INFLUENCE OF THE WIDENING OF A TRIBUTARY ON CONFLUENCE MORPHOLOGY: PRELIMINARY RESULTS

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ABSTRACT: The aim of this work is to obtain a better understanding of the morphological development in alpine river confluences, where special attention is given to the widening of the tributary channel and its influence on bed morphology evolution. Hydraulic tests have been performed in an experimental set-up of asymmetric confluent channels connected in an angle of 90° . The main and post-confluence channels are 3.6 m respectively 4.9 m long while the tributary channel is 4.8 m long. Two different configurations have been considered: one with all channels 0.50 m wide and other where the tributary has been widened (1.0 m wide) over a length of 1.0 m just upstream the confluence. Experiments have been carried out under mobile bed conditions where sediments (poorly sorted grain distribution) are feed on the tributary channel. Preliminary results considering a discharge ratio ($Q_{main\ channel}/Q_{tributary}$) equal to 1 have shown that the influence of the widening of the tributary is quite important on the morphological development in a confluence zone. The enlarged zone is characterized by high variability on water depths, flow velocities and particle size distribution, showing that widening the tributary channel can be adequate to improve ecological values on confluence zones in the framework of river rehabilitation projects.

1 INTRODUCTION

During the past few centuries, river training works have been applied in many industrialized countries to improve flood protection of cultivated and urban land. To represent this concept, there is a dictum by the river engineer Johann Tulla in 1815: "As a rule, no stream or river needs more than one bed" (Ettema, 2007). Based on this "Tulla's law", numerous channels have being channelized in Europe and other places.

Most of these river training works underestimated or did not even foresee ecological impacts (Havinga et al., 2005).

From the end of the 20th century, "river rehabilitation" has been a concept widely employed by environmental professionals and authorities (Reichert et al., 2007). The purpose is to recover the vital space required for the rivers which were degraded by human interventions, linking the sustainable use of rivers and wetlands with human well-being (Nakamura et al., 2006).

In fluvial networks, confluences are inevitable and produce significant flow changes. Its presence is

important for the ecological connectivity, flood control, navigation, water quality, etc. A good understanding of the flow patterns, sediment transport as well as the bed morphology development is necessary to successfully accomplish river rehabilitation project in these zones.

The knowledge of confluence streams acquired over the last 60 years has been extremely important in understanding the complex three-dimensionality of flow. Several authors (Taylor, 1944, Webber & Greated, 1966; Ramamurthy et al., 1988; Schwalt & Hager, 1995, Gurram et al., 1997; Hsu et al., 1998; Coelho, 2003) have proposed one-dimensional models to estimate the influence in water depths upstream of the confluences for different angles, discharge ratios, channel widths, slope and flow regimes.

Even if one-dimensional analyses provide important information about the flow, the applicability of these approaches may be limited by the three-dimensional behaviour of the flow patterns in confluences. Over the last 30 years, the synergy between laboratory tests and field measurements, supported by computational dynamic models has provided

valuable information about flow dynamics in confluences.

Best (1985) proposes a descriptive model of flow dynamics at confluences based on laboratory flume tests, as shown in Figure 1. The confluence presents six different zones: flow deflection, flow stagnation, flow separation, maximum velocity, shear layer and flow recovery.

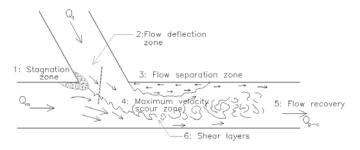


Figure 1. Descriptive model of flow dynamics at channel confluence (Biron et al., 1996 based on Best, 1985)

The flow stagnation zone is created by a deflection of both streams at the junction and is associated with an increase of pressure and flow depth and a decrease of flow velocities and shear stresses.

The direction change of the tributary creates a separation zone due to the flow detachment from the inner wall in the post confluence and its reattachment further downstream (Best & Reid, 1984). The geometry of the separation zone is significant because it delimits the width of the postconfluence channel and is an area of reduced pressure and circulating flow where sediments tend to accumulate. The maximum velocity zone is created after the junction of the flows at the contracted cross section besides the separation zone. Shear layers are formed along the contact of the stagnated areas of the fluid and the flow outside, and are characterized by high turbulence intensities and shear stresses and also the presence of wellorganized flow structures (Rhoads & Sukhodolov, 2004). This turbulence intensity is responsible for the creation of a mixing layer (Biron et al., 1996). A flow recovery zone, at the end of the mixing layer where confluent streams become one single channel is found downstream of the junction.

Despite the advances in understanding the complex three-dimensionality of flow and in connecting flow structure to the fluvial dynamics of confluences, there are relatively few studies of bed morphology at channel confluences and none which attempt to quantify the nature of sediment transport within the junction. This lack of knowledge can jeopardize confluence rehabilitation projects.

The abrupt meeting of two channels each having independent flow and sediment discharge regimes creates unique erosional and depositional

environments with consequent changes in channel morphology at confluences (Best 1986). As suggested by Rhoads and Kenworthy (1995), bed morphology of small confluences is highly responsive to changes in momentum flux ratio and changes dramatically its configuration in response to hydrological events. Bed load sediment transport and bed morphology at low lands have been observed and measured in medium-size (Lane & Richards, 1998; Rhoads & Sukhodolov, 2001) and for large-scale river confluences (Rhoads, 2005; Parsons et al., 2007). However, no detailed description of these parameters could be found at alpine confluences.

Concerning experimental data, only few works can be found in the literature. Mosley (1976) performed a laboratory study in a wide flume (1.3 m) with mobile bed for symmetrical and asymmetrical planforms and sediment discharge input. Ashmore & Parker (1983) extended their analyses to self formed confluences with a simple geometry similar to Mosley based on experimental tests in two wide channels (1.3 m and 2.3 m).

Best (1986) was the first author to present a detailed description of the bed morphology, based on experimental tests in small channels (0.15 m wide) in asymmetrical confluences (where the post-confluence channel is a continuation of the main channel) with bed concordance and different angles. In his description, he characterized the bed morphology of confluences in three distinct elements, as illustrated in Figure 2:

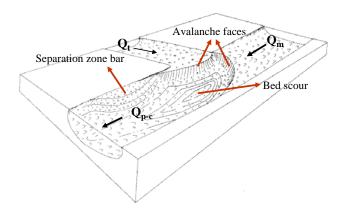


Figure 2. Schematic bed morphology of an asymmetric confluence with bed concordance (based on Bristow et al., 1993)

- Avalanche faces, formed at the mouth of each confluence channel;
- Bed scour, the region into the avalanche faces dip. Velocity at these zones can reach 1.6 times the upstream flow velocity (Roy & Roy, 1988).
 Scour holes can also be part of the explanation of bed discordance, where a small step can appear by erosion at the reattachment point because of

- high velocities and turbulent stresses acting on the bed;
- Separation zone bar, formed in the postconfluence channel having its origin in the formation of the separation flow zone.

Recently, Ghobadian & Bajestan (2007) presented non-dimensional relationships for predict scour hole and point bar dimensions based on the discharge ratio (Q_t/Q_{p-c}), the confluence angle (α), the Densimetric Froude Numbers (F_g).

The present research project aims to increase the understanding of the flow structure and morphological development in alpine confluences through systematic experimental laboratory tests in order to provide practical recommendations for confluence projects in the framework of river rehabilitation. In this project, special attention is given to the widening of the tributary channel.

Five different discharge ratios will be considered for a reference geometry configuration (main, tributary and post-confluence channels with same width) and three different tributary widening. Experiments will be performed in mobile bed conditions and sediment discharge will be introduced at the tributary channel. Only asymmetric confluences will be considered.

This project should provide an answer to the following questions:

- How does bed morphology develop in a confluence zone and what are the bed-forming events for a given configuration?
- What are the consequences on bed morphology, sediment transport and flow regime in the main channel when the tributary channel is widened?
- Which are the optimal combinations of geometry, angle and channel widths for sediment transport and flood control for a given scenario?
- What is the confluence geometry that provides the greatest benefit in environmental terms?

The objective of this paper is to present the results of preliminary experimental tests performed for the reference configuration and one tributary widening for one discharge ratio, performed at the framework of the research project.

2 EXPERIMENTAL SET-UP

Laboratory experiments have been performed in a confluence, adjustable for testing several configurations. The main channel is 8.5 m long, 0.50 m wide and 0.80 m deep. A second channel (tributary), 5 m long, 0.50 m wide and 0.50 m deep is connected with an angle of 90°, 3.60 m downstream of the inlet of the main channel (Figure 3). The tributary channel has the possibility of being widened in its downstream reach.

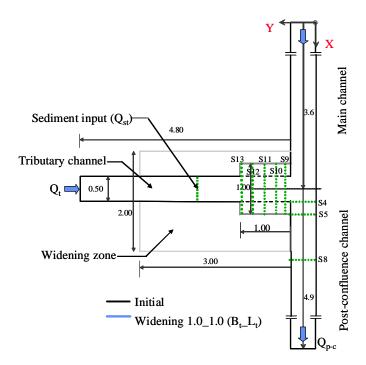


Figure 3. Laboratory set-up

Poorly-sorted sediments (gradation coefficient σ =4.15) is used for the bed constitution and the solid discharge. The material is a mixture of sand 0-4 mm (80%) and gravel 4-8 mm (20%). As shown in Figure 4, the d_{50} is equal to 0.82 mm and d_{90} equal to 5.7 mm.

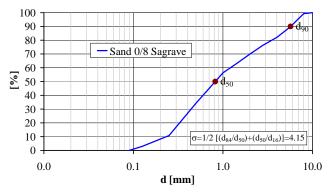


Figure 4: Grain size distribution considered for laboratory experiments

The initial bed at the tributary, main and postconfluence channels are concordant and the bed is horizontal.

A constant discharge of 10 l/s has been considered for both main and tributary channels, totalizing 20 l/s at the post-confluence channel.

This discharge have been chosen in order to maintain the ratio between channel width (B) and water depth (h) larger than 5, in order to assure the bi-dimensionality of the flow (Yalin, 1971). The flow water level downstream of the post-confluence channel (x=8.50 m) is maintained constant (initial water depth equal to 10 cm). Full development of the flow is theoretically guaranteed upstream of the confluence in both main and tributary channels

(Schlichting, 1968). However, at the currently phase of the project, no velocity profiles have been measured to validate the above criteria.

From preliminary calculations of sediment transport, the solid discharge (Q_{st}) that was introduced at the tributary channel was 0.1 l/min. The sediment input was done at y=2.25 m (Figure 3). The selection of this value is based on calculations with the Smart & Jaeggi (1983) formula, for a flow discharge of 10 l/s, a d_{50} equal to 0.82 mm and an equilibrium bed slope of 0.5%. It is accepted that the final bed slope will be higher than 0.5% because an armoring layer ($d>d_{50}$) is expected at the tributary channel.

Experiments have been performed during 34 hours for the reference configuration and 45 hours for the widened configuration.

During the experiments, bed evolution and water levels have been measured manually by means of a limnimeter. In total, 13 cross sections, 8 at the postconfluence and 5 at the tributary channels spaced by 25 cm are considered. In addition, topographic mapping using cotton threads have been performed with a vertical resolution of approximately 0.5 cm. Measurements have been performed at the beginning of the tests and after 4.5, 6.5, 10, 15.5, 21.5, 27.5 and 34 hours for both reference and widened configurations. For the widened condition, experiments have been extended and additional measurements are performed after 40 and 45h. The total duration of the tests is longer than that considered by previously experimental confluence works (Best, 1988, Ghobadian & Bajestan, 2007).

Surface flow measurements have been performed using Large-Scale Particle Image Velocimetry technique (LSPIV) at the end of each test. Plastic particles (d_m = 3.4 mm; ρ =960 kg/m³) have been introduced in both main and tributary channels and the position of these particles is recorded by a digital camera. Image treatment is then performed with the FlowManager software. Detailed description of this technique can be found in Kantoush et al. (2008).

3 RESULTS

3.1 Bed evolution

The tributary bed evolution has been analysed. At the beginning of the test, the tributary channel flows in fluvial regime (Fr~0.14) and the stream can not carry the input sediment.

First, the excess of sediment is deposed at the section where sediments are introduced, and the flow condition continues to change until a local bed equilibrium slope is reached over which the flow can transport the supplied sediment. The front of

deposited sediments moves forward until reaching the channel confluence. The progression of the aggradation front is shown in Figure 5 for the reference configuration.

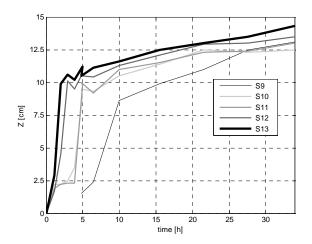


Figure 5. Bed evolution at the centre of cross sections 9 (x=3.6, y=0.58), 10 (x=3.6; y=0.75), 11 (x=3.6, y=1.0), 12(x=3.6, y=1.25) and 13 (x=3.6, y=1.5) for the reference condition

Figure 5 shows a quickly bed level increase for all cross sections. This increase corresponds with the aggradation front displacement until the confluence zone, which occurs approximately after 10h. Once the front arrives at the confluence, the tributary bed level still increases, but the average bed slope seems not changing with time.

For the widened configuration, the front displacement has the same behaviour until reaching the widened zone. After this point, the deposed sediments create an elevated channel at the widened zone until the confluence. The filling up of the outer and inner banks starts to increase when the front reaches the confluence. Figure 6 presents the bed morphology of the widened tributary after 11.5h.

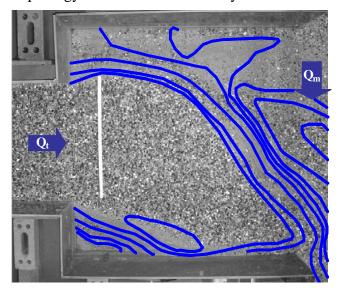


Figure 6. Tributary bed morphology for the widened configuration after 11.5h

The bed evolution at the post-confluence channel is presented at the cross section 8 (x=4.85) at Figure 7 for the reference configuration and at Figure 8 for the widened configuration.

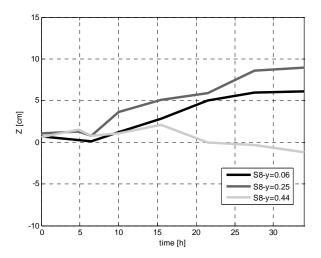


Figure 7. Bed evolution at cross sections 8 at the outer bank (x=4.85, y=0.06), centre (x=4.85; y=0.25) and inner bank (x=4.85; y=0.44) for the reference configuration

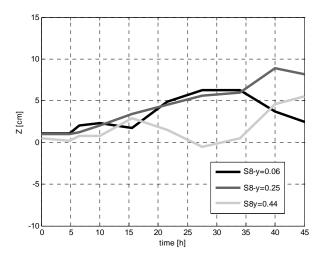


Figure 8. Bed evolution at cross sections 8 at the outer bank (x=4.85, y=0.06), centre (x=4.85; y=0.25) and inner bank (x=4.85; y=0.44) for the widened configuration

From theses figures it can be confirmed that the equilibrium state was not yet reached at the end of the tests. Nevertheless, flow and bed configuration descriptions have been performed for both considered configurations in order to understand the general behaviour of the bed morphology evolution in a confluence zone. For further experiments, tests will be performed until achieving the bed equilibrium state.

3.2 Flow behaviour and bed morphology at the of the tests

As explained above, with the input of a solid discharge at the tributary channel, the initial flow configuration has changed considerable. The bed level and slope have increased, but the flow remains in fluvial regime.

For the reference condition, the Froude Number $(Fr=U/(gh)^{1/2})$ at the tributary is equal to 0.86 and Fr=0.12 and 0.68, respectively for main and post-confluence channels. Table 1 summarizes the principal flow parameters as channel discharge (Q), average flow velocity (U), average flow depth (h) and Froude Number for the reference configuration.

	Reference configuration		
_	Main	Tributary	Post-Confluence
Q (l/s)	10.00	10.00	20.00
h (m)	0.14	0.04	0.07
<i>U</i> (m/s)	0.14	0.53	0.56
Fr	0.12	0.86	0.68

Table 1. Flow conditions for the reference configuration

As suggested by Rhoads and Kenworth (1995), flow conditions in confluence zones are strongly influenced by momentum flux ratio M_r ($\rho QU_{main}/\rho QU_{tributary}$). For the present study, this value is about 0.25 for the initial condition.

The estimation of average flow conditions at the widened tributary is quite complicated. Due to the high variation of the bed morphology and the different specific flow discharge, average values are not representative. Anyway, the momentum flux ratio for this configuration is slightly higher than that for the reference configuration.

The average surface flow velocities are presented in Figure 9 for the reference configuration and in Figure 10 for the widened configuration. The final bed morphology for the reference and widened configurations are presented at the Figure 11.

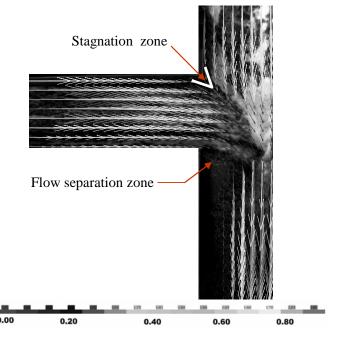


Figure 9. Average surface flow velocities at the confluence zone for the reference configuration

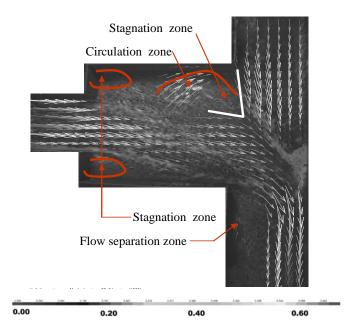


Figure 10. Average surface flow velocities at the confluence zone for the widened configuration

For both configurations, the high velocity zone occurs at the outer bank at the post-confluence channel. This behaviour is due to the low value of M_r . Flow from the main channel plunges to the bed at this point and the surface flow is composed by water from the tributary. As this phenomenon is in 3D, surface flow measurements are not able to describe them.

The tributary flow "pushes" the main flow to the outer bank of the confluence zone. This behaviour explains the fact that the maximum scour zone is located near the outer bank of the post-confluence channel and not as proposed by Best (1986) and presented in Figure 2.

The maximum scour depth for the initial condition is located at cross section S4. This cross section corresponds to the extension of the inner corner of the confluence into the post-confluence channel. If the tributary is widened, the maximum erosion depth decreases, but is still occurring at the extension of the right corner of the tributary (cross section S5 - x=4.1). However, the erosion zone is larger for the widened configuration.

The maximum deposition for both conditions occurs at the centre of the separation zone downstream of the inner confluence corner. For both cases, as expected, well-sorted fine grains are found at the separation zone.

The stagnation zone is much larger for the widened condition. The deposition occurring at the outer site of the widened tributary is quite important and is due to the circulation zone developed at this region with the tributary widening. Water depths at this zone are relatively small and some dry regions could be observed. One preliminary conclusion can be that

the left widening of the tributary is not efficient, once this region will be filled of sediments with the time.

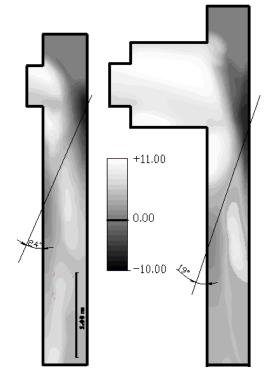


Figure 11. Final bed configuration at the post-confluence channel for the reference (left) and widened (right) configurations

For both configurations, a lateral inclination of the bed is observed at the tributary channel due to the formation of a preferential channel, characterized by higher surface flow velocities. This preferential channel is responsible for formatting an armoring layer at this zone, with diameters fairly higher than those found at the circulation zone, as shown in Figure 12.

For the widened configuration, flow stagnation zones are observed at the upstream right and left corners. These zones have been filled up with fine grain size sediments, and could be considered dry at the end of the runs.

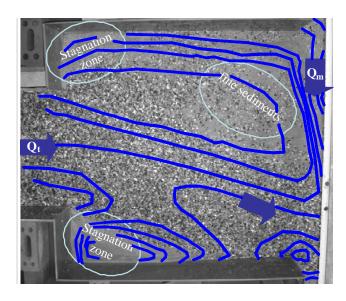


Figure 12. Plan view of the widened tributary at the end of the test (45h)

Avalanches faces are observed at the mouth of both main and tributary channels for the two considered conditions. Sediments carried from the tributary channel are deposed in the main channel, thus the avalanche faces edge is found inside this channel. The upstream part of the post-confluence channel is characterized by non uniform width-distribution of velocity and water depth as presented in Figure 13.

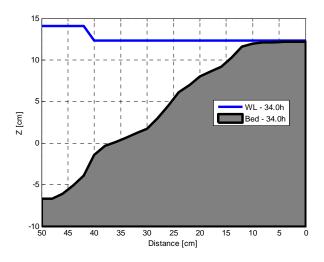


Figure 13. Final bed configuration and water level at the cross section S4 for the reference configuration

If it is considered as an initial condition for the post-confluence channel, this situation tends to evolve to a flat bed and a horizontal water level. The passage between the non-uniform cross section to the uniform one is characterized by lateral oscillation that decreases with the distance. This phenomenon can explain the bed evolution at the post-confluence zone that is characterized by alternated depositional /erosion zones. It is interesting to note that for the widened configuration, the oscillation is less important (19°x24° for the reference configuration), as can be seen in Figure 11.

4 CONCLUSIONS

Preliminary tests in the framework of a research project related to the morphology of alpine confluences have been discussed here.

The experimental tests performed in a 90° confluent flumes with movable bed conditions have shown quite significant differences compared to other experimental studies in confluences zones with same confluent width ratio. These differences, mainly found at the position of the maximum erosion zone and the flow and bed morphology at the post-confluence channel are probably due to the low value of momentum ratio between the main and tributary channels and the poorly-sorted grain size characteristics of the used sediment material.

In view of the influence of the widening of the tributary channel, important conclusions can be noted. The filling up of the left side of the widening zone is an indication that this zone does not contributes to the flow, once the tributary channel is widened. The same occurs with the right corner of the tributary widening, which is filled with small grain size sediments. However, the widened zone is characterized by high variability on water depths, velocities and particle size distribution. Because of the river ecology degradation generated by river training works in the past, this type of solution would be adequate to improve ecological value of the confluences in river rehabilitation.

It is important to note that even if the tests have been performed during a considerable time compared to other confluence studies, they have not been enough long to achieve flow and bed morphology equilibrium state.

5 ACKNOWLEDGEMENTS

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