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Real estate and portfolio risk: an analysis based on copula functions

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This article examines the risk-return trade-off of a mixed-asset portfolio that includes real estate using copula functions. In particular, it analyses the role of direct as opposed to securitised real estate in terms of diversification when the dependence structure is modelled by an appropriate copula. The empirical analysis is conducted using Swiss data for the period 1987–2003. It is shown that a better portfolio diversification is obtained with indirect than with direct real estate. This finding has important practical consequences for asset allocation decisions.

Keywords: Real estate finance; portfolio diversification; copulas; tail dependence modelling

1 Indroduction

The beneficial effects of real estate inclusion in an institutional portfolio have been well dcumented in the literature. Hoesli, Lekander & Witkiewicz (2004), based on portfolio allocation models in a mean-variance framework, showed that the inclusion of real estate in a mixed-asset portfolio enables the portfolio's standard deviation to be reduced by 10–20%. In these studies, co-variances or correlation coefficients between financial assets and real estate have a significant impact on the amount of risk reduction achieved through diversification. Indeed, several studies, such as that of Liang, Chatrath & McIntosh (1996), concluded that real estate returns are weakly (positively or negatively) correlated to those of stocks and bonds.

The assumption of multivariate normally distributed observations lies at the heart of traditional portfolio models. In this framework, where each asset is characterised by the mean and variance of its return, the natural tool and most applied concept for quantifying dependence structure between real estate and financial asset returns, corresponding to the way in which several random variables are interlinked, is Pearson's correlation coefficient. However, since Fama (1965), we know that stock returns are leptokurtic and as, we explain in the next step, the linear correlation or Pearson's correlation coefficient is not an accurate dependency measure in a non-Gaussian world.

These distributional characteristics of asset returns have led to suggestions that portfolio allocation models, including real estate, should use a risk measure other than variance, and a structure of dependence other than correlation coefficients. Bond and Patel (2003) proposed the use of a semi-variance risk measure; Hamelink and Hoesli (2004) used the maximum drawdown risk measure. With very few exceptions in real

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estate literature, empirical analysis of the dependence structure relies on correlation coefficients. However, new methodologies have been applied to quantify dependence between real estate and financial assets. For example, Chaudhry, Myer & Webb (1999), using cointegration techniques, found that stocks tend to have an inverse long-term relationship with real estate.

In the financial literature, copulas have been applied to model the inter-dependence between financial assets. The copula approach, introduced in the financial context by Embrechts, McNeil & Straumann (1999), allows a joint distribution of multiple variables to be defined describing the totality of the dependence structure. Copulas have been used in a number of studies in various forms. For instance, Di Clemente and Romano (2004) presented a methodology for optimising the credit risk of a loan portfolio adopting a copula-based approach. Geman and Kharoubi (2003), using copula functions, showed that the diversification effect attributed to hedge funds is overestimated because of the non-normality of marginal distribution and the non-Gaussian dependence structure.

Although this is the first time a copula-based approach has been implemented to evaluate the risk-return trade-off of a mixed-asset portfolio including direct real estate, Knight, Lizieri & Satchell (2006) also used a copula function in a real estate framework. Their study examined the links between real estate and equity markets during extreme events, assuming the copula without performing a test to select the best copula. Using the Joe–Clayton copula they found that direct real estate is unrelated to the equity market, whereas the tail dependence between securitised real estate and stocks is greater in the lower than in the upper tail.

Our objective is to gauge the risk-return trade-off of a mixed-asset portfolio that includes real estate, taking into account the non-normality of marginal distributions and an appropriate structure of dependence. Special attention is devoted to the question of whether direct real estate is a portfolio diversifier compared with securitised real estate.

This paper is structured as follows. Section 2 outlines the deficiencies of Pearson's correlation coefficient as a measure of dependency. Section 3 discusses the assumption of normality concerning the data under consideration. Section 4 presents a copula-based framework for quantifying dependence. Section 5 shows the results regarding parameters and copula choice. This latter section additionally examines both the relative contribution of marginal distributions and dependence structure in risk estimation, together with the role of direct and indirect real estate as portfolio diversifiers. Finally, Section 6 contains some concluding remarks.

2 Linear correlation as dependence measure

This section discusses the validity of the linear correlation as a dependence measure. It is worth noting that the dependence between two time series could be quantified with a covariance, but since the number representing covariance depends on the measurement unit of the data, it is nonsense to compare covariances among data sets having different units.

Thus, owing to interpretation problems, the dependence is usually described by Pearson's correlation coefficient that addresses this issue by normalising the covariance to the product of the standard deviations of the variables, creating a dimensionless quantity. Pearson's correlation coefficient assesses the extent to which random variables are linearly correlated and can be interpreted as the percentage of variation of a variable that can be attributed to another one. The linear correlation coefficient between two random variables X_1 and X_2 is commonly denoted by $\rho(X_1, X_2)$

$$p(X_1, X_2) = \frac{Cov(X_1, X_2)}{\sqrt{\sigma^2(X_1)\sigma^2(X_2)}} \quad (1)$$

where $Cov(X_1, X_2)$ is the covariance between X_1 and X_2, and $\sigma^2(X_1)$. $\sigma^2(X_2)$ denote the variance of X_1 and X_2. The linear correlation coefficient has the following properties: for all (X_1, X_2) it ranges between −1 and 1 and in the case of independent random variables $\rho(X_1, X_2) = 0$.

We must be cautious in the interpretation of the correlation coefficient, which, despite its popularity, has serious deficiencies. First, variances have to be finite for Pearson's correlation measure to be defined. We know, of course, that this is not always the case in finance in which long tails are commonly observed. Second, this measure is founded on the assumption of multivariate normally distributed observations. Indeed, a zero correlation coefficient means independence between variables in a Gaussian framework exclusively. Thus, this measure is ill-suited for capturing a non-linear dependence relationship. Third, correlation is not an invariant measure under increasing non-linear transformations. This last deficiency originates from the computational formula of the coefficient of correlation which is defined in relation to the volatilities of the various series. Hence, it is not possible to assess dependence independently of volatilities.

To suggest an alternative statistical tool as a measure of dependence between real estate and financial assets, the univariate distributional characteristics of the data must be studied. In the next section, we will determine whether the assets under scrutiny respect the normality assumption of Pearson's correlation coefficient and whether the real estate index differs from other market indices.

3 Risk characteristics of assets: empirical evidence

This section is devoted to the empirical study of statistical properties of Swiss market data. The dataset comprised quarterly stocks, bonds, securitised real estate and direct real estate for the period ranging from the first quarter of 1988 through to the last quarter of 2008. All series were based on nominal total returns, i.e. they include the capital gain and income return components. The Swiss stock and bond[1] indices are taken from Bloomberg. As a proxy for the Swiss stock and bond market, we used respectively the SPI and the Citigroup indices.[2]

The Swiss securitised real estate index is the net wealth-weighted index from Rüd Blass, which is constructed on the basis of data pertaining to the largest Swiss real estate mutual funds. In Switzerland, real estate mutual funds and property companies are the two vehicles of investment in securitised real estate where shares are traded on a stock exchange. To track direct real estate, a hedonic index for income-producing property is taken from the IAZI/CIFI.

3.1 Behaviour of index returns

For practical reasons we will divide each closing value by its value in the previous quarter and then compute the logarithm of this ratio; we will therefore work with continous compunding returns. In this section we will focus on the distributional characteristics of returns. For the financial and real estate indices we have estimated a number of characteristics. These include the average return, standard deviation, maximum, minimum, skewness and kurtosis. The descriptive statistics of the three series are presented in Table 1.

Comparing the return and volatility of real estate indices with the return and volatility of financial assets, we find that the securitised real estate index is characterised by a higher return and a higher volatility than the direct real estate and bond index but by a lower return and volatility than the stock index. As expected, direct real estate exhibits favourable risk and return characteristics compared with stocks but unfavourable compared with bonds. This feature is due to the impact of the bearish real estate period of the late 1980s–early 1990s. In contrast, the bullish real estate market of the 1970s is not included in the sample period, since the IAZI/CIFI index was not available prior to 1988. The third distributional characteristic is referred to as skewness. Both stocks and direct real estate show significant negative skewness but securitised real estate and bonds are positive. Moreover, all the indices demonstrate fat tails, namely a higher percentage of very low and very high returns than would be expected with a normal distribution. Kurtosis is particularly high with regard to securitised real estate, implying that extreme returns on securitised real estate have a high probability of occurrence. Thus, the distribution of securitised real estate returns is highly peaked and positively skewed. The results suggest that the returns are not normally distributed. The Jarque–Bera normality test confirms the suggestion as, with the exception of the bond index, all series fail to pass the Jarque–Bera normality test at the level of significance of 5%.

We can therefore reject the multivariate normality assumption underlying Markowitz's portfolio model (1952), and conclude that, in this framework of heavy-tailed distributions, copula functions are more suitable for modelling tail dependence than correlation coefficients.

In the next section, the main properties of copulas and the methodology followed in the paper will be presented.

4 Properties of copulas

The goal of this section is to link marginal distributions to form a joint distribution that better models reality than the normal distribution. A copula is the selected instrument to construct a multivariate distribution where the univariate margins can be separated from the dependence structure represented by a copula. More specifically, a copula is a multivariate distribution function with uniform marginals. If U_1, U_2, …, U_n are uniform random variables, then the function C from $[0, 1]^n$ into [0, 1] defined by

Table 1. Quaterly descriptive statistics of the data.

	Mean	S.D.	Max.	Min.	Skew.	Kurt.	J–B (*p*-value)
Direct real estate	0.0073	0.0201	0.0648	−0.0578	−0.1016	1.3027	0.0400
Securitised real estate	0.0150	0.0465	0.1938	−0.1039	0.6812	3.4116	0.0000
Bonds	0.0110	0.0206	0.0638	−0.0313	0.0127	−0.4185	0.8175
Stocks	0.0285	0.0943	0.1971	−0.2895	−1.1837	2.4220	0.0000

$$P[U_1 \le u_1,\ U_2 \le\ u_2, \cdots, U_n \le\ u_n] = C(u_1, u_2, \cdots,\ u_n) \tag{2}$$

is a copula whose properties are:

- $\forall u_i \in\ [0, 1]$ and $i \in\ \{1, \ldots, n\}$, $C(1, \ldots, 1, u_i, 1, \ldots, 1) = u_i$,
- $\forall u_i \in\ [0, 1]$, $C(u_1, \ldots, u_n) = 0$ if at least one of the u_i equals zero,
- $C(u_1, \ldots, u_n)$ is increasing in each component of u_i.

When applying Sklar's theorem, we may define the copula function in terms of the different cumulative distribution functions, such that:

$$F[x_1, x_2, \cdots, x_n) = C\,(F_1(x_1), F_2(x_2), \cdots,\ F_n(x_n)) \tag{3}$$

with $F_1(x_1)$, $F_2(x_2)$, …, $F_n(x_n)$ being continuous uniform random variables and $F(x_1, x_2, \ldots, x_n)$ taken as the multivariate cumulative distribution function evaluated at $x_1, x_2, \ldots, x_n$. Moreover, Sklar provides proof of the 'unique copula'. Indeed, if marginal distributions are continous, then the copula is unique, but if there are discontinuities in one or more marginals then there is more than one copula representation for marginal distributions. From the joint cumulative distribution function, it is possible to determine the joint density. Let f be the density of joint distribution F:

$$f(x_1,\ x_2, \cdots,\ x_n) \ = \ c(F_1(x_1),\ F_2(x_2), \cdots F_n(x_n)) \prod_{i=1}^{n} f_i(x_i) \tag{4}$$

where f_i is the univariate density function associated to the cumulative F_i and c is the density of the copula given by the expression

$$c(u_1, u_2, \cdots,\ u_n) \ = \ \frac{\partial^n C(u_1, u_2, \cdots, u_n)}{\partial u_1 \partial u_2 \cdots \partial u_n}. \tag{5}$$

4.1 Copulas used

In Section 3, we showed that real estate and financial assets are characterised by heavy tails. The question of dependence between these tails involves the concept of tail dependence which measures the probability of extreme events occurring jointly. Dependencies between extreme returns of same sign are an important issue for portfolio management because of their negative impact on diversification strategies geared to acquiring assets that may be used to counterbalance the losses of other portfolio elements. Thus, in the presence of tail dependence, diversification strategies become less effective since the benefits of the diversification may be lower at times when they are needed most.

Since it is important to determine whether it is more probable that extreme events will occur independently or simultaneously, and since this issue cannot be resolved with the Pearson's correlation coefficient but requires the employment of copula functions, this study investigates five parametric copulas. The copulas included in this empirical investigation were selected according to their tail characteristics and since economic agents are particularly averse to extreme negative returns, we ensured that some of the copulas model the dependence between these returns.

(1) The function given by:

$$C(u_1,\ u_2,\cdots,\ u_n)\ =\ (u_1^{-\alpha}+u_2^{-\alpha}+\cdots+u_n^{-\alpha}-n+1)^{\frac{-1}{\alpha}} \tag{6}$$

with α > 0 belongs to the Clayton (1978) family and it is sometimes referred to as the Cook–Johnson copula.

(2) Gumbel (1960) copula:

$$C(u_1,\ u_2,\cdots,\ u_n)\ =\ \exp(-[(-\ln u_1)^{\alpha}+(-\ln u_2)^{\alpha}+\cdots+(-\ln u_n)^{\alpha}]^{\frac{-1}{\alpha}} \tag{7}$$

with α ≥ 1.

(3) Frank (1979) copula:

$$C(u_1,\ u_2,\cdots,\ u_n)\ =-\frac{1}{\alpha}\ln(1+\frac{(\exp^{-\alpha u_1}-1\ (\exp^{-\alpha u_2}-1)\cdots(\exp^{-\alpha u_n}-1)}{(\exp^{-\alpha}-1)^{n-1}}) \tag{8}$$

with α ≠ 0.

(4) Gaussian copula:

$$C(u_1,\ u_2,\cdots,\ u_n)\ =\ \Phi_p^n\left[\Phi^{-1}(u_1),\Phi^{-1}(u_2),\cdots,\Phi^{-1}\ (u_n)\right] \tag{9}$$

with ϕ^n_ρ is an n-variate normal distribution, ρ is the correlation matrix and ϕ^{-1} is the inverse of the univariate standard normal distribution.

(5) Student-*t* copula:

$$C(u_1,\ u_2,\cdots,\ u_n)\ =\ t_{v,\Sigma}^n\left[t_v^{-1}(u_1),t_v^{-1}(u_2),\cdots,t_v^{-1}\ (u_n)\right] \tag{10}$$

where $t_{v,\Sigma}^n$ is an n-variate Student-t distribution with v degrees-of-freedom and shape parameter matrix Σ. t_v^1 is the inverse of the univariate Student-t distribution.

The Frank copula exhibits neither lower nor upper tail dependence like the Gaussian copula unless $\rho = 1$. The Gumbel copula has an asymptotic upper dependence but has no lower-tail dependence; the Clayton copula has an asymptotic lower dependence but no upper-tail dependence, while Student-*t* copula has symmetric tail dependence.

Figure 1 illustrates the diversity of density forms obtained with the copulas used.[3]

4.2 Estimation of copulas

Copulas can be estimated in three ways, by using fully parametric,[4] semi-parametric and non-parametric methods. The first two methods are explained in Shih and Louis (1995) and Genest, Ghoudi & Rivest (1995) respectively, while the third method is described in Fermanian and Scaillet (2002). In this study we have adopted the semi-parametric approach, known as 'conditional maximum likelihood'. This is a two-step procedure in which we first estimated the marginal distribution using the non-parametric empirical distribution function, and then estimated the parameter of the copula using the maximum likelihood method.

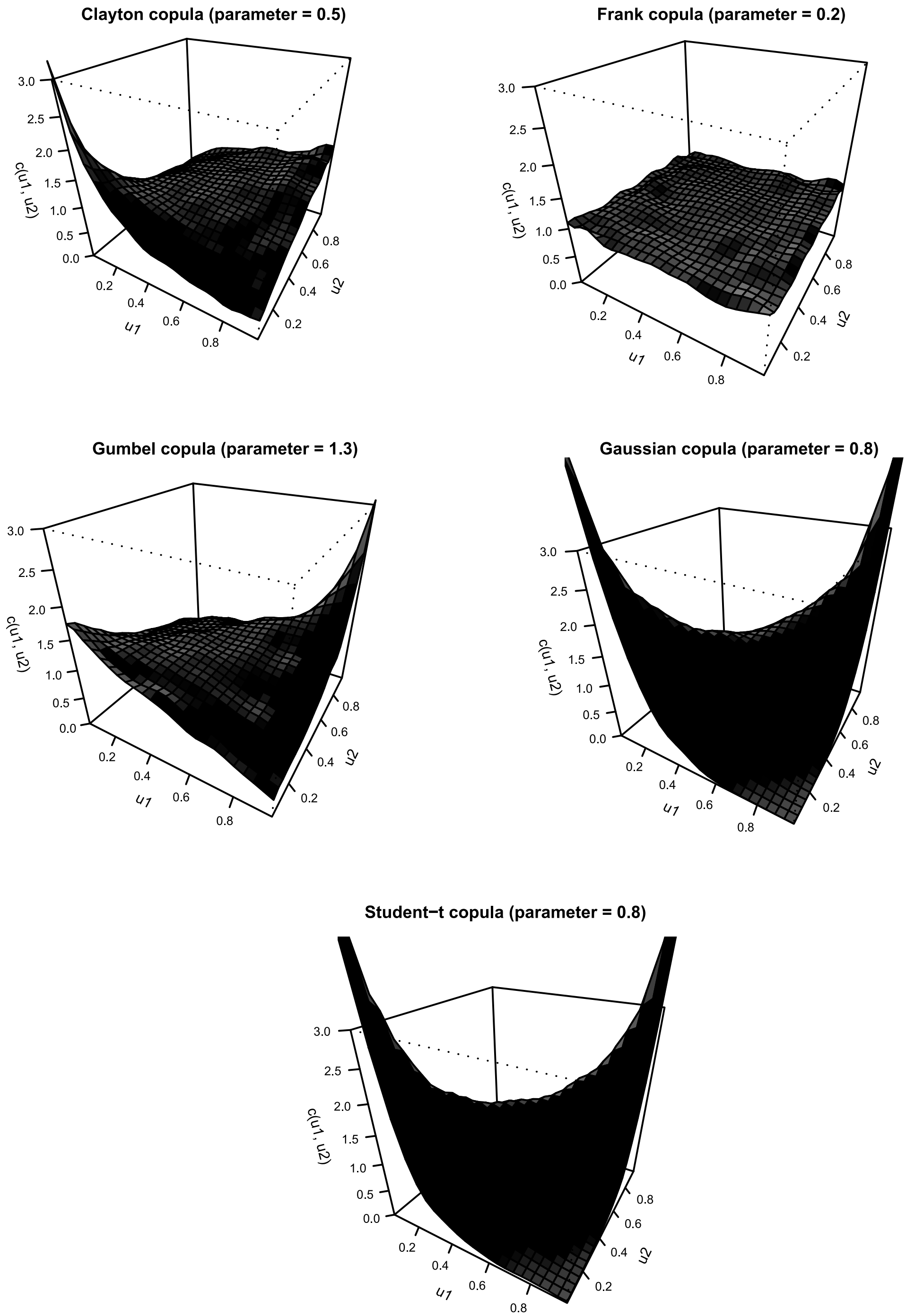

Figure 1. Graphical representation of copula density.

4.2.1 Non-parametric estimation of marginals. The unknown marginal distribution functions are constructed with the empirical distribution function defined by:

$$\hat{F}_n(x_n) = \frac{1}{T}\sum_{t=1}^{T} 1_{\{x_n^t < x_n\}} \tag{11}$$

where $1_{\{\cdot\}}$ is the indicator function which is equal to 1 if the argument is true and 0 if it is false. After obtaining the empirical marginal distribution function, we performed the usual probability transformation. Indeed, the data $\{x_1^t, x_2^t, \ldots, x_n^t\}_{t=1}^T$ were transformed into corresponding empirical distributions $F_n(x_n)$ to obtain standard uniform marginal distributions

$$\hat{u}_n^t = \hat{F}_n(x_n^t) \tag{12}$$

4.2.2 Parametric estimation of copulas. We applied the likelihood approach to estimate the parameters of copulas. The empirical likelihood function of the copula is:

$$L(\alpha; \hat{u}_1^t, \cdots, \hat{u}_n^t) = \prod_{t=1}^{T} c(F_1(x_1^t), \cdots, \hat{F}_n(x_n^t); \alpha). \tag{13}$$

Equivalently and for maximisation reasons, we were working with the log-likelihood function defined as follows:

$$l(\alpha; \hat{u}_1^t, \cdots, \hat{u}_n^t) = \prod_{t=1}^{T} \ln c(\hat{F}_1(x_1^t), \cdots, \hat{F}_n(x_n^t); \alpha). \tag{14}$$

The estimator of the copula could then be calculated by maximising the log-likelihood using the equation:

$$\hat{\alpha} = \arg\max \sum_{t=1}^{T} \ln c(\hat{F}_1(x_1^t), \cdots, \hat{F}_n(x_n^t); \alpha). \tag{15}$$

The advantage of this approach compared with a parametric estimate of the marginals is that we avoided the mis-specification of the marginals, which could have a significant impact on parameter estimation.

4.3 Copulas selection

A common assumption of financial models is the Gaussian distribution. In Section 3, we showed that the marginals are not normally distributed. Now we will investigate whether the dependence structure between real estate and financial assets is Gaussian.

Parametric copula selection is an open question in the literature and many tests have recently been proposed. Genest, Rémillard & Beaudoin (2007) divided the literature on the copula selection into three groups. Firstly, the procedures that can be applied for testing specific dependence structure, such as the Gaussian copula (Malevergne and Sornette, 2003). Second, the statistics that can be used to test the goodness-of-fit of any class of copulas but require strategic choices for their use as, for example, the specification of arbitrary parameters or kernels (Scaillet, 2007). Third, the tests applicable to all copula functions and requiring no strategic choice for their use, known as ‘blanket tests’.

To find the most fitting copula, we used the ‘blanket test’ procedure from Genest and Rémillard (2008), based on the empirical process comparing the empirical copula with a parametric estimate of the copula derived under the null hypothesis H_0: $C \in C_0$ for some class C_0 of copulas. Approximate p-values for the test statistic were obtained using the parametric bootstrap.

5 Empirical results

In this section, we assess the dependence structure of a portfolio containing four variables, namely bonds, stocks, direct and securitised real estate, using the methodology described in the previous section and quantify the impact of non-Gaussian assumptions on the risk.

Parameter estimates of the Clayton, Gumbel and Frank copulas are listed in Table 2 while the parameters of the Gaussian and Student-t copula are shown in Table 3; the estimates of their standard error are in parentheses. Table 4 shows the results of the goodness-of-fit test for copulas under consideration. In spite of being unable to reject the null hypothesis for the copulas under study, we conclude that the Student-t copula provides the best fit to model the dependence structure between bonds, stocks and direct and securitised real estate. Indeed, the p-value of the goodness-of-fit test is greater for the Student-t copula than for the other copula functions.

Table 2. Estimated parameters of Clayton, Gumbel and Frank copula for the portfolio containing bonds, stocks, direct and securitised real estate.

Clayton	Gumbel	Frank
0.1272 (0.0779)	1.0739 (0.0441)	0.6549 (0.3159)

Table 3. Estimated parameters of Gaussian and Student-t copula for the portfolio containing direct and securitised real estate bonds and stocks.

Gaussian copula	Direct real estate	Securitised real estate	Bonds	Stocks
Direct real estate		–0.1880 (0.1146)	–0.1666 (0.1167)	0.0386 (0.1207)
Securitised real estate			0.4209 (0.0902)	0.3686 (0.0965)
Bonds				0.1902 (0.1129)
Stocks				
Student-t copula with $v = 5$	**Direct real estate**	**Securitised real estate**	**Bonds**	**Stocks**
Direct real estate		–0.1327 (0.1147)	–0.1426 (0.1172)	0.0865 (0.1168)
Securitised real estate			0.4140 (0.0985)	0.3285 (0.1059)
Bonds				0.1554 (0.1173)
Stocks				

Table 4. Goodness-of-fit test of Clayton, Gumbel, Frank, Gaussian and Student-*t* copula.

	Statistic	*p*-value
Clayton	0.0430	0.1963
Gumbel	0.0377	0.3081
Frank	0.0389	0.3701
Gaussian	0.0321	0.4260
Student-*t*	0.0268	0.6928

Table 5. Comparison of quaterly conditional value-at-risk estimates.

		Three months CVaR at 99% confidence level		
Margins	Copula	Low-risk allocation (%)	Medium-risk allocation (%)	High-risk allocation (%)
Non-parametric	Gaussian	−1.766	−11.235	−25.435
Non-parametric	Student-*t*	−1.873	−11.669	−25.671
Gaussian	Student-*t*	−3.924	−9.586	−19.959

This finding provides a new perspective for market integration studies which investigate how real estate behaves as compared with stocks and bonds. Previous studies considered that real estate and financial assets are either integrated or segmented. In addition, the focus of past work was almost exclusively on the degree of integration among these assets, with little attention given to the stability of the relationship across markets conditions. It is often mentioned that direct real estate is segmented whereas securitised real estate is integrated with financial assets. Because the Student-*t* copula exhibits weak dependence in normal market conditions and strong dependence in extreme market conditions, we can argue that real estate and financial assets are both integrated and segmented depending on the market situation. More precisely, the relationship between real estate and financial assets is non-linear over the market period since real estate is integrated with stocks and bonds during periods in which markets are turbulent, and segmented at other times.

We discuss below some reasons that could explain why the kind of real estate considered in this study, and financial assets thrive and crash simultaneously. Real estate and financial markets are becoming increasingly integrated as a result of deregulation and market liberalisation, thereby facilitating investment and divestment. Generally, real estate investors will borrow money from banks to leverage their investments. If the price of these investments surges far enough, the bank will increase the loan and the investors will buy, for example, securities to invest their money. By contrast, if the value of these investments drops far enough, the bank calls in the loan and the real estate investors have to sell securities to refund it. The dependence, during extreme markets, between real estate and financial assets is fed with this deregulated linkage between the banking and real estate industry. Another element favouring cross-asset dependence derives from the herd behaviour often found in financial and real estate markets. This occurs when many people simply follow the actions of others, rather than acting according to a personal information-based strategy. Cross-asset dependence increases when abnormal returns occur among a given asset class. In such situations people lose confidence and tend to succumb to the temptation simply to follow what appears to be the prevailing market consensus. Such herding behaviour is further amplified by new information technologies broadcasting the same information to a large number of people.

Such findings will have important practical consequences for asset allocation decisions since the Student-*t* copula implies an effective portfolio diversification, when the returns fluctuate around the centre of the distribution, but a reduction in benefits arising from the portfolio diversification during extreme markets. The next section examines the optimal portfolio allocation when the best fitting copula is taken into account and analyses the allocation discrepancy existing when the Student-*t* copula is used instead of the Gaussian copula.

5.1 Risk-return analysis by Monte Carlo simulations

Risk-return analysis in a non-Gaussian world cannot be carried out using any risk measure. Artzner, Delbaen, Eber & Heath (1999) established the axioms of coherence. A coherent risk measure satisfies four properties, namely monotonicity, positive homogeneity, translation invariance and sub-additivity. The diversification effect is closely linked to the latter of these. Contrary to the conditional value-at-risk (CVaR),[5] the commonly used value-at-risk (VaR) is not a coherent risk measure because it is not sub-additive. Sub-additivity is important because it makes sure that the diversification principle of portfolio theory holds. Indeed, a sub-additive measure would always return a lower risk measure for a diversified portfolio than a non-diversified portfolio.[6]

Consequently, to gauge the risk-return trade-off of a mixed-asset portfolio that includes real estate, we generated scenarios. The simulations were done with the help of the best fitting copula as well as the non-parametric marginal distributions. Then, we minimised CVaR subject to a given level of return and the constraints that the asset weights were all non-negative and added up to one. The confidence level used in the CVaR function was 95%. In addition, the same simulation and optimisation procedures have been performed with the Gaussian copula.

Table 6 contains the asset allocation for the low-, medium- and high-risk case where portfolios are optimised in mean-CVaR space. The low-risk portfolio has the highest exposure to direct real estate (84.8%) and a significant allocation to bonds (15.2%). For the medium- and high-risk portfolio, the weight of direct real estate was equal to zero, suggesting that as risk increases, the likelihood of an asset with a return as low as direct real estate entering the optimal portfolios is unlikely. The medium-risk portfolio has the highest exposure to bonds (51.9%), with a significant allocation to stocks (34.1%) and a lower weighting of securitised real estate (14.0%). This portfolio is the only one that is dominated by bonds, reflecting the low return of direct real estate and the higher risk of the other two options. Clearly, stocks dominate the high-risk portfolio at 85.52%, since, in the long range, stocks are more risky but out-perform bonds and direct property. Surprisingly, for the high-risk portfolio, securitised real estate is 14.48%.

When the analysis is altered to consider the Gaussian instead of the Student-*t* copula, the optimal mean-CVaR portfolios demonstrate results with somewhat varying allocations. The main result is that the allocation to real estate is reduced when the Gaussian copula is used instead of the best fitting copula. The Gaussian copula underestimates the risk associated with real estate and financial assets, by not considering the likelihood of a simultaneous crash, and then neglects the importance of direct real

Table 6. Optimal portfolio composition.

Return (%)	Risk (%)	Direct RE[a] (%)	Securitised RE (%)	Bonds (%)	Stocks (%)
Mean-CVaR portfolio with non-parametric margin and Student-t copula					
0.80	3.34	84.76	0.00	15.24	0.00
1.80	8.83	0.00	14.00	51.90	34.10
2.70	21.51	0.00	14.48	0.00	85.52
Mean-CVaR portfolio with non-parametric margin and Gaussian copula					
0.80	3.20	80.04	0.00	19.96	0.00
1.80	9.49	0.00	9.47	53.31	37.22
2.70	22.72	0.00	9.33	0.00	90.67
Mean-SD portfolio with Gaussian margin and copula					
0.80	2.78	87.97	5.22	6.81	0.00
1.80	3.30	7.16	11.58	53.15	28.10
2.70	8.85	0.00	0.00	3.27	96.73

[a]No illiquidity premium for property.

estate for the low-risk portfolio and securitised real estate for the medium- and high-risk portfolio. Thus, the assumption of normality can be an important source of error for fund managers.

Table 6 contains as well the optimal allocation of portfolios resulting from Markowitz assumptions, which assumed multivariate normal distribution,[7] and an optimization done in a mean-standard deviation (mean-SD) space. For low and medium level of risk, typically those portfolios of interest to institutional investors, the optimal holding in direct real estate under Markowitz's assumptions is much higher than what is obtained under mean-CVaR with Student-*t* copula and non-parametric marginals. The over-weighting of real estate stems from three causes: the optimization in mean-SD space, the normality of marginal distributions and the dependence structure itself. This last result helps reconcile the discrepancy between optimal allocations to real estate suggested by academic studies and actual allocations by institutional investors, which is far below the academic findings (Hoesli, Lekander & Witkiewicz, 2003).

5.2 Diversification effects of direct versus securitised real estate investments

To examine which type of real estate is more profitable in a portfolio, we adopted the methodology described previously by comparing two optimal portfolios constructed with the most fitting copula and non-parametric marginal distributions for the following:

(1) Direct real estate, bonds and stocks.
(2) As above, except with direct real estate replaced by securitised real estate.

In both cases, the Student-*t* copula provides the best fit to model the dependence structure. The efficient frontiers are traced out in Figure 2. It is striking to note that inclusion of securitised real estate in place of direct real estate has a measurable benefit. Indeed, Figure 2 shows that investors would have obtained greater benefit by including securitised real estate rather than direct real estate. Thus, we can conclude

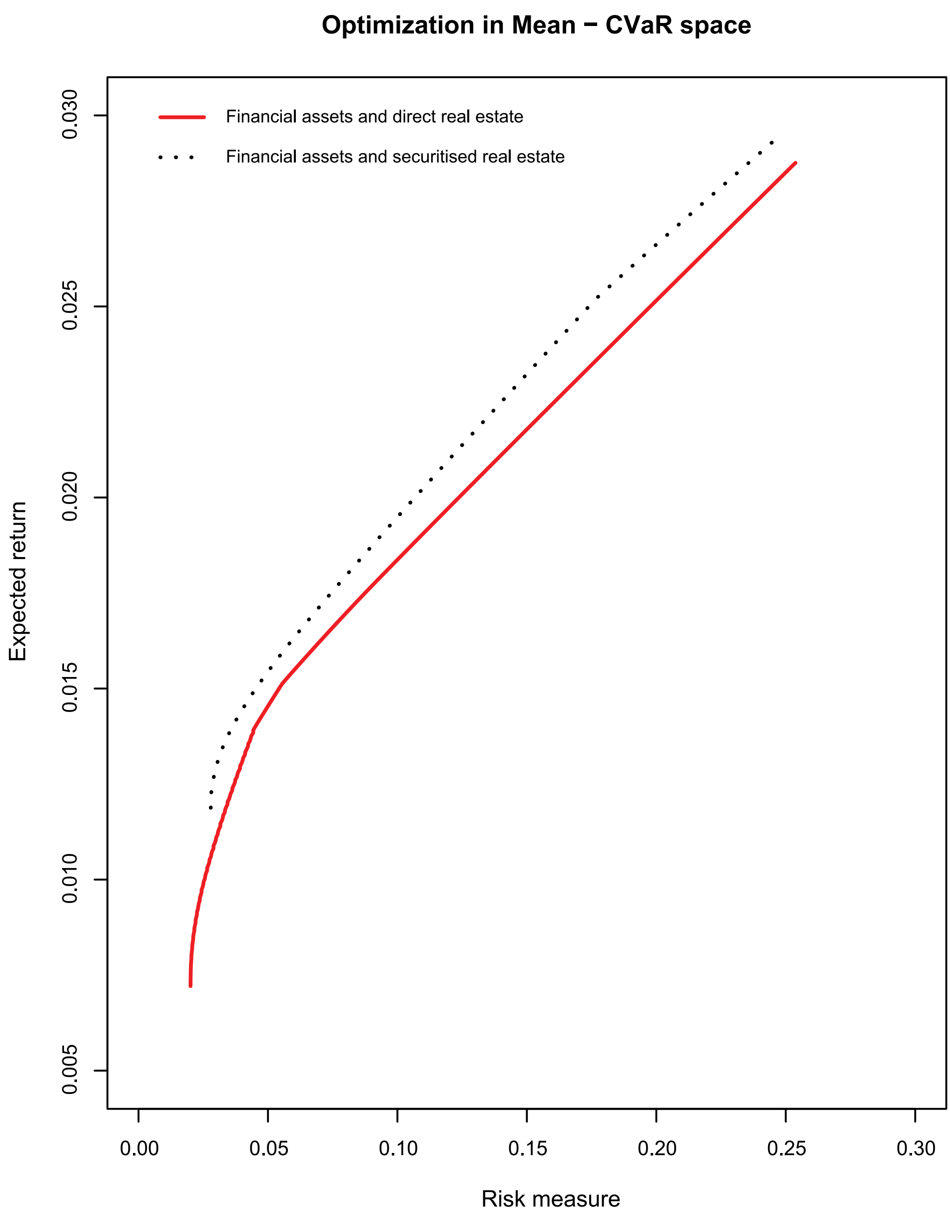

Figure 2. Efficient frontiers with Student-*t* copula.

that securitised real estate is a better diversifier than direct real estate when normality assumptions are dismissed. Moreover, if the in-depth knowledge of local legislation required for managing direct real estate and the time necessary for selling a property are included in the analysis, the benefit from including securitised real estate instead of direct real estate should be even greater than that which is observed in Figure 2.

The superiority of securitised real estate, on which we focus, to direct real estate, in a mixed asset portfolio derives from the local institutional and legal context. Swiss real estate mutual funds are governed by the Federal Law on Mutual Funds of October

19, 1994 and its Ordinances. The law (Article 66.2) allows for requesting redemption at the close of the annual accounting period, provided a 12-month notice period has been given. The redemption price (Article 80) will be the monetary counterpart of the net asset value (NAV) based on regular appraisals of the properties by independent real estate experts (Article 64.2). The market price of real estate mutual fund shares can theoretically fluctuate below, at, or above the redemption price. The downside risk is linked to the market situation in the direct real estate market, while the upside opportunity follows partially the stock market. This particularity of Swiss real estate mutual funds explains both the superiority of real estate mutual funds compared with direct real estate in a mixed asset portfolio and why the distribution of real estate mutual funds is positively skewed.

5.3 Dependence impact on the risk

Thus far, we have been modelling the dependence structure between bonds, stocks, direct and securitised real estate, analysing its impact on the portfolio allocation and examining which type of real estate is more profitable in a portfolio. However, these results do not provide us with any opportunity to quantify the importance of the dependence structure in terms of risk in comparison with a Gaussian copula. To quantify this relationship, 100 000 Monte Carlo simulations were run with the objective of computing the CVaR for portfolios weighted according to the results found previously in a mean-CVaR space when the most fitting copula and non-parametric marginal distributions are used.

The results in Table 5 demonstrate the importance of the structure of dependence on the CVaR. Compared to an appropriate dependence structure, the Gaussian copula underestimates the risk by 10.7, 43.4 and 23.6 basis points respectively for the low-, medium- and high-risk portfolio. However, the Gaussian assumption made on the marginal distributions has a greater impact on the CVaR than on the dependence structure and strongly reduces the CVaR. Indeed the difference in terms of risk between a Student-*t* copula with non-parametric marginals and a Student-*t* copula with Gaussian marginals is –94.9, –208.3 and –571.2 basis points respectively for the low, medium and high-risk portfolio. The underestimation due to Gaussian marginals is explained by the fact that the series are leptokurtic (see Section 3). As shown above, the underestimation due to Gaussian assumptions distorts the optimal portfolio choice.

6 Conclusion

In this report, it is shown that the Gaussian assumption made on marginal distributions and dependence structure leads to misleading inferences in terms of risk, and distorts the optimal level of diversification.

Since the data under scrutiny are not Gaussian, we modelled the relationship between financial assets and real estate returns for the Swiss market using copula functions. We found that the dependence structure between bonds, stocks, direct and securitised real estate is best modelled by a Student-*t* copula. We have examined the optimal portfolio allocation when the best fitting copula is taken into account. In addition, we have analysed the allocation discrepancy existing when the Student-*t* copula and non-parametric marginals are used, instead of the Gaussian copula, in a mean-CVaR space, and when the Student-*t* copula and non-parametric marginals are used instead of the multivariate normal distribution in a mean-SD space. The results reveal that direct real estate is a good portfolio diversifier only for low-risk portfolios, whereas securitised real estate plays an important diversification role for medium and high-risk portfolios.

The essential purpose of this paper has been to calculate the effect of non-normality on the risk of a portfolio including real estate and the efficient frontiers where risk is defined as the CVaR. Our results show that dependence structure has less influence on portfolio risk than marginal distributions and that Gaussian assumptions underestimate the risk. Moreover, separating the influence of direct and indirect real estate, we have found that securitised real estate diversifies a portfolio better than direct real estate. Similar methodology will be applicable for further research in other countries and for other periods of time.

Notes on contributor/s

Philippe Thalmann received his PhD in Economics from Harvard. He is a Professor at the Swiss Federal Institute of Technology in Lausanne and teaches building and housing economics to students in Architecture. He also teaches to students in Engineering and Sciences, mainly Environmental Economics and Economics of Sustainable Development. His current research interests are concentrated in the economics of the natural and the built environment. He is working on issues of the housing and the real estate markets, such as tenure choice, affordability and valuation.

Notes

1. Bond index contained domestic and international bonds of excellent quality denominated in Swiss francs for all maturities.
2. The Bloomberg ticker symbols for the stock and bond indices are respectively SPI and SBSZL.
3. The graphic representation of copula density is constructed with a beta kernel and based on Monte Carlo simulation of u_1 and u_2.
4. This method is known under the term 'inference functions for margins' (IFM).
5. The CVaR describes the expected loss conditional on the losses exceeding or equal to the VaR and the VaR is the maximal portfolio loss for a given confidence level over a specified time horizon.
6. A risk measure ψ is sub-additive if the following is true: $\psi(x_1 + x_2) \leq \psi(x_1) + \psi(x_2)$
7. Normal copula with normal marginals is equivalent to a multivariate normal distribution.

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