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Influence of Flexural Reinforcement on Shear Strength of Prestressed Concrete Beams. Paper by Elias I. Saqan and Robert J. Frosch

Discussion by Miguel Fernández Ruiz, Stefano Campana, and Aurelio Muttoni
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The paper presents the experimental results of a series of nine prestressed concrete beams without stirrups failing in flexure and in shear. Some theoretical considerations are also proposed on the basis of a theory previously developed by the authors with respect to shear strength in reinforced concrete members (Tureyen and Frosch 2003). The innovative design of the tests as well as the well-documented data presented by the authors have allowed the discussers to investigate a number of aspects with respect to shear strength in prestressed members. A series of independent conclusions and interpretations derived from this analysis may complete those proposed by the authors.

As shown by the tests of the paper, beams are developing shear (diagonal) cracking at a given load level. Such cracking, however, does not lead to failure of the specimens, and load can be significantly increased before failure. For two specimens (V-4-0 and V-4-0.93), the increase meant that yielding of the flexural reinforcement was reached and bending was governing for the strength. For the other seven specimens, failure also developed in shear, but at a load 42% higher on average than the shear cracking load. The increase on the failure load with respect to the shear cracking load can, in the discussers' opinion, be explained and calculated accounting for the different regions of Kani's valley (Kani et al. 1979).

Figure 12(a) shows a sketch of Kani's valley and its two governing regimes. The ascending branch (named "crack propagation" in the figure, see Point A) is due to a sudden propagation of a flexural crack as it develops through the theoretical compression strut carrying shear. Such failure (disabling the teeth action as proposed by Kani) is followed by a total loss of load-carrying capacity of the member and is often named diagonal shear failure. The descending branch (named "direct struting" in the figure) has a different nature. Flexural cracks may reach the location of the theoretical compression strut carrying shear and develop through it (Point B in Fig. 12(a)) but they do not progress in an unstable manner. Instead, once such inclined cracks have developed, they can widen progressively as the load increases. A typical crack pattern illustrating this case is plotted in Fig. 12(b)

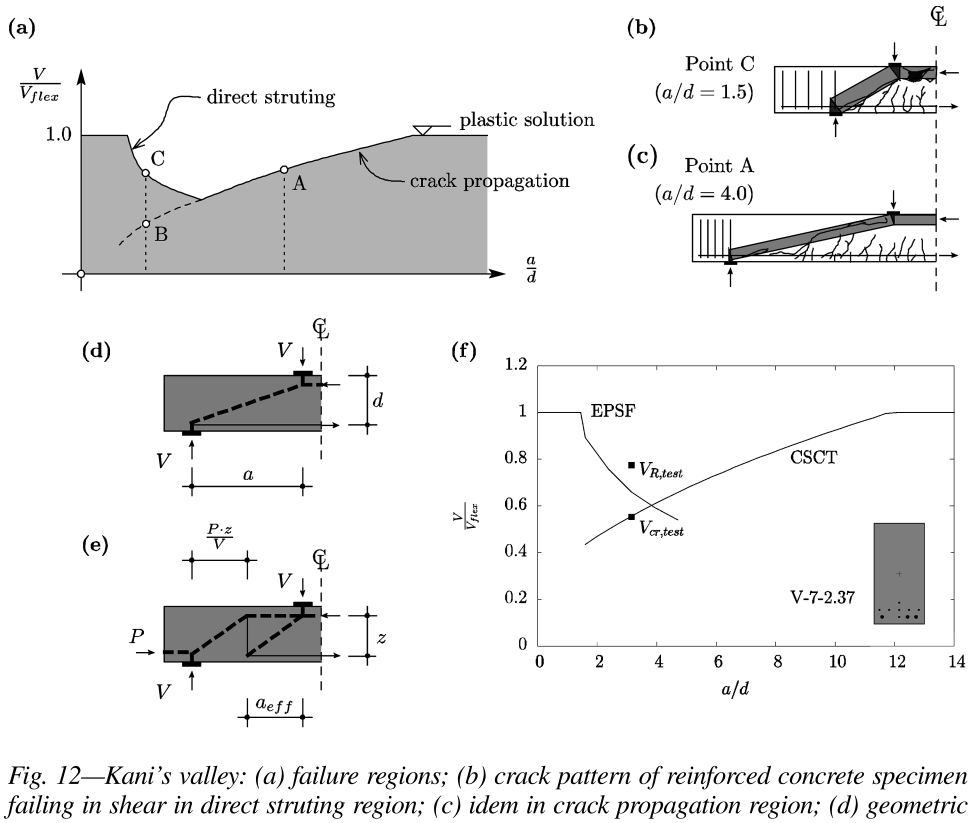

Fig. 12—Kani's valley: (a) failure regions; (b) crack pattern of reinforced concrete specimen failing in shear in direct struting region; (c) idem in crack propagation region; (d) geometric shear span in a reinforced concrete beam; (e) effective shear span for a prestressed member; and (f) CSCT and EPSF results for Specimen V-7-2.37 compared with test results.

Table 5—Shear cracking loads V_{cr} and failure loads V_R for all specimens*

Specimen	$V_{cr,test}$, kN (kips)	$V_{R,test}$, kN (kips)	$V_{cr,calc}$ (CSCT), kN (kips)	$V_{cr,test}/V_{cr,calc}$	$V_{R,calc}$ (EPSF), kN (kips)	$V_{R,test}/V_{R,calc}$
V-4-0	205 (46.0)	244 (54.8)	200 (44.7)	1.04	204† (45.6)	1.20
V-4-0.93	265 (59.5)	334 (75.2)	250 (55.9)	1.06	300† (67.1)	1.11
V-4-2.37	296 (66.5)	367 (82.4)	301 (67.3)	0.98	367 (82.1)	1.00
V-7-0	256 (57.5)	370 (83.1)	225 (50.3)	1.14	276 (61.7)	1.34
V-7-1.84	298 (67.0)	484 (109)	295 (66.0)	1.01	354 (79.2)	1.37
V-7-2.37	305 (68.5)	428 (96.1)	307 (68.7)	0.99	363 (81.2)	1.18
V-10-0	287 (64.5)	406 (91.2)	242 (54.1)	1.19	292 (65.3)	1.39
V-10-1.51	300 (67.5)	440 (99.0)	295 (66.0)	1.02	351 (78.5)	1.25
V-10-2.37	322 (72.5)	440 (99.0)	318 (71.1)	1.01	369 (82.5)	1.19
			Average	1.05		1.23
			COV	0.07		0.10

*Calculations performed with geometric and mechanical properties given in paper and assuming average values of yield strength for prestressing and ordinary reinforcement (f_{py} = 1690 MPa [245 ksi] and f_y = 496 MPa [71.9 ksi] estimated on the basis of Mirza and MacGregor [1979]).

†Flexural failures.

where it can be noted that inclined cracking affects only a limited region of the compression strut. Failure happens when bending becomes governing (for rather short shear spans) or when the opening of these inclined cracks is such that the strength of the compression strut is severely limited failing in shear (Point C in Fig. 12(a)). Large scatter in the strength of members governed by direct struting is typically observed. This is due to the fact that strength depends primarily on the positions of the cracks affecting the compression strut, which are rather random.

Looking at the experimental results of the paper, the discussers think that the tested specimens were, in fact, governed by the second type of behavior (direct struting). This conclusion is supported by two facts:

1. It was experimentally observed that propagation of shear cracks was not unstable, and load could be increased significantly after diagonal cracking; and

2. The beams had a geometric shear span of 3.3 according to the authors. This value corresponds approximately, for ordinary reinforced beams, to the limit between unstable crack propagation and direct struting regions. In prestressed members, however, the effective shear span is smaller than the geometric one (Muttoni and Fernández Ruiz 2008). This is due to the fact that prestressing does not allow flexural cracks to develop close to the support region, as shown in Fig. 12(e). The effective shear span a_{eff} can be calculated in this case (eccentric prestressing) as

$$a_{eff} = a - \frac{P}{V}z$$

where a is the geometric shear span, P is the prestressing force, V is the shear force, and z is the flexural lever arm of the member. For preliminary estimates, this equation can be simplified further by replacing z with d (effective depth).

The theoretical model used by the authors to investigate the test results seems, in the discussers' opinion, only valid for members failing in the unstable crack propagation regime. In fact, this is what the authors are doing by comparing their predictions to the load leading to development of first diagonal cracking. This approach leads to reasonable (rather safe) estimates for diagonal cracking load (term V_c of ACI 318-08) but clearly underestimates the actual failure load. In this sense, it should be noted that first diagonal cracking load is not directly related to the actual failure load.

A more accurate analysis of the results can, on the basis of the previous considerations, be performed accounting for both regimes and determining the characteristic points (refer to Points B and C in Fig. 12(a)). This can, for instance, be accomplished using some theoretical models proposed by the discussers, such as the critical shear crack theory (CSCT) (Muttoni and Fernández Ruiz 2008) for the crack propagation regime and the continuous elastic-plastic stress fields (EPSF) (Fernández Ruiz and Muttoni 2007) for the direct struting regime. The results of analyses using these two theories are shown in Table 5. It can be noted that pretty accurate estimates of the shear force leading to crack propagation is obtained using the CSCT, with an average value of the measured-to-predicted load of 1.05 and a coefficient of variation (COV) of 7%. The fitting to test results is even better than with the approach followed by the authors (Table 4), which seems to be (if discussers have correctly understood the theory proposed by the authors) a simplified design formula that neglects some phenomena (such as size effect) that may have a significant influence on actual strength. Regarding strength, a significant increase beyond diagonal cracking load is obtained using EPSF, with an average increase of 20% in the failure load for specimens failing in shear (Fig. 12(f)) and predicting flexural failures for Specimens V-4-0 and V-4-0.93 (as observed in the tests). The estimates of the failure load are safer than for first shear cracking. This is logical because scatter in this failure regime is significantly larger as previously discussed and this is accounted for in the EPSF (see also value of COV).

To conclude, the discussers would like to highlight with this discussion that shear failures have to be carefully investigated accounting for their various failure regions. This is particularly important in prestressed members as the amount and layout of prestressing may significantly shorten the effective shear span.

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Influence of Flexural Reinforcement on Shear Strength of Prestressed Concrete Beams. Paper by Elias I. Saqan and Robert J. Frosch

Discussion by Andor Windisch

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The authors tested nine specimens to evaluate the influence of the area of prestressing and mild reinforcement on the shear behavior of structural concrete members. Similar to ACI 318-08 (and to the originators of the shear code provisions, Zwoyer and Siess [1954] and Sozen et al. [1959]), they assume that the concrete contribution of shear is that shear force when the principal tensile stress reaches the tensile strength of concrete. They propose a lower bound equation (Eq. (4)) that consists of the depth of the neutral axis. The analysis is performed on a cracked section. It is the discusser's opinion that it is contradictory that a force causing cracking be calculated on a cracked section.

Reporting on the structural behavior of the specimens, the authors write "...the structure transitioned from behaving as a flexure beam to a tied arch...As evident from the cracking patterns, the structure resembled a strut-and-tie model." The discusser would like to ask the following questions:

1. What do the strut-and-tie models look like in case of the crack patterns shown in Fig. 4 at beams without transverse reinforcement, that is, without transverse ties? and

2. Can the strut-and-tie models and the tied arches coexist?

Figure 6 performs the development of the neutral axis depth as a function of the moment, where the values of neutral axis corresponding to the measured strength at the formation of the primary crack (V_{test}) and the calculated shear strength (V_{calc}), respectively, as it is due from a fair lower bound estimation, $V_{test} > V_{calc}$. Nevertheless, the neutral axis depth belonging to V_{test} is always greater than the depth belonging to V_{calc}, that is, the development of the neutral axis depth and of the concrete contribution goes in opposite directions. This means that Eq. (4) is valid in a single point only. How can the proper neutral axis depth be found? Moreover, in two cases (V-4-0 and V-4-0.93), the longitudinal reinforcement yielded prior to the formation of the primary shear crack. It is not clear how the proper neutral axis depth shall be calculated. Is this assuming yielding of reinforcement or not? The authors should point out the impact of the different bond characteristics of the prestressing steel (strand) and of the mild reinforcing bars.

The authors interpret the failures as diagonal tension failure. How can an arch fail in diagonal tension? How may a diagonal tension failure mode be characterized with the neutral axis depth?

The discusser appreciates that, according to Eq. (4), the concrete contribution of shear is related to the neutral axis, that is, the concrete compression zone that must be considered as the real source of the shear contribution. In 2009, we must finally move away from the interpretation of the originators in the 1950s. The formation of the primary shear crack might be a safe lower bound value, nevertheless, it is physically not sound. The failure patterns shown in Fig. 4 reveal unambiguously that the failures occurred as compression-shear failures of the compression zones.

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Influence of Flexural Reinforcement on Shear Strength of Prestressed Concrete Beams. Paper by Elias I. Saqan and Robert J. Frosch

Discussion by W. L. Gamble

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Equation (2) is attributed to the wrong sources. The data behind it are mostly in the cited two papers, but not the specific equation. The equation, without the V_d term, was first published in a discussion on the ACI Committee 326 report on shear and diagonal tension (Sozen and Hawkins 1962). Further information on the development of the equation is given by Olesen et al. (1967).

Equation (2) is actually a simplified form of the one suggested by Olesen et al. (1967)

$$V_{ci} = 0.6 b_w d \sqrt{f_c'} + \frac{M_{cr}}{\frac{M}{V} - \frac{d}{2}} + V_d$$

In this implementation, M_{cr} is the cracking moment in excess of the dead load moment, and M and V are the live load moment and shear at the section considered, respectively. This is consistent with the 2008 ACI Code definitions.

The $d/2$ term was dropped when the equation was introduced into the 1971 ACI Code, probably for the sake of simplicity. The consequence of this varies along a span, and it is probably somewhat different for cases of uniformly distributed loads and moving concentrated loads (trucks).

With respect to Eq. (4), a clarification is needed. Is c the elastic or plastic (ultimate) neutral axis depth? They can be quite different, especially for lightly reinforced members. The discussion that cites the low axial stiffness of fiberreinforced polymer materials would seem to imply that this is an elastic neutral axis depth because Young's modulus ordinarily is not a factor in the plastic neutral axis position. If it is elastic, it needs a different notation than c, such as k_d, which was used with allowable stress design. It would be fair to note that if $c = 0.4d$, which is a fairly common value for steel-reinforced members, Eq. (4) reverts to the current ACI 318 shear value.

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AUTHORS' CLOSURE

Closure to discussion by Fernández Ruiz et al.

The authors would like to thank the discussers for their kind words regarding the testing program. They provide excellent discussion and insight regarding the third stage of behavior for loading beyond the formation of the critical shear crack. While this stage was not the focus of the paper as previously discussed, this discussion definitely complements the work. The authors also agree that the effective shear span for prestressed members is different than that of reinforced members because prestressing increases the region of uncracked concrete from the support. This difference greatly influences overall structural behavior and must be carefully considered in developing testing programs evaluating shear strength.

Closure to discussion by Windisch

The authors would like to thank the discusser for his comments and hope that this response provides clarification. First, it is questioned how a force causing cracking can be calculated on a cracked section. There appears to be confusion between the shear force that causes flexural cracking at the section under consideration, which is the basis of Eq. (11-10) in ACI 318-08 (Eq. (2)), and the shear force that forms the primary shear crack as discussed in the paper. As explained, the beams experienced three distinct stages. The first stage is up to flexural cracking where the beam behaves linear-elastic, and the section is uncracked. In the second stage, the beam is cracked, and calculations are based on a cracked section analysis. This stage ends at the formation of a primary shear crack. Failure is assumed to occur when the principal stress in the compression zone of the cracked section reaches the tensile strength of the concrete. This concept was first introduced in a paper by Tureyen and Frosch (2003). Therefore, the shear force discussed in the paper is not the shear that causes the section to crack but rather the shear force that causes a crack to form in the compression zone of a cracked section. Although the beams were able to carry higher loads beyond the formation of the primary shear crack, the authors believe that the behavior of the beams after formation of the primary shear crack changed significantly and that the concrete contribution to shear should be limited to the load-causing formation of the primary shear crack. In the third stage, the beams behaved as a tied arch. The statement in the paper regarding resembling a strut-and-tie model was in referral to the tied arch where the compression flows from the load to the supports with the tension reinforcement serving as the tie.

The discusser points out that in Fig. 6, the neutral axis depth belonging to V_{test} is always greater than the depth belonging to V_{calc}. It appears that the discusser is confused, as Fig. 6 shows that the neutral axis depth corresponding to V_{test} is less than the neutral axis depth corresponding to V_{calc} in all cases. Therefore, the measured shear that caused the primary shear to form was higher than that calculated. V_{calc} was calculated based on a lower bound value of $K = 5$. As discussed in the paper, the applied load was increased in the analytical model until the applied shear at any location along the beam was greater than the strength calculated by Eq. (4). Therefore, the neutral axis depth and resulting shear strength are determined as part of the analysis procedure. The analysis procedure also determines if the reinforcement (mild and prestressed) is yielded or not. It should be noted that perfect bond is assumed in the cracked section analysis for both the mild and prestressing reinforcement. Further details on the analysis and design of prestressed members using this approach are available in Wolf and Frosch (2007).

The authors agree with the discusser that the final failure mode was compression-shear, and this was stated in the paper. The classification of the failure mode as diagonal tension was intended for formation of the critical shear crack. While the final mode of failure was compression-shear and did occur at loads significantly higher than that at formation of the primary shear crack, there are substantial physical reasons why the shear strength associated with this event was considered important in the 1950s and should be considered today. As discussed in the paper, a significant change in overall behavior and in particular stiffness of the beam is observed following formation of the primary shear crack. In addition, significant widening of the shear crack was observed, which would not be considered desirable structural behavior. Finally, if stirrups were present, it is expected that they would contribute significantly to the strength following formation of the shear crack. While behavior beyond the formation of the primary shear crack is of importance, it is for these reasons that the concrete contribution of shear strength, V_c, was considered at the formation of the primary shear crack rather than at the failure load of the beam.

Closure to discussion by Gamble

The authors would like to thank the discusser for providing a clearer view of the historical development of Eq. (2). With respect to Eq. (4), this equation was introduced in an earlier paper by Tureyen and Frosch (2003), and the details behind its development are available in this paper. To clarify, c is the depth of the neutral axis at the section considered and is calculated based on a cracked section analysis. For a reinforced concrete beam, the neutral axis is calculated based on the elastic cracked section for which it is appropriate to use $c = kd$ where k is a function of the reinforcement percentage ρ and the modular ratio n. It should be noted that, along the beam length, the depth of the neutral axis is constant considering the elastic cracked section. As mentioned by the discusser, Eq. (4) reverts to the current ACI 318 shear value of $V_c = 2\sqrt{f_c'}b_w d$ for the case where $k = 0.4$. For values of k less than 0.4, however, Eq. (4) suggests that shear strengths lower than that provided by $V_c = 2\sqrt{f_c'}b_w d$ may result. For a prestressed concrete beam, the depth of the neutral axis c is not constant along the beam length but rather varies with the magnitude of the applied moment at the section considered. In fact, the neutral axis depth c for prestressed members can vary significantly depending on the amount of steel present (prestressed and mild) and the effective steel stress resulting in significant changes in the shear strength.