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Tokamak Plasma Equilibria and Axisymmetric Stability with a Zero Total Toroidal Current

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Several tokamak experiments have reported the existence of plasma equilibria with a zero total toroidal current in their AC operations. By taking a plasma profile model based on linear functions of the poloidal flux function ψ, a variety of equilibrium configurations with a zero total toroidal current have been found.

The MHD_N0 stability code [1] is able to compute axisymmetric mode stability of non-nested tokamak equilibria with arbitrary island topologies. Both equilibrium and stability codes use the same unstructured triangular grids.

The plasma shape and profile effects on the existence and stability of zero total current equilibria were investigated. Stable equilibria against axisymmetric (n=0) ideal MHD modes were discovered with a conducting wall close enough to the plasma boundary.

1 Zero current equilibria

To model axisymmetric equilibria we use the standard representation of the magnetic field $\vec{B} = \nabla\psi\times\nabla\phi + f\nabla\phi$ and toroidal current density $j_\phi = Rp' + ff'/R$. Special choices of the flux functions p', ff' in the Grad-Shafranov equation

$$-R^2\nabla\cdot\left(\frac{\nabla\psi}{R^2}\right) = R^2p' + ff', \tag{1}$$

allow us to obtain equilibria with zero total toroidal current, $\int j_\phi dS = 0$.

In [2] an analytic solution of the equilibrium equation with the following flux functions was proposed:

$$p' = -A_1, \quad ff' = A_2 + \alpha^2\psi, \tag{2}$$

for a special choice of rectangular plasma cross-section.

In [3] linear terms in ψ were added to p' and quadratic terms in ψ were added to ff' functions in (2). The Grad-Shafranov equation was numerically solved for arbitrary plasma cross-section and a wide variety of zero toroidal current configurations has been obtained. However, even in a restricted class of equilibria (2), when the poloidal flux function can be easily obtained as a solution of a linear equation for ψ, experimentally relevant configurations can be obtained [3]. Then the functions of toroidal field and pressure can be found:

$$f = (B_0^2R_0^2 + 2A_2\psi + \alpha^2\psi^2)^{1/2}, \quad p = p_0 - A_1\psi, \quad p_0 = -\min(-A_1\psi), \tag{3}$$

where B_0 is vacuum magnetic field in the plasma center $R = R_0$ and $p \geq 0$. The values of plasma current in the high field side (HFS) half of the plasma I_{in} and toroidal beta are defined as follows:

$$I_{in} = \int_{R \le R_0} j_\phi dS, \quad \beta_v = \frac{2 \int p dV/V}{B_0^2}. \tag{4}$$

With normalizations $B_0 = 1$ and $a = 1$, where a is plasma minor radius, they give normalized plasma current $I_{in} = \mu_0 I[A]/(a[m]B_0[T])$ and beta $\beta_v = 2\mu_0 < p >_v [Pa]/(B_0^2[T])$ in SI units.

For the purpose of modeling the CT-6B tokamak with zero total toroidal current the following parameters were chosen in [3]: $R_0 = 0.45m, a = 0.125m$, current in the HFS half of the plasma $I = 300A, B_0 = 0.45T$ and $\beta_v = 5.73 \times 10^{-5}$. It corresponds to a value of the normalized current of $I_{in} = 0.0067$. In Fig.1 the solution of the equilibrium problem (1),(2) with circular plasma boundary cross-section is presented.

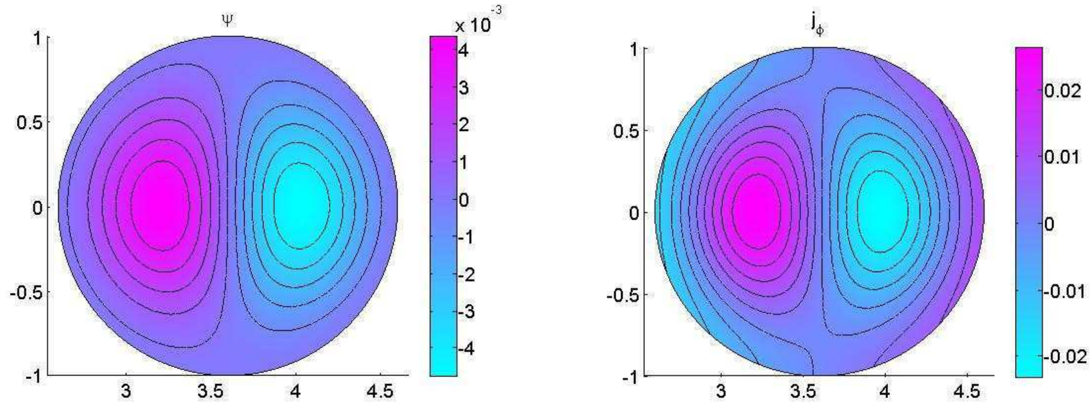


Figure 1. Level lines of poloidal flux and toroidal current density for the equilibrium with parameters A_1 = -0.0064, A_2 = -0.0832, α^2 = 23.4641; I_{in}=0.0067, $\beta_v = 5.73 \times 10^{-5}$. Circular cross-section.

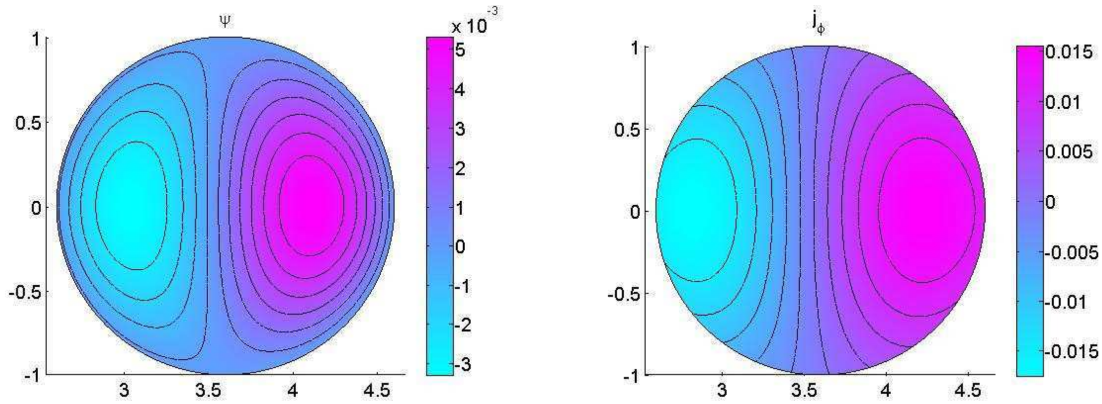
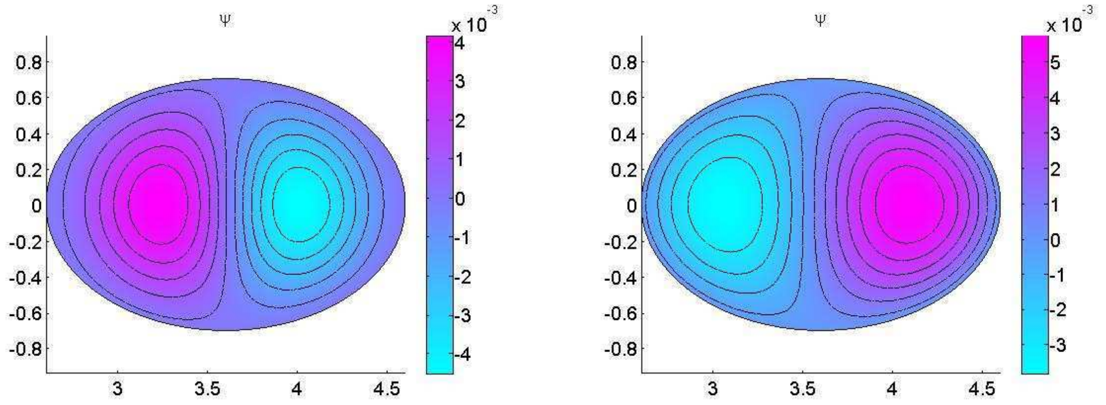


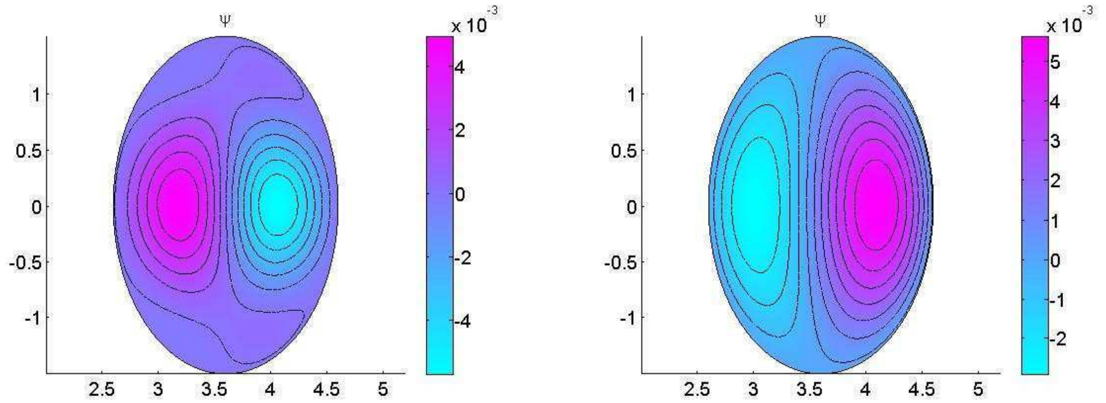
Figure 2. Level lines of poloidal flux and toroidal current density for the equilibrium with parameters A_1 = -0.0064, A_2 = -0.0832, α^2 = 6.934, I_{in}=-0.014, $\beta_v = 5.23 \times 10^{-5}$

Fixing the values of parameters A_1 and A_2 but varying the value of α^2 another equilibrium with zero toroidal current can be found for lower $\alpha^2 = 6.9340$ (Fig.2). The sign and the value of normalized current are both different for the new equilibrium. Re-normalization of the equilibrium to the reference value of $I_{in} = 0.0067$ results in a lower beta $\beta_V = 1.31 \times 10^{-5}$. Similarly, pairs of equilibria can be found for elliptical plasma cross-sections with elongations 0.7 and 1.5 (Figs.3,4).



(a) (b)

Figure 3. Level lines of ψ for equilibria with parameters A_1 = -0.0064, A_2 = -0.0883, a) α^2 = 27.5456; I_{in}=0.0067, $\beta_v = 5.73 \times 10^{-5}$; b) α^2 = 11.1670; I_{in}=-0.013, $\beta_v = 6.21 \times 10^{-5}$. Elliptic cross-section with elongation 0.7



(a) (b)

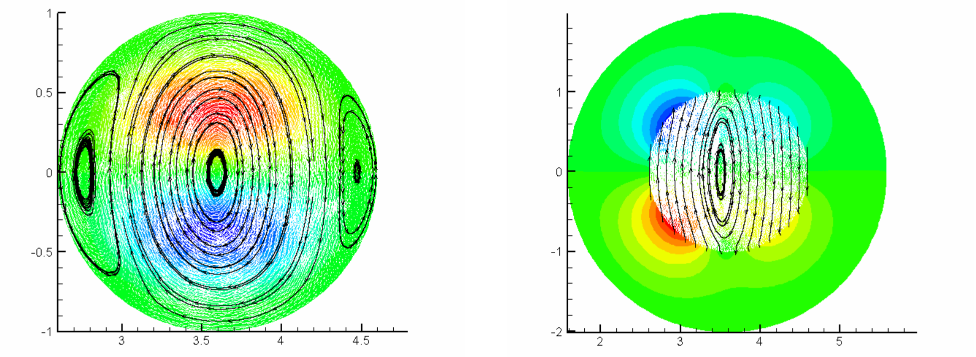
Figure 4. Level lines of ψ for equilibria with parameters A_1 = -0.0052, A_2 = -0.0691, a) α^2 = 19.5604; I_{in}=0.0067, $\beta_v = 5.73 \times 10^{-5}$; b) α^2 = 6.0935; I_{in}=-0.0175, $\beta_v = 4.06 \times 10^{-5}$. Elliptic cross-section with elongation 1.5

2 Axisymmetric stability

The ideal MHD axisymmetric stability of the equilibria was studied with the MHD_N0 code on unstructured grids [1]. The values of growth rates squared, $-\omega^2$, are normalized by the squared Alfven poloidal frequency $\omega_{Ap}^2 = \max(\psi^2)/(a^4R^2\rho) \approx B_p^2/(\mu_0 a^2\rho)$ assuming the constant plasma density ρ.

The equilibria with higher α^2 for each presented pair (i.e. shown in Figs.1, 3(a) and 4(a)) are unstable against ideal axisymmetric modes even with a conducting wall at the plasma (fixed boundary), Fig.5,a. The flow pattern is characteristic to the tilt mode as in [1] for dipole equilibrium configurations.

The equilibria with lower values of α^2 (or lower beta for the same reference HFS current) shown in Figs.2, 3(b) and 4(b) are stable with fixed boundary condition. The free boundary growth rates for the equilibria with different elongations versus wall radius a_w/a are given in Fig.6. The corresponding flow patterns are demonstrated in Fig.5,b and Fig.7. Note that for elongated cross-section there is a second free boundary unstable mode corresponding to a plasma vertical displacement in addition to the most unstable tilt mode.



(a) (b)

Figure 5. Arrow plot (white) and streamlines (black) of the plasma displacement vector projected onto the $\phi = \text{const}$ *plane* $\vec{\xi}_{pol}$ *. Filled contours show the toroidal component of the electric field* E_ϕ*. a) equilibrium from Fig.1,* $-\omega^2/\omega_{Ap}^2 = 7.04$*, fixed boundary; b) equilibrium from Fig.2,* $-\omega^2/\omega_{Ap}^2 = 1.16$*, free boundary,* $a_w/a = 2$

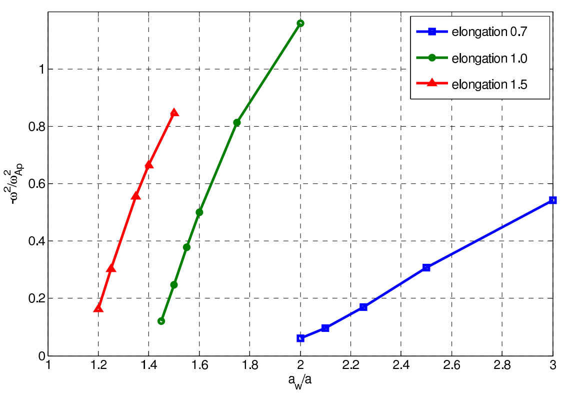
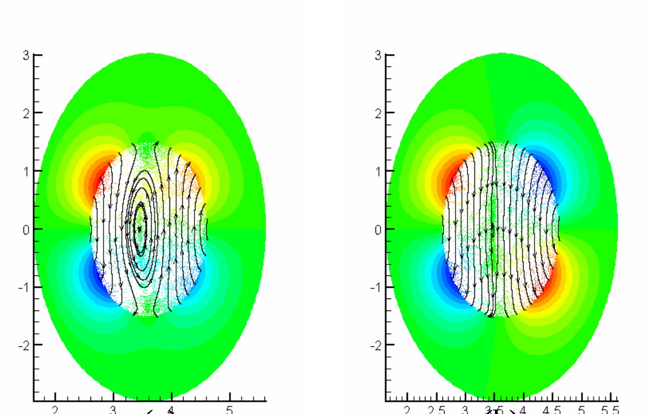


Figure 6. Squared growth rates $-\omega^2/\omega_{Ap}^2$ *for the free boundary* $n = 0$ *mode versus wall radius* a_w/a *for the equilibria with different elongations from Figs.2, 3(b) and 4(b)*



(a) (b)

Figure 7. Two unstable modes for elongated equilibrium from Fig.4,b, $a_w/a = 2$*;*
a) $-\omega^2/\omega_{Ap}^2 = 1.31$*; b)* $-\omega^2/\omega_{Ap}^2 = 0.77$

3 Discussion

Axisymmetric MHD stability calculations provide an additional insight into the modeling of zero current equilibria and tokamak AC operation. In particular, zero current equilibria stable against $n = 0$ ideal modes with a conducting wall at finite distance from the plasma boundary were found in the class described by the linear equation. The question about the number of parameters in the toroidal current density representation needed to match experimentally relevant values of current density and beta while keeping the equilibrium stable against $n = 0$ fixed boundary modes is still open. Keeping the values of α^2 as low as possible seems to be a useful guideline for choosing an experimentally relevant stable configuration.

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