 10^{-8}$, where $m_{\text{sol}} \simeq 0.01$ eV is the solar-neutrino mass difference, M is the mass of the heavier sterile neutrino, v is the Higgs vev. With the use of general results of [25] one can find that the entropy production factor can be in the region $1 < S < 2$. Requiring that sterile neutrinos constitute all the dark matter we find that $f_1 \simeq (4\text{–}5) \times 10^{-11}$ for $m_I \simeq 300$ MeV and that their mass m_s should be in the interval $m_s \simeq 16$–20 keV. Quite amazingly, the keV scale for the sterile neutrino mass follows from observed dark matter abundance and from inflaton self-coupling, fixed by the observations of fluctuations of the CMB, provided the inflaton mass is in the GeV region. The average momentum of sterile neutrinos, accounting for the dilution factor S, can be as small as $0.6 p_T$ in this case. For the inflaton mass $m_I \sim 100$ GeV, we find $f_1 \simeq 10^{-10}$, leading to the sterile neutrino mass $m_s \sim \mathcal{O}(10)$ MeV.

A sterile neutrino in this mass range is perfectly consistent with all cosmological and astrophysical observations. As for the bounds on mass versus active–sterile mixing coming from X-ray observations of our galaxy and its dwarf satellites [29, 30], they are easily satisfied since the production mechanism of sterile neutrinos discussed above has nothing to do with the active–sterile neutrino mixing leading to the radiative mode of sterile neutrino decay. Note that for small enough $F_{\alpha 1}$ the dark matter sterile neutrino cannot explain the pulsar kick velocities [31] and the early reionization[32]. As for the limits coming from Lyman-α forest considerations [33] $m_s > \langle p \rangle / p_T \times m_{\text{Lyman}}$ where $m_{\text{Lyman}} \simeq 15.4$ keV [34], for $S = 2$ one gets $m_s > 10$ keV. These values are comfortably within the mass interval discussed above.

Having fixed the Yukawa coupling constant f_1 for the lightest neutrino, we can estimate the constants f_2 and f_3. First, these constants must be nearly equal, so as to achieve the amplification of CP-violating effects necessary for baryogenesis in the νMSM [2]. Second, the mass of the heavier sterile neutrinos should be roughly in the interval 1–20 GeV, the lower bound comes from the requirement that their decays do not spoil the big bang nucleosynthesis [2, 20, 25] while the upper bound comes from the requirement that the processes with lepton number non-conservation must be out of thermal equilibrium at the electroweak scale [2]. Depending on the vev of the inflaton, we arrive at $f_2 \simeq f_3 \lesssim 10^{-4}$. These values are too small to lead to lepton number violating processes, which could spoil the baryogenesis mechanism via neutrino oscillations, provided the decays $\chi \to 2N_{2,3}$ are kinematically forbidden. They are also too small to spoil the flatness of the inflaton potential.

In conclusion, we constructed a minimal model that provides inflation, gives a candidate for a dark matter particle, explains the baryon asymmetry of the Universe, being consistent with neutrino oscillations. It contains only light particles, in the keV range for a dark matter sterile neutrino, and in the GeV range for two other degenerate neutrinos and the inflaton, which makes it to be potentially testable in laboratory experiments. The electroweak scale in the model is related to the vacuum expectation value of the inflaton. One can go even further and speculate that the Planck scale may be generated by a similar mechanism. Indeed, if the interaction of inflaton with gravity is also scale invariant, $L_G = \frac{1}{g^2}\chi^2 R$, where R is the scalar curvature, the Planck scale will be given by $M_{\rm Pl} \sim \frac{\langle\chi\rangle}{g}$, requiring very small $g \sim 10^{-14}$–10^{-11}. If true, the inflation in this theory may resemble the pre-Big Bang scenario, proposed in [35]. In this case an estimate of the inflaton self-coupling β may be no longer valid and the values of the inflaton mass and sterile neutrino mass derived in this paper may be considered as indicative only. If the requirement of the scale invariance is given up, the relation between electroweak scale, sterile neutrino masses and inflaton mass disappears. However, the fact that the sterile neutrino dark matter abundance can be determined by the interaction with inflaton rather than with the fields of the Standard Model remains in force.

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