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Strong Sphalerons and Electroweak Baryogenesis

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Abstract

We analyze the spontaneous baryogenesis and charge transport mechanisms suggested by Cohen, Kaplan and Nelson for baryon asymmetry generation in extended versions of electroweak theory. We find that accounting for non-perturbative chirality-breaking transitions due to strong sphalerons reduces the baryonic asymmetry by the factor $(m_t/\pi T)^2$ or α_W, provided those processes are in thermal equilibrium.

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The popular mechanisms for electroweak baryogenesis discussed in the literature are the spontaneous baryogenesis mechanism and the charge transport mechanism, both suggested by Cohen, Kaplan and Nelson [1, 2, 3]. It has been advocated in [1, 2, 3] that these mechanisms produce an asymmetry parametrically larger that the mechanism associated with the decay of topologically non-trivial fluctuations of the gauge and Higgs fields during the electroweak phase transition, considered in refs. [4, 5](for earlier suggestions, see refs. [6, 7, 8]). In the latter case, the asymmetry produced is proportional to the square of the ratio of the top-quark mass to the temperature [4], while for the first two mechanisms it was argued that this suppression is absent [1, 2, 3]. The key observation made in [1, 2, 3] is that in the presence of a non-zero fermionic hypercharge density, ρ_Y, the equilibrium value of the baryonic charge, generally speaking, is not equal to zero:

$$\langle B\rangle = A\rho_Y, \tag{1}$$

where A is a number determined by the particle content of the theory and $A \neq 0$ in the limit $m_t \to 0$.

In this paper, we show that under some circumstances A may be zero in the massless quark limit. Moreover, we will argue that this is most probably the case for the two Higgs doublet theory, considered in [1, 2].

In this model, in addition to the KM CP-violation, there is an explicit CP-violation in the Higgs sector. The vevs of the Higgs fields at zero temperature have the form:

$$\langle\phi_1\rangle = (0, v_1 e^{i\theta}),\ \langle\phi_2\rangle = (0, v_2), \tag{2}$$

where θ is the relative phase, which cannot be rotated away by a gauge transformation. Due to explicit CP-violation in the Higgs potential this phase is space-dependent inside bubble walls forming at the electroweak phase transition (it would be time-dependent for a spinodial decomposition phase transition). Since quarks are getting their masses from interactions with the Higgs fields, the most important interaction is the top-quark coupling[3],

$$L_Y = f_t \bar{Q}_3 U_3 \phi_1. \tag{3}$$

Here Q_i are the left fermionic doublets, U_i, D_i are right quark fields, and i is the generation index. The bottom-quark couplings can be safely ignored provided the thickness of the domain wall l_0 or the duration of the spinodial decomposition phase transition τ_0 is smaller than the rate of chirality-changing transitions associated with the bottom Yukawa coupling; that is, if

$$l_0 < (\alpha_s f_b^2 T)^{-1} \sim \frac{10^4}{T}, \tag{4}$$

where T is the temperature. This is always the case for current estimates of the domain wall thickness, ranging from $40/T$ [9] to $1/T$ [10]. The same statement is obviously true for lighter quarks.

[3]We assume that the first Higgs field couples only to up-quarks, while the second is coupled only to down-quarks. This assumption is, however, inessential in what follows.

We will clarify our point first within the context of the spontaneous baryogenesis mechanism. Let us take for simplicity the spinodial decomposition phase transition. As in [1], we assume that the rate of chirality-flipping transitions for the top quark is larger than the typical inverse-time scale τ_0^{-1} of the variation of the phase θ during this transition. Now, following [1], we make a hypercharge rotation of the fermionic fields in such a way that time-dependence disappears from the Yukawa coupling (3). This change of variables introduces the following modification of the effective lagrangian:

$$L_{eff} = L + \dot{\theta} Y_F, \tag{5}$$

where Y_F is a fermionic hypercharge operator,

$$Y_F = \sum_{i=1}^{3} \left[\frac{1}{3} \bar{Q}_i \gamma_0 Q_i + \frac{4}{3} \bar{U}_i \gamma_0 U_i - \frac{2}{3} \bar{D}_i \gamma_0 D_i - \bar{L}_i \gamma_0 L_i - 2 \bar{E}_i \gamma_0 E_i \right]. \tag{6}$$

We want to check whether in thermal equilibrium with respect to fermionic number non-conserving processes in a theory with effective action (5) the baryonic number is nonzero. To determine the number A defined in (1), the following standard procedure must be used (see *e.g.* ref. [11]). One has to define the *complete* set of conserved charges X_i and construct the most general equilibrium density matrix with the help of those charges, introducing chemical potentials μ_i for each of them. The partition function is

$$Z = Tr \exp \left[-\frac{1}{T} (H - \dot{\theta} Y_F - \sum_i \mu_i X_i) \right], \tag{7}$$

while the density matrix is

$$\rho = \frac{1}{Z} \exp \left[-\frac{1}{T} (H - \dot{\theta} Y_F - \mu_i X_i) \right]. \tag{8}$$

Now, since X_i are conserved operators, their average must be equal to zero. This requirement fixes the chemical potentials μ_i through the system of equations

$$\frac{\partial}{\mu_i} Z = 0. \tag{9}$$

Then the baryonic number is just

$$\langle B \rangle = Tr[B\rho]. \tag{10}$$

Let us define the set of conserved numbers. It is sufficient to consider only SU(3) gauge singlets. We start from purely fermionic currents. In a model with massless neutrinos (no right-handed neutrinos!), the total number of different fermionic currents is 15, the number of fermionic degrees of freedom. They are: 3 left quark currents, $\bar{Q}_i \gamma_\mu Q_i$, 6 right quark currents, $\bar{U}_i \gamma_\mu U_i$ and $\bar{D}_i \gamma_\mu D_i$, 3 left leptonic currents, $\bar{L}_i \gamma_\mu L_i$ (L_i is the left leptonic doublet), and 3 right leptonic currents, $\bar{E}_i \gamma_\mu E_i$. Not all of these currents are conserved. One has to take into account the following processes:

(i) Perturbative chirality-changing transitions due to Yukawa interactions (3). The rate of these processes is estimated to be $\tau_Y \sim 30/T$ [3]. This decreases the number of conserved currents by one: instead of the pair $\bar{Q}_3\gamma_\mu Q_3$ and $\bar{U}_3\gamma_\mu U_3$, we have a conserved linear combination $\bar{Q}_3\gamma_\mu Q_3 + \bar{U}_3\gamma_\mu U_3$.

(ii) Non-perturbative chirality-breaking transitions due to strong interactions. It is well known that the quark axial vector current has an anomaly and therefore is not conserved. The rate of chirality non-conservation at high temperatures Γ_{strong} is connected with the rate of topological transitions in QCD ("strong" sphalerons, [12]),

$$\frac{\partial Q_5}{\partial t} = -\frac{12 \cdot 6}{T^3}\Gamma_{strong} Q_5, \tag{11}$$

where Q_5 is the axial charge. The factor of 12 comes from the total number of quark chirality states, the factor of 6 from the relation between the asymmetry in quark number density and the chemical potential,

$$\int \frac{d^3k}{(2\pi)^3}[n_F(\epsilon,\mu) - n_F(\epsilon,-\mu)] = \frac{\mu T^2}{6}(1 - \frac{3m^2}{2\pi^2 T^2}), \tag{12}$$

where $n_F(\epsilon,\mu) = 1/(\exp(\frac{\epsilon-\mu}{T}) + 1)$ is the Fermi distribution, $\epsilon^2 = k^2 + m^2$. The rate of the strong sphaleron transitions is related to the rate of weak sphaleron transitions through

$$\Gamma_{strong} = \frac{8}{3}(\frac{\alpha_s}{\alpha_W})^4 \Gamma_{sph} = \frac{8}{3}\kappa(\alpha_s T)^4, \tag{13}$$

where κ is the usual parameter characterizing the strength of the electroweak sphaleron transitions in the unbroken phase. The characteristic time of these transitions is therefore

$$\tau_{strong} = \frac{1}{192\kappa\alpha_s^4 T}. \tag{14}$$

Using the conservative lower bound on κ derived in lattice simulations [13], $\kappa > 0.5$ (see also discussion in ref. [14]), we obtain a conservative bound $\tau_{strong} < 100/T$. With $\kappa \sim 20$, so estimated by a different method in ref. [15], we obtain $\tau_{strong} \sim 2.5/T$. Therefore, the rate of strong sphaleron transitions is comparable to or even larger than the rate of chirality-flip transitions through the Yukawa coupling of the top quark. Hence, these processes must be taken into account. This decreases the number of conserved currents by one[4].

(iii) Anomalous fermion number non-conservation must be taken into account [17]. It decreases the number of conserved currents by one.

[4]Note that influence of strong sphalerons on a different mechanism for electroweak baryogenesis has been discussed also in ref. [16].

Therefore, a complete set of conserved anomaly-free fermionic currents can be represented as

$$\bar{Q}_i\gamma_\mu Q_i + R - L_L,\ \ i = 1, 2, \tag{15}$$
$$\bar{Q}_3\gamma_\mu Q_3 + \bar{U}_3\gamma_\mu U_3 + R/2 - L_L, \tag{16}$$
$$\bar{L}_i\gamma_\mu L_i - L_L/3, i = 1, 2, \tag{17}$$
$$\bar{E}_i\gamma_\mu E_i, i = 1, 2, 3, \tag{18}$$
$$\bar{D}_i\gamma_\mu D_i - R/2, i = 1, 2, 3, \tag{19}$$
$$\bar{U}_1\gamma_\mu U_1 - \bar{U}_2\gamma_\mu U_2, \tag{20}$$

where

$$L_L = \sum_{i=1}^{3} \bar{L}_i\gamma_\mu L_i,\ \ R = \sum_{i=1}^{2} \bar{U}_i\gamma_\mu U_i. \tag{21}$$

In addition to the quantum numbers associated with these currents, there exists a conserved operator containing scalar fields. This is the familiar hypercharge

$$Y = Y_F + Y_s, \tag{22}$$

where

$$Y_s = -i[\phi_1^\dagger \mathcal{D}_0\phi_1 - (\mathcal{D}_0\phi_1)^\dagger\phi_1] - i[1 \to 2] \tag{23}$$

is the scalar field contribution. Finally, one should add the third component of the weak isospin operator T_3. As in [1, 3], we assume that the classical motion of the scalar field does not introduce any non-zero hypercharge or T_3 density. Therefore the hypercharge density is equal to zero for a uniform spinodial decomposition phase transition.

Now one can check that, in the massless quark approximation, eqs. (7, 8, 9, 15 - 20) imply that the equilibrium value of the baryonic charge is zero, $\langle B\rangle = 0$. In order to get this result, we used, following ref. [18], the particle spectrum of the *unbroken* phase rather than the broken one.

The reader may wonder why we did not use the electric charge operator and the physical particle spectrum of the broken phase instead of Y and T_3 [5]. The reason is that the use of the *physical* spectrum in such a calculation would correspond to a computation of the free energy of the system in the broken phase in the one-loop approximation in a *unitary gauge*. It is known [19, 20, 21], that perturbation theory in a unitary gauge is a very delicate thing (*e.g.*, it does not converge [21]). In particular, it gives an incorrect result for the critical temperature of the phase transitions in gauge theories.

It is clear that for sufficiently small vacuum expectation values of the Higgs field, all effects associated with spontaneous symmetry breaking are just the mass corrections which can be neglected in the high-temperature approximation in any *renormalizable gauge*. In the worst case, the expansion parameter may be $(v/T)^2$. In our case, the vacuum expectation value of the Higgs field v is bounded from above by the requirement

[5] Actually, the zero result is reproduced also with Q instead of T_3 and Y and the particle spectrum of the broken phase.

that the sphaleron processes are sufficiently fast, so that $v < g_W T$. Therefore, the use of the particle spectrum of the unbroken phase is perfectly justified.

The zero result is also reproduced when fermionic number non-conservation is out of thermal equilibrium. Then one can write an equation for the baryonic number evolution [18] (see also refs. [1, 2, 3])

$$\frac{\partial B}{\partial t} = -N^2 \frac{\Gamma_{sph}}{T} \frac{\partial F}{\partial B} = -9 \frac{\Gamma_{sph}}{T} \mu_B, \tag{24}$$

where $F(B)$ is the free energy of the system characterized by zero values of all conserved charges but non-zero baryonic density B, Γ_{sph} is the rate of sphaleron transitions, and $N = 3$ is the number of fermionic generations. In our case,

$$\frac{\partial F}{\partial B} = \mu_B \sim B, \tag{25}$$

so that B stays zero all the time.

We stress that the inclusion of strong sphalerons was essential for this result. Strong sphalerons have the physical effect of mantaining the same chemical potential for left- and right-handed baryonic numbers. If instead one neglects these processes, then the system of charges (20) has to be supplied with an extra charge, say R, while the density matrix will contain an additional chemical potential. One can check that if one chooses this set of conserved charges, the result is indeed non-zero. Which solution has to be used? The answer depends on the ratio of the typical time scales. If $\tau_0 \ll \tau_{strong}$, then strong sphalerons are irrelevant, and the asymmetry indeed does not contain Yukawa coupling suppression provided that τ_Y is smaller than the typical time scale τ_0 of the Higgs phase change in the spontaneous baryogenesis mechanism. If, on the contrary, $\tau_0 > \tau_{strong}$ or $\tau_0 \sim \tau_{strong}$, then the asymmetry vanishes in the massless approximation.

In the original calculation of Cohen, Kaplan and Nelson [1, 2] a non-zero result for the baryonic asymmetry was obtained, since strong sphaleron transitions have been neglected and only $B - L$, Q, and $B_3 - \frac{1}{2}(B_1 + B_2)$ have been included as conserved charges. However, in presence of strong sphalerons transitions, their solution corresponds to non-zero averages for some of the approximately *conserved* charges defined in eqs. (15 - 20).

Is the conclusion that $\langle B \rangle = 0$ fatal for the spontaneous baryogenesis mechanism? In fact, this is not the case when quark mass corrections are included. Mass corrections can be taken into account in perfect analogy with the case of the leptonic asymmetries discussed in refs. [22, 18] (for a later discussion see ref. [23]). To make the computation simpler (and to make clearer why we previously obtained zero), we observe that in fact the number of independent chemical potentials (14) can be reduced to a smaller number (6) with the use of flavour symmetry. Namely, our lagrangian has quite a large global symmetry group $SU(2)_Q$ x $SU(3)_D$ x $SU(2)_U$ x $SU(3)_L$ x $SU(3)_E$, where the group labels denote the fields on which the corresponding group is acting. The conserved charges invariant under this symmetry are

$$A_1 = \sum_{i=1}^{2} \bar{Q}_i \gamma_\mu Q_i + 2R - 2L_L, \tag{26}$$

$$A_2 = \bar{Q_3}\gamma_\mu Q_3 + \bar{U_3}\gamma_\mu U_3 + R/2 - L_L, \tag{27}$$

$$A_3 = \sum_{i=1}^{3} \bar{D}_i\gamma_\mu D_i - 3R/2, \tag{28}$$

$$A_4 = \sum_{i=1}^{3} \bar{E}_i\gamma_\mu E_i, \tag{29}$$

hypercharge Y and weak isospin T_3. We denote the corresponding chemical potentials by $\mu_i,\ i = 1,\ ...\ , 4$, μ_Y and μ_T. They are to be found from the equations

$$\langle A_i \rangle = \langle Y \rangle = \langle T_3 \rangle = 0. \tag{30}$$

Let us first solve this system neglecting the mass of the top quark. In order to show that the baryonic density is zero, one can consider only the total baryonic density and the left-handed leptonic density. The total baryonic density is

$$\langle B \rangle = 4[(\dot{\theta} + \mu_Y) + 2\mu_1 + \mu_2]\frac{T^2}{6}, \tag{31}$$

while the total left leptonic density is

$$\langle L_L \rangle = -6[(\dot{\theta} + \mu_Y) + 2\mu_1 + \mu_2]\frac{T^2}{6}. \tag{32}$$

Then from $\langle B - L_L \rangle = 0$, it follows $\langle B \rangle = 0$. For future reference, we give here the complete solution of eq. (30) in the massless approximation:

$$\mu_1 = -\frac{4}{21}(\dot{\theta} + \mu_Y),\ \mu_2 = -\frac{13}{21}(\dot{\theta} + \mu_Y),\ \mu_3 = \frac{11}{21}(\dot{\theta} + \mu_Y), \tag{33}$$

$$\mu_4 = 2(\dot{\theta} + \mu_Y),\ \mu_T = 0,\ (\dot{\theta} + \mu_Y) = \dot{\theta}\frac{14n_s}{9 + 14n_s}, \tag{34}$$

where n_s is the number of scalar doublets.

The exact proportionality between baryonic number and left-handed leptonic number is, however, an artifact of the massless approximation. Using eq. (12), the baryonic density becomes

$$\langle B \rangle = -\frac{3m^2}{20\pi^2 T^2}\rho_Y = -\frac{9n_s}{10(9 + 14n_s)\pi^2}m^2\dot{\theta}, \tag{35}$$

where ρ_Y is the fermionic hypercharge density. This is our final result for the equilibrium density of the baryonic number in the background of a scalar field with slowly changing phase.

If strong sphalerons were not in equilibrium, we should add a new approximately conserved charge, for instance $A_5 \equiv R$, to our previously defined set of charges, see eqs.(26)–(29). Following the same procedure used to obtain eq.(35), we would find:

$$\langle B \rangle = \frac{n_s}{6 + 11n_s}T^2\dot{\theta}. \tag{36}$$

Therefore the presence of strong sphalerons leads to a suppression factor $\frac{126}{185}(\frac{m}{\pi T})^2$, where we have taken $n_s = 2$; this corresponds numerically to a suppression of about $3 \cdot 10^{-2}$ for $f_t \sim 1$, $v(T) \sim g_W T$.

The approximation that anomalous baryon number violation is in equilibrium is unlikely to be correct. Thus, following ref. [1], we consider the evolution equation for the baryon asymmetry density, eq. (24). We can compute μ_B using the same procedure followed above, adding B to the set of conserved charges and imposing, besides eq. (30), the further constraint $\langle B \rangle = 0$. We find, if strong sphalerons are in equilibrium,

$$\mu_B = \frac{9n_s}{4(9+14n_s)} \frac{m^2}{\pi^2 T^2} \dot{\theta}, \tag{37}$$

and, if strong sphalerons are not in equilibrium,

$$\mu_B = -\frac{2n_s}{3(1+2n_s)} \dot{\theta}. \tag{38}$$

The mass suppression present when strong sphalerons are in equilibrium amounts to a factor $\frac{135}{296}(\frac{m}{\pi T})^2 \sim 2 \cdot 10^{-2}$.

We can now consider the case of baryogenesis with the charge transport mechanism. As shown in ref. [2], the CP-violating interactions of the fermions with the moving thin bubble wall lead to a non-vanishing flux of particles carrying some quantum number X. If, as in the original work of ref. [2], we identify X with the hypercharge, then it is easy to realize that $\langle B \rangle = 0$, when strong sphalerons are in equilibrium and all fermions are massless. The calculation is analogous to the one for spontaneous baryogenesis, with the only difference that, instead of the source $\dot{\theta}$ we have to consider a non-vanishing Y density. However, as shown by Khlebnikov [24], the Debye screening prevents the transport of the gauged charge Y over distances larger than about $2/T$.

Therefore the recipe proposed in ref. [25] is to identify as the transported charges X only global charges which are orthogonal to gauge charges. We then define

$$B' = B + \alpha_B Y, \qquad A'_i = A_i + \alpha_i Y, \qquad i = 1, ..., 4, \tag{39}$$

with α_B and α_i chosen such that B' and A'_i are orthogonal to Y, *i.e.*, in a massless quark approximation:

$$-\alpha_B = -\frac{1}{8}\alpha_1 = -\frac{1}{4}\alpha_2 = \frac{2}{9}\alpha_3 = \frac{2}{3}\alpha_4 = \frac{1}{10+n_s}. \tag{40}$$

By introducing chemical potentials for the charges (26)–(29), Y, and B, we can write the asymmetry density for each species of particles as:

$$\rho = \frac{T^2}{6} \left(q_Y \mu_Y + q_B \mu_B + \sum_i q_{A_i} \mu_i \right). \tag{41}$$

The chemical potentials μ_Y, μ_B, and μ_i are then computed from the equations

$$\langle Y \rangle = 0, \quad \frac{\langle B \rangle}{\alpha_B} = \frac{\langle A_i \rangle}{\alpha_i} = n_x \neq 0, \tag{42}$$

which correspond to a situation in which the gauged charge Y is screened, but the Y components of the global charges can carry a non-vanishing density n_x. The solution of eq. (42) yields $\mu_B = 0$ and therefore anomalous baryon number violating interactions are unable to generate any baryon asymmetry. One can also check that $\mu_B \neq 0$, if strong sphalerons are not in equilibrium.

The cancellation in the case of charge transport baryogenesis may seem more severe than in the case of spontaneous baryogenesis since the relevant processes occur in the *unbroken phase*, where the Higgs field condensate is absent. There are two types of corrections which save the situation. Just as in the leptonic case considered in [18], Yukawa and gauge radiative corrections are important. The equation for the asymmetry number density of each chiral fermion in the unbroken phase, including radiative corrections, can still be written as in eq.(12), with the replacement:

$$m^2 \to \left[C_s g_s^2 + (C_W + \frac{Y^2}{4} \sin^2\theta_W) g_W^2 + \frac{f^2}{2} \right] \frac{T^2}{4}, \tag{43}$$

where C_s is $\frac{4}{3}$ for quarks and 0 for leptons, C_W is $\frac{3}{4}$ for weak doublets and 0 for singlets, Y is the hypercharge quantum number, and f is the Yukawa coupling, taken to be non-vanishing only for the top quark. The solution of eq. (42) now gives:

$$\mu_B = \frac{9}{64\pi^2(9+14n_s)} (3f_t^2 + 2g_W^2 \sin^2\theta_W) \frac{n_x}{T^2}, \tag{44}$$

which, as noticed above, vanishes in the limit of negligible radiative corrections. The result of eq. (42) would correspond to a reduction of the baryonic asymmetry in comparison with [2, 3] by the factor $\sim \frac{27(1+2n_s)}{128\pi^2(9+14n_s)} f_t^2 \sim 3 \cdot 10^{-3}$, if other effects are absent. These more important effects are associated with electroweak corrections which break the degeneracy between left and right top quarks in the unbroken phase resulting in a net baryonic current J_B (see refs. [26, 14]) related to the hypercharge current J_Y through

$$J_B = \frac{2}{3} \frac{p_L - p_R}{p_L + p_R} J_Y, \tag{45}$$

where p_L and p_R are the momenta of incident left and right quarks corresponding to the same energy. Now, p_L and p_R are different since left quarks interact with SU(2) gauge fields while right quarks do not. With the use of eq. (43) we get a larger baryonic chemical potential,

$$\mu_B \simeq \frac{\alpha_W \pi}{16} \frac{n_x}{\bar{p}^2}, \tag{46}$$

where $\bar{p} > m_t$ is a typical transverse momentum of the quarks contributing to charge transport. So the final estimate of the suppression of the asymmetry in this case is a factor of about 10^{-2}. Therefore, the charge transport mechanism (as well as the "topological" mechanism [4, 5]) can still explain the baryonic asymmetry since, according to the estimate of ref. [2], n_B/s can be as large as 10^{-7}. With strong sphalerons taken into account, this number should be converted to 10^{-9}, which is still larger than observation.

To conclude, we have shown that the baryonic asymmetry in the two Higgs doublet model produced by any of the mechanisms considered in [1, 2, 3] contains a parametric suppression associated with the top Yukawa coupling or electroweak coupling due to the existence of strong sphalerons. Strong sphalerons have not been taken into account in a number of papers on electroweak baryogenesis and may change their conclusions.

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