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Punching Tests of Slabs with Low Reinforcement Ratios. Paper by Stefano Guadalini, Oliver Burdet, and Aurelio Muttoni

Discussion by Dimitrios Theodorakopoulos and Narayan Swamy

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The authors are to be congratulated for presenting the results of a test series and providing valuable information on the punching shear behavior of slabs in which the geometry of the slab and the reinforcement ratio were varied. It is noted that scanty experimental data from slabs reinforced with low reinforcement ratios are currently available, although low steel ratios are usually used in practice. As a secondary objective, the authors also analyzed their test results according to the critical shear crack theory[9] and various design codes.

The discussers would like to offer the following comments and contribution:

1. Doubling the steel ratio from 0.75% (PG-11) to 1.50% (PG-1), the test punching load was increased by 34% (1023/763 = 1.34). This increase is in agreement with test results reported in the literature. On the other hand, doubling the steel ratio from 0.75% (Slab PG-7) to 1.50% (Slab PG-6), there was no increase in punching strength (241 kN for Slab PG-7 as compared to 238 kN for Slab PG-6). Given that, in each pair of slabs mentioned previously, the steel reinforcement ratio was the only variable, how are these test results explained?

2. According to the authors' critical shear crack theory, the percentages of increase in punching shear strength in each pair of slabs mentioned previously were 23.3% (841/682 = 1.233) and 17.3% (231/197 = 1.173), respectively. How might these deviations from the test results be explained?

3. The discusser's design model for punching shear[22,23] of plain steel-reinforced concrete slabs has been applied to predict the ultimate strength of test slabs presented in the paper. The basic equations are given in the following

$$V_{calc} = \frac{1}{2}0.234f_{cu}^{2/3}(4r+12d)\xi_s\frac{2\alpha_s\lambda_s}{1+\alpha_s\lambda_s}d \tag{4a}$$

where $f_{cu} = (f_c'/0.80)$ is the cube concrete strength; $f_{cu} = (f_c'/0.80)$ is the cube concrete strength; $\xi_s = (100/d)^{1/6}$ (d in mm) is the size effect factor; $\alpha_s = \rho_s f_y/0.145 f_{cu}$; and λ_s is the steel stress f_s divided by the steel yield stress f_y, where

$$\lambda_s = \begin{cases} 1.60 - 0.75\alpha_s & \text{for } 0.20 < \alpha_s \le 0.50 \\ 1.35 - 0.25\alpha_s & \text{for } 0.50 \le \alpha_s \le 1.00 \\ 1.20 - 0.10\alpha_s & \text{for } 1.00 \le \alpha_s \le 2.50 \\ 1.30 - 0.14\alpha_s & \text{for } 2.50 \le \alpha_s \le 5.00 \end{cases} \tag{4b}$$

It is obvious that the value of $\lambda_s = f_s/f_y$ calculated from Eq. (4b) must not exceed the value of f_u/f_y in slabs with low reinforcement ratios.

It is to be noted that the design model[22,23] has been based on the discusser's theoretical analysis[24] and employs no fitting factors to match the trend of the available steel reinforced slab test results reported in the literature.

Table 4 shows the comparison between the observed strengths of all 11 slabs of the paper and the discusser's design model predictions (V_t/V_{calc}).

It can be concluded that:

1. The calculated punching failure loads were found to be in very good agreement with those reported from experiments.

2. The statistics of all 11 slabs, average ratio V_t/V_{calc} = 1.065, and coefficient of variation of 8.4% (0.089/1.065)

Table 4—Observed ultimate strengths compared with design predictions[22,23]

Slab	r, mm	d, mm	ρ, %	f_u/f_y,* MPa	f_c'/f_{cu},† MPa	V_t, kN	α_s	λ_s‡ Eq. (4b)	λ_s‡ f_u/f_y	V_{cal}, kN	V_t/V_{cal}
PG-1	260	210	1.50	573/656	27.6/34.5	1023	1.72	1.028	—	1045	0.979
PG-2b	260	210	0.25	552/612	40.5/50.6	440	0.19	—	1.109	365	1.205
PG-4	260	210	0.25	541/603	32.2/40.2	408	0.23	—	1.115	373	1.094
PG-5	260	210	0.33	555/659	29.3/36.6	550	0.35	—	1.187	495	1.111
PG-10	260	210	0.33	577/648	28.5/35.6	540	0.37	—	1.123	490	1.102
PG-11	260	210	0.75	570/684	31.5/39.4	763	0.75	1.163	—	833	0.916
PG-3	520	456	0.33	520/607	32.4/40.5	2153	0.29	—	1.167	1876	1.148
PG-6	130	96	1.50	526/607	34.7/43.4	238	1.25	1.075	—	268	0.888
PG-7	130	100	0.75	550/623	34.7/43.4	241	0.66	1.133	—	212	1.137
PG-8	130	117	0.28	525/586	34.7/43.4	140	0.23	—	1.116	131	1.069
PG-9	130	117	0.22	525/586	34.7/43.4	115	0.19	—	1.116	108	1.065
									Average ratio		1.065
									Standard deviation		0.089

*Actual yield and ultimate steel stresses.
†$f_{cu} = f_c'/0.80$.
‡λ_s = min[Eq. (4b), f_u/f_y].
Note: 1 in. = 25.4 mm; 1 MPa = 145 psi; 1 kN = 0.2248 kips.

indicate that the design model for punching shear compares favorably to the authors'[9] theory and other design codes.

3. Also, the same conclusion can be drawn from the statistics (not shown herein) of the results of tests where the flexural capacity V_{flex} was not reached (Slabs PG-1, PG-11, PG-3, PG-6, and PG-7), V_t/V_{calc} = 1.014, standard deviation = 0.122.

4. With regard to slabs reinforced with low reinforcement ratios (Slabs PG-2b, PG-4, PG-5, PG-10, PG-3, PG-8, and PG-9) the average ratio V_t/V_{calc} = 1.114, and especially the low standard deviation (0.049) (not shown herein), verify the ability of the design equation, Eq. (4a), to provide good predictions not subject to any limitation as far as the geometry, the material properties, and steel ratio are concerned.[22]

REFERENCES

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AUTHORS' CLOSURE

The authors thank the discussers for their interest in the paper. With respect to Questions 1 and 2: The question of the discusser is very pertinent. It can be easily explained on the basis of the scatter of test results (refer to Fig. 15(a)). As explained in the paper, the shear strength is rather sensitive to the location of the critical shear crack, which has a somewhat random nature. We have had the opportunity to check that the strength of Specimen PG-1 was, in fact, rather high while performing another test series. In this additional series (whose results have recently been submitted for publication to this journal) another test was performed on a slab with identical geometric and mechanical properties than Slab PG-1 (ρ = 1.50%). The measured failure load was, however, 974 kN (218 kips) (for a concrete f_c = 34 MPa [4900 psi]) instead of 1023 kN (230 kips) measured in Slab PG-1 (f_c = 27.6 MPa [4000 psi]).

3. The authors find the model proposed by the discussers interesting. Similar accuracy than the one obtained using the critical shear crack theory is obtained. Considering the influence of the flexural reinforcement ratio on the punching shear strength (as does EC-2) is an implicit way of considering the deformation (and opening) of critical shear cracks. On the basis of the critical shear crack theory, the authors have also identified other parameters, such as slenderness, that show a similar effect.[9]