

The ν MSM, Dark Matter and Baryon Asymmetry of the Universe

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Abstract

We show that the extension of the standard model by three right-handed neutrinos with masses smaller than the electroweak scale (the ν MSM) can explain simultaneously dark matter and baryon asymmetry of the universe and be consistent with the experiments on neutrino oscillations. Several constraints on the parameters of the ν MSM are derived.

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Introduction.— The canonical Minimal Standard Model (MSM) [1], despite being extremely successful in particle physics, cannot accommodate experimental data on neutrino oscillations [2] simply because neutrinos are exactly massless in the MSM and thus do not oscillate. In addition, the MSM does not contain any particle physics candidate for cold dark matter and cannot explain the baryon asymmetry of the universe.

The simplest renormalisable extension of the standard model, consistent with neutrino experiments, contains \mathcal{N} right-handed $SU(2)\times U(1)$ singlet neutrinos N_I ($I = 1, \dots, \mathcal{N}$) with the most general gauge-invariant interactions described by the Lagrangian:

$$\delta\mathcal{L} = \bar{N}_I i\partial_\mu \gamma^\mu N_I - F_{\alpha I} \bar{L}_\alpha N_I \Phi - \frac{M_I}{2} \bar{N}_I^c N_I + h.c., \quad (1)$$

where Φ and L_α ($\alpha = e, \mu, \tau$) are respectively the Higgs and lepton doublets, and both Dirac ($M_D = F\langle\Phi\rangle$) and Majorana (M_I) masses for neutrinos are introduced. We have taken a basis in which the mass matrices of charged leptons and right-handed neutrinos are real and diagonal, and F is a matrix with elements $F_{\alpha I}$.

In addition to quite a large number of dimensionless Yukawa couplings, this model contains \mathcal{N} dimensionful parameters - the Majorana masses of right-handed fermions. The neutrino oscillation experiments cannot fix these new scales, as the masses and mixing angles of active neutrinos contain only specific combinations of M_D and M_I , coming from the diagonalization of the complete mass matrix.

In this paper we propose to choose these unknown mass parameters to be of the order of the electroweak scale or below. In other words, we will assume that the model defined in (1) is a true low energy theory up to the Planck (or, say, grand-unified) scale. Moreover, we fix \mathcal{N} to be 3, keeping the number of right-handed neutrinos to be equal to the number of fermionic generations. The specific model in this parameter range will be called below the “ ν MSM”, underlying its minimal character and the fact that no new energy scale is introduced.

The aim of the present work is to demonstrate that the ν MSM with a particular choice of parameters, consistent with the data on neutrino masses and mixing, can explain simultaneously the dark matter and baryon asymmetry of the universe.

General properties of the ν MSM.— The ν MSM contains 18 new parameters in comparison with the MSM. Three of them are the Majorana masses, while another 15 are hidden in the Yukawa matrix $F_{\alpha I}$ and can be chosen as 3 diagonal Yukawa couplings, 6 mixing angles and 6 CP-violating phases in the following way:

$$F = \tilde{K}_L f_d \tilde{K}_R^\dagger, \quad (2)$$

where

$$f_d = \text{diag}(f_1, f_2, f_3), \quad \tilde{K}_L = K_L P_\alpha, \quad \tilde{K}_R^\dagger = K_R^\dagger P_\beta. \quad (3)$$

The diagonal matrices for Majorana phases are

$$P_\alpha = \text{diag}(e^{i\alpha_1}, e^{i\alpha_2}, 1), \quad P_\beta = \text{diag}(e^{i\beta_1}, e^{i\beta_2}, 1), \quad (4)$$

and the KM-like mixing matrix K_L is given by

$$K_L = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{L23} & s_{L23} \\ 0 & -s_{L23} & c_{L23} \end{pmatrix} \begin{pmatrix} c_{L13} & 0 & s_{L13}e^{-i\delta_L} \\ 0 & 1 & 0 \\ -s_{L13}e^{i\delta_L} & 0 & c_{L13} \end{pmatrix} \begin{pmatrix} c_{L12} & s_{L12} & 0 \\ -s_{L12} & c_{L12} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad (5)$$

where $c_{Lij} = \cos(\theta_{Lij})$ and $s_{Lij} = \sin(\theta_{Lij})$. The expression for K_R is written in full analogy with the replacement of L to R . We fix the indices in such a way that “1” corresponds to the lightest right-handed neutrino and “3” to the heaviest. The new light neutral fermions can naturally be called the “dark” (N_D), for the dark matter candidate, “clear” (N_C) and “bright” (N_B) sterile neutrinos.

As the Majorana masses are assumed to be of the order of the electroweak scale or below, the model can only be consistent with the neutrino experiments if the Yukawa couplings are very small, $f_i^2 \sim O(m_\nu M_I/v^2)$, where m_ν are the masses of active neutrinos and $v = 174$ GeV is the VEV of the Higgs field. At the same time, in the interesting parameter range discussed below, the Majorana masses are much larger than the Dirac masses, so that the see-saw formula [3] for the masses of the active neutrinos is applicable. The active neutrino mixing matrix [4] is coming then from the diagonalization of the see-saw mass.

The ν MSM and dark matter.— For small f_i the lifetime of a right-handed neutrino may exceed the age of the universe; in this case the ν MSM provides a particle physics candidate for the warm dark matter [5]-[9]. The allowed mass range of the corresponding sterile neutrino is severely restricted as

$$2 \text{ keV} \lesssim M_I \lesssim 5 \text{ keV} , \quad (6)$$

where the lower bound comes from the cosmic microwave background and the matter power spectrum inferred from Lyman- α forest data [10], and the upper bound is given by the radiative decays of sterile neutrinos in dark matter halos limited by X-ray observations [11].

In the recent paper [12] we have shown that the minimal number of sterile neutrinos, which can explain the dark matter in the universe, is $\mathcal{N} = 3$. In this case only one sterile neutrino can be the dark matter. We identify it with the lightest one N_1 , i.e. N_1 is the “dark” sterile neutrino. In addition, the observed mass density of the dark matter leads to the following constraint on the parameters of the model:

$$(M_D^\dagger M_D)_{11} \simeq m_0^2, \quad (7)$$

where $m_0 = \mathcal{O}(0.1)$ eV. In terms of Yukawa couplings and mixing angles it reads:

$$f_1^2 c_{R12}^2 c_{R13}^2 + f_2^2 c_{R13}^2 s_{R12}^2 + f_3^2 s_{R13}^2 \simeq 3.3 \cdot 10^{-25}. \quad (8)$$

In other words, at least one of the couplings f_i must be of order $6 \cdot 10^{-13}$. We choose it to be f_1 . Though being much smaller than the Yukawa constants in the charged lepton sector, this value does not contradict to anything and is stable against loop corrections in the ν MSM.

The dark matter constraint allows us to fix the absolute values of the active neutrino masses as follows: the mass of the lightest active neutrino should lie in the range $m_1 \leq m_\nu^{\text{dm}} = \mathcal{O}(10^{-5})$ eV, while other masses are $m_3 = [4.8_{-0.5}^{+0.6}] \cdot 10^{-2}$ eV and $m_2 = [9.05_{-0.1}^{+0.2}] \cdot 10^{-3}$ eV ($[4.7_{-0.5}^{+0.6}] \cdot 10^{-2}$ eV) in the normal (inverted) mass hierarchy [12].

There are further constraints on Yukawa couplings coming from Big Bang Nucleosynthesis (BBN) and from the absolute values of neutrino masses. Since the second and third active neutrinos are much heavier than the first, one must have $f_{2,3} \gg f_1$. This means that these neutrinos equilibrate before BBN and spoil its predictions, unless they decay before BBN. This leads to the constraint [13] $M_{2,3} > 1$ GeV, and correspondingly, to $f_2^2 > 3 \cdot 10^{-16}$, $f_3^2 > 2 \cdot 10^{-15}$, where the specific numbers were derived assuming small mixing angles θ_{Lij} and

normal hierarchy. Together with eq. (8), this tells that the mixing angles θ_{R12} and θ_{R13} must be small, with

$$s_{R12} < 3.3 \cdot 10^{-5} \left(\frac{\text{GeV}}{M_2} \right)^{1/2} \quad \text{and} \quad s_{R13} < 1.4 \cdot 10^{-5} \left(\frac{\text{GeV}}{M_3} \right)^{1/2}. \quad (9)$$

For the case of inverted hierarchy the conclusion on the smallness of s_{R12} and s_{R13} is the same.

We would now like to see whether this model can account for the baryon asymmetry of the universe in a specified parameter range.

The ν MSM and baryon asymmetry.— In considering the problem of the baryon asymmetry we take the most conservative point of view and assume the validity of the standard Big Bang theory well above the electroweak scale. Since the right-handed neutrino Yukawa coupling constants are small, N 's are out of thermal equilibrium at high temperatures and may come into thermal equilibrium at smaller temperatures. In any case, the lightest sterile neutrino, playing the role of dark matter, never equilibrates. Because of this we will assume that initial concentrations of right-handed neutrinos are equal to zero.

To the best of our knowledge there has been only one study of baryon production in the model (1) with small Majorana masses¹. Namely, in ref. [13] Akhmedov, Rubakov and Smirnov (ARS in what follows) proposed an interesting idea that the baryon asymmetry can be generated through CP-violating sterile neutrino oscillations. For small Majorana masses the total lepton number of the system, defined as the lepton number of active neutrinos plus the total helicity of sterile neutrinos, is conserved and equal to zero during the universe's evolution. However, because of oscillations the lepton number of active neutrinos becomes different from zero and gets transferred to the baryon number due to rapid sphaleron transitions [15]. Roughly speaking, the resulting baryon asymmetry is equal to the lepton asymmetry at the sphaleron freeze-out [16], with the exact relation in [17].

According to the ARS computation, the produced lepton asymmetry is entirely expressed through the Majorana masses M_I of sterile neutrinos, Yukawa couplings f_i and the mixing matrix K_R and can easily be of the required order of magnitude. However, the dark matter and BBN constraints put severe limitations on the phase space of the ν MSM and we found

¹ Note that the Majorana neutrinos with masses much larger than the electroweak scale give rise to thermal leptogenesis [14].

that no choice of the parameters, consistent with (8,9), can lead to the observed baryon asymmetry of the universe, if the ARS equations are used. This can be understood in the following way. The CP-violating effects must be proportional to the Jarlskog determinant [18] related to the matrix K_R , $J = c_{R13}^2 c_{R12} c_{R23} s_{R13} s_{R12} s_{R23} \sin \delta_R$. With the constraints on the mixing angles discussed above, the value of J is at most 10^{-10} . Other factors, such as the total number of degrees of freedom, make the asymmetry predicted by ARS formula well below the observed value in the region of parameter space we are interested in. However, we have reached a different conclusion. Below we will reconsider the neutrino oscillations in the early universe and identify the similarities and crucial differences with ARS.

In general terms, the evolution of possible lepton asymmetries can be found with the use of kinetic equations for a complete neutrino density matrix [19] - [21]. In our case this is a 12×12 matrix with components describing the mixing of active neutrinos and anti-neutrinos as well as the sterile neutrinos of different helicity states:

$$\rho = \begin{pmatrix} \rho_{LL} & \rho_{L\bar{L}} & \rho_{LN} & \rho_{L\bar{N}} \\ \rho_{\bar{L}L} & \rho_{\bar{L}\bar{L}} & \rho_{\bar{L}N} & \rho_{\bar{L}\bar{N}} \\ \rho_{NL} & \rho_{N\bar{L}} & \rho_{NN} & \rho_{N\bar{N}} \\ \rho_{\bar{N}L} & \rho_{\bar{N}\bar{L}} & \rho_{\bar{N}N} & \rho_{\bar{N}\bar{N}} \end{pmatrix}, \quad (10)$$

where different entries in ρ are the 3×3 matrices describing the neutrino states in different sectors (by \bar{N} we denote the negative helicity state of the sterile neutrinos). This density matrix satisfies the kinetic equation [21] which can be written in the form

$$i \frac{d\rho}{dt} = [H, \rho] - \frac{i}{2} \{\Gamma, \rho\} + \frac{i}{2} \{\Gamma^p, 1 - \rho\}, \quad (11)$$

where $H = k(t) + H^0 + H_{int}$ is the Hermitian effective Hamiltonian incorporating the medium effects on neutrino propagation [22], $k(t) \sim T$ is the neutrino momentum, Γ and Γ^p are the destruction and production rates correspondingly. Following ARS, we will use the Boltzmann statistics and replace the last term in (11) by $i\Gamma^p$.

If the system is in thermal equilibrium, the equilibrium density matrix, given by $\rho^{eq} = \exp(-H/T)$ must satisfy eq. (11), which gives the relation $\Gamma^p = \frac{1}{2} \{\Gamma, \rho^{eq}\}$ ².

² For numerical estimates ρ^{eq} can be safely replaced by $\exp(-k/T)$, as was done in ARS analysis, since the corrections to ρ^{eq} coming from H^0 and H_{int} are small.

In the leading approximation (all Yukawa couplings f_i are neglected) the Hamiltonian H^0 is diagonal and can be written as

$$H^0 = \text{diag}(H_{LL}^0, H_{\bar{L}\bar{L}}^0, H_{NN}^0, H_{\bar{N}\bar{N}}^0) \quad (12)$$

with, in turn [23],

$$H_{LL}^0 = H_{\bar{L}\bar{L}}^0 = \frac{T^2}{k(t)} \left[\frac{3g_W^2 + g_Y^2}{32} \text{diag}(1, 1, 1) + \frac{1}{8} \text{diag}(h_e^2, h_\mu^2, h_\tau^2) \right], \quad (13)$$

where the first term comes from the electroweak gauge correction to the active neutrino propagator, and the second from the Higgs correction. h_e^2 , h_μ^2 and h_τ^2 are the charged lepton Yukawa couplings. For the sterile neutrinos we have

$$H_{NN}^0 = H_{\bar{N}\bar{N}}^0 = \frac{1}{2k(t)} \text{diag}(M_1^2, M_2^2, M_3^2). \quad (14)$$

Because of the structure of H^0 the consideration of oscillating neutrinos can be simplified significantly. To remove the trivial time dependence of the density matrix, make a change

$$\rho = U(t) \tilde{\rho} U^\dagger(t), \quad (15)$$

where $U = \exp(-i \int_0^t dt' H^0(t'))$, which brings us to the evolution equation in the interaction picture. Now, as we will see later (and in accordance with ARS) the most important region for leptogenesis is $T_L \simeq (\Delta M^2 M_0)^{\frac{1}{3}}$, where ΔM^2 is the typical value of the Majorana mass squares and $M_0 \simeq 7 \cdot 10^{17}$ GeV appears in the time-temperature relation, $t = \frac{M_0}{2T^2}$. At this time most of the off-diagonal elements of density matrix $\tilde{\rho}$ undergo very rapid oscillations because of $U(t)$ and decouple from the system. These elements are listed below: ρ_{LN} , $\rho_{L\bar{N}}$, $\rho_{\bar{L}N}$, $\rho_{\bar{L}\bar{N}}$, ρ_{NL} , $\rho_{N\bar{L}}$, $\rho_{\bar{N}L}$, and $\rho_{\bar{N}\bar{L}}$; they can be safely put to zero. In addition, all non-diagonal parts of ρ_{LL} and $\rho_{\bar{L}\bar{L}}$ can be neglected as well. In physics terms, the oscillations between L and N are strongly suppressed since these particles have very different effective masses. The same is true for $L_\alpha \rightarrow L_\beta$, $\alpha \neq \beta$ oscillations. On the other hand, the non-diagonal parts of ρ_{NN} and $\rho_{\bar{N}\bar{N}}$ must be kept, as the corresponding exponentials in $U(t)$ are of order 1. Another four non-diagonal entries of the density matrix, $\rho_{L\bar{L}}$, $\rho_{\bar{L}L}$, $\rho_{N\bar{N}}$, and $\rho_{\bar{N}N}$ can be removed as they include the processes with lepton number nonconservation in the active sector and helicity-flip in the sterile sector, suppressed by the small Yukawa couplings and small mass to temperature ratio for N 's [13]. As a result, the

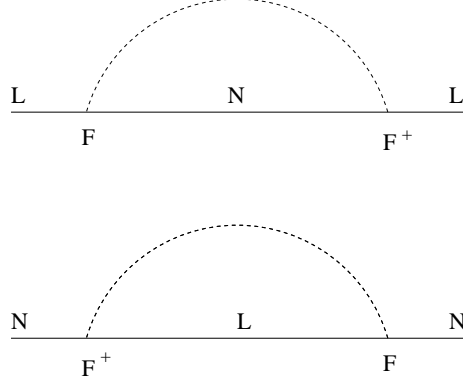


FIG. 1: The diagrams contributing to the real potential and rate Γ in the kinetic equation.

kinetic equation contains the density matrices ρ_{NN} and $\rho_{\bar{N}\bar{N}}$ where coherent quantum effects are essential, and the diagonal parts of ρ_{LL} and $\rho_{\bar{L}\bar{L}}$, describing concentrations of active neutrinos and anti-neutrinos. The latter quantities were not considered by ARS.

Now we are ready to write our kinetic equations. We will do that for L and N only, as the equations for \bar{L} and \bar{N} are given by CP-conjugation. The medium effects follow from computation of real and imaginary parts of the two-point Greens functions on Fig. 1³. The equation for the sterile neutrinos is

$$i\frac{d\tilde{\rho}_{NN}}{dt} = [H_{int}^N(t), \tilde{\rho}_{NN}] - \frac{i}{2}\{\Gamma^N(t), \tilde{\rho}_{NN} - \tilde{\rho}_{NN}^{eq}\} + \frac{\sin\phi}{8} T U^\dagger(t) F^\dagger (\rho_{LL} - \rho_{LL}^{eq}) F U(t). \quad (16)$$

For the diagonal part of the active neutrino density matrix we have:

$$i\frac{d\rho_{LL}}{dt} = \text{diag} \left[\frac{\sin\phi}{8} T F U(t) (\tilde{\rho}_{NN} - \tilde{\rho}_{NN}^{eq}) U^\dagger(t) F^\dagger - \frac{i}{2}\{\Gamma^L(t), \rho_{LL} - \rho_{LL}^{eq}\} \right]. \quad (17)$$

Here the following notation is introduced:

$$H_{int}^N(t) = \frac{T}{8} U^\dagger(t) K_R f_d^2 K_R^\dagger U(t), \quad \Gamma^N(t) = \sin\phi H_{int}^N(t), \quad \Gamma^L(t) = \frac{\sin\phi}{8} T K_L f_d^2 K_L^\dagger, \quad (18)$$

where $\tilde{\rho}_{NN}^{eq}$ and ρ_{LL}^{eq} are the diagonal equilibrium density matrices for sterile and active neutrinos respectively, and $\sin\phi \simeq 0.02$ [13] is the ratio of the absorptive part of the diagrams to their real part. These equations are the basis for the analysis of the time evolution of the asymmetries as a function of the parameters of the ν MSM.

³ Note that the absorptive parts of these diagrams are in fact not suppressed by the Majorana neutrino masses, as was stated by ARS, but by the effective mass of the Higgs boson. However, this effect leads to the same numerical estimates of the absorption rates as in [13] (see [24]).

The equation (16) with the third term omitted coincides with that of ARS. The first term in (16) describes the oscillations of the sterile neutrinos, whereas the second is responsible for absorption and creation of them. The first term in (17) describes the transfer of leptonic number from sterile neutrinos to the active ones. It comes from the first diagram of Fig. 1 and incorporates the change in Γ due to the presence of the CP-breaking medium, coming from non-trivial $\tilde{\rho}_{NN}$. The trace of the second term of (16) is exactly equal to the trace (with a minus sign) of the first term in (17), ensuring the exact conservation of the lepton number. The second term in (17) describes the absorptive processes with active neutrinos and ensures that the system eventually thermalizes. The third term in (16) is a counterpart of this one and takes into account the transfer of the active lepton number into the sterile sector. It comes from the change in Γ due to the presence of asymmetries in active neutrinos.

Our equations have a rich CP-violating structure. In addition to the CP-violating phase in the K_R matrix, they contain 3 extra phases, δ_L and α_1, α_2 . Note that the Majorana phases β_1, β_2 do not appear in the limit of the small Majorana masses we are interested in. The equations (16,17) are to be solved with initial condition $\rho_{LL} - \rho_{LL}^{eq} = 0|_{t=0}$ (no lepton-flavour asymmetry at the beginning) and $\tilde{\rho}_{NN} = 0|_{t=0}$ (no sterile neutrinos in the initial state).

For sufficiently small Yukawa couplings, when all sterile neutrinos are still out of thermal equilibrium at the freeze-out of the sphaleron transitions, the solution can be found perturbatively. This is realized provided $f_i^2 < 2 \times 10^{-14}$ [13]. This regime requires the relatively light sterile neutrinos, as from the see-saw formula, and with the use of the absolute values of the active neutrino masses one gets $M_{2,3} < 12$ GeV. For these masses the processes with lepton number non-conservation induced by the Majorana neutrino masses are safely out of thermal equilibrium.

For the higher Majorana masses and larger Yukawa couplings the system can be solved numerically. It is clear, however, that if both N_C and N_B are already thermalized at the sphaleron freeze-out, no baryon asymmetry is produced, since the production of the N_D is highly suppressed and at that time no neutrino can carry the lepton asymmetry. This leads to the requirement that all the sterile neutrino masses must be smaller than the electroweak scale, exactly in accordance with our definition of the ν MSM.

Since the aim of our paper is not the analysis of the complete parameter space of the ν MSM but rather the existence proof of a possibility of simultaneous explanation of neutrino

oscillations, dark matter and baryon asymmetry, we will consider here the perturbative regime only (we checked, however, that numerical solution coincides with the perturbative one in the limit of its validity).

In the region of parameter space where the dark matter constraint is satisfied the generation of the lepton asymmetry occurs as follows. First, the sterile neutrinos are created in a CP-invariant state (the Jarlskog determinant is very small for K_R !) due to the second term in eq. (16). However, their density matrix is non-trivial and contains non-diagonal terms. Because of that and because of the presence of CP-violating phases in \tilde{K}_L , this leads to generation of CP-asymmetries in the active neutrino flavours, due to the first term in (17). Finally, this asymmetry is partially transferred into the total active (or sterile, with the opposite sign) asymmetry.

Now we are ready to solve eqns. (16) and (17) with the use of perturbation theory in f_i^2 . For this end they can be rewritten in the integral form, as

$$\tilde{\rho}_{NN} = -i \int_0^t dt' (\text{right-hand-side of eq.(16)}) , \quad (19)$$

$$\rho_{LL} = \rho_{LL}^{eq} - i \int_0^t dt' (\text{right-hand-side of eq.(17)}) \quad (20)$$

and then solved iteratively. We will present below the leading contributions only and refer for the details of calculation to [24].

Let us first discuss the asymmetries in active neutrino flavours induced by the non-thermal density matrix of sterile neutrinos. This effect comes from the first term in (17) and results in a second order effect in $\Delta_L = \rho_{LL} - \rho_{\bar{L}\bar{L}}$:

$$\frac{d\Delta_L}{dt} = \frac{\sin \phi}{8} T \text{diag} (F \rho_{NN} F^\dagger - F^* \rho_{\bar{N}\bar{N}} F^T) . \quad (21)$$

To leading order in perturbation theory the elements of ρ_{NN} come from eq. (16) by neglecting the third term on the right-hand side.

As in the ARS analysis, the asymmetries are produced most effectively at the typical temperature $T_L \simeq (\Delta M^2 M_0)^{1/3}$ and take constant values in the later evolution. Then, when $T_L \gg T_W \sim 100$ GeV, we obtain the following expression

$$(\Delta_L)_{\alpha\alpha}|_{T_W} = \frac{\pi^{\frac{3}{2}} \sin^2 \phi}{12 \cdot 3^{\frac{1}{3}} \Gamma(\frac{5}{6})} \sum_{I>J} \delta_{IJ}^\alpha \frac{M_0^{\frac{4}{3}}}{(\Delta M_{IJ}^2)^{\frac{2}{3}}} , \quad (22)$$

where the effective CP violation parameter is given by

$$\delta_{IJ}^\alpha = \text{Im} \left[F_{\alpha I} (F^\dagger F)_{IJ} F_{J\alpha}^\dagger \right] . \quad (23)$$

It can be seen that the total lepton number of active neutrinos is zero, i.e. $\delta_{IJ}^e + \delta_{IJ}^\mu + \delta_{IJ}^\tau = 0$. Since the mixing angles s_{R12} and s_{R13} are very small, as shown in (9), we can safely put them to zero in what follows, $s_{R12} = s_{R13} = 0$. In this case we obtain

$$(\Delta_L)_{\alpha\alpha}|_{TW} = a^\alpha \frac{\pi^{\frac{3}{2}} \sin^2 \phi}{48 \cdot 3^{\frac{1}{3}} \Gamma(\frac{5}{6})} \frac{f_2 f_3 (f_3^2 - f_2^2) M_0^{\frac{4}{3}}}{(\Delta M_{32}^2)^{\frac{2}{3}}}, \quad (24)$$

where a^α are given by

$$\begin{aligned} a^e &= -4s_{R23}c_{R23} [s_{L12}s_{L13}c_{L13} \sin(\delta_L + \alpha_2)], \\ a^\mu &= 4s_{R23}c_{R23} [s_{L12}s_{L13}c_{L13}s_{L23}^2 \sin(\delta_L + \alpha_2) - c_{L12}c_{L13}s_{L23}c_{L23} \sin \alpha_2], \\ a^\tau &= 4s_{R23}c_{R23} [s_{L12}s_{L13}c_{L13}c_{L23}^2 \sin(\delta_L + \alpha_2) + c_{L12}c_{L13}s_{L23}c_{L23} \sin \alpha_2]. \end{aligned} \quad (25)$$

It is clearly seen that the phases δ_L and α_2 in the mixing matrix \tilde{K}_L are crucial for the asymmetry in the active neutrino flavours⁴. Furthermore, it should be noted that the observed large mixing angles in the atmospheric and solar neutrino oscillations indicate that these parameters are not suppressed but can be $\mathcal{O}(1)$. As an illustration of numerics, we shall take the typical values of the Dirac Yukawa couplings to be

$$f_2^2 = \frac{m_{\text{sol}} M_2}{v^2}, \quad f_3^2 = \frac{m_{\text{atm}} M_3}{v^2}, \quad (26)$$

where $m_{\text{sol}} = \sqrt{\Delta m_{\text{sol}}^2} \simeq 9.1$ meV and $m_{\text{atm}} = \sqrt{\Delta m_{\text{atm}}^2} \simeq 51$ meV. Then the asymmetries are:

$$\begin{aligned} (\Delta_L)_{\alpha\alpha}|_{TW} &= a^\alpha \frac{\pi^{\frac{3}{2}} \sin^2 \phi}{48 \cdot 3^{\frac{1}{3}} \Gamma(\frac{5}{6})} \frac{m_{\text{sol}}^{\frac{1}{2}} m_{\text{atm}}^{\frac{3}{2}} M_2^{\frac{1}{2}} M_3^{\frac{3}{2}} M_0^{\frac{4}{3}}}{v^4 (\Delta M_{32}^2)^{\frac{2}{3}}}, \\ &\simeq 10^{-6} a^\alpha \left(\frac{10^{-6}}{\Delta M_{32}^2 / M_3^2} \right)^{\frac{2}{3}} \left(\frac{M_3}{10 \text{GeV}} \right)^{\frac{2}{3}}. \end{aligned} \quad (27)$$

This equation shows that sizable flavour asymmetries can be generated when the heavier “bright” and “clear” neutrinos are highly degenerate in mass. In fact, these flavour asymmetries are partially converted into the baryon asymmetry of the universe even if the total lepton asymmetry is zero, due to effects associated with the masses of the charged leptons [16] (for an exact computation of the mass corrections see [25]). However, the conversion

⁴ The CP-violating phases which can be found experimentally from active neutrino oscillations depend also on β_2 and thus cannot be used to fix uniquely δ_L and α_2 .

rate is suppressed by the factor $\simeq \frac{5}{13\pi^2} \frac{m^2}{T^2}$ and asymmetries of $(\Delta_L)_{\tau\tau} \sim \mathcal{O}(0.01)$ are required to account for the observed baryon asymmetry, if this mechanism is used.

However, this is not the end of the story. The above asymmetries in the active neutrino flavours trigger the generation of asymmetries in the sterile neutrino sector in a very efficient way. This phenomenon is due to the third term in (16). Indeed, the asymmetries $\Delta_N = \rho_{NN} - \rho_{\bar{N}\bar{N}}$ are generated as

$$\frac{d\Delta_N}{dt} = \frac{\sin \phi}{8} T (F^\dagger \Delta_L F). \quad (28)$$

Since Δ_L takes constant values after the production time, this equation can be easily solved. At the electroweak temperature the asymmetries in the sterile neutrinos are

$$(\Delta_N)_{II}|_{T_W} = \frac{\sin \phi}{8} \frac{M_0}{T_W} (F^\dagger \Delta_L|_{T_W} F)_{II}, \quad (29)$$

where $\Delta_L|_{T_W}$ is defined in (22). We should stress that these asymmetries are generated at higher order in f_i^2 compared with Δ_L (third versus second order); however the production processes continue below the temperature T_L and thus receive a huge enhancement factor⁵ of the order of M_0/T_W . Since the “dark” sterile neutrino only possesses small Yukawa couplings as shown in (7), the asymmetries are generated in $(\Delta_N)_{22}$ and $(\Delta_N)_{33}$.

Let us estimate the trace of Δ_N , which is crucial for the baryon asymmetry of the universe, in the specific parameter choice discussed above. We find from (27) that

$$\text{Tr} \Delta_N|_{T_W} = \frac{\pi^{\frac{3}{2}} \sin^3 \phi}{384 \cdot 3^{\frac{1}{3}} \Gamma(\frac{5}{6})} \sum_{\alpha, I} a^\alpha |F_{\alpha_I}|^2 \frac{m_{\text{sol}}^{\frac{1}{2}} m_{\text{atm}}^{\frac{3}{2}} M_2^{\frac{1}{2}} M_3^{\frac{3}{2}} M_0^{\frac{7}{3}}}{v^4 T_W (\Delta M_{32}^2)^{\frac{2}{3}}}. \quad (30)$$

Further simplifications come about as we notice that the successful asymmetry generation requires a significant mass degeneracy in M_2 and M_3 (see below). In order to explain the observed active neutrino mass pattern in the normal hierarchy, considered below for numerical estimates, one should have a certain hierarchy in Yukawa couplings $f_3^2 \gg f_2^2$ ($\gg f_1^2$), which gives at the leading order

$$\sum_{\alpha, I} a^\alpha |F_{\alpha_I}|^2 = f_3^2 \delta_{\text{CP}}, \quad (31)$$

⁵ Note that when $M_0/T_W \rightarrow \infty$ the perturbative approximation breaks down and one must solve the equations exactly. It is clear, however, what happens in this limit: the system will eventually equilibrate because of the second terms in (16,17) and all asymmetries will go away. However, before this time the effect is accumulating, as all sterile neutrinos are out of thermal equilibrium.

where

$$\begin{aligned} \delta_{\text{CP}} = 4s_{R23}c_{R23} & \left[s_{L12}s_{L13}c_{L13} \left((c_{L23}^4 + s_{L23}^4)c_{L13}^2 - s_{L13}^2 \right) \cdot \sin(\delta_L + \alpha_2) \right. \\ & \left. + c_{L12}c_{L13}^3 s_{L23}c_{L23} (c_{L23}^2 - s_{L23}^2) \cdot \sin \alpha_2 \right]. \end{aligned} \quad (32)$$

We can see that the CP violation parameter δ_{CP} can be $\mathcal{O}(1)$. Finally, the total asymmetry in the sterile neutrino sector is given by

$$\text{Tr}\Delta_N|_{T_W} = \delta_{\text{CP}} \frac{\pi^{\frac{3}{2}} \sin^3 \phi}{384 \cdot 3^{\frac{1}{3}} \Gamma(\frac{5}{6})} \frac{m_{\text{sol}}^{\frac{1}{2}} m_{\text{atm}}^{\frac{5}{2}} M_2^{\frac{1}{2}} M_3^{\frac{5}{2}} M_0^{\frac{7}{3}}}{v^6 T_W (\Delta M_{32}^2)^{\frac{2}{3}}}. \quad (33)$$

The total lepton number conservation tells us that $\text{Tr}\Delta_N|_{T_W} + \text{Tr}\Delta_L|_{T_W} = 0$ and, therefore, that the lepton asymmetry in the active neutrino sector is generated⁶. It is partially converted into the baryon asymmetry due to the rapid sphaleron conversion as $\Delta B = -\frac{28}{79} \text{Tr}\Delta_L|_{T_W} = +\frac{28}{79} \text{Tr}\Delta_N|_{T_W}$ [16, 17]. This completes the computation of the baryon asymmetry of the universe in the ν MSM satisfying the dark matter constraint. Accounting for the entropy factor at the electroweak temperature, the baryon to entropy ratio is obtained as

$$\frac{n_B}{s} = 7 \cdot 10^{-4} \text{Tr}\Delta_N|_{T_W} = 2 \cdot 10^{-10} \delta_{\text{CP}} \left(\frac{10^{-6}}{\Delta M_{32}^2/M_3^2} \right)^{\frac{2}{3}} \left(\frac{M_3}{10\text{GeV}} \right)^{\frac{5}{3}}. \quad (34)$$

This shows that the correct baryon asymmetry of the universe $\frac{n_B}{s} \simeq (8.8 - 9.8) \times 10^{-11}$ is generated when the heavier sterile neutrinos with the masses, say, 10 GeV are degenerate to one part in 10^6 . This looks like a strong fine tuning but may also indicate the intriguing mass relation $|M_3 - M_2| \sim M_1$. In this mass range the decays of heavier sterile neutrinos induce no significant entropy dilution. The leptogenesis temperature for degenerate neutrinos is rather low, $T_L \sim 10^4$ GeV. Note that with this low temperature the right-handed electron is always thermalized [26] so that the equilibrium formulas of [16, 17] can be used. Moreover, if the inverted hierarchy pattern is chosen for the active neutrino masses, even stronger degeneracy is needed, since an extra suppression of the baryon asymmetry is coming from the small difference between f_2 and f_3 , required in this case.

⁶ Equally, the generation of a net lepton number in the sector of active neutrinos can be understood as follows. The asymmetries (22) in active neutrino flavours are the subject of dissipation described by the second term in (17). Since Γ^L does depend on neutrino flavour, these asymmetries evolve differently, leading to the total lepton asymmetry.

Conclusions.— Let us summarize the obtained results. The ν MSM with three right-handed neutrinos with masses smaller than the electroweak scale is the simplest and the most economical extension of the Minimal Standard Model. It shares with the MSM its advantages (renormalisability and agreement with most particle physics experiments) and its fine-tuning problems (the gauge hierarchy problem, flavour problem, etc). However, unlike the MSM, the ν MSM can explain simultaneously three different phenomena, observed experimentally, namely neutrino oscillations, dark matter, and baryon asymmetry of the universe. The parameter-space of the model is rather constrained (the heavier neutrinos are required to be quite degenerate in mass, the Yukawa coupling f_1 and mixing angles θ_{R12} and θ_{R13} must be very small), which, however, makes this model experimentally testable. The analysis of possible experimental signatures of the ν MSM goes beyond the scope of the present letter⁷. We would just like to mention that on the astrophysical side, the best signal would be the γ radiation coming from the decay of dark neutrinos. On the particle physics side, as all the sterile neutrinos are relatively light, one could imagine that all parameters of the ν MSM, in particular the CP-violating phases, will be determined one day (this is clearly a very hard, if not impossible experimental challenge because of the smallness of the Yukawa couplings). One would then be able to test the baryogenesis formula and in particular its sign.

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⁷ It would be interesting to see, for example, whether the ν MSM can also explain in the suggested parameter range the LSND anomaly [27] and the pulsar kick velocities [28].

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