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Baryon and lepton number violation rates across the electroweak crossover

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Abstract

We point out that the results of many baryogenesis scenarios operating at or below the TeV scale are rather sensitive to the rate of anomalous fermion number violation across the electroweak crossover. Assuming the validity of the Standard Model of electroweak interactions, and making use of previous theoretical work at small Higgs masses, we estimate this rate for experimentally allowed values of the Higgs mass ($m_H = 100...300$ GeV). We also elaborate on how the rate makes its appearance in (leptogenesis based) baryogenesis computations.

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1. Introduction

The scenario of thermal leptogenesis [1] relies on anomalous baryon + lepton number violation [2], which is very rapid at temperatures above the electroweak scale [3], to convert the original lepton asymmetry into an observable baryon asymmetry. Usually the temperature range where the lepton asymmetry generating source terms are active, is much above the electroweak scale. In this case the anomalous processes have ample time to operate, and their precise rate is not important. In fact, the conversion factors are simple analytic functions [6, 7], for which various limiting values were derived already long ago [4, 5].

However, baryon asymmetry generation may also be a low temperature phenomenon, in which CP-breaking source terms are active down to the electroweak scale; for recent examples, see Refs. [8]–[16]. In this case the temperature dependence of the anomalous rate does play an important role. This is even more so for the large (Standard Model like) Higgs masses that are currently allowed by experiment [17]: the electroweak symmetry gets "broken" through an analytic crossover rather than a sharp phase transition [18, 19], whereby the anomalous rate also decreases only gradually.

To allow for a precise study of generic scenarios of this type, it is the purpose of this note to collect together all the relevant rate equations, such that systematic errors from this part of the computation can be brought under reasonable control. We reiterate the baryon and lepton violation rate equations in Sec. 2, estimate the anomalous "sphaleron" rate as a function of the Higgs mass and temperature in Sec. 3, and summarise in Sec. 4.

2. Baryon and lepton number violation rates

To zeroth order in neutrino Yukawa couplings, the Standard Model allows to define three global conserved charges:

$$X_i \equiv \frac{B}{n_{\rm G}} - L_i \;, \tag{2.1}$$

where B is the baryon number, L_i the lepton number of the $i^{\rm th}$ generation, and $n_{\rm G}$ denotes the number of generations. Given some values of X_i, a system in full thermodynamic equilibrium at a temperature T and with a Higgs expectation value $v_{\rm min}$ (suitably renormalised and in, say, the Landau gauge), contains then the baryon and lepton numbers [6]

$$B \equiv B_{\rm eq} \equiv \chi\Big(\frac{v_{\rm min}}{T}\Big) \sum_{i=1}^{n_{\rm G}} X_i \;, \quad L_i \equiv L_{i,\rm eq} \equiv \frac{B_{\rm eq}}{n_{\rm G}} - X_i \;, \tag{2.2}$$

$$\chi(x) = \frac{4[5 + 12n_{\rm G} + 4n_{\rm G}^2 + (9 + 6n_{\rm G})x^2]}{65 + 136n_{\rm G} + 44n_{\rm G}^2 + (117 + 72n_{\rm G})x^2} \;. \tag{2.3}$$

These relations hold up to corrections of order $\mathcal{O}((X_i/VT^3)^2)$ from the expansion in small chemical potentials, $\mathcal{O}((hv_{\rm min}/\pi T)^2)$ from the high-temperature expansion, as well as $\mathcal{O}(h^2)$ from the weak-coupling expansion, where h is a generic coupling constant.

If we deviate slightly from thermodynamic equilibrium, the baryon and lepton numbers evolve with time. A non-trivial derivation [4] yields the equations [4, 6, 20]

$$\dot{B}(t) = -n_{\rm G}^2\,\rho\Big(\frac{v_{\rm min}}{T}\Big)\frac{\Gamma_{\rm diff}(T)}{T^3}[B(t)-B_{\rm eq}]\;, \quad \dot{L}_i(t) = \frac{\dot{B}(t)}{n_{\rm G}}\;, \tag{2.4}$$

$$\rho(x) = \frac{3[65+136n_{\rm G}+44n_{\rm G}^2+(117+72n_{\rm G})x^2]}{2n_{\rm G}[30+62n_{\rm G}+20n_{\rm G}^2+(54+33n_{\rm G})x^2]}\;. \tag{2.5}$$

In the literature the factor $n_{\rm G}^2\,\rho(v_{\rm min}/T)$ is often replaced with the constant $13n_{\rm G}/4$, which indeed is numerically an excellent approximation. The term $\Gamma_{\rm diff}(T)$ is called the Chern-Simons diffusion rate, or (twice) the sphaleron rate, and is defined by

$$\Gamma_{\rm diff}(T) \equiv \lim_{V,t\to\infty}\frac{\langle Q^2(t)\rangle_T}{Vt}\;, \tag{2.6}$$

where $Q(t) \equiv \int_0^t {\rm d}t' \int_V {\rm d}^3\mathbf{x}'\, q(x') \equiv N_{\rm CS}(t) - N_{\rm CS}(0)$ is the topological charge, and $N_{\rm CS}(t)$ is the Chern-Simons number. The expectation value in Eq. (2.6) is to be evaluated in a theory without fermions [4]. Corrections to Eq. (2.5) are of the same type as those to Eq. (2.3).

For practical purposes, it is useful to eliminate the conserved charges X_i from the equations, and write just a coupled system for $B(t), L_i(t)$. Defining

$$\gamma \equiv n_{\rm G}^2\,\rho\Big(\frac{v_{\rm min}}{T}\Big)\Big[1-\chi\Big(\frac{v_{\rm min}}{T}\Big)\Big]\frac{\Gamma_{\rm diff}(T)}{T^3}\;, \quad \eta \equiv \frac{\chi(v_{\rm min}/T)}{1-\chi(v_{\rm min}/T)}\;, \tag{2.7}$$

and introducing sources $f_i(t)$ for the lepton numbers, we can convert Eqs. (2.2), (2.4) to

$$\dot{B}(t) = -\gamma(t)\Big[B(t)+\eta(t)\sum_{i=1}^{n_{\rm G}} L_i(t)\Big]\;, \tag{2.8}$$

$$\dot{L}_i(t) = -\frac{\gamma(t)}{n_{\rm G}}\Big[B(t)+\eta(t)\sum_{i=1}^{n_{\rm G}} L_i(t)\Big] + f_i(t)\;. \tag{2.9}$$

These equations can easily be integrated, if we know the temperature dependence of $v_{\rm min}/T$ and the time evolution of T. The solution is particularly simple if we make use of the fact that η is, to a reasonable approximation, a constant, $\eta(t) \simeq 0.52 \pm 0.03$. In this case linear combinations of Eqs. (2.8), (2.9) yield independent first order equations for $B(t) - L(t)$ and $B(t) + \eta L(t)$, where $L(t) \equiv \sum_{i=1}^{n_{\rm G}} L_i(t)$. Denoting $\omega(t';t) \equiv \exp[-(1+\eta)\int_{t'}^t {\rm d}t''\,\gamma(t'')]$ and $f(t) \equiv \sum_{i=1}^{n_{\rm G}} f_i(t)$, the solution reads

$$B(t) = \frac{1}{1+\eta}\bigg\{\Big[B(t_0)+\eta L(t_0)\Big]\omega(t_0;t)+\eta\Big[B(t_0)-L(t_0)\Big]-\eta\int_{t_0}^t {\rm d}t'\, f(t')\Big[1-\omega(t';t)\Big]\bigg\}\;. \tag{2.10}$$

A further simplification follows by noting that $\omega(t';t)$ varies very rapidly with the time t' around a certain $t' \sim t_*$, from zero at $t' < t_*$ to unity at $t' > t_*$, while $f(t')$ is a slowly varying function of time. Assuming furthermore that $B(t_0) = L(t_0) = 0$, we obtain

$$B(t) \approx \frac{-\eta}{1+\eta}\int_{t_0}^{t_*}{\rm d}t'\, f(t') = -\chi\int_{t_0}^{t_*}{\rm d}t'\, f(t')\;, \tag{2.11}$$

where the "decoupling time" can be defined as $t_* \equiv t_0 + \int_{t_0}^{t} \mathrm{d}t' \, [1-\omega(t';t)]$. Thus, if $f(t') \neq 0$ around the time t_*, the baryon asymmetry generated depends sensitively on t_*, and it is important to know the function $\omega(t';t)$, determined by $\gamma(t'')$, quite precisely.

The equations that we have written were formally derived in Minkowski spacetime. They are easily generalised to an expanding background, however: their form remains invariant if we simply replace the total (comoving) baryon and lepton numbers B, L_i by number densities over the entropy density $s(T)$: $B \to n_B \equiv B/[a^3 s(T)]$, $L \to n_L \equiv L/[a^3 s(T)]$, where a^3 is a comoving volume element. Furthermore, it is often convenient to replace time derivatives with temperature derivatives via

$$\frac{\mathrm{d}}{\mathrm{d}t} = -\frac{\sqrt{24\pi}}{m_{\mathrm{Pl}}} \frac{\sqrt{e(T)}}{\mathrm{d}[\ln s(T)]/\mathrm{d}T} \frac{\mathrm{d}}{\mathrm{d}T} \;, \tag{2.12}$$

where $e(T)$ is the energy density; we assumed the Universe to be flat ($k=0$); and we ignored the cosmological constant. Both $s(T) = p'(T)$ and $e(T) = Ts(T) - p(T)$ follow from the thermodynamic pressure $p(T)$ which is known to high accuracy [21], but can in practice be reasonably well approximated with the ideal gas formula $p(T) \approx g_* \pi^2 T^4/90$, with $g_* \simeq 106.75$.

In many baryogenesis scenarios, the source terms $f_i(t)$ in Eq. (2.9) are approximated by Boltzmann-type equations for the various left-handed and right-handed neutrino number densities. Collecting the number densities to the matrices $\mathbf{n}_L, \mathbf{n}_R$, respectively, with the normalization $\mathrm{Tr}\,[\mathbf{n}_L] = n_L$, a concrete realization of Eqs. (2.8), (2.9) could then read

$$\dot{n}_B(t) = -\gamma(t) \Big\{ n_B(t) + \eta(t) \mathrm{Tr}\,[\mathbf{n}_L(t)] \Big\} \;, \tag{2.13}$$

$$\dot{\mathbf{n}}_L(t) = -\frac{\gamma(t)}{n_{\mathrm{G}}} \Big\{ n_B(t) + \eta(t) \mathrm{Tr}\,[\mathbf{n}_L(t)] \Big\} \mathbb{1} + \mathcal{F}_L[\mathbf{n}_R, \mathbf{n}_L, t] \;, \tag{2.14}$$

$$\dot{\mathbf{n}}_R(t) = \mathcal{F}_R[\mathbf{n}_R, \mathbf{n}_L, t] \;, \tag{2.15}$$

with functionals $\mathcal{F}_L, \mathcal{F}_R$ that need to be determined for the specific model in question.

3. Chern-Simons diffusion rate

An essential role in the rate equations (2.13)–(2.15) is played by the function $\gamma(t)$ whose time dependence is, via Eq. (2.7), dominantly determined by $\Gamma_{\mathrm{diff}}(T)$, defined in Eq. (2.6). We now collect together the current knowledge concerning $\Gamma_{\mathrm{diff}}(T)$ in the Standard Model.

At high temperatures (in the "symmetric phase") the Chern-Simons diffusion rate is purely non-perturbative, and needs to be evaluated numerically. So-called classical real-time simulations [22] produce $\Gamma_{\mathrm{diff}}(T) = (25.4 \pm 2.0)\alpha_w^5 T^4$ [23], where the number 25.4 is in fact the value of a function containing terms like $\ln(1/\alpha_w)$ [24], at the physical α_w.

At lower temperatures, the rate is traditionally written in the form [25]

$$\Gamma_{\mathrm{diff}}(T) = 4T^4 \frac{\omega_-}{g v_{\mathrm{min}}} \left(\frac{\alpha_w}{4\pi}\right)^4 \left(\frac{4\pi v_{\mathrm{min}}}{gT}\right)^7 \mathcal{N}_{\mathrm{tr}}(\mathcal{N}\mathcal{V})_{\mathrm{rot}}\; \kappa\; \exp\!\left(-\frac{E_{\mathrm{sph}}}{T}\right) . \tag{3.1}$$

Here g is the SU(2) gauge coupling, $\alpha_w = g^2/4\pi$; ω_- might generically be called the dynamical prefactor, and is related to the absolute value of the negative eigenvalue of the fluctuation operator around the sphaleron solution; $\mathcal{N}_{\rm tr}(\mathcal{N}\mathcal{V})_{\rm rot}$ are normalisation factors related to the zero-modes of the fluctuation operator; κ contains the contributions of the positive modes; and $E_{\rm sph}$ is the energy of the saddle-point configuration (the sphaleron) [26].

Most of the factors appearing in Eq. (3.1) have been evaluated long ago. In particular, $E_{\rm sph}$ can be found in Ref. [27] for the bosonic sector of the SU(2)×U(1) Standard Model, while fermionic effects were clarified in Ref. [28]. The zero-mode factors and (the naive version of) ω_- were evaluated in Refs. [29, 30], while κ was determined numerically in Refs. [31, 32].

Unfortunately, it as not *a priori* clear how accurate the corresponding results are. Indeed, Eq. (3.1) has an inherently 1-loop structure, but it is known from studies of the electroweak phase transition that 2-loop effects, parametrically suppressed only by the infrared-sensitive expansion parameter $\mathcal{O}(hT/\pi v_{\rm min})$, are large in practice [33, 34]. Moreover, the naive definition of ω_- through the negative eigenvalue does not appear to be correct [35].

A reliable determination of $\Gamma_{\rm diff}$ can again be obtained by numerical methods, employing real-time classical simulations. Of course classical simulations are not exact either, but they do contain the correct infrared physics, and should thus only suffer from infrared-safe errors of the type mentioned below Eq. (2.3). Thus, classical simulations allow in principle to incorporate the dominant higher order effects, as well as a correct treatment of ω_-.

In the "broken symmetry phase", large-scale classical simulations have been carried out in Ref. [36]. Unfortunately, they only extend up to Higgs masses around $m_H = 50$ GeV, and were only carried out for certain temperatures (there are some results also at larger Higgs masses but with less systematics [37]). While we have not carried out any new simulations, we do make use of the observation [36] that the discrepancy between the numerical results, and a certain analytical recipe, of the type reiterated below, appears to be independent of the Higgs mass. We thus extend the analytical recipe to large Higgs masses, and add to these results a (small) constant correction factor, extracted from Ref. [36]. In practice, the steps are as follows:

(i) We employ the (resummed) 2-loop finite-temperature effective potential $V(v)$ in Landau gauge, as it is specified in Ref. [38]. Effects of the hypercharge group U(1) need to be taken into account only at 1-loop level, as demonstrated in Ref. [19]. The potential is parametrised by the zero-temperature physical quantities m_W, m_Z, $m_{\rm top}$, m_H, $\alpha_s(m_Z)$, G_F; their values (apart from m_H) are taken from Ref. [17].

We remark that although this is formally a higher order effect, the effective potential does depend on the scale parameter $\bar\mu$ of the $\overline{\rm MS}$ scheme. One may thus consider various choices of $\bar\mu$. We follow a strategy similar to Ref. [34] and write $V(v)-V(0) = \int_0^v {\rm d}v'\, \partial V(v')/\partial v'|_{\bar\mu=\bar\mu(v')}$, where the scale is chosen as $\bar\mu(v) \equiv \Delta\sqrt{3\lambda_{\rm eff} v^2}$, where $\lambda_{\rm eff}$ is the scalar coupling of the dimensionally reduced theory [38] and Δ is a constant. We consider $\Delta \equiv 1.0$ as the "reference value", while variations in the range $\Delta = 0.25...4.0$ indicate the magnitude of uncertainties.

(ii) To avoid threshold singularities at small v related to the Higgs and Goldstone masses, we replace the exact 2-loop potential by a polynomial fit around the broken minimum:

$$\frac{\mathrm{Re}[V(v) - V(0)]}{T^4} = \sum_{n=2}^{4} b_n \, (\hat{v} - \hat{v}_{\mathrm{min}})^n + \mathcal{O}((\hat{v} - \hat{v}_{\mathrm{min}})^5) \; , \tag{3.2}$$

where $\hat{v} \equiv v/T$. We carry out the fit in the range $v = (0...1.5)\, v_{\mathrm{min}}$. Only values $v \le v_{\mathrm{min}}$ are needed for the sphaleron solution, but including some larger values allows for a better fit of the curvature around the minimum. We have considered other fit forms as well and find that the errors introduced through the fitting are insignificant compared with other error sources.

(iii) We compute the sphaleron energy E_{sph}/T for this potential. We assume that the use of the 2-loop potential rather than the tree-level potential takes care of the factor κ in Eq. (3.1), which we thus set to unity. At 1-loop level this can to some extent be demonstrated explicitly [32], but what is more important for us is that any possible errors from this approximation are compensated for by step (v) below. The effect of the U(1) group is treated perturbatively [26], which is an excellent approximation [27]. We use an effective finite-temperature Weinberg-angle $\tan^2(\theta_W)_{\mathrm{eff}} \approx 0.315$ [19].

(iv) We determine the zero-mode factors $\mathcal{N}_{\mathrm{tr}}$, $(\mathcal{N}\mathcal{V})_{\mathrm{rot}}$ and the dynamical factor ω_-, as described in Ref. [30], except that every appearance of the tree-level $\lambda(h^2-1)^2/4g^2$ is replaced by the 2-loop potential $V(hv_{\mathrm{min}})/g^2_{\mathrm{eff}} v^4_{\mathrm{min}}$. We also determine the effective gauge coupling g_{eff} of the dimensionally reduced theory [38], and use g_{eff} instead of g in Eq. (3.1). The effect of the zero-mode factors and ω_- is to effectively decrease E_{sph}/T by about 15%, or by 3...10 in absolute units.

(v) Finally we add a correction from Ref. [36], which we assume to be a constant:

$$\Gamma^{\mathrm{(full)}}_{\mathrm{diff}} \equiv \Gamma^{\mathrm{(i)\text{-}(iv)}}_{\mathrm{diff}} \exp\Big[-(3.6 \pm 0.6)\Big] \; . \tag{3.3}$$

This correction is in most cases subleading compared with those in step (iv), and goes in the opposite direction. It may be noted that there is some latitude with respect to which gauge is used for the evaluation of the prefactors appearing in Eq. (3.1) [25, 30], but since $\Gamma^{\mathrm{(full)}}_{\mathrm{diff}}$ is gauge-independent, the non-perturbative correction factor compensates for this as well.

(vi) Finally, since we rely on an extrapolation of the non-perturbative correction factor to larger Higgs masses, we assign a generous overall uncertainty to Γ_{diff}, in the range

$$\left| \delta \ln \left[\frac{\Gamma_{\mathrm{diff}}(T)}{T^4} \right] \right| \approx 2.0 \; . \tag{3.4}$$

This amounts to roughly three times the error in Eq. (3.3). We stress that even though the Higgs masses leading to Eq. (3.3) are much smaller than we consider, the values of v_{min}/T

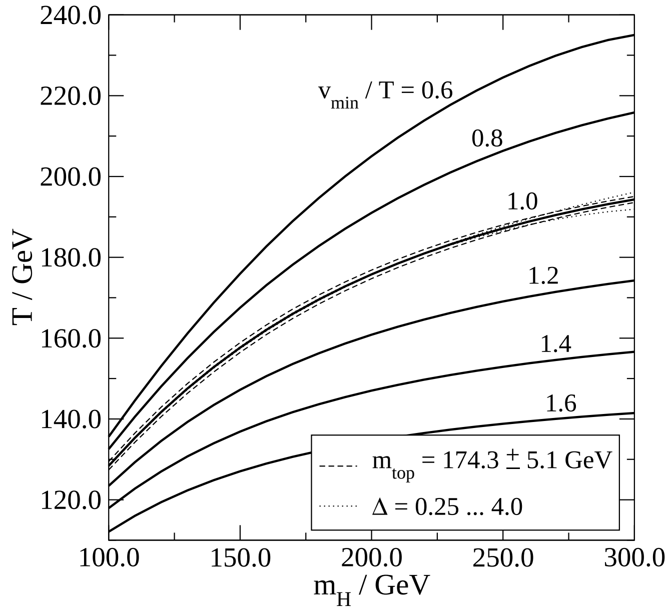

Figure 1: *The temperatures for which specific values of v_{min}/T (in Landau gauge) are reached, as a function of the Higgs mass m_H. For $v_{min}/T = 1.0$ we also show the effects of the variations $m_{top} = 174.3 \pm 5.1$ GeV (dashed lines) and $\Delta = 0.25...4.0$ (dotted lines).*

are similar, and thus the bulk of the effect in Eq. (3.3) should still remain intact, at least in the physically most plausible range 100 GeV $\leq m_H \leq$ 200 GeV.

In Fig. 1, we show the location of the minimum of the 2-loop effective potential. We only consider values for which the infrared sensitive expansion parameter $hT/\pi v_{\rm min}$ remains reasonably small. For higher temperatures, the corresponding rate $\Gamma_{\rm diff}$ extrapolates smoothly to the symmetric phase value [37], like standard thermodynamic observables [18, 19, 39].

The rates $\Gamma_{\rm diff}$ are displayed in Fig. 2, with assumed uncertainties of the order in Eq. (3.4). For practical applications, we note that in the range 100 GeV $\leq m_H \leq$ 200 GeV and for T such that $-\ln[\Gamma_{\rm diff}(T)/T^4] \approx 30...50$, the results can within our uncertainties be approximated by

$$-\ln\left[\frac{\Gamma_{\rm diff}(T)}{T^4}\right] \approx \sum_{i,j\geq 0}^{i+j\leq 2} c_{ij} \left(\frac{m_H - 150\ {\rm GeV}}{10\ {\rm GeV}}\right)^i \left(\frac{T - 150\ {\rm GeV}}{10\ {\rm GeV}}\right)^j , \tag{3.5}$$

with the coefficients

$$\begin{aligned} &c_{00} = 39.6\ , \quad c_{10} = 3.52\ , \quad c_{01} = -7.09\ , \\ &c_{20} = -0.376\ , \quad c_{11} = 0.421\ , \quad c_{02} = 0.170\ . \end{aligned} \tag{3.6}$$

Given $\Gamma_{\rm diff}(T)/T^4$, we can finally estimate the decoupling time t_* and/or the corresponding decoupling temperature T_*, needed in Eq. (2.11). In the limit that $\Gamma_{\rm diff}(T)/T^4$ changes very

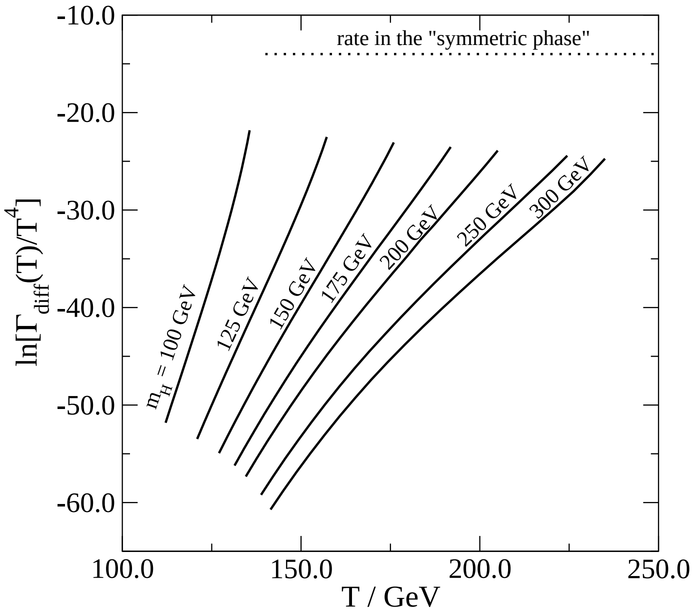

Figure 2: $\ln[\Gamma_{diff}(T)/T^4]$ *as a function of the Higgs mass and temperature. The overall error is estimated in Eq. (3.4). The dotted horizontal line indicates the value which all curves approach at large* T*. The values in the range 100 GeV* $\leq m_H \leq$ *200 GeV can be roughly approximated by Eq. (3.5). Note that the rate falls off more slowly at large Higgs masses.*

rapidly with T, the solution is given by the equation $n_{\rm G}^2\,\rho\,\Gamma_{\rm diff}(T_*)/T_*^3 = H(T_*)$, where $H(T)$ is the Hubble rate defined through $H^2(T) = 8\pi e(T)/3m_{\rm Pl}^2$. Writing

$$\ln\left[\frac{\Gamma_{\rm diff}(T)}{T^4}\right] = \ln\left[\frac{\Gamma_{\rm diff}(T_*)}{T_*^4}\right] + A(T-T_*) + \mathcal{O}((T-T_*)^2)\;, \tag{3.7}$$

corrections to this leading order approximation are of relative order $\mathcal{O}(1/AT_*)$, which according to Eqs. (3.6) is in the one percent range, and thus subdominant compared with other error sources. The leading order solution is shown in Fig. 3.

Comparing Fig. 3 with Fig. 1, it is seen that T_* corresponds to values $v_{\rm min}/T = 1.0...1.2$. At the same time, the rate of change of $\Gamma_{\rm diff}$ is less abrupt (A is smaller) at large Higgs masses, and a sudden decoupling is a less precise approximation. This can be seen in Fig. 4, where the full function $1-\omega(t';t)$ appearing in Eq. (2.10) is plotted.

4. Summary and conclusions

The main contents of this note are the baryon and lepton number rate equations shown in Eqs. (2.7)–(2.9), as well as the "sphaleron rate" $\Gamma_{\rm diff}(T)/T^4$ that enters these equations,

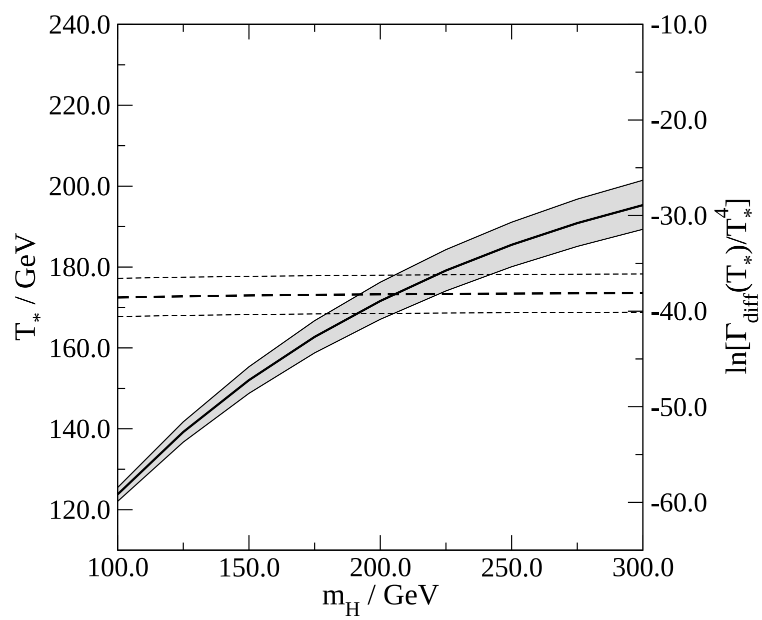

Figure 3: *The solid line indicates the decoupling temperature T_* as defined in the text (assuming a constant $g_* \simeq 106.75$), with an error band following from changing $\Gamma_{diff}(T_*)/T_*^4$ within the range of Eq. (3.4). The dashed lines show the corresponding anomalous rate.*

shown in Fig. 2 and in Eq. (3.5). With this knowledge, and given that the factors χ, ρ, η are to a fairly good approximation constants, the equations can be integrated in closed form, leading to Eq. (2.10). An even simpler estimate for the baryon number generated in a given scenario can be obtained from Eq. (2.11), where t_* corresponds to the temperature T_* shown in Fig. 3. On the other hand, the most precise results can be obtained by integrating Eqs. (2.7)–(2.9) numerically down to temperatures shown in Fig. 2. All of these equations are model-independent in form; the specific model enters through the source terms f_i.

The biggest uncertainties of our estimates for $\Gamma_{\rm diff}(T)/T^4$ originate from the fact that systematic numerical studies have only been carried out at fairly small Higgs masses [36, 37]. If a Standard Model like Higgs particle is found at the LHC, there is certainly a strong motivation for repeating the numerical studies at the physical value of the Higgs mass, in order to remove the corresponding error source (Eq. (3.4)) from our estimates.

Acknowledgements

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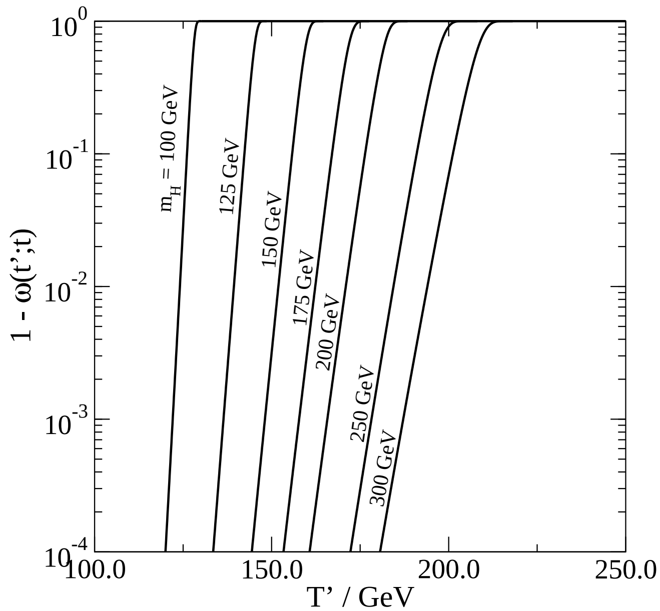

Figure 4: *The function $1 - \omega(t'; t)$ appearing in Eq. (2.10), as a function of the temperature T' corresponding to the time t' (the final moment t is fixed to the point where $T = 100$ GeV). We indicate temperatures instead of times, because this significantly reduces the dependence on the constant $g_* \simeq 106.75$, which has non-negligible radiative corrections [21]. This figure can be used to gauge the accuracy of the sudden decoupling approximation shown in Fig. 3.*

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