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Capital Structure and Asset Prices: Some Effects of Bankruptcy Procedures*

We examine the impact of the U.S. bankruptcy procedure on the valuation of corporate securities and capital structure decisions. We provide closed-form solutions for corporate debt and equity values when defaulting firms can either liquidate their assets or renegotiate outstanding debt under court protection. We show that the possibility to renegotiate the debt contract (i) has an ambiguous impact on leverage choices and (ii) increases credit spreads on corporate debt. The sharing rule of cash flows during bankruptcy has a large impact on optimal leverage. By contrast, credit spreads on corporate debt show little sensitivity to this very parameter.

I. Introduction

The modeling of default is instrumental in determining the values of corporate securities and the firm's financing decisions since it affects firm value and its sharing among claimholders. Because of the limited liability principle, default is the shareholders' decision and can be endogenized by identifying the default policy that maximizes equity value. This point, first made by Black and Cox (1976), is developed in more recent works by Mello and Parsons (1992), Leland (1994), Goldstein, Ju, and Leland (2001), and Morellec (2001). In most of these models, default leads to an immediate liquidation of the firm's assets. In practice, however, a firm in financial distress can either liquidate its assets under Chapter 7 of the U.S. Bankruptcy Code or try to renegotiate outstanding debt in terms that are more affordable.

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The possibility of out-of-court renegotiations has been recognized by a stream of models, including those of Anderson and Sundaresan (1996), Mella-Barral and Perraudin (1997), and Fan and Sundaresan (2000). These papers incorporate, under the assumption of costless renegotiation, debt reliefs granted to firms in financial distress. Since liquidation is costly and bondholders bear liquidation costs, there is room for strategic default, that is, situations in which the firm pays less than the contractual debt service. Strategic default occurs each time the state variable—typically the value of the firm's assets—falls below some endogenous default threshold. In a rational expectations model, bondholders anticipate the opportunistic behavior of shareholders and reflect the associated wealth extraction in the pricing of corporate debt.

While contingent claims models of out-of-court renegotiations provide a useful framework to analyze private workouts, their applicability to court-supervised debt renegotiation is more limited. Indeed, renegotiations under Chapter 11 of the U.S. Bankruptcy Code differ from private workouts in a number of dimensions that affect the renegotiation process and, hence, the values of corporate securities. To our knowledge the only two contingent claims models that analyze the impact of Chapter 11 on debt values are those by Franks and Torous (1989) and Longstaff (1990). These both model Chapter 11 as the right to extend (once) the maturity date of the debt (a zero-coupon bond). The longer this extension privilege, the more valuable it is to shareholders and hence the larger the credit spread on corporate debt. Unfortunately, these articles do not analyze optimal leverage nor the decision to go bankrupt.

This article analyzes the impact of Chapter 11 on asset prices, capital structure choices, and the decision to default. For so doing, the framework we develop captures the following features of the U.S. bankruptcy procedure (see White 1996):

1. Bankruptcy filings can be initiated either voluntarily by managers or involuntarily by creditors. However, the U.S. bankruptcy law discourages involuntary filings by leaving the choice of the chapter filing—liquidation under Chapter 7 or reorganization under Chapter 11—to the manager.
2. Bankruptcy filings under Chapter 11 imply the automatic stay of assets that prevents bondholders from appropriating firm value by liquidating the firm's assets; Chapter 11 also imposes time and cost constraints on the renegotiation process.
3. The bankruptcy filing provides the defaulting firm with an observation period (also known as the exclusivity period) during which the court guarantees its survival; at the end of the observation period, the court decides whether the firm continues as a going concern or not.

Section II presents the valuation framework. In our setting, default can lead either to liquidation of the firm's assets or to renegotiation of the debt contract. In bankruptcy, the firm incurs costs of financial distress, and the cash flows it generates are shared among claimholders. Moreover, for firms that choose to renegotiate their claims under the court's protection, the default date is the starting point of a period during which the parties involved in the process (the claimholders and the court) observe the evolution of the value of the firm's assets. The firm emerges from financial distress if the value of its assets shows signs of recovery during the observation period. Otherwise, liquidation is pronounced at the end of the period.

In Section III we analyze the main implications of the model for asset prices and capital structure decisions. We show that the possibility to file for Chapter 11 and to renegotiate debt contracts has an ambiguous effect on firms' leverage choices. By contrast, the impact of debt renegotiation on shareholders' default incentives and credit spreads is unambiguous. Specifically, debt renegotiation encourages early default and increases credit spreads on corporate debt. Moreover, both the time and costs constraints associated with the renegotiation process affect the magnitude of these effects. The analysis in this article also reveals that the quantitative impact of claimholders' bargaining power on corporate financing decisions is important. By contrast, credit spreads on corporate debt show little sensitivity to this very parameter.

II. Chapter 11 and Asset Prices

This section presents the valuation framework. In order to emphasize the effects of Chapter 11 on asset prices and capital structure decisions, we take as benchmarks the models developed by Leland (1994) and Fan and Sundaresan (2000). Consequently, the model relies on the following assumptions.

Assumption 1. Assets are continuously traded in arbitrage-free and complete markets.

Assumption 2. The default-free term structure is flat with an instantaneous riskless rate r, at which investors may lend and borrow freely.

Assumption 3. Management acts in the best interests of shareholders.

Assumption 4. The firm has issued perpetual debt with contractual coupon payment c.[1]

Assumption 5. Corporate taxes are paid at a rate τ, and full offsets of corporate losses are allowed.[2]

Assumption 6. Upon liquidation, the firm incurs costs that represent a proportion α of the value of remaining assets. The proceeds from liquidation are distributed according to absolute priority.

Assumption 7. The value of the firm's assets $(V_t)_{t\geq 0}$ is independent of capital structure choices and is governed, under the risk-neutral measure Q,

1. Collin-Dufresne and Goldstein (2001) propose a model where leverage follows an exogenous mean-reverting process. Goldstein et al. (2001) analyze endogenous leverage changes.

2. The tax shield provided by debt financing is the only motive for issuing debt in the present model. Morellec (2004) develops a model of leverage choices where debt financing also increases firm value by constraining management's choice of an investment policy.

by the process

$$dV_t = (r - \delta)V_t dt + \sigma V_t dW_t. \quad (1)$$

In equation (1), δ is the firm's payout rate, σ is the constant volatility of asset returns, and $(W_t)_{t\geq 0}$ is a standard Brownian motion defined on the probability space $(\Omega, \mathcal{F}, Q)$.

Although the firm may operate its assets forever, it can also choose to default on its debt obligations. Within our framework, default leads either to liquidation of the firm's assets (under Chapter 7 of the U.S. Bankruptcy Code) or to renegotiation of the debt contract. While firms can renegotiate their debt privately, we focus hereafter on Chapter 11 filings, that is, court-supervised debt renegotiation. (We show below that our model encompasses those of Leland [1994] and Fan and Sundaresan [2000], which apply, respectively, to Chapter 7 filings and private workouts.)

Chapter 11 differs from Chapter 7 and private workouts in a number of dimensions that affect shareholders' incentives to default and, hence, the values of corporate securities. Key among these differences is the treatment of claimholders in default. When default leads to a Chapter 7 filing, the firm's assets are liquidated, and no renegotiation is possible. By contrast, private workouts allow claimholders to renegotiate their claims after default without any time constraint. Chapter 11 filings typically lie in between these two alternatives. Upon default, the court grants the firm a period of observation during which the firm can renegotiate its claims. At the end of this period, the court decides whether the firm continues as a going concern or not.

Empirical studies show that most firms emerge from Chapter 11. Only a few firms (5%, according to Gilson, John, and Lang [1990] and Weiss [1990]), and between 15% and 25%, according to Morse and Shaw [1988]) are eventually liquidated under Chapter 7 after filing Chapter 11. Why do some firms recover while some others do not? It is generally acknowledged (see Wruck 1990 or White 1996) that there exist two types of defaulting firms. First, firms that are economically sound promptly recover under Chapter 11. Default was only due to a temporary financial distress. Second, firms that are economically unsound keep on losing value under Chapter 11. Their financial indicators remain in the red. In that same spirit, we assume hereafter that the firm is liquidated only if the value of its assets reaches a level V_B—indicative of financial distress—and remains below that same level for a time interval larger than the period of observation. This specification for default and liquidation implies that firms emerging from Chapter 11 do not bear real economic costs (see Maksimovic and Phillips [1998] for empirical evidence). It also captures the fact that a large proportion of firms that emerge from Chapter 11 reenter bankruptcy in the following years (see, e.g., Hotchkiss 1995 or Gilson 1997).

In order to determine the value of corporate securities in this setting, we define the following random variables:

$$g_t^{V_B} = \sup\{s \leq t : V_s = V_B\}, \quad (2)$$

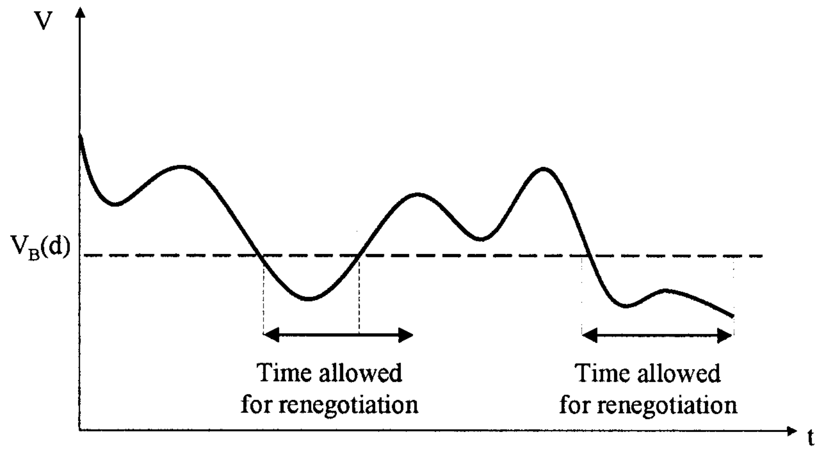

FIG. 1.—The liquidation criterion

$$\theta^{V_B} = \inf\{t \geq 0 : (t - g_t^{V_B}) \geq d, V_t \leq V_B\}, \quad (3)$$

where $g_t^{V_B}$ is the last time before t that the value of the firm's assets reaches the default threshold and θ^{V_B} is the liquidation time, that is, the first time the value of the firm's assets has spent d units of time consecutively below the default threshold. Figure 1 illustrates our modeling of the liquidation date. In our setup, shareholders hold a Parisian down-and-out call option on the firm's assets. That is, shareholders have a residual claim on the cash flows generated by the firm's assets unless the value of these assets reaches the default threshold and remains below that threshold for the exclusivity period.[3]

We assume in what follows that claimholders anticipate the same length d of the observation period, reflecting the balance of power between the court, the manager, and claimholders. The following examples are of special interest since they pertain to previous contributions of the literature.

EXAMPLE 1. When $d = 0$, default leads to an immediate liquidation and the liquidation date is described by the first passage time T^{V_B} of the state variable to V_B: $T^{V_B} = \inf\{t \geq 0 : V_t = V_B\}$. Thus when $d = 0$, we get as a special case the standard modeling of default and liquidation (see Leland 1994).

EXAMPLE 2. When $d \to +\infty$, default never leads to liquidation, and we get as a special case the standard modeling of default and renegotiation (see Anderson and Sundaresan 1996 or Fan and Sundaresan 2000).

In order to examine the implications of debt renegotiation for the values of corporate securities and the firm's default decision, we have to determine

3. To check whether this modeling of default and liquidation is consistent with the available empirical evidence, we carry out a Monte Carlo simulation. Details are reported in app. A.

the impact of a successful renegotiation on the cash flows accruing to claimholders. This impact depends on the value of the firm's assets triggering default, the costs of renegotiation, and the sharing rule for the cash flows from the firm's assets during strategic default.

As discussed above, we presume that the firm renegotiates its debt obligations whenever the value of its assets falls below a constant threshold V_B. In particular, we follow Fan and Sundaresan (2000, p. 1062) by considering that, when the firm is in default, "borrowers stop making the contractual coupon and start servicing the debt strategically until the firm's asset value goes back above the threshold again." In our framework, we first evaluate corporate securities for a given renegotiation boundary and then determine the selected threshold endogenously by maximizing equity value. The sharing rule for the cash flows from assets in place during default typically results from a bargaining game between firm claimants. Again, we follow Fan and Sundaresan (2000) by considering a Nash bargaining game. We denote the bargaining power of shareholders by η and the bargaining power of bondholders by $1 - \eta$. In contrast to previous models, we also incorporate in our analysis proportional costs borne by the firm during the renegotiation process, which we denote by φ. This assumption captures the fact that, even though the costs implied by private workouts generally are low (0.32% of the value total assets in the study by Gilson, John, and Lang [1990]), Chapter 11 filings are associated with large costs of financial distress (see Andrade and Kaplan 1998) that may affect shareholders' default decision.

Using the above set of assumptions, we can determine the allocation of the renegotiation surplus between firm claimants as follows. Assumption 6 implies that, if default leads to liquidation, equity is worthless upon default, while the value of bondholders' claims is $(1 - \alpha) V_B$. By contrast, if default leads to renegotiation, firm value is equal to $v(V_B)$ and is shared between firm claimants. Denoting the sharing rule for firm value upon default by θ, the incremental value accruing to shareholders is $\theta v(V_B) - 0$ and the incremental value to bondholders is $(1 - \theta) v(V_B) - (1 - \alpha) V_B$.[4] Therefore, the sharing rule for the renegotiation surplus upon default satisfies

$$\theta^* = \arg\max \left\{[\theta v(V_B)]^\eta [(1 - \theta) v(V_B) - (1 - \alpha) V_B]^{1-\eta}\right\}, \quad (4)$$

the solution to which is

$$\theta^* = \eta\left[1 - \frac{(1 - \alpha)V_B}{v(V_B)}\right]. \quad (5)$$

4. When the debt contract is renegotiated privately, bondholders are better off not liquidating the firm whenever $(1 - \theta) v(V_B) \geq (1 - \alpha) V_B$. When the firm files Chapter 11, the automatic stay of assets prevents bondholders from liquidating the firm's assets. However, adoption of the renegotiation plan requires that each participant receive a payoff that exceeds the liquidating value of its claims. Therefore, bondholders' payoff has to exceed $(1 - \alpha) V_B$, which, in turn, implies the above specification for the bargaining game.

The solution to the bargaining game between bondholders and shareholders can be interpreted as follows. The surplus associated with debt renegotiation is equal to $v(V_B) - (1-\alpha)V_B$. Therefore, equation (5) shows that the outcome of the renegotiation process allocates this surplus between financial stakeholders according to their bargaining power. Specifically, shareholders get

$$\theta^* v(V_B) = \eta[v(V_B) - (1-\alpha)V_B], \tag{6}$$

and bondholders get

$$(1-\theta^*)v(V_B) = (1-\eta)[v(V_B) - (1-\alpha)V_B] + (1-\alpha)V_B. \tag{7}$$

Equation (7) shows that the value of bondholders' claim upon default is equal to the liquidation value of the firm ($(1-\alpha)V_B$) plus a fraction $(1-\eta)$ of the surplus associated with renegotiation. This, in turn, implies that if shareholders have no bargaining power ($\eta = 0$), bondholders' claim is equal to firm value upon default. An equally important result is that the surplus shareholders extract from renegotiation increases with those very parameters that increase the difference between the ongoing value of the firm and its liquidation value. In particular, we show below that shareholders' surplus increases with the exclusivity period d, liquidation costs α, and the corporate tax rate τ and decreases with costs of financial distress φ and the volatility of returns on the firm's assets σ.

Equations (6) and (7) demonstrate that the value of shareholders' claim upon default depends on firm value. Before valuing corporate securities, it will thus be necessary to determine firm value. Within our model, firm value is equal to the value of the firm's unlevered assets plus the present value of the tax advantage of debt less the present value of costs of financial distress and liquidation costs. Assumption 1 ensures that there exists a unique probability measure Q under which the price of any asset, discounted at the riskless rate, is a martingale (see Harrison and Pliska 1981). Therefore, we can write firm value as

$$v(d,c) = \mathbb{E}_Q\left\{\int_0^{\theta^{V_B}} du \cdot e^{-ru}\left[\delta V_u + \tau c 1_{V_u > V_B} - \varphi V_u 1_{V_u < V_B}\right]\right\} - \alpha \mathbb{E}_Q\left\{e^{-r\theta^{V_B}} V_{\theta^{V_B}}\right\},$$

where 1_ω is the indicator function of the event ω.

This equation can be interpreted as follows. The first term on the right-hand side accounts for the cash flows generated by the firm until liquidation. This term reflects the fact that (i) costs of financial distress are incurred by the firm whenever it files Chapter 11 (third term in the square brackets) and (ii) tax benefits are akin to a security that pays a constant coupon τc as long as the firm has not declared Chapter 11 (second term).[5] By contrast, liquidation costs are borne only when the firm files Chapter 7 (second term on the right-hand side).

Solving this equation yields the following result.

PROPOSITION 1. When claimholders take into account the possibility of Chapter 11 filings, firm value satisfies for a given default threshold V_B:

$$v(d,c) = V + \frac{\tau c}{r} - \left[\frac{\varphi}{\delta}(\delta A(d) - C(d))V_B + \alpha V_B C(d) + \frac{\tau c}{r}B(d)\right]\left(\frac{V}{V_B}\right)^{-\xi}, \quad (8)$$

with $b = (1/\sigma)(r - \delta - \sigma^2/2)$, $\lambda = \sqrt{2r + b^2}$, and $\xi = (1/\sigma)(b + \lambda)$. In equation (8), we have

$$A(d) = \frac{1}{\lambda}\left[\frac{1}{\lambda + b + \sigma} + \frac{1}{\lambda - b - \sigma}\frac{\Phi(-\lambda\sqrt{d})}{\Phi(\lambda\sqrt{d})}\right],$$

$$B(d) = \frac{\lambda - b}{2\lambda} + \frac{\lambda + b}{2\lambda}\frac{\Phi(-\lambda\sqrt{d})}{\Phi(\lambda\sqrt{d})},$$

$$C(d) = \frac{\Phi\left[-(\sigma + b)\sqrt{d}\right]}{\Phi(\lambda\sqrt{d})},$$

and

$$\Phi(x) = 1 + x\sqrt{2\pi}\exp\left(\frac{x^2}{2}\right)\mathcal{N}(x),$$

with $\mathcal{N}$ the standard normal cumulative distribution function.

Proof of proposition 1. See appendix B.

The next step is to solve for the equity and debt values. For doing so, it is necessary to determine the impact of Chapter 11 filings on the cash flows accruing to claimholders. The above discussion shows that, within the present model, Chapter 11 filings affect the payoffs accruing to bondholders in two ways. First, Chapter 11 filings change the average liquidation time and firm value upon liquidation. In particular, the payoff accruing to bondholders upon liquidation (i.e., to be received at time θ^{V_B}) is $(1 - \alpha)V_{\theta^{V_B}}$. Second, bondholders lose part of the interest payments each time the firm files for Chapter 11. Specifically, if we denote by $s(V_t)$ the strategic debt service when the firm is in Chapter 11, coupon payments are defined at any time u, $0 \le u \le \theta^{V_B}$, by

$$c1_{V_u > V_B} + s(V_u)1_{V_u < V_B},$$

5. The treatment of the tax benefits of debt in default is complex and usually very idiosyncratic. The appendix in Gilson (1997) provides a detailed discussion of the general case. We follow Fan and Sundaresan (2000) by presuming that the firm does not benefit from the tax shield in bankruptcy. This choice of modeling allows for a direct comparison of our results with theirs.

whereas dividend payments are defined by

$$\left[\delta V_u - (1-\tau)c\right]1_{V_u > V_B} + \left[(\delta - \varphi)V_u - s(V_u)\right]1_{V_u < V_B}.$$

The payoffs to shareholders in and outside bankruptcy allow for a determination of the values of corporate securities. In particular, when claimholders anticipate the possibility of Chapter 11 filings, the values of corporate debt and equity are given by

$$D(d,c) = \mathbb{E}_Q\left\{\int_0^{\theta^{V_B}} du \cdot \mathrm{e}^{-ru}\left[c1_{V_u > V_B} + s(V_u)1_{V_u < V_B}\right]\right\}$$
$$+ \ (1-\alpha)\mathbb{E}_Q\left[e^{-r\theta^{V_B}} V_{\theta^{V_B}}\right],$$

and

$$E(d,c) = \mathbb{E}\left(\int_0^{\theta^{V_B}} du \cdot e^{-ru}\left\{\left[\delta V_u - (1-\tau)c\right]1_{V_u > V_B}\right.\right.$$
$$\left.\left. + \ [(\delta - \varphi)V_u - s(V_u)]1_{V_u < V_B}\right\}\right).$$

In order to solve these equations and determine the values of corporate securities, we proceed as follows. The value of equity can be written as the value of the perpetual stream to the current flow of income $(\delta V_u - (1-\tau)c)$ plus the option to default on the firm's debt obligation. This implies that, using the strong Markov property of Brownian motion, we can write equity value as

$$E(d,c) = V - \frac{(1-\tau)c}{r} + aV^{-\xi}, \tag{9}$$

where a is a constant and the first two terms on the right-hand side represent the unlimited liability value of equity. Consider now the value of the option to default. The sharing rule for the surplus upon default implies that the value of equity at the default time satisfies

$$E(V)\mid_{V=V_B} = \eta\left[v(V)\mid_{V=V_B} - (1-\alpha)V_B\right], \tag{10}$$

where firm value is given by equation (8). Combining equations (9) and (10) gives the value of equity for a given default threshold V_B. The value of corporate debt is then given by

$$D(V) = v(V) - E(V).$$

Solving these equations leads to the following proposition.

PROPOSITION 2. When claimholders take into account the possibility of

Chapter 11 filings, the values of corporate debt and equity satisfy for a given default threshold V_B

$$D(d,c) = \frac{c}{r}\left[1 - \left(\frac{V}{V_B}\right)^{-\xi}\right] + (1-\alpha)V_B\left(\frac{V}{V_B}\right)^{-\xi} + (1-\eta)R(d)\left(\frac{V}{V_B}\right)^{-\xi}, \tag{11}$$

and

$$E(d,c) = V - V_B\left(\frac{V}{V_B}\right)^{-\xi} - \frac{(1-\tau)c}{r}\left[1 - \left(\frac{V}{V_B}\right)^{-\xi}\right] + \eta R(d)\left(\frac{V}{V_B}\right)^{-\xi}, \tag{12}$$

where the renegotiation surplus at the time of default $R(d)$ satisfies

$$R(d) = \alpha V_B(1 - C(d)) - \frac{\varphi}{\delta}(\delta A(d) - C(d))V_B + \frac{\tau c}{r}(1 - B(d)),$$

and the factors $A(d)$, $B(d)$, and $C(d)$ are defined in proposition 1.

The closed-form expressions reported in proposition 2 for the values of corporate debt and equity can be interpreted as follows. The values of corporate debt and equity are equal to a perpetual entitlement to the current flow of income (respectively given by c and $\delta V_t - (1-\tau)c$) plus the change in values occurring in default. Because this change in values occurs if and when the firm defaults on its debt obligations, it has to be weighted by the Arrow Debreu price of this event ($(V/V_B)^{-\xi}$).

It is noteworthy that these expressions extend those obtained by Leland (1994) and Fan and Sundaresan (2000) to incorporate the possibility of a positive but finite maximum period of renegotiation. Indeed, when $d = 0$ or $\eta = 0$, equations (11) and (12) reduce to equations (7) and (13) in Leland (1994). These equations hold for situations in which (i) default leads to an immediate liquidation of the firm's assets or (ii) shareholders have no bargaining power and hence do not extract any surplus from renegotiation. When $d \to +\infty$ and renegotiation is costless ($\varphi = 0$), equations (11) and (12) are similar to those provided by Fan and Sundaresan (2000) where default never leads to liquidation.

Within our framework, we can also derive the probability of liquidation. This probability is given in the following proposition.

PROPOSITION 3. The probability of liquidation $P_L(d)$ obeys

$$P_L(d) = \left(\frac{V}{V_B}\right)^{1-[2(\mu-\delta)/\sigma^2]} \frac{\Phi\left(-\frac{\mu-\delta-\sigma^2/2}{\sigma}\sqrt{d}\right)}{\Phi\left(\frac{\mu-\delta-\sigma^2/2}{\sigma}\sqrt{d}\right)}, \tag{13}$$

where $\mu > \delta + \sigma^2/2$ stands for the mean total return on the firm's assets under the physical probability measure and $P_L(d)$ equals the product of the physical probability of default and the physical probability of liquidation, given default.

Proof of proposition 3. See appendix B.

The expression reported in proposition 3 for the liquidation probability again generalizes those of Leland (1994) and Fan and Sundaresan (2000) to account for a strictly positive but finite maximum period of renegotiation. Indeed, when $d = 0$, we have $\Phi(0) = 1$, and this probability reduces to the probability that the process (1) ever reaches V_B in the future. When $d \to +\infty$, the second term on the right-hand side of equation (13) tends to zero, and so does the probability of liquidation.

III. Applications and Analysis

We now turn to the implications of the model for the decision to default, credit spreads, and optimal leverage. While most of the results are derived analytically, part of the analysis relies on numerical examples. Input parameter values for these numerical simulations are set as follows:[6] the initial value of the firm's assets, $V = 100$; the riskless interest rate, $r = 6\%$; the net tax advantage of debt, $\tau = 20\%$; liquidation costs, $\alpha = 40\%$; costs of financial distress, $\varphi = 3\%$; the constant payout rate, $\delta = 5\%$; the exclusivity period, $d = 2$; and the mean total rate of return on V, $\mu = 13.5\%$. For each simulation, the coupon is set so as to maximize equity value at the time of debt issuance. In addition, we consider that the bargaining power is balanced equally between firm claimants, that is, $\eta = 0.5$.

A. *The Timing of Default*

When the decision to default is endogenous, the timing of default depends on the default threshold selected by shareholders. This default threshold can

6. These parameters roughly reflect a typical Standard & Poor's 500 firm. The net tax advantage of debt takes into account the corporate tax rate, the personal tax rate on equity income, and the tax rate on bond income. Liquidation costs are defined as the firm's going concern value minus its liquidation value, divided by its going concern value. Using this definition, Alderson and Betker (1995) and Gilson (1997) report liquidation costs equal to 36.5% and 45.5% for the median firm in their samples. Costs of financial distress lie between 3% and 7.5% of firm value 1 year before bankruptcy in the empirical studies by Warner (1977), Weiss (1990), or Betker (1997). The average time period between the indication of financial distress and its resolution is between 2 and 3 years for firms that renegotiate their claims under Chapter 11 of the U.S. Bankruptcy Code (see Franks and Torous 1989; Gilson 1997; or Helwege 1999). Also, Weiss and Wruck (1998) report that Eastern Airlines remained almost 2 years in Chapter 11 before filing Chapter 7.

be determined by characterizing the default policy that maximizes equity value. Using equation (12), we get the following result.

PROPOSITION 4. When claimholders take into account the possibility of Chapter 11 filings, the default threshold selected by shareholders is given by

$$V_B = \frac{\xi}{\xi + 1} \frac{c[1 - \tau + \eta\tau(1 - B(d))]}{r - r\eta[\alpha(1 - C(d)) - \frac{\varphi}{\delta}(\delta A(d) - C(d))]}. \qquad (14)$$

Proof of proposition 4. See appendix B.

The expression provided in proposition 4 for the default threshold extends those obtained by Leland (1994) and Fan and Sundaresan (2000) to incorporate the possibility of Chapter 11 filings. Indeed, when default leads to an immediate liquidation, we have $d = 0$ and equation (14) reduces to equation (14) in Leland:

$$V_B(d) = \frac{\xi}{\xi + 1} \frac{c(1 - \tau)}{r}.$$

This equation reveals that, if default leads to liquidation, it is optimal for shareholders to default when equity is worthless and the default threshold does not (directly) depend on bankruptcy costs.

When default never leads to liquidation and renegotiation is costless, we have $d \to \infty$ and $\varphi = 0$. Equation (14) then reduces to the expression determined by Fan and Sundaresan (2000) for private workouts:

$$V_B(\infty) = \frac{\xi}{\xi + 1} \frac{c(1 - \tau + \eta\tau)}{r(1 - \eta\alpha)},$$

where $V_B(\infty) \equiv \lim_{d\to\infty} V_B(d)$. More generally, proposition 4 reveals that the default threshold should depend on the maximum length of the renegotiation process, d. This result is not surprising since this parameter reflects the extent to which claimants can bargain over firm value.

The closed-form expression of the default threshold reported in proposition 4 allows us to compare the default threshold associated with Chapter 11 filings with those associated with private workouts or Chapter 7 filings. Specifically, using the inequalities

$$0 \leq \eta\tau[1 - B(d)] \leq \eta\tau,$$

and

$$r(1 - \eta\alpha) \leq r\left\{1 - \eta\left[\alpha(1 - C(d)) - \frac{\varphi}{\delta}(\delta A(d) - C(d))\right]\right\} \leq r,$$

we immediately get the following result.

PROPOSITION 5. The default threshold selected by shareholders satisfies for any $d \geq 0$ and $c \geq 0$:

$$V_B(0) \leq V_B(d) \leq V_B(\infty).$$

In addition, a sufficient condition for the threshold $V_B(d)$ to be lower than the liquidity threshold c/δ is

$$\delta \leq r(1 - \eta\alpha).$$

Proposition 5 characterizes the default policy that maximizes equity value for a given—exogenous—debt level c. Specifically, this proposition reveals that the default threshold associated with private workouts ($V_B(\infty)$) is greater than the default threshold associated with Chapter 11 filings ($V_B(d)$), which in turn is greater than the default threshold associated with Chapter 7 filings ($V_B(0)$). If default leads to liquidation, the decision to default is irreversible and it is optimal for shareholders to default when equity is worthless. When it is possible to renegotiate the debt contract, the decision to default no longer is irreversible and shareholders have incentives to default earlier in order to extract concessions from bondholders. Because the liquidation probability decreases with the length of the observation period, the shareholders' incentives to default early increase with d, inducing a negative relation between the anticipated length of the observation period and the default threshold V_B.[7]

An additional implication of proposition 5 is that, for most input parameter values, the borrower will default on its debt obligations in the region where the firm is cash constrained ($V_B < c/\delta$). This implies that equity will be issued to finance the contractual coupon during this period when V belongs to the interval $[V_B, c/\delta]$. To explore the determinants of the region of asset values where the firm is cash constrained, we can examine the impact of the input parameter values on its relative size defined by

$$\frac{c/\delta}{V_B} \equiv \frac{\xi + 1}{\xi} \frac{r - r\eta[\alpha(1 - C(d)) - \frac{\varphi}{\delta}(\delta A(d) - C(d))]}{\delta[1 - \tau + \eta\tau(1 - B(d))]}. \tag{15}$$

This expression reveals that the relative size of the interval $[V_B, c/\delta]$ decreases with those very parameters that increase shareholders' incentives to default early. In other words, the relative size of this interval (i) decreases with liquidation costs α, the length of the renegotiation process d, and shareholders'

7. While we do not address explicitly the choice between private workouts and Chapter 11 in this article, the above argument suggests that firms that renegotiate their debt privately will tend to have a higher going-concern value at the time of default. Gilson, John, and Lang (1990) find, in a sample of 169 financially distressed companies, that firm value at the time of default is related to the choice between private vs. legally supervised renegotiation. In particular, they document that firms with relatively high going-concern value are more likely to restructure privately. Our predictions are consistent with their results. However, our model suggests that the observed correlation is explained by the reverse causality. If shareholders have rational expectations, they anticipate the way financial distress will be resolved (see Gilson et al. [1990] for empirical evidence) and thus revise the default threshold accordingly.

TABLE 1 Comparative Statics of the Model

Scenario/ Exclusivity Period	Leverage Ratio	Default Threshold	Liquidation Probability	Credit Spread
Base case:				
$d = 0$	56.80	34.66	3.20	102
$d = 3$	62.88	47.48	2.14	140
$d = 5$	63.83	49.34	1.59	146
$\sigma = 15\%$:				
$d = 0$	62.66	42.56	.36	65
$d = 3$	64.96	53.20	.18	84
$d = 5$	65.50	55.59	.11	88
$r = 7\%$:				
$d = 0$	59.96	38.85	4.63	93
$d = 3$	65.08	53.10	3.09	133
$d = 5$	66.03	55.26	2.30	140
$\alpha = 50\%$:				
$d = 0$	53.34	32.09	2.49	96
$d = 3$	58.18	44.77	1.77	132
$d = 5$	59.22	46.98	1.36	139
$\varphi = 1\%$:				
$d = 0$	56.80	34.66	3.20	102
$d = 3$	64.47	51.10	2.72	152
$d = 5$	66.29	54.99	2.26	166

NOTE.—Reported are the optimal leverage ratio (in %), the optimal default threshold, the probability of liquidation (in %), and the credit spread (in basis points). Parameters for the base case are the riskless interest rate $r = 6\%$, the firm's payout rate $\delta = 5\%$, the volatility of the firm's assets $\sigma = 20\%$, the tax advantage of debt $\tau = 20\%$, liquidation costs $\alpha = 40\%$, costs of financial distress: $\varphi = 3\%$, shareholders' bargaining power $\eta = 0.5$, and the mean total rate of return on the firm's assets $\mu = 13.5\%$. The value of the firm's underlying assets is $V = 100$, and the default level V_B is determined endogenously. We examine deviations from the base case by respectively setting $\sigma = 15\%$, $r = 7\%$, $\alpha = 50\%$, and $\varphi = 1\%$.

bargaining power η, and (ii) increases with the corporate tax rate and costs of financial distress φ. Equation (15) also shows that the size of this region of asset values where the firm is cash constrained depends on the volatility of returns on assets in place, the firm's dividend yield, and the risk-free rate. In particular, because more volatility implies a greater value for the option of waiting to default, additional volatility increases the relative size of the region where the firm issues equity to finance the contractual coupon.

Additional implications concerning the behavior of the default threshold at optimal leverage are reported in table 1. As shown in this table, the default threshold, and thus equity value at the time of default, increases with the corporate tax rate τ and decreases with shareholders' bargaining power η, liquidation costs α, costs of financial distress φ, firm risk σ, and the total payout rate to claimholders δ. It is noteworthy that some of these effects go in the opposite direction to that predicted by equation (14) for a given coupon payment. In other words, these effects are essentially driven by the impact of these very parameters on optimal leverage, as we now illustrate.

B. Optimal Leverage

Before turning to the analysis of leverage decisions, it is important to make a clear distinction between the value of equity ex ante—at the time of debt issuance—and ex post—after debt has been issued. The value of equity ex post is given by the present value of the cash flows accruing to shareholders after the debt has been sold (see eq. [12]). The value of equity ex ante is given by the sum of the value of equity ex post and the market value of debt at the time it is issued. As a result, although the default threshold typically is selected ex post to maximize equity value, optimal leverage is determined ex ante to maximize firm value.

Within the present model, investment policy is fixed and capital structure decisions result from a trade-off between the tax advantage of debt and bankruptcy costs. Optimal leverage is defined by

$$L(d, c^*) = \frac{D(d, c^*)}{v(d, c^*)},$$

where c^* is the value-maximizing coupon payment. The following proposition relates this coupon payment to the various parameters of the model.

Proposition 6. When claimholders take into account the possibility of Chapter 11 filings, the value-maximizing coupon payment satisfies

$$c^* = V\left((\xi + 1)\left\{B(d)\beta^{\xi} + \frac{r}{\tau}\beta^{\xi+1}\left[\alpha C(d) + \frac{\varphi}{\delta}(\delta A(d) - C(d))\right]\right\}\right)^{-1/\xi}, \quad (16)$$

where

$$\beta = \frac{\xi}{\xi + 1}\frac{1 - \tau + \eta\tau(1 - B(d))}{r - r\eta[\alpha(1 - C(d)) - \frac{\varphi}{\delta}(\delta A(d) - C(d))]}.$$

Equation (16) reveals that the bargaining power of shareholders η, liquidation costs α, the corporate tax rate τ, and costs of financial distress φ affect the value-maximizing coupon payment. In particular, leverage should increase with corporate tax rate τ (the benefit of debt) and decrease with liquidation costs α and costs of financial distress φ (the costs of debt). In addition, the larger the bargaining power of shareholders, the greater the cost of debt and hence the lower optimal leverage. These effects are illustrated in table 1 and in figure 2*c*, which report the firm's leverage choices in different economic environments.

While it is not possible to determine analytically the impact of debt renegotiation on the debt level selected by shareholders, it is clear from equation (16) that optimal leverage depends on the exclusivity period. In other words, one of the predictions of the present model is that the treatment of claimholders in default has an impact on the capital structure selected ex ante by firms. Figure 2 plots optimal leverage, credit spreads, and the selected default threshold as a function of the exclusivity period for different levels of shareholders' bargaining power. In this figure, we consider three different scenarios: a take-it-or-leave-it offer by bondholders ($\eta = 0$), a take-it-or-leave-it offer by share-

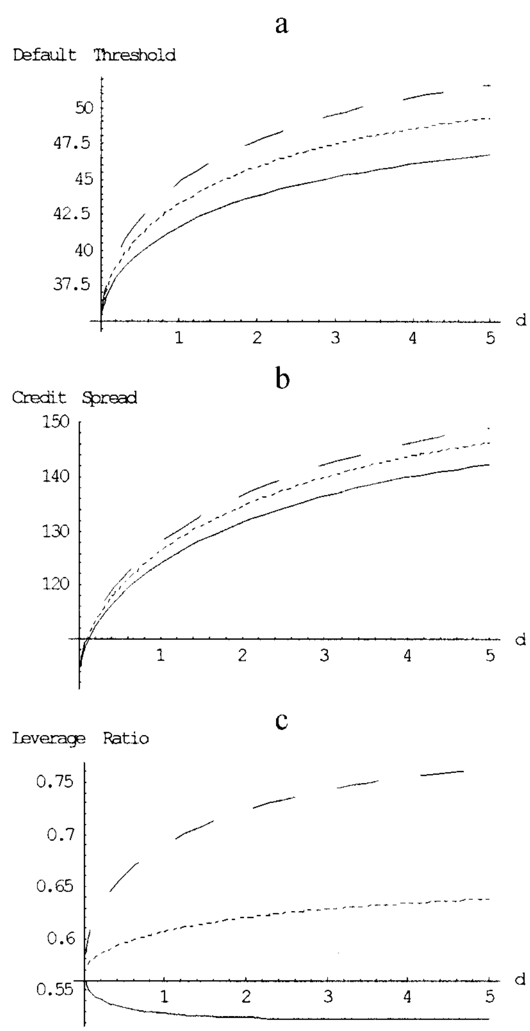

FIG. 2.—Shareholders' bargaining power and corporate debt. Parts *a*, *b*, and *c* plot, respectively, the equilibrium default threshold, credit spreads (in basis points), and optimal leverage as a function of the exclusivity period for various shareholders' bargaining powers. Parameters are set as in the base case. Shareholders' bargaining power is $\eta = 0$ (long-dashed line), $\eta = .5$ (short-dashed line), and $\eta = 1$ (thin line).

holders ($\eta = 1$), and a game where the bargaining power is balanced equally between firm claimants ($\eta = 0.5$).

In our base case, optimal leverage is roughly 57% when liquidation immediately follows default. Allowing for renegotiation affects optimal leverage, either increasing it or decreasing it, as figure 2*c* illustrates. Allowing for renegotiation after default reduces the present value of liquidation costs. On the other hand, every observation period is associated with costs of financial distress and tax shield reductions that make debt less attractive. Figure 2*c* shows that the impact of these two effects on optimal leverage strongly depends on shareholders' bargaining power. If debtors have most of the bargaining power, the first effect dominates and leverage decreases as the constraints put on the renegotiation process decrease. By contrast, if creditors have most of the bargaining power, the second effect dominates and leverage increases with the length of the exclusivity period. Overall, the analysis suggests that the bargaining power of claimholders in default has a large impact on ex ante leverage choices. It also shows that bankruptcy procedures may help explain the wide variety of debt levels observed in practice.

C. Credit Spreads

For any promised coupon payment c, credit spreads on corporate debt are defined by

$$y(d,c) = \frac{c}{D(d,c)} - r.$$

While this equation can be solved in closed form, the analytical expression of the credit spreads is complex and does not provide any economic intuition. Therefore, we do not report this expression here and instead base the analysis of credit spreads on numerical examples. Before proceeding to this analysis, it should be noted that the level of credit spreads generated by our model is consistent with observed quantities. Using input parameter values that are in line with U.S. current economic conditions, table 1 reports credit spreads ranging from 65 to 166 basis points. On the empirical side, Duffee (1998), for instance, reports credit spreads between 67 and 184 basis points on a sample of 2,814 noncallable bonds quoted from 1985 to 1995.

Let us now turn to analyzing the determinants of the levels of credit spreads. Consider first the impact of the exclusivity period on credit spreads. Figure 2*b* indicates that credit spreads increase with the length of the exclusivity period. Within the present model, the possibility to file for Chapter 11 has two major effects on credit spreads. First, the probability of liquidation decreases with d. Liquidation costs, which are at the expense of creditors (ex post), are therefore less imminent. Second, Chapter 11 filings involve observation periods during which the debt service is potentially reduced. Simulations results indicate that the second effect dominates at optimal leverage so that credit spreads increase with d.

Consider next the impact of shareholders' bargaining power. Interestingly, figure 2*b* shows that credit spreads at optimal leverage display little sensitivity to the allocation of bargaining power between claimholders. This result arises within the present model because of two distinct effects of this sharing rule on credit spreads. First, the default threshold selected by shareholders decreases with η (see fig. 2*a*). Second, shareholders' bargaining power increases the cost of debt and thus makes debt less attractive (see fig. 2*c*). These two effects partially offset each other and make credit spreads vary only by a few basis points when η changes. This behavior of credit spreads sharply contrasts with that of leverage (see fig. 2*c*): in our model, a change in claimholders' balance of power significantly affects leverage ratios but has a very limited impact on credit spreads.

Finally, data in table 1 also reveal that credit spreads are negatively correlated with the riskless interest rate, which is now a well-documented behavior of corporate bonds. In addition, credit spreads are decreasing with liquidation costs at optimal leverage. This result arises because higher liquidation costs induce shareholders to lower the optimal coupon, which in turn reduces the default threshold. These two effects combined decrease the riskiness of debt. Other standard comparative statics apply, and so we do not report them.

IV. Conclusion

This article presents a contingent claims model that captures some of the fundamental features of Chapter 11 and investigates their impact on corporate financing decisions. In our setting, Chapter 11 filings allow shareholders to benefit from the automatic stay of assets during a fixed period of time granted by the court. Defaulting on the debt contract implies costs of financial distress to be paid during the observation period. In return, it avoids immediate liquidation and redefines the payoffs that, respectively, accrue to both shareholders and bondholders. In this context, the decision to default depends on the degree of protection offered to shareholders by Chapter 11 pondered by the inevitable costs of financial distress.

The model shows that the possibility to file for Chapter 11 and to renegotiate debt contracts has an unambiguous effect on both shareholders' default incentives and credit spreads. Specifically, debt renegotiation encourages early default and increases credit spreads on corporate debt. By contrast, the impact of debt renegotiation on firms' leverage choices is ambiguous. In particular, the model shows that, when debtors (creditors) have most of the bargaining power, we can expect leverage to decrease (increase) with the constraints put on the renegotiation process. The analysis in this article also reveals that the quantitative impact of claimholders' bargaining power on corporate financing decisions is important. By contrast, credit spreads on corporate debt show little sensitivity to this very parameter. Finally, although simulations have been calibrated with respect to U.S. estimates for Chapter 11 modeling, our framework is flexible enough to capture other bankruptcy procedures that involve observation periods with the automatic stay of assets.

Appendix A

Monte Carlo Simulations

To generate firm value sample paths, the base case parameters (see the discussion in Sec. III) are used: $V = 100$, $\mu = 13.5\%$, $\delta = 5\%$, and $\sigma = 20\%$. Other parameters are $d = 2$, $r = 6\%$, $\alpha = 40\%$, $\varphi = 3\%$, $\tau = 20\%$, and $\eta = 0.5$. The optimal default threshold is $V_B = 45.8891$. To choose the appropriate time step, we first remark that bankruptcy can only be declared with the board's approval. In a sample of 1,382 observations for 307 firms over the years 1990–94 (or 90.1% of the maximum possible observations in a sample of the 350 largest firms that are listed in the Forbes compensation survey for 1992), Vafeas (1999) documents that board meeting frequency was seven times a year on average and exceeded 20 times a year for only one firm. In addition, Vafeas finds that board meeting frequency increases as firm value deteriorates. We therefore report simulations with time steps ranging from weekly (52 times a year) to two-monthly observations (six times a year). For every simulation, we have generated 100,000 paths over a 10-year period.

Results are summarized in tables A1 and A2. Table A1 gives the number of firms (among the 100,000) that experienced one or several Chapter 11 filings without being liquidated at the end of the 10-year period (i.e., firms for which the process V_t reached the default barrier at least once over the 10-year period). For instance, with a monthly time step, we find that 2,180 firms defaulted over the 10-year period without being liquidated. In addition, this table sorts this total number of nonliquidated defaulting firms into five subsets according to the number of times the firm defaulted. For instance, in this subsample of 2,180 firms, 540 defaulted twice without being liquidated. Table A2 reports similar statistics for firms that experienced one or several Chapter 11 filings with the last one leading to liquidation.

Observe that the simulated liquidation rate of defaulting firms is robust across various time steps (between 13.9% and 15.3%) and is in line with the rates reported in the empirical literature. Also, while repeated Chapter 11 filings are a bit numerous for some board meeting frequencies we have selected, they remain reasonably limited. For example, with a monthly time step, among the 2,533 firms that defaulted at least once in 10 years, 1,986 firms (78%) have had less than four filings.

TABLE A1 **Monte Carlo Simulations for Chapter 11 Filings and Liquidation Results for Defaulting Firms Not Liquidated**

Time Step	Firms That Defaulted	Firms Sorted by Number of Times They Defaulted				
		1	2	3	4	≥ 5
1/6	1,810	878	486	242	119	85
	(100)	(48.5)	(26.8)	(13.4)	(6.6)	(4.7)
1/12	2,180	811	540	350	211	268
	(100)	(37.2)	(24.8)	(16.0)	(9.7)	(12.3)
1/24	2,469	648	540	373	292	616
	(100)	(26.2)	(21.9)	(15.1)	(11.8)	(25.0)
1/52	2,727	474	406	395	298	1,154
	(100)	(17.4)	(14.9)	(14.5)	(10.9)	(42.3)

NOTE.—The numbers in parentheses represent the ratio of the number of firms that defaulted 1, 2, 3, 4, and 5 or more times to the total number of firms that defaulted over the 10-year period.

TABLE A2 Monte Carlo Simulations for Chapter 11 Filings and Liquidation Results for Defaulting Firms Liquidated

		Firms Sorted by Number of Times They Defaulted				
Time Step	Firms Liquidated	1	2	3	4	≥ 5
1/6	326	151	100	51	14	10
	(100)	(46.3)	(30.7)	(15.6)	(4.3)	(3.1)
1/12	353	110	105	70	28	40
	(100)	(31.2)	(29.8)	(19.8)	(7.9)	(11.3)
1/24	402	117	82	63	49	91
	(100)	(29.1)	(20.4)	(15.7)	(12.2)	(22.6)
1/52	466	91	87	66	52	170
	(100)	(19.5)	(18.7)	(14.2)	(11.1)	(36.5)

NOTE.—The numbers in parentheses represent the ratio of the number of liquidated firms that defaulted 1, 2, 3, 4, and 5 or more times before being liquidated to the total number of liquidated firms.

Appendix B

Proofs of Propositions 1, 3, and 4

Proof of proposition 1. Firm value is given by

$$v(d,c) = \mathbb{E}_Q\left\{\int_0^{\theta^{V_B}} du \cdot e^{-ru}\left[\delta V_u + \tau c 1_{V_u>V_B} - \varphi V_u 1_{V_u<V_B}\right]\right\} - \alpha \mathbb{E}_Q\left\{e^{-r\theta^{V_B}} V_{\theta^{V_B}}\right\}.$$

The first term on the right-hand side of this equation can be decomposed as follows

$$\mathbb{E}_Q\left\{\int_0^{\theta^{V_B}} du \cdot e^{-ru}\delta V_u 1_{V_u>V_B}\right\} + \mathbb{E}_Q\left\{\int_0^{\theta^{V_B}} du \cdot e^{-ru}\left[\tau c 1_{V_u>V_B} + (\delta - \varphi) V_u 1_{V_u<V_B}\right]\right\}.$$

Writing $Z_u = W_u + bu$ with $b = (r - \delta - \sigma^2/2)/\sigma$ and $a = (1/\sigma)\ln(V_B/V) < 0$ yields

$$\mathbb{E}_Q\left\{\int_0^{\theta^{V_B}} du \cdot e^{-ru} V_u 1_{V_u>V_B}\right\} = \mathbb{E}_Q\left\{\int_0^{\theta^a} du \cdot V e^{\sigma Z_u} 1_{W_u^b>a} e^{-ru}\right\}.$$

Note that $a < 0$ since the firm is not in default at the initial date. Applying the Girsanov theorem with the density

$$\forall t \geq 0,\ \left.\frac{dQ}{dP}\right|_{\mathcal{F}_t} = e^{bZ_t - (b^2/2)t},$$

we get

$$V\mathbb{E}_P\left\{\int_0^{\infty} du \cdot e^{(\sigma+b)Z_u} e^{-(\lambda^2/2)u} 1_{Z_u>a}\right\} - V\mathbb{E}_P\left\{\int_{\theta^a}^{\infty} du \cdot e^{(\sigma+b)Z_u} e^{-(\lambda^2/2)u} 1_{W_u^b>a}\right\},$$

where $\lambda = \sqrt{2r + b^2}$ and $(Z_t)_{t\geq 0}$ is a standard Brownian motion under P. Let us now rely on the following lemma (see, e.g., Karatzas and Shreve [1991, p. 272], for a derivation).

LEMMA 1. If $\varphi : \mathbb{R} \to \mathbb{R}$ is a piecewise continuous function with

$$\int_{-\infty}^{\infty} dy \, |\varphi(x+y)| e^{-|y|\sqrt{2\alpha}} < \infty; \ \forall x \in \mathbb{R}$$

for some constant $\alpha > 0$ and $(W_t, t \geq 0)$ is a standard Brownian motion, the resolvent operator of Brownian motion $K_\alpha(\varphi)$ is defined by

$$K_\alpha(\varphi) \stackrel{\triangle}{=} \mathbb{E}\left\{\int_0^\infty dt \cdot e^{-\alpha t}\varphi(W_t)\right\} = \frac{1}{\sqrt{2\alpha}}\int_{-\infty}^{\infty} dy \cdot \varphi(y) e^{-|y|\sqrt{2\alpha}}.$$

Using lemma 1, we can write the first integral as

$$\mathbb{E}_P\left\{\int_0^\infty du \cdot e^{(\sigma+b)Z_u} e^{-(\lambda^2/2)u} 1_{Z_u > a}\right\} = \int_a^\infty dy \frac{1}{\lambda} e^{-|y|\lambda} e^{(\sigma+b)y}.$$

After simplifications, we finally obtain, with $\xi = (b+\lambda)/\sigma$,

$$\mathbb{E}_P\left\{\int_0^\infty du \cdot \delta V e^{(\sigma+b)Z_u} e^{-\lambda u} 1_{Z_u > a}\right\} = V - \frac{\delta V_B}{\lambda^2 + (\sigma+b)\lambda}\left(\frac{V}{V_B}\right)^{-\xi}.$$

Thanks to the equality in law between $\theta^a(Z)$ and $\theta^0(Z) + T^a(Z)$ for two independent copies and the independence of $T^a(Z)$ and $\theta^0(Z)$, we can write

$$\begin{aligned}
&\mathbb{E}_P\left\{\int_{\theta^a}^\infty du \cdot V e^{(\sigma+b)Z_u} e^{-(\lambda^2/2)u} 1_{Z_u > a}\right\} \\
&= V\mathbb{E}_P\left\{e^{-(\lambda^2/2)T^a(Z_u)} \int_{\theta^0}^\infty du \cdot e^{(\sigma+b)(Z_u+a)} e^{-(\lambda^2/2)u} 1_{Z_u > 0}\right\} \\
&= V e^{(\sigma+b)a - |a|\lambda} \mathbb{E}_P\left\{e^{-(\lambda^2/2)\theta^0} \int_0^\infty du \cdot e^{(\sigma+b)Z_{u+\theta^0}} e^{-(\lambda^2/2)u} 1_{Z_{u+\theta^0} > 0}\right\}.
\end{aligned}$$

In order to solve this equation, we will use the following lemma due to Chesney, Jeanblanc, and Yor (1997).

LEMMA 2. If $(W_t)_{t\geq 0}$ is a Brownian motion starting from zero, the Laplace transform of the first time a negative Brownian excursion lasts more than d is

$$\forall \gamma \geq 0, \ \mathbb{E}_Q[\exp(-\gamma\theta^0(W))] = 1/\Phi\left(\sqrt{2\gamma d}\right),$$

where $\Phi(x) = \int_0^{+\infty} z \exp(zx - \frac{z^2}{2}) dz = 1 + \sqrt{2\pi} x \exp(\frac{x^2}{2}) \mathcal{N}(x)$.

Using lemma 2 and the distribution of Z_{θ^0} (see Chesney et al. 1997), we have

$$\begin{aligned}
&\mathbb{E}_P\left\{\int_{\theta^a}^{\infty} du V e^{(\sigma+b)Z_u} e^{-(\lambda^2/2)u} 1_{Z_u>a}\right\} \\
&= \frac{V_B(V_B/V)^{\xi}}{\Phi(\lambda\sqrt{d})}\int_{\Re_-} P(Z_{\theta^0}\in dx)e^{(\sigma+b)x}\mathbb{E}_P\left\{\int_0^{\infty} du\cdot e^{(\sigma+b)Z_u}e^{-(\lambda^2/2)u}1_{Z_u>-x}\right\} \\
&= \frac{-V}{\Phi(\sqrt{2\lambda d})}\left(\frac{V_B}{V}\right)^{\xi+1}\int_{\Re_-} dx\frac{x}{d}e^{(\sigma+b)x-x^2/2d}\int_{-x}^{\infty} dy\frac{1}{\lambda}e^{(\sigma+b-\lambda)y} \\
&= \frac{V_B}{\lambda}\left(\frac{V}{V_B}\right)^{-\xi}\frac{\Phi(-\lambda\sqrt{d})}{\Phi(\lambda\sqrt{d})}\frac{1}{\lambda-\sigma-b}.
\end{aligned}$$

This last result completes the computation of the first expectation.

Consider next the term

$$(\delta-\varphi)\mathbb{E}_Q\left\{\int_0^{\theta^{V_B}} du\cdot e^{-ru}V_u 1_{V_u<V_B}\right\}.$$

This term can be decomposed into

$$\mathbb{E}_Q\left\{\int_0^{\theta^{V_B}} du\cdot e^{-ru}V_u\right\} - \mathbb{E}_Q\left\{\int_0^{\theta^{V_B}} du\cdot e^{-ru}V_u 1_{V_u>V_B}\right\}.$$

Using the same line of reasoning as in the previous calculation, it is then easy to show that

$$\mathbb{E}_Q\left\{\int_0^{\theta^{V_B}} du\cdot e^{-ru}V_u\right\} = \frac{1}{\delta}\left[V - \frac{\Phi(-(\sigma+b)\sqrt{d})}{\Phi(\lambda\sqrt{d})}V_B\left(\frac{V}{V_B}\right)^{-\xi}\right].$$

Then the last term satisfies

$$\mathbb{E}_Q\left\{\int_0^{\theta^{V_B}} du\cdot e^{-ru}1_{V_u>V_B}\right\} = 1 - \left[\frac{\lambda-b}{2\lambda} + \frac{\lambda+b}{2\lambda}\frac{\Phi(-\lambda\sqrt{d})}{\Phi(\lambda\sqrt{d})}\right]\left(\frac{V}{V_B}\right)^{-\xi}.$$

Finally, the liquidation value of the firm is given by

$$(1-\alpha)\mathbb{E}_Q\left[e^{-r\theta^a}V_{\theta^a}\right].$$

Applying the Cameron-Martin-Girsanov theorem using the Radon-Nykodim density defined above, we get

$$\mathbb{E}_Q\left[e^{-r\theta^a}V_{\theta^a}\right] = V_B\mathbb{E}_P\left[e^{ba}e^{-(r+b^2/2)\theta^a}e^{-(\sigma+b)m_1\sqrt{d}}\right],$$

where m_1 is the Brownian meander taken at time 1.[8] Under P, the meander m_1 and θ^a are independent. Hence, using lemma 2, the equality in law between $\theta^a(Z)$ and $\theta^0(Z) + T^a(Z)$, the independence of $T^a(Z)$ and $\theta^0(Z)$, and the law of the Brownian meander, we get

$$\mathbb{E}_Q\left[e^{-r\theta^a}V_{\theta^a}\right] = V_B\left(\frac{V}{V_B}\right)^{-\xi}\frac{\Phi(-(b+\sigma)\sqrt{d})}{\Phi(\lambda\sqrt{d})}.$$

Proof of proposition 3. The Laplace transform of the liquidation time is given by

$$\mathbb{E}\left[\exp(-\gamma\theta^a(Z))\right] = \mathbb{E}\left[\exp(-\gamma\theta^a(Z))\cdot 1_{\theta^a(Z)<\infty}\right].$$

Therefore, the probability of liquidation, P_L, satisfies

$$P_L(d) = P(\theta^a(Z)<\infty) = \lim^{\gamma\to 0}\mathbb{E}[\exp(-\gamma\theta^a(Z))].$$

Applying lemma 2, we get, with $a = (1/\sigma)\ln(V_B/V)$ and $b = (1/\sigma)[\mu - \delta - (\sigma^2/2)] > 0$,

$$P_L(d) = \exp(2ab)\frac{\Phi(-b\sqrt{d})}{\Phi(b\sqrt{d})}.$$

Proof of proposition 4. The default threshold selected by shareholders is the one that maximizes equity value

$$E(d,c) = V - V_B\left(\frac{V}{V_B}\right)^{-\xi} - \frac{(1-\tau)c}{r}\left[1 - \left(\frac{V}{V_B}\right)^{-\xi}\right]$$
$$+ \eta\left[\alpha V_B(1-C(d)) - \frac{\varphi}{\delta}(\delta A(d) - C(d))V_B + \frac{\tau c}{r}(1-B(d))\right]\left(\frac{V}{V_B}\right)^{-\xi}.$$

The first-order condition with respect to V_B gives

$$0 = -(\xi+1)aV_B^{\xi} + \xi b V_B^{\xi-1},$$

with

$$a = V^{-\xi}\left\{1 - \eta\left[\alpha(1-C(d)) - \frac{\varphi}{\delta}(\delta A(d) - C(d))\right]\right\},$$

$$b = V^{-\xi}\frac{[1-\tau+\eta\tau(1-B(d))]c}{r}.$$

The solution to this equation is reported in proposition 2. The second-order condition

8. If we fix a real number t, the Brownian meander is the process

$$\left(m_u = \frac{1}{\sqrt{t-g_t}}\,|W_{g_t+u(t-g_t)}|,\ 1 \geq u \geq 0\right).$$

gives

$$\xi\left[-(\xi + 1)aV_B^{\xi-1} + \xi bV_B^{\xi-2}\right] - \xi bV_B^{\xi-2} \leq 0.$$

The first term on the right-hand side corresponds to the first-order condition and is equal to zero. The second term is negative if $b \geq 0$, which is immediate for $\tau \leq 1$ and $\eta \geq 0$.

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