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Statistical Analysis of Structural Glass Strength

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Abstract

In contrast to most other building materials the strength of glass is not a material parameter, but rather a combination of the material treatment and the load position effects. Structural glass elements are produced with heat treatment when compressive residual stresses are created on the surface. The loading position of glass specimen is also significant due to variations in material quality. These two parameters have an effect on the failure stress of glass specimens tested in bending. In this study, the importance of these two parameters is analyzed using a series of experimental data that was previously published. The data is utilized as a source for statistical analysis using the method of sweeping, least square algorithm and finally analysis of variance.

Keywords: glass, structural design, testing, sweeping, least square algorithm, analysis of variance

1. *Introduction*

Conventionally glass is used as a window element. In order to increase the use of glass as a load bearing element, further studies are required. Glass is a challenging building material due to its brittleness and the difficulty of defining its strength-related parameters.

The strength of glass is not a constant material parameter, but is rather dependent on the processing quality and the damage on the glass surface. Euronorm provides specifications for the characteristic strength of different types of treated glass (i.e. annealed, heat strengthened and fully tempered glass). All these three treatments are referred to soda-lime silicate float glass with different levels of residual stresses due to a heat treatment [1]. Annealed glass has no residual stresses and fully tempered glass has the highest level of residual stresses. Treatment has an effect on the strength and the fragmentation of element in failure.

There exists a significant controversy in the literature on the approach used to attain the Euronorm values [1]. The failure stress values differ considerably between specimens when they are tested in standing or lying position.

The aim of this study is to analyze statistically the experimental data previously published [2]. The experiments studied the influence of the treatment and the position on failure behavior.

Sweeping and least square method with dummy variable approach is used to evaluate the effect of the factors. The outcomes of the two analyses are compared. In both cases, averaged values and direct experiment data are compared.

2. *Experimental data*

The data used for statistical analysis was obtained from an experimental study, where several specimens were loaded until failure in a four-point bending measuring the failure stress [2].

*In alphabetical order

Specimens tested were glass elements of 1000×100×10 mm in size. The tests were performed with two loading positions and three treatments. Of these specimens one third was fully tempered, one third heat strengthened and one third was in the original, annealed state. Of each group of specimen half were tested standing and half lying, resulting in six groups of data [2].

The experimental results are presented in Table 1 [2]. The outcome of the experiments demonstrates a significant spread in the failure stress values. Lying annealed glass has a failure stress range between 25.8 MPa and 58.6 MPa, while standing annealed glass has a range of 21.2 MPa to 39.1 MPa. The interval is even larger for heat strengthened glass: Lying heat strengthened fails at a stress varying from 58.8 MPa to 167 MPa and standing heat strengthened from 54.9 MPa to 95.8 MPa. Finally, the failure stress of lying fully tempered glass differ between 96.1 MPa to 205.1 MPa and that of standing fully tempered glass vary within the interval of [75.6, 122.1] MPa. The aim of study is to statistically model these experimental data in order to understand the effect of different treatments and specimen position in glass failure prediction.

Table 1. 4 point bending failure stress of all specimens in MPa [2].

Test	Annealed lying	Annealed standing	Heat strenghened lying	Heat strenghened standing	Fully tempered lying	Fully tempered standing
1.0	25.8	21.2	58.8	54.9	96.1	72.6
2.0	28.3	21.4	65.0	56.1	107.7	74.5
3.0	30.7	23.2	71.6	57.1	120.2	76.7
4.0	31.2	23.7	74.5	57.3	130.9	80.5
5.0	32.1	23.9	81.4	57.7	133.6	85.1
6.0	36.1	24.1	81.6	59.3	135.3	88.4
7.0	36.6	24.2	84.2	59.6	139.2	89.1
8.0	37.7	24.6	85.9	64.3	145.5	89.8
9.0	38.4	24.7	90.7	66.2	146.8	91.0
10.0	38.8	24.9	95.7	68.9	147.1	92.2
11.0	39.9	24.9	99.6	69.5	147.2	92.5
12.0	40.9	25.5	99.9	70.1	147.3	95.0
13.0	41.4	25.6	103.1	72.9	147.7	96.2
14.0	41.7	25.6	104.1	74.0	153.0	96.7
15.0	42.6	26.2	106.3	74.6	156.2	97.3
16.0	46.7	26.7	111.7	75.7	166.4	99.7
17.0	47.3	27.4	119.7	76.0	172.3	100.2
18.0	47.7	27.8	125.6	78.2	182.8	102.8
19.0	47.9	31.6	128.1	79.6	184.1	103.4
20.0	53.8	32.7	133.4	82.2	186.7	103.5
21.0	54.3	33.5	149.1	95.8	191.4	103.6
22.0	54.5	38.2	154.6		191.5	107.4
23.0	55.8	39.1	167.0		197.0	109.4
24.0	58.6				205.0	122.1
25.0					205.1	
Average	42.0	27.0	104.0	69.0	157.4	94.6

3. *Statistical analysis*

The statistical approaches used are Sweeping with two factors (i.e. treatment and position) and Least square algorithm with dummy variables applied for five factors.

3.1 Sweeping

3.1.1 Description of the method

Sweeping is applied for two types of matrices. In both cases, the analysis is made for two factors including three treatments and two positions for each case. The three treatments are annealing (X1), heat strengthened (X2) and fully tempered (X3). The positions are lying (X4) and standing (X5).

The cases are:

1. A 2x3 Sweeping matrix using average values of the experimental data, see Table 2.

2. A 40x3Sweeping matrix using 20 first values of the experimental data, see Table 1.

Table 2. Matrix using average values calculated from 20 first data points.

	X1	X2	X3
X4	39.3	96.0	147.3
X5	25.5	67.7	91.4

The decomposition of the matrix includes calculating first the mean value of the data and then separating the column effect (treatment), row effect (position) and the residual. The interaction effect cannot be included due to insufficient degree of freedom. The same procedure is carried out for the 40x3 matrix taking into account the effect of interactions.

The effect of interaction is computed only for the matrix with the first 20 values.

3.1.2 Results

The results were analyzed using analysis of variances (ANOVA). When only average values are used, treatment and position parameters have high probabilities (17% and 23%) of randomness, see Table 3.

Table 3. ANOVA results for the analysis using average values.

Source	**ss**	**df**	**ms**	**F**	**P**
Data	46048	6			
Mean	36379	1			
Treatment	7608	2	3804	16.6	17.10%
Position	1603	1	1603	7	23.02%
Residual	459	**2**	229	1	
Total	46048	6			

In comparison with the treatment parameter, the position parameter has a higher randomness probability, see Table 3. The graphical analysis of variance shown in Figure 1 further illustrates the findings.

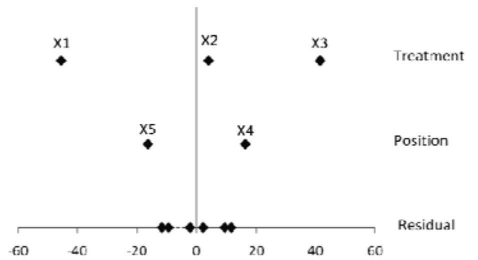

Figure 1. Graphical ANOVA for the sweeping with average values.

Results of graphical ANOVA indicate that the treatment values vary more than those for position.

In the second case for 3x40 matrix, the probability of randomness is less than 0.0001% (Table 4).

Table 4. ANOVA for 3x40 matrix.

Source	ss	df	ms	F	P
Data	944623	120			
Mean	727571	1			
Treatment	152154	2	76077	366.4	< 0.0001%
Position	32056	1	32056	154.4	< 0.0001%
Interaction	9172	2	4586	22.1	< 0.0001%
Residual	23671	114	208	1	
Total	944623	120			

ANOVA presents that the results are accurate and that all the factors influence the results.

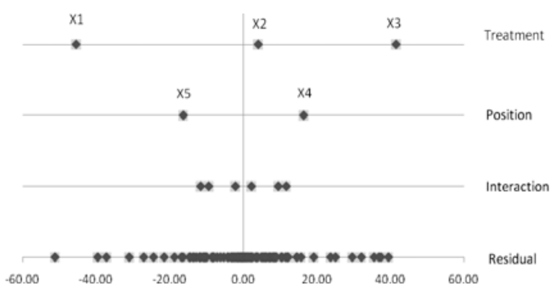

Figure 2: Graphical ANOVA of the 3x40 matrix analysis.

The graphical ANOVA illustrates the real situation of variation between factors (X1-X5) (Figure 2). As a result, there exists larger variation in values concerning the treatment than the position.

The graphical ANOVA give similar results when presented for mean values.

3.2 Least square method

3.2.1 Description of the method

In the least square algorithm, the coefficients of the model are estimated applying the following expression:

$$\vec{\beta} = \left(X^T X\right)^{-1} X^T \vec{Y} \quad (1)$$

where $\vec{\beta}$ stands for the model coefficients, X stands for the model matrix and $\vec{Y}$ stands for the experimental results. The columns of the model matrix should be linearly independent in order to ensure that $X^T X$ has an inverse matrix.

In the created model, the "1" shows that the effect is present on experiment. If the effect is not present, the situation is represented with a "0" in the model matrix. The factors are lying position (X1), standing position (X2), annealing (X3), heat strengthening (X4) and fully tempering (X5).

The least square algorithm is applied for the mean values of each kind of experiments (model with 6 tests) and using 20 first values of the experimental data (model with 120 tests). The following table shows the model matrix and the results for the average case.

Table 5. Model matrix for 5 factors and results (average case).

Test	I	Model matrix X					Results Y
		X1	X2	X3	X4	X5	
1	1	1	0	1	0	0	42.0
2	1	1	0	0	1	0	104.0
3	1	1	0	0	0	1	157.4
4	1	0	1	1	0	0	26.7
5	1	0	1	0	1	0	69.0
6	1	0	1	0	0	1	94.7

The columns of the model matrix are not independent. Factors X2 and X5 are dummy variables because $X2 = -X1$ and $X5 = -(X3 + X4)$.

Therefore, the least square algorithm must be applied only to X1, X3-X5 and X4-X5 for this particular case. The interactions between X1, (X3-X5) and (X4-X5) are also considered in the analysis. Table 6 shows the correct model matrix and the corresponding coefficients.

Table 6. Model matrix and coefficients of average case modeled with least square algorithm

Test	I	Model matrix X					Results
		X1	X3-X5	X4-X5	X1*(X3-X5)	X1*(X4-X5)	Y
1	1	1	1	0	1	0	42.0
2	1	1	0	1	0	1	104.0
3	1	1	-1	-1	-1	-1	157.4
4	1	0	1	0	0	0	26.7
5	1	0	0	1	0	0	69.0
6	1	0	-1	-1	0	0	94.7
Coeff.	β0	β1	β2	β3	β12	β13	
	63.47	37.67	-36.77	5.53	-22.37	-2.67	

Three models are evaluated:

1. Constant model: $\hat{Y} = \beta_0 + \varepsilon$
2. Linear model:

$$\hat{Y} = \beta_0 + \beta_1 X1 + \beta_2 (X3 - X5) + \beta_3 (X4 - X5) + \varepsilon$$

3. Linear model with interactions:

$$\hat{Y} = \beta_0 + \beta_1 X1 + \beta_2 (X3 - X5) + \beta_2 (X4 - X5) + \beta_{12} X1(X3 - X5) + \beta_{23} X1(X4 - X5) + \varepsilon$$

In a similar vein, the least square algorithm is applied with individual results for first 20 rows of

Table 1. The coefficients are presented on Table 7.

Table 7. Coefficients for model with 120 experiments

β0	β1	β2	β3	β12	β13
61.52	32.69	-36.03	6.19	-18.90	-4.35

The model decomposition is not orthogonal. Therefore, a correction of the sum squares must be carried out in order to make the ANOVA more precise [3,4].

3.2.2 Results

The results of the preformed analysis of variance are shown in Table 8 and Table 9 for model of the average values and model of 120 experiments, respectively.

Table 8. ANOVA analysis for average values

Model parts	SS	DF	MS	F	p
Data	51797	6			
Constant	40640	1	40640	18.2	0.7957%
Residual 1	11157	5	2231		
	51797				
Constant	40640	1	40640	107.5	0.1914%
Linear	10023	2	5011	13.3	3.2406%
Residual 2	1134	3	378		
	51797				
Constant	40640	1	40640	8.1E+11	0.0001%
Linear	10023	2	5011	1.0E+11	0.0002%
Interaction	1134	2	567	1.1E+10	0.0007%
Residual 3	5.0.E-08	1	5.E-08		
	51797				

Results show that the linear model with interactions has the lowest p value. Thus, the best model to predict the average values of the experiments is the linear model with interactions.

Table 9. ANOVA analysis for 120 tests

Model parts	SS	DF	MS	F	p
Data	944623	120			
Constant	727571	1	727570.6	398.9	<< 0.0001%
Residual 1	217052	119	1824.0		
	944623				
Constant	727571	1	727570.6	2008.8	<< 0.0001%
Linear	175039	3	58346.2	161.1	<< 0.0001%
Residual 2	42014	116	362.2		
	944623				
Constant	727571	1	727570.6	3504.0	<< 0.0001%
Linear	175039	3	58346.2	281.0	<< 0.0001%
Interaction	18343	2	9171.6	44.2	<< 0.0001%
Residual 3	23671	114	207.6		
	944623				

When the analysis is performed with the same factors for a set of 120 experiments, every layer for all models have p values close to 0 due to high degrees of freedom of residuals. This indicates that when data is acquired by a large number of experiments, all of the three tested models are adequate to explain the experiments.

An ANOVA analysis using the same parameters is also performed (Table 10). This analysis shows that the mean term and factors X1, X3-X5 and their interactions are significant on these experiments, as the probability for them to be random is much smaller than 0.0001%. The effect of the factor X4-X5 and its interaction with X1 have a big probability of being random. Therefore the analysis of the models should be performed without these factors in order to provide a precise estimation.

Table 10. ANOVA of a parametric model.

Effects	Coef.	SS	DF	MS	F	p
Constant	61.52	454190	1	454190	330.4	<< 0.0001%
X1	32.69	128223	1	128223	93.3	<< 0.0001%
X3-X5	-36.03	155751	1	155751	113.3	<< 0.0001%
X4-X5	6.19	4595	1	4595	3.3	7.011%
X1*(X3-X5)	-18.90	42880	1	42880	31.2	< 0.0001%
X1*(X4-X5)	-4.35	2274	1	2274	1.7	20.097%
Residual	-	156709	114	1375	-	-

4. *Conclusions*

A statistical analysis is performed to evaluate significance of treatment and position factors in glass failure tests. Two types of statistical methods are used and the results are compared: Sweeping and Least square algorithm

As a result of the sweeping analysis, the treatment has more variation than the position. The analysis made for average values reveal randomness in treatment (17%) and position (23%). When ANOVA is made for 20 values (3x40 matrix), the randomness of the effects are very small (p< 0.0001%). The results of graphical ANOVA are similar for both cases, showing larger variation in treatment than in position, see Figure 1 and Figure 2.

As expected, Least square analysis provides statistical information similar to the results obtained via Sweeping. In Least square analysis three models are compared: linear model with interactions, linear model without interactions and constant model. For average values, the linear model with interactions fits better than the other models to the experimental data. This is because the linear model with interactions has smaller randomness in all model layers compared to the constant or linear models. However, for the direct experimental data (with 120 tests), all the applied models fit adequately as the probability of randomness is less than 0.0001%. The analysis of variance for factors in least square algorithm with direct experimental data reveals that the X4-X5 and its interaction with X1 have a high probability of randomness (7% and 20%). Results indicate that the position of the specimen has very small randomness (p<0.0001%) while one of the treatments, heat strengthening, has some randomness ($p_{X4\text{-}X5}$=7% and $p_{X1*(X4\text{-}X5)}$ =20%).

In both methods (Sweeping and Least square method), there is no significant difference between the results obtained. The results are dependent on the number of experimental values used in the analysis. Therefore, there is a difference between results obtained by using average values and direct experimental data within same analyses. If only average values are used, ANOVA indicates that the factors have high randomness. On the contrary, when the direct experimental data values are applied, the effects of the factors have very small randomness.

In order to provide a more precise statistical model to explain the glass failure, an analysis with further factors should be conducted. That is to say, the two factors used are insufficient to explain the spread in the data. The statistical analysis would provide more precise outcomes with this improvement of the experimental data.

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References

[1] prEN 13474-1, *Glass in building - Design of glass panes Part 1: General basis of design*, 1999.

[2] Veer F.A., Louter P.C., Bos F.P. *The strength of architectural glass*. Challenging Glass. Conference on Architectural and Structural Applications of glass.-Faculty of Architecture, Delft University of Technology.- May 2008.- PP419 – 428.

[3] Box, G.E., Hunter, J.S. and Hunter, W.G. *Statistics for Experimenters, Design, Innovation, and Discovery*. A John Wiley & Sons, Inc., Publication, 2005.

[4] Fürbringer,J-M, *Design of experiments*, 2005.