15th International Workshop on Dynamics and Control
IWDC 2009
J. Rodellar and E. Reithmeier (Eds)

Dynamic analysis and vibration control of an active tensegrity structure

N. Bel Hadj Ali and I. F. C. Smith

Applied Computing and Mechanics Laboratory
Ecole Polytechnique Fédérale de Lausanne (EPFL)
ENAC/IS/IMAC, Station 18, 1015 Lausanne, Switzerland
E-mail: nizar.belhadjali@epfl.ch; ian.smith@epfl.ch

Keywords: Vibration control, Tensegrity, Modal analysis, Multi-objective optimization.

Abstract. *Tensegrities are lightweight structures composed of cables and struts. Stability is provided by the self-stress state between tensioned and compressed elements. They present attractive solutions for controllable and smart structures as often small amounts of energy are needed to meet control requirements. Being lightweight structures, tensegrity systems are sensitive to dynamic loading. In spite of much research related to geometry, form-finding and architecture of tensegrity structures, few studies have focused on dynamic behavior and control. Also, few experimental studies have been observed to be of practical significance. Results are mainly tested numerically on small, simple and symmetrical tensegrity models.*

This paper extends ten years of research work on quasi-static control to perform dynamic analyses and study vibration control of a full-scale active tensegrity structure. Vibration modes of the structure are identified experimentally and compared with those determined through a finite element model. Laboratory testing is carried out for multiple self-stress levels and for different excitation frequencies. Measurements and numerical simulations confirm that the dynamic behavior of the structure is closely related to its degree of self-stress. These results indicate the potential to adjust the natural frequencies of the structure to meet vibration control requirements.

A multi-objective vibration control strategy is proposed. Vibration control is carried out by modifying the self-stress level of the structure through small movement of active struts in order to shift the natural frequencies away from excitation. Stochastic search is used to identify good control commands enabling reduction of structural response to acceptable levels at minimum control cost. Vibrations are thus reduced by small changes in self-stress level through active struts. These results provide further progress towards robust adaptive structures.

1 Introduction

Tensegrities are spatial, reticulated and lightweight structures that are composed of struts and tendons. Stability is provided by the self-stress state between tensioned and compressed elements. Tensegrities have received significant interest among scientists and engineers in fields such as architecture, civil engineering and aerospace applications. Among different traditional approaches, the tensegrity concept is one of the most promising for active and deployable structures. When used for structural applications, tensegrity systems might be subjected to dynamic loading such as those caused by wind, impact or earthquakes. Being lightweight structures, tensegrities are particularly sensitive to dynamic loading and thus likely to present significant vibration levels.

In spite of much research related to geometry, form-finding and architecture of tensegrity structures, few studies have focused on dynamic behavior. Sultan et al. [1] derived linearized dynamic models for two classes of tensegrity structures and showed that a linear kinetic frictional damping at joints is sufficient to ensure stability of these configurations. Murakami and Nishimura [2, 3] presented a set of procedures for characterizing static and dynamic response of tensegrity modules. Masic and Skelton [4] used a linearized dynamic model to enhance the dynamic control performance of a tensegrity structure. Dubé et al. [5] presented a comparative study between experimental tests and numerical simulations carried out on a tensegrity minigrid considering static as well as dynamic loading. Recently, Tan and Pellegrino [6] investigated the nonlinear vibration of a cable-stiffened pantographic deployable structure and showed that the system resonant frequencies are related to the level of active cable pretension. All studies cited so far aimed to find a dynamic model of tensegrity structures and to predict their behavior. Most studies are either analytical or numerical, rarely both. Also, experimental studies rarely included full-scale structures.

Research into active control of tensegrity structure was initiated in the mid 1990s. Tensegrities are attractive solutions for controllable and smart structures as often, small amounts of energy are needed to change the shape of tensegrity structures [7]. Experimental work that explored the active tensegrity potential was carried out by Fest et al. [8] on a five-module active tensegrity structure. A quasi-static control strategy based on stochastic search is first proposed to satisfy serviceability criterion [9]. The control strategy is then extended to take into account additional robustness objectives [10]. Djouadi et al. [11] developed an active control algorithm for vibration damping of tensegrity structures intended to spatial applications. Kanchanasaratool and Williamson [12] used a constrained particle dynamic model to investigate feedback shape control for a tensegrity module with three actuated bars and nine passive strings. Chan et al. [13] presented an experimental study of active vibration control of a three-stage tensegrity structure. Active damping is performed on a small scale tensegrity structure using local integral force feedback and acceleration feedback control. Averseng and Crosnier [14] introduced a vibration control approach based on robust control. They presented experimental validation done with a tensegrity plane grid of 20 m^2 where an actuation system is connected to the supports. de Jager and Skelton [7] have investigated placement of sensors and actuators to control vibrations on a planar tensegrity structure made up of three units. Ganesh Raja and Narayanan [15] presented a theoretical analysis of vibration control of a two module tensegrity structure under random excitations using optimal control theory and piezoelectric actuators.

Few experimental studies have been observed to be of practical significance. Results are mainly tested numerically on small, simple and symmetrical tensegrity models. Neither modal identification nor experimental testing under dynamic loads for multiple self-stress levels could be found in the literature. Structures are much simpler than would be needed for practical applications. Furthermore, no study has examined attenuation of dynamic vibrations using active control of a large scale tensegrity structure.

This paper investigates the vibration control of a full-scale active tensegrity structure, extending ten years of research work on quasi-static control to perform dynamic analyses and control. Natural vibration modes of the tensegrity structure are identified experimentally and compared with those determined through a finite element model. Dynamic behavior is experimentally investigated through testing under dynamic excitation. Laboratory testing is carried out for multiple self-stress levels and for different excitation frequencies. The dynamic behavior of the structure is also numerically simulated. Vibration control is then carried out by modifying the self-stress level of the structure through contractions and elongations of active struts in order to shift the natural frequencies away from excitation. Stochastic search is used to identify good control commands enabling reduction of structural response to acceptable levels at minimum control cost.

2 Dynamic analysis of tensegrity structures

Tensegrity systems can be regarded as a special class of spatial and reticulated structures. They are closely coupled structures that often display geometrically nonlinear behaviour. Different nonlinear dynamic models of theses structures are available in the literature [2-4, 16]. However, research into tensegrity dynamics showed that a linearized dynamic model around an equilibrium configuration offers a good approximation of the nonlinear behaviour of simple tensegrity structures.

The linearized equation of motion of a tensegrity structure at an equilibrium configuration is as follow:

$$\mathbf{M}\,\ddot{u} + \mathbf{C}\,\dot{u} + \mathbf{K_T}\,u = \mathbf{F} \tag{1}$$

Where: **M**, **C** and $\mathbf{K_T}$ are the mass, damping and tangent stiffness matrices, respectively. **F** is the applied load vector. u, $\dot{u}$ and $\ddot{u}$ are respectively vectors of nodal displacement, velocity and acceleration. The tangent stiffness matrix $\mathbf{K_T}$ is decomposed into the linear stiffness matrix $\mathbf{K_E}$, commonly used for small-deformation truss analyses, and the geometrical stiffness matrix $\mathbf{K_G}$ induced by self-stresses.

$$\mathbf{K_T} = \mathbf{K_E} + \mathbf{K_G} \tag{2}$$

For the development of a finite element model of the tensegrity structure, each element in the structure is characterized by the following mass and stiffness matrices [17]:

$$\mathbf{K_E} = \left(\frac{EA}{L}\right).\begin{bmatrix} I_0 & -I_0 \\ -I_0 & I_0 \end{bmatrix} \quad ; \quad \mathbf{K_G} = \left(\frac{T}{L}\right).\begin{bmatrix} I_3 & -I_3 \\ -I_3 & I_3 \end{bmatrix} \tag{3}$$

$$\mathbf{M} = \left(\frac{m}{6}\right).\begin{bmatrix} 2I_3 & -I_3 \\ -I_3 & 2I_3 \end{bmatrix} \tag{4}$$

$$I_0 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad ; \quad I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{5}$$

Where: E is the elastic modulus; A is the member area; L is the length of the member and T is the axial load. Mass and stiffness matrices are first formulated in a local coordinate system where x axis is along the element axis. The global mass and stiffness matrices **M** and $\mathbf{K_T}$ are obtained by adding up contributions from the individual elements expressed in a global coordinate system.

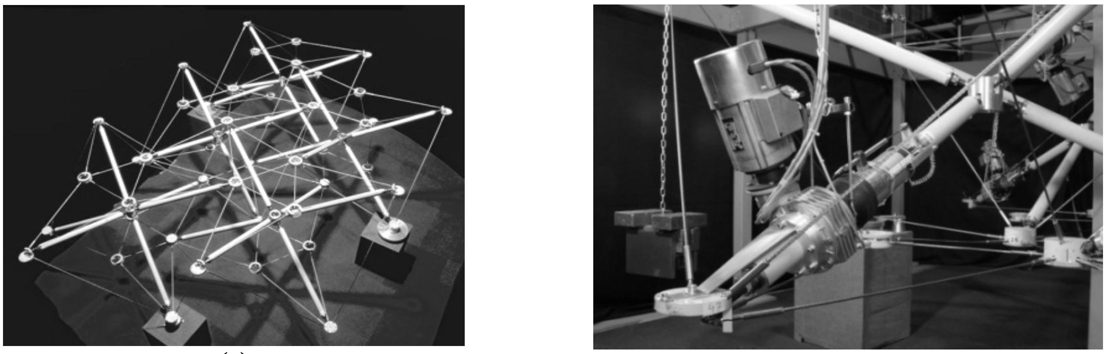
The modal analysis of the tensegrity structure is conducted by neglecting the damping matrix and the vector of applied forces in Eq.1. The generalized eigenproblem (Eq. 6) is then obtained considering a small harmonic motion of the form: $u = \bar{u}\ sin(\omega t)$, where ω is the angular frequency and $\bar{u}$ is the amplitude vector.

$$\mathbf{K_T}\,\bar{u} = \omega^2 \mathbf{M}\,\bar{u} \tag{6}$$

The spectral decomposition of matrix $\mathbf{M^{-1}K}$ then yields the natural frequencies and corresponding mode shapes of the finite element model (FEM) of the structure.

3 Modal analysis and vibration experiments

The structure that is used for experimental testing is composed of 5 modules and rests on three supports (Fig. 1(a)). It covers a surface area of $15m^2$, has a height of 1.20m and has a distributed dead load of $300N/m^2$. It is composed of 30 struts and 120 tendons. Struts are fiber reinforced polymer tubes of 60mm diameter and 703mm2 cross section. Tendons are stainless steel cables of 6mm in diameter. In each module, struts converge toward a central node where connection is provided by contact compression in a steel ball. This topology was proposed to limit buckling lengths, thereby allowing for more slender compression elements than more traditional tensegrities [18]. The structure rests on three supports that allow statically determinate support conditions. The structure is also equipped with ten active struts placed in in-line pairs in each module (Fig. 1(b)). Actuated struts are used for strut length adjustment controlling by the way the self-stress state in the tensegrity structure. Vertical displacements of the structure top surface nodes are measured with inductive displacement sensors.



(a) (b)

Figure 1 : Five module tensegrity structure (a) and one of the ten active struts (b).

3.1 Experimental modal analysis

Preliminary modal tests were conducted to determine the natural frequencies and mode shapes of the tensegrity structure. Free vibration tests employed a single mass that was suspended from a node on the top surface of the structure. Displacement measurements began once the load was suddenly removed. Ten tests were carried out with two initial loads at five nodes such that all modes of interest were excited. Vertical displacements were measured at 7 nodes of the top surface of the structure.

Modal identification analysis of the tensegrity structure was performed using the Frequency Domain Decomposition technique (FDD). The FDD technique consists of decomposing the system response into a set of single degree of freedom systems, each corresponding to an individual mode, through a decomposition of the spectral density function matrix [19]. Natural frequencies as well as damping ratios for the first five modes are displayed in Table 1.

	Experimental results		FEM results
Mode	Frequency [Hz]	Damping Ratio [%]	Frequency [Hz]
Mode 1	3.07	2.63	3.056
Mode 2	3.51	1.60	3.484
Mode 3	3.91	1.40	3.947
Mode 4	5.02	2.30	5.027
Mode 5	5.67	1.19	5.658

Table 1 : Natural frequencies and damping ratios of the tensegrity structure.

Experimentally identified natural frequencies are compared with those determined by the FEM

(Table 1). Experimental and analytical results match within a few percent for the first five natural frequencies. Therefore, the linearized dynamic model offers a good approximation of the nonlinear behavior of the five module tensegrity structure.

3.2 Vibration experiments

This testing involved exciting the tensegrity structure and measuring the vibration response. A single point dynamic loading was applied using an electro-mechanic shaker and vertical displacement measurements were taken at the top surface nodes of the structure.

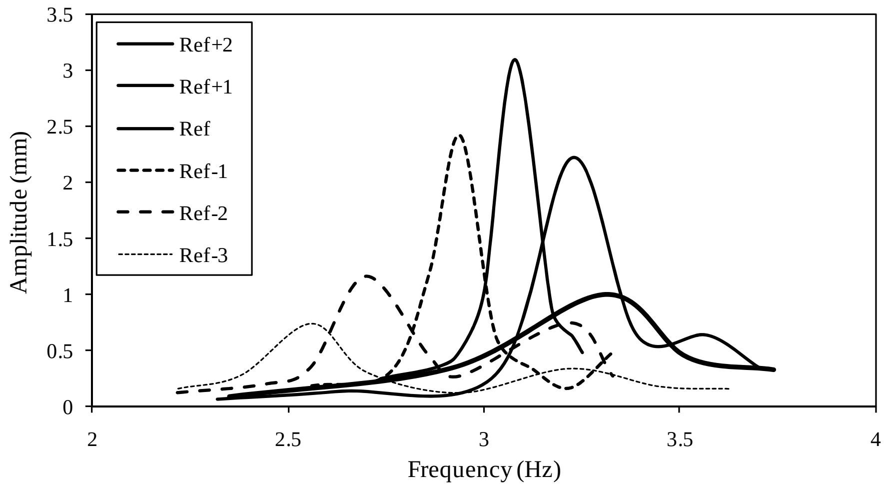


Figure 2 : Vibration amplitude at node 39 for different excitation frequencies and stress levels.

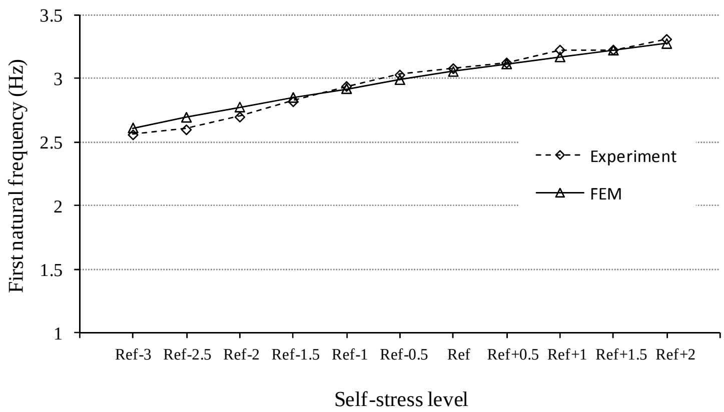


Figure 3 : Evolution of the first natural frequency with respect to self-stress level.

Vibration tests were performed for different self-stress levels in order to identify the relationships between the tensegrity self-stress level and its dynamic behavior. The tensegrity self-stress level was controlled through elongations and contractions of active struts. Stress levels were varied around a reference self-stress level (Ref) through increments of millimeter elongations and contractions. For example, (Ref+1) denotes the stress level induced by a one mm elongation of the active struts from the reference stress level. Excitation tests have been performed with frequencies running between 1.5 and 4.0 Hz. The variation of response amplitude at one of the tensegrity top surface nodes (node 39) according to the excitation frequencies and self-stress levels is displayed in Figure 2. Amplitude peaks in Figure 2 correspond to the first natural frequencies of the structure for the self-stress levels that were studied. Figure 2 shows that amplitude peaks change with respect to self-stress level. Decreasing the active strut lengths has the effect of reducing the natural frequency of the first resonance mode. These results confirm that, as observed for other configurations, the dynamic response of this tensegrity structure is closely related to its self-stress level. The evolution of the first natural frequency of the structure with respect to the degree of self-stress is displayed in Figure 3. FEM results confirmed experimental ones and it is shown that experimental and analytical results match well for the different self-stress levels.

4 Vibration control

Experimental measurements and numerical simulations have confirmed that the dynamic behavior of the five module active tensegrity structure is closely related to its degree of self-stress. These results indicate the potential to adjust the natural frequencies of the structure to meet vibration control requirements. Under a given excitation loading, response amplitudes may be attenuated through shifting natural frequencies away from the excitation. This can be carried out by modifying the self-stress level of the tensegrity structure through active strut movements.

A general objective of vibration control is to reduce structural response resulting from initial disturbances to acceptable levels with a minimum control cost. Control objective can be achieved by finding a set of strut positions defining a self-stress level configuration that shifts the natural frequencies away from a given excitation frequency. In addition, it is important to achieve this objective in an optimal manner leading to least perturbation of the geometry and the stiffness of the structure. The vibration control task can thus be stated as a multi-objective optimization problem. A first objective function measures the distance between the excitation frequency and the nearest natural frequency of the structure under a particular self-stress level. The control cost is taken into consideration in a second objective function. Control cost is evaluated through the sum of active strut adjustments which has to be minimized. This is a simple manner to guaranty that vibration control will be done with least perturbation of both geometry and stiffness of the tensegrity structure.

Let $\mathbf{x}^{\mathbf{t}} = [x_1, x_2, ..., x_{10}]$ be the vector of active strut movements. The vibration control problem can be stated as follows:

$$Max\ F_1(x) = \left| f_n(x) - f_{ex} \right| \text{ and } Min\ F_2(x) = \sum_{i=1}^{10} [x_i]^2 \tag{7}$$

Subject to

$$g_{x,\max} = x_{i,\max} - x_i \geq 0, \qquad \forall i = 1,...,10 \tag{8}$$

$$g_{x,\min} = x_i - x_{i,\min} \geq 0, \qquad \forall i = 1,...,10 \tag{9}$$

Where f_{ex} is the excitation frequency and f_n is the nearest resonance frequency of the structure to excitation frequency. Natural frequencies are calculated under current self-stress level defined after applying active strut adjustments. Equations 8 and 9 represent the constraints on decision variables. We assume that each active strut adjustment x_i is limited to values running between $x_{i,\,min}$ and $x_{i,\,max}$.

The number of active struts and the discrete strut moves define the space of possible solutions. With ten active struts, it is impossible to generate and test every possible solution due to the combinatorial nature of the task. Stochastic search is therefore useful for this situation. Stochastic methods sample the solution space using special strategies. Although there is no guaranty of reaching a global optimum, near optimal solutions are usually sufficient for control applications.

This optimization task was addressed using Probabilistic Global Search Lausanne (PGSL). The PGSL technique is based on the assumption that sets of better solutions are more likely to be found in the neighborhood of sets of good solutions and, therefore, intensifies search in regions that contain sets of good values. Search is driven by probability density functions [20].

In this study, the methodology for multi-objective vibration control includes two phases. First, the multi-objective problem is solved using PGSL optimization. A set of solutions is generated and then filtered so that only Pareto optimal solutions are considered. Second, an outranking relation is employed to select a compromise control solution. Outranking is performed using the PROMOTHEE method (Preference Ranking Organization METHod for Enrichment Evaluation) [21].

A 3 Hz vertical excitation force was applied to one of tensegrity top surface nodes. The frequency of the excitation force was selected to be close to the first natural frequency of the tensegrity structure. Active strut movements were limited to ± 3 mm and the precision range of each move in steps of ± 0.1 mm. Control solutions are found through optimization employing the PGSL algorithm. For this purpose, the first objective function (F_1) is optimized while the second objective function F_2 is transformed into inequality constraint (Eq.10).

$$F_2(x) = \sum_{i=1}^{10} \left[x_i\right]^2 \leq \varepsilon \tag{10}$$

By changing the bound ε of the new constraint, we obtained 30 solutions of our problem using the PGSL algorithm. Dominated solutions were eliminated and only eleven solutions are considered in the Pareto optimum set. The control command was then identified using the PROMOTHEE II method. This outranking strategy was applied using linear preference functions and the same weight (w_1=w_2=1) for the two objective functions.

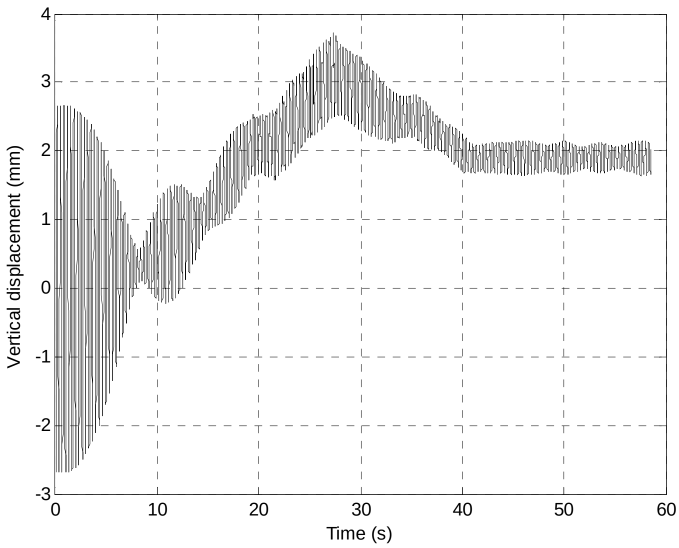


Figure 4: Vertical displacement of node 39 for uncontrolled and controlled configurations.

The control solution was applied to the tensegrity structure for experimental validation. To illustrate control results, the time history of the vertical displacement at one of the tensegrity top surface nodes (node 39) for controlled and uncontrolled configurations is displayed in Figure 4. Displacement amplitude is reduced by 90% after control. The application of the control command on the structure by adjusting lengths of the ten active struts took less than 40 seconds. Note that controlling the structure results in geometry changes leading node 39 to move 1.9mm away from its initial position. Vertical displacements caused by control application are less than 5mm for all structure nodes. In the same time, vibration control results in a maximum variation of about 17% for element internal forces.

6 Conclusions

Active tensegrity structures are reusable structural systems that are capable of reacting to their environment. In this paper, we focus on the dynamic behavior and the vibration control of a five module active tensegrity structure. The control strategy adopted in this tensegrity structure is capable of meeting vibration control objectives. Experimental as well as numerical results confirmed that natural frequencies can be shifted when the self-stress level in the tensegrity structure is modified. Vibration control is formulated as a multi-objective optimization problem. Control commands are identified using stochastic search through PGSL and PROMOTHEE outranking strategy. The capacity of the active control system to attenuate vibrations by shifting values of natural frequencies away from excitation is demonstrated. These results are expected to provide further progress leading to more robust adaptive civil engineering structures.

References

[1] Sultan C, Corless M, Skelton RE. Linear dynamics of tensegrity structures. Engineering Structures. 2002;24(6):671-85.
[2] Murakami H. Static and dynamic analyses of tensegrity structures. Part 1. Nonlinear equations of motion. International Journal of Solids and Structures. 2001;38(20):3599-613.
[3] Murakami H. Static and dynamic analyses of tensegrity structures. Part II. Quasi-static analysis. International Journal of Solids and Structures. 2001;38(20):3615-29.
[4] Masic M, Skelton RE. Selection of prestress for optimal dynamic/control performance of tensegrity structures. International Journal of Solids and Structures. 2006;43(7-8):2110-25.
[5] Dubé JF, Angellier N, Crosnier B. Comparison between experimental tests and numerical simulations carried out on a tensegrity minigrid. Engineering Structures. 2008;30(7):1905-12.
[6] Tan GEB, Pellegrino S. Nonlinear vibration of cable-stiffened pantographic deployable structures. Journal of Sound and Vibration. 2008;314(3-5):783-802.
[7] de Jager B, Skelton RE. Input-output selection for planar tensegrity models. Control Systems Technology, IEEE Transactions on. 2005;13(5):778-85.
[8] Fest E, Shea K, Smith IFC. Active Tensegrity Structure. Journal of Structural Engineering. 2004;130(10):1454-65.
[9] Domer B, Smith IFC. An Active Structure that Learns. Journal of Computing in Civil Engineering. 2005;19(1):16-24.
[10] Adam B, Smith IFC. Tensegrity Active Control: Multiobjective Approach. Journal of Computing in Civil Engineering. 2007;21(1):3-10.
[11] Djouadi S, Motro R, Pons JC, Crosnier B. Active Control of Tensegrity Systems. Journal of Aerospace Engineering. 1998;11(2):37-44.
[12] Kanchanasaratool N, Williamson D. Modelling and control of class NSP tensegrity structures. International Journal of Control. 2002;75:123-39.
[13] Chan W, Arbelaez D, Bossens F, Skleton RE. Active vibration control of a three-stage tensegrity structure. *SPIE 11th Annual International Symposium on Smart Structures and Materials*. San Diego, California, USA 2004.
[14] Averseng J, Crosnier B. Static and dynamic robust control of tensegrity systems. Journal of The International Association for Shell and Spatial Structures. 2004;45(146):169-74.
[15] Ganesh Raja M, Narayanan S. Active control of tensegrity structures under random excitation. Smart Materials and Structures. 2007;16:809-17.
[16] Skelton RE, Pinaud JP, Mingori DL. Dynamics of the shell class of tensegrity structures. Journal of the Franklin Institute. 2001;338(2-3):255-320.
[17] Kebiche K, Kazi-Aoual MN, Motro R. Geometrical non-linear analysis of tensegrity systems. Engineering Structures. 1999;21(9):864-76.
[18] Paronesso A, Passera R. The cloud of Yverdon. *IASS 2004 Symposium, International Association for Shell and Spatial Structures*. Montpellier: Editions de l'Espérou 2004.
[19] Brincker R, Zhang L, Andersen P. Modal identification of output-only systems using frequency domain decomposition. Smart Materials and Structures. 2001;10(3):441-5.
[20] Raphael B, Smith IFC. A direct stochastic algorithm for global search. Applied Mathematics and Computation. 2003;146(2-3):729-58.
[21] Brans J-P, Mareschal B. Promethee Methods. *Multiple Criteria Decision Analysis: State of the Art Surveys* 2005:163-86.