Modeling strategies for spatial extremes; A case study in extreme fire sizes

K.F. Turkman and M.A. Amaral Turkman
CEAUL and DEIO, FCUL, University of Lisbon

Research partially supported by
FSB-IMA and CIB- Centre Interfacultaire Bernoulli, EPFL.

Conditional vs Unconditional models

- Random effect approach to modeling spatial extremes: Conditional independence. Ease in inference.
- Modeling unconditional distributions by Max-stable processes: correct asymptotic arguments, but difficulties in inference.
- Should we model the conditional distributions or the unconditional distributions by an extreme value family?
- Ideally both! Correct asymptotics and ease in inference.
- Such max-stable processes exist?
  Fougéres-Nolan-Rootzén(2009): Stable mixtures of EV distributions.
- Full conditional specification of Heffernan and Tawn for areal models: Do full conditionals define a unique joint distribution?

Models

1. max-stable processes with Frechét margins
   $Y(s) = \mu(s) + \frac{\sigma(s)}{k(s)}\left[Z_1^{k(s)}(s) - 1\right]$
2. max-stable processes with Gumbel margins
   $Y(s) = \sigma(s)Z_2(s) + \mu(s)$
3. $Z_1(s)$ and $Z_2(s)$ are respectively the unit Frechét and unit Brown-Resnick max-stable processes. Many possible variations in the canonical representations.
4. Areal Fougères-Nolan-Rootzén model:
   $X(i,j) = \mu(i,j) + \sigma\log(H(i,j)) + G(i,j)$,
   $H(i,j)$ latent MA-process composed of positive α-stable r.v´s, $G(i,j)$ iid $Gumbel(0,\sigma)$

Inference

1. Semi-parametric methods: estimation of the exponent measure of the limiting max-stable process.
2. Composite likelihood methods: Approximations based on bivariate densities.

- Our objective: Devise fully Bayesian Hierarchical methods in fitting max-stable processes.
- Will report inference for Brown-Resnick processes .
- Apply these inferential methods to a data set which needs non-standard modeling strategies.

BHM

- Data: $Y(s_i, t), i = 1, ..., K, t = 1, ..., N$. Y represents the block maxima data (annual, monthly etc), observed at K fixed sites.

- Assume that the data are N iid realization of the max-stable process
  $Y(s) = \sigma(s)Z(s) + \mu(s)$,

- $Z(s) = \vee_{i=1}^{\infty}[(u_i + V_i(s) - c_1(s)/2]$,
  $\{u_i\}$ is a realization of a point process on real line with intensity $e^{-y}dy$, $V_i(s)$ are independent copies of a 0-mean, intrinsic stationary Gaussian process with variogram $\gamma(h)$ and variance $c_1(s)$.

Latent process

- We take $V_i(s)$ to be a isotropic, Gaussian process with the exponential decay covariance function $r(h) = c_1 e^{-\left(\frac{|h|}{c_2}\right)^v}$, $c_1 > 0$, $v \in (0, 2]$, $c_2 > 0$.
- and the corresponding variogram $\gamma(h) = 2c_1(1 - e^{-\left(\frac{|h|}{c_2}\right)^v})$
- Can use instead, intrinsic stationary Gaussian process with $\gamma(h) = (|h|/c)^v$, $c > 0$, $v \in (0, 2)$.
- With such intrinsic process, we get asymptotical independence of $Z(s_1)$, $Z(s_2)$, for $|s_1 - s_2| \to \infty$. Pairwise dependence measure: $\rho(h) = 2(1 - \Phi(\sqrt{\gamma(h)}/2))$

Bivariate

- Bivariate distributions of $Z(s)$ are given by Kabluchko et al(2009):
- $$\begin{aligned} & P(Z(s_1) \leq z_1, Z(s_2) \leq z_2) \\ = \quad & \exp\left\{ -e^{-z_1} \Phi(\sqrt{\gamma(h)/2} + \frac{z_2 - z_1}{\sqrt{\gamma(h)}}) \right\} \\ \times \quad & \exp\left\{ -e^{-z_2} \Phi(\sqrt{\gamma(h)/2} + \frac{z_1 - z_2}{\sqrt{\gamma(h)}}) \right\} \end{aligned}$$
- $f_{Z(s_1),Z(s_2)}(z_1, z_2)$,the corresponding density.

Composite

- Hierarchical model based on composite likelihood:
- $$\begin{aligned} & \log \mathcal{L}(y(s_i,t), i=1,...,K, t=1,...,N|\boldsymbol{\Theta}) \\ = & \sum_{t=1}^{N} \log \mathcal{L}(y(s_i,t), i=1,...,K|\boldsymbol{\Theta}) \\ \sim & \sum_{t=1}^{N} \sum_{i<j} \log \mathcal{L}(y(s_i,t), y(s_j,t)|\boldsymbol{\Theta}) \\ = & \sum_{t=1}^{N} \sum_{i<j} \log f_{Z(s_1),Z(s_2)}(g(y(s_i,t), g(y(s_j,t)) \frac{1}{\sigma(s_i)\sigma(s_j)} \end{aligned}$$
- $g(y(s_i,t) = \frac{1}{\sigma(s_i)}[y - \mu(s_i)]$
- $\boldsymbol{\Theta}$ all model parameters.

Parameters

- Parameters:
- $\sigma(s) = \sigma$, $\mu(s) = \mu$ with vague priors
- Can introduce, if justified, extra temporal and/or extra spatial structures through the parameters:

$$\sigma(s,t) = \sigma + \eta_1(s) + \zeta_1(t),$$

$$\mu(s,t) = \mu + \eta_2(s) + \zeta_2(t)$$

Priors

- Priors for the dependence parameters of the Brown-Resnick max-stable process:
- $c_1 \sim U(a, b)$
- $\phi = c_2^{-1} \sim U(\phi_{min}, \phi_{max})$
- ϕ_{min}, ϕ_{max} are functions of the minimum and maximum observed distances
- $v \sim ?$ Difficult to specify priors. Convergence problems if too few sites. We fixed it at $v = 1$.
- Time consuming, even for moderately large number of sites.
- Need alternative, less time consuming methods.

Convergence

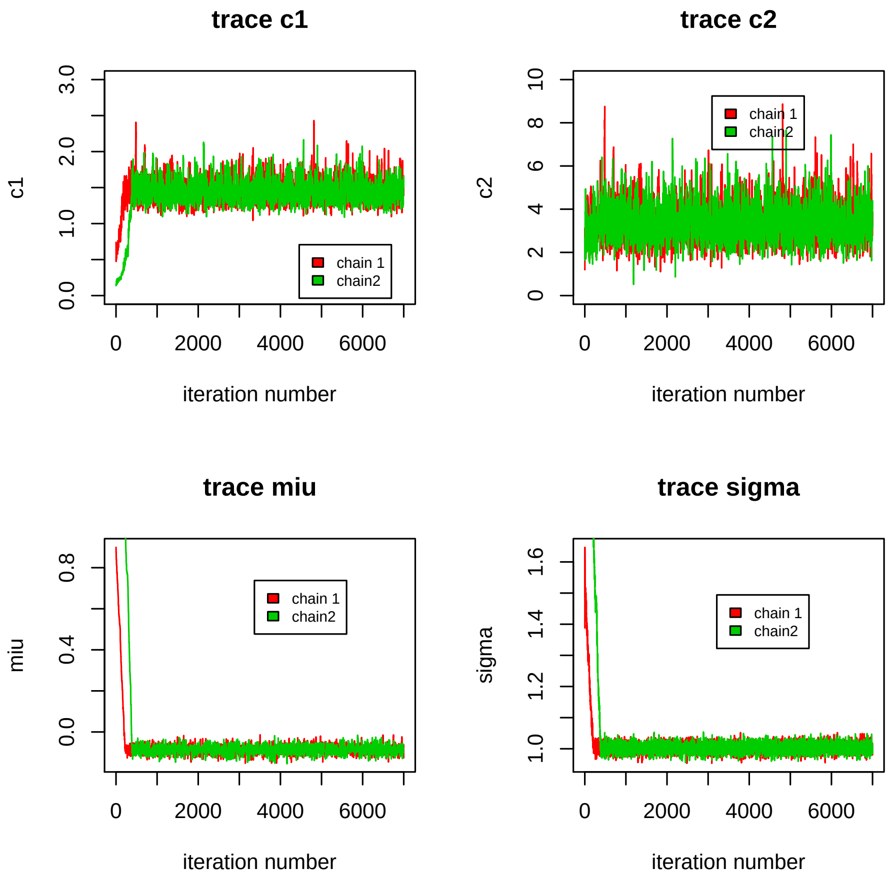


Figure 1: Traces

Posterior

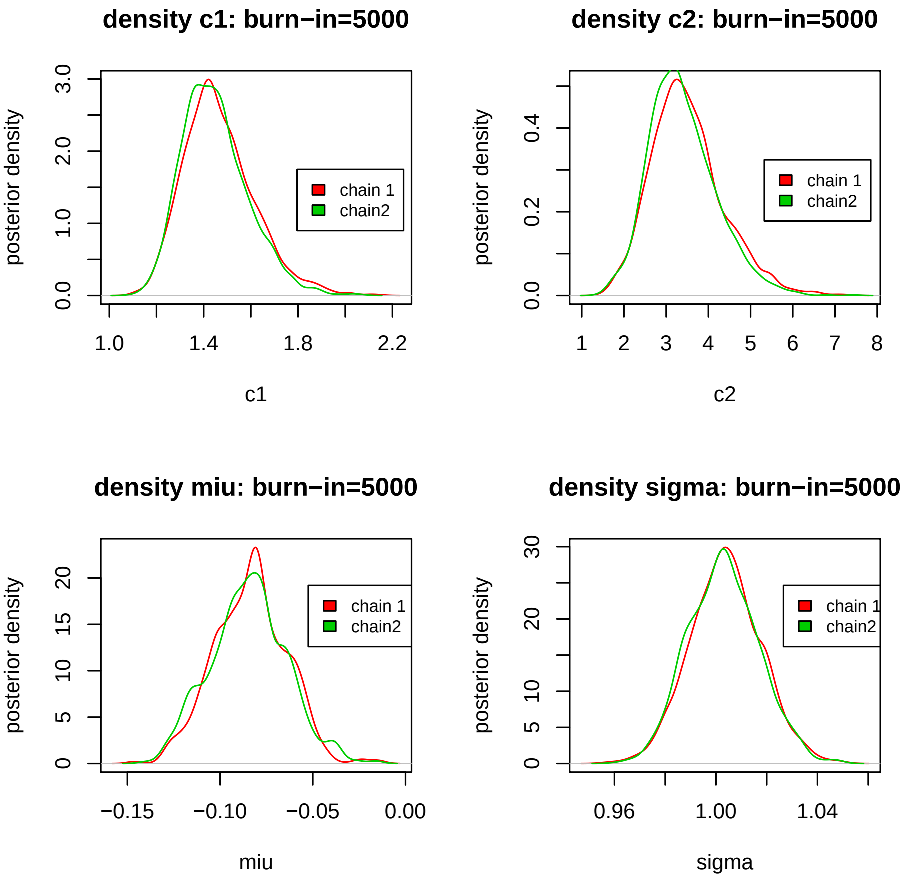


Figure 2: posterior densities

Alternative

- Alternative, simulation based hierarchical model
- Basic trick: Replace the likelihood by

$$Y(s,t) = \sigma(s)Z(s,t) + \mu(s) + \epsilon(s,t)$$

- The error terms $\epsilon(s,t)$ are iid random variables 0 mean and with very large precision $\tau \sim 200$, so that effectively, we sample from the max-stable process.
- Simulate Z conditional on V and u from
- $Z_t(s)\sim \bigvee_{i=1}^{20}[u_{it}(s) + V_{it}(s) - c_1/2]$
- $u_{it} = -\log(E_{1t} + ... + E_{it})$, E_{jt} iid replicates of $exp(1)$.
- with the previous isotropic structure for $V(s)$.

Tuning

- Not very good results. Large precision $\Rightarrow$ sampling in a very narrow region of parameter space, small precision $\Rightarrow$ not sampling from max-stable process.
- Model for ϵ? Need a model for ϵ which is compatible with conditional and unconditional distributions of Y.
- Fougerés-Nolan-Rootzén results suggests taking $\epsilon \sim$ Exponential-stable.

Predictions

How to estimate $P((Y(s_1^*), ..., Y(s_k^*)) \in A|\mathcal{D})$

1. Fit composite likelihood based hierarchical model to data $\mathcal{D}$
2. Get samples $\mathbf{\Theta}_i$ from the joint posterior distribution of the model parameters $\mathbf{\Theta} = (c_1, c_2, \mu, \sigma)$
3. For every i, Simulate the max stable process $Y(s)$ locations $s_1^*, ..., s_k^*$ conditional on $\mathbf{\Theta}_i$. These samples are from the predictive distribution of $(Y(s_1^*), ..., Y(s_k^*))$.
4. Can estimate $P((Y(s_1^*), ..., Y(s_k^*)) \in A|\mathcal{D})$ using empirical methods,namely the number of times these simulated samples hit A.
5. Credible intervals for these estimates? Possible for marginal probabilities.
6. Spatial krigging of extremes: Composite likelihood arguments for the conditional distributions.

Data

- The data consist of 34.345 records of wildfires observed in Portugal between 1975 and 2005.

  1975 -1983 fires above 35 hectares, 1984-2005 fires above 5 hectares

- Fire perimeters were mapped from satellite imagery, About 170 satellite images, acquired annually after the end of the summer fire season, were analyzed over the 31-year period.

- no covariate information is available for this study.

- Important factors that generate extreme fires: Synoptic meteorological conditions acting on large spatial distances, local conditions (land topology, vegetation etc) acting on shorter distances.

Fires

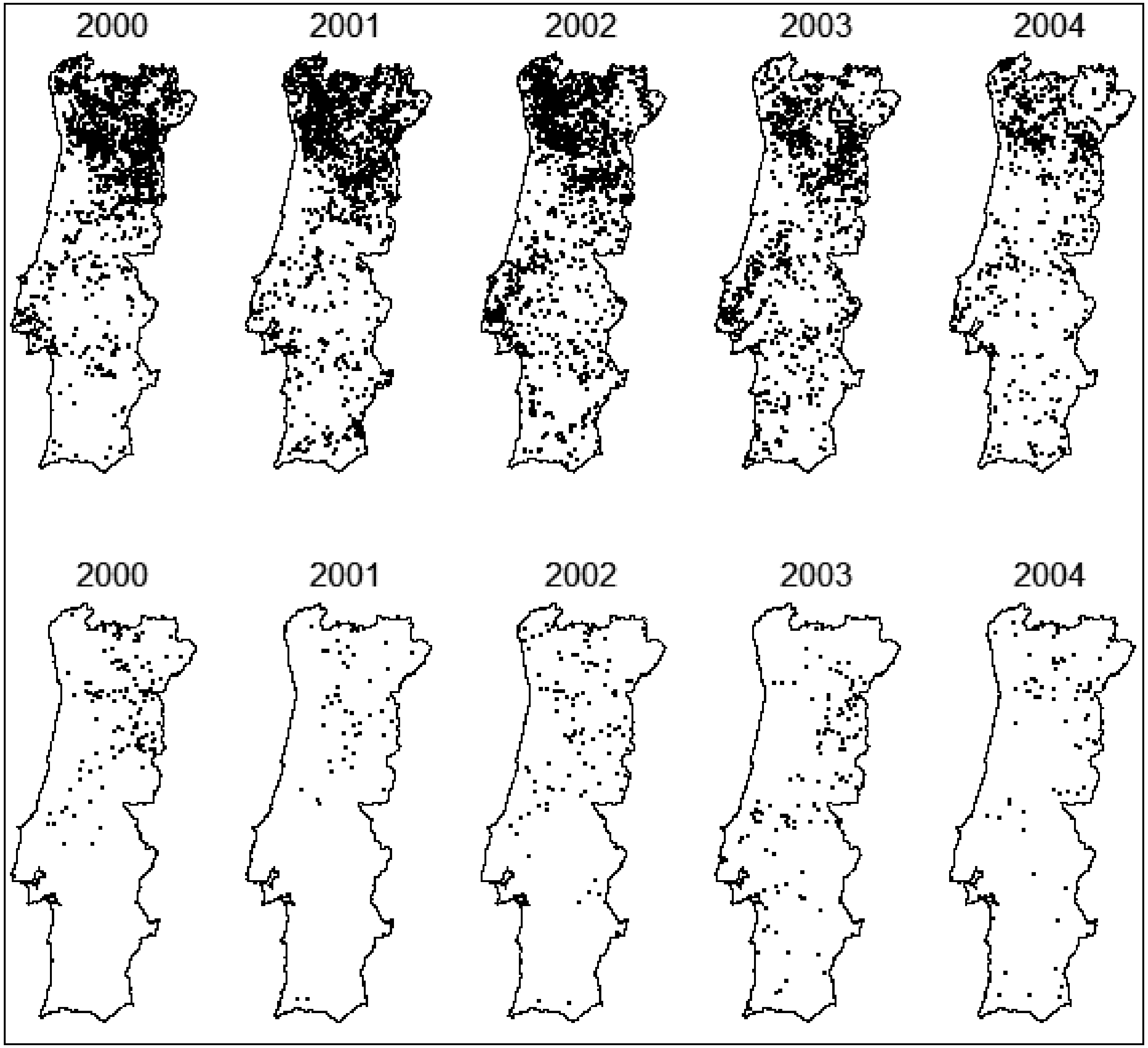


Figure 3: Locations of fires above 5 hectares (top) and locations of fires above 250 hectares (bottom)

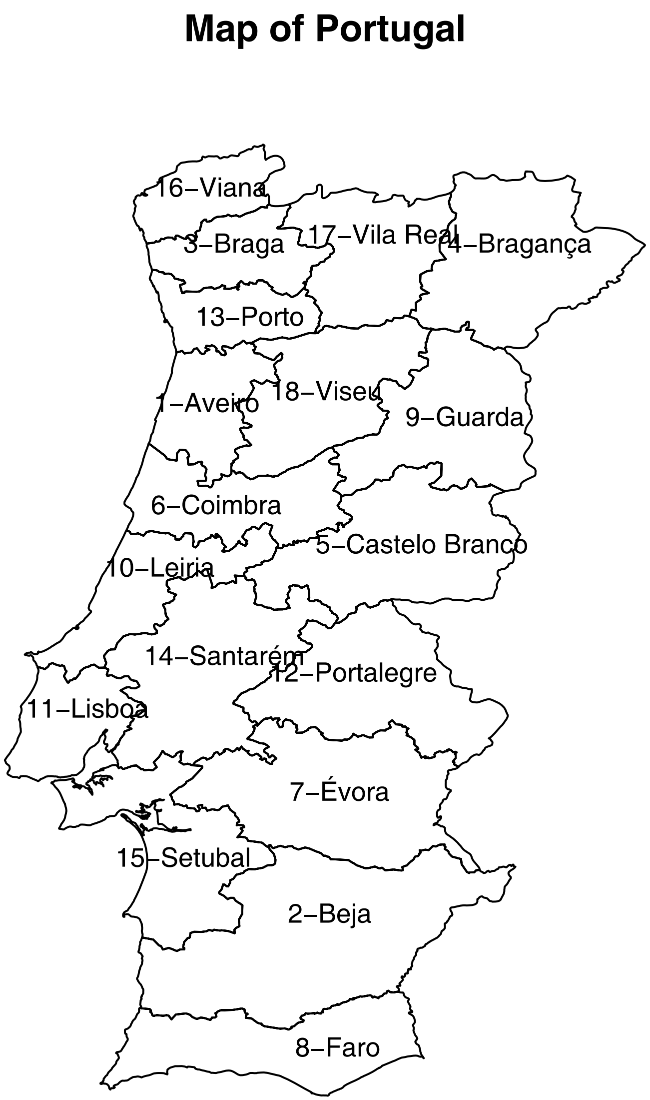
Dependence

- Data is heavy tailed, consistent with fire size distributions with infinite variance.
- Question: which conditions are more important in generating large fires: meteorological conditions or local conditions? How can one answer this without covariate information?
- Model these two set of conditions by two latent random factors, having short and long range dependence structures.
- Large fires are also time dependent. Need to put this structure in the model too. Can the model capture the growth cycle of fire prone vegetation? Is there any temporal structure in the large fires?

Conditional Independence

- Reasonable assumption: if one is able to eliminate the effect of all meteorological and local variations, the occurrence of large fires in space are independent.
- A hierarchical GPD model was devised. Local and global spatial dependence structures are introduced as latent random factors through the model parameters.
- long range spatial dependence is introduced as a CAR model linking 18 administrative regions. Short term dependence due to local topological conditions are introduced as a local moving average term.
- How to fit a max stable process to this data when there are no fixed observation sites? Use spatial block maxima instead of time-block maxima as data.

Areal



Map of Portugal
16–Viana
3–Braga
17–Vila Real
4–Bragança
13–Porto
1–Aveiro
18–Viseu
9–Guarda
6–Coimbra
5–Castelo Branco
10–Leiria
14–Santarém
12–Portalegre
11–Lisboa
7–Évora
15–Setubal
2–Beja
8–Faro

Max-stable model

- We look at the log transformed data and fit a max stable process with Gumbel margins.
- $Y(s_i, t) =$ the (log) largest annual fire in each of the 18 administrative region, observed during year t. Fit a max-stable process to the data $Y(s_i, t), i = 1, ..., 18, t = 1, ..., 31$
- $Y(s_i, t) = \sigma(s_i, t) Z_t(s_{i,t}) + \mu(s_i, t)$
- $\log(\sigma(s, t)) = \sigma_0 + \delta_1(t)$
- $\mu(s, t) = \mu_0 + \delta_2(t)$
- $Z_t(s_{i,t})$ are 31 independent,identical replicates of a Brown-Resnick process, observed at different locations at each year, $\delta_i(t)$ latent, temporal factors.

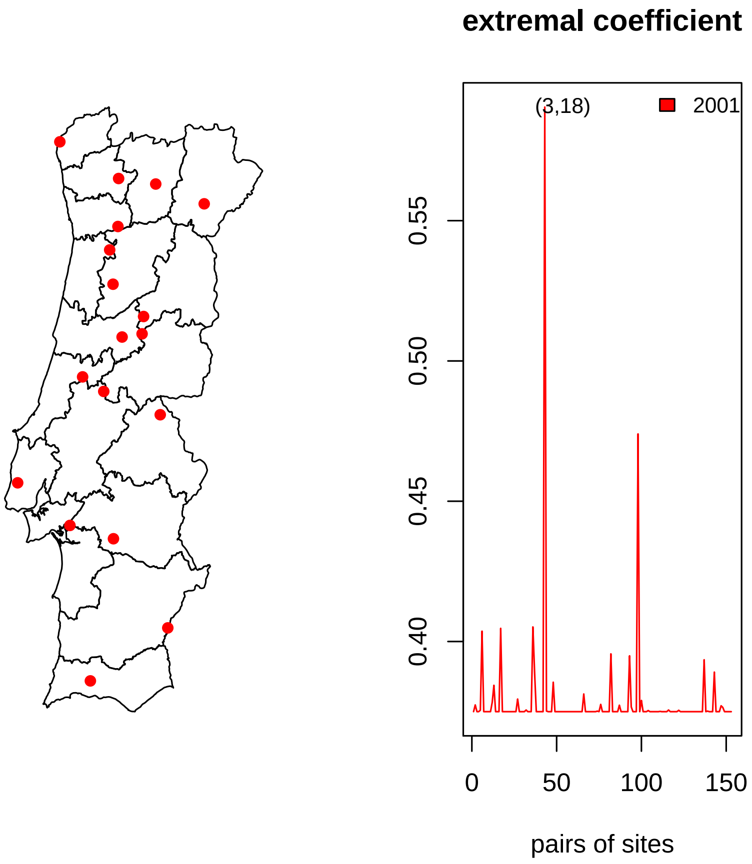


Figure 5: Traces

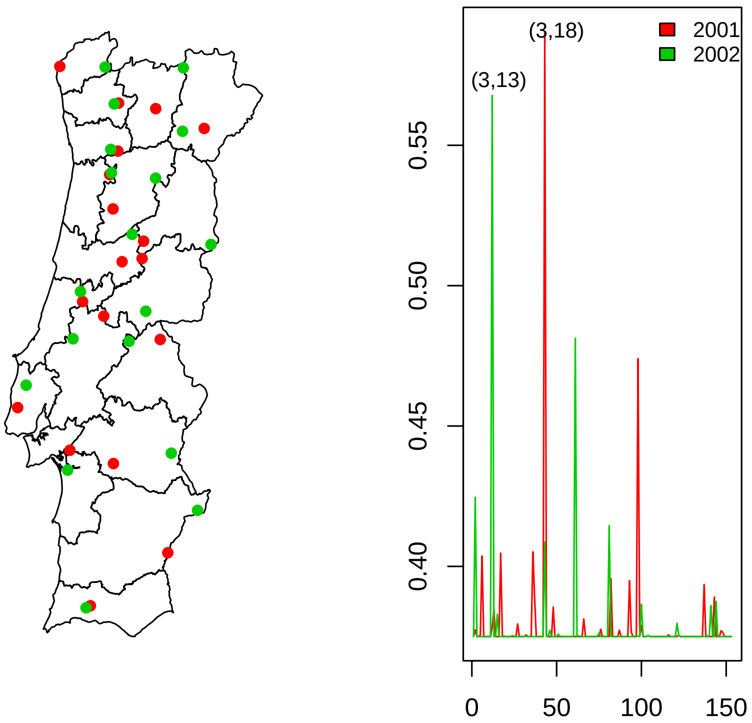


Figure 6: Traces

extremal coefficients

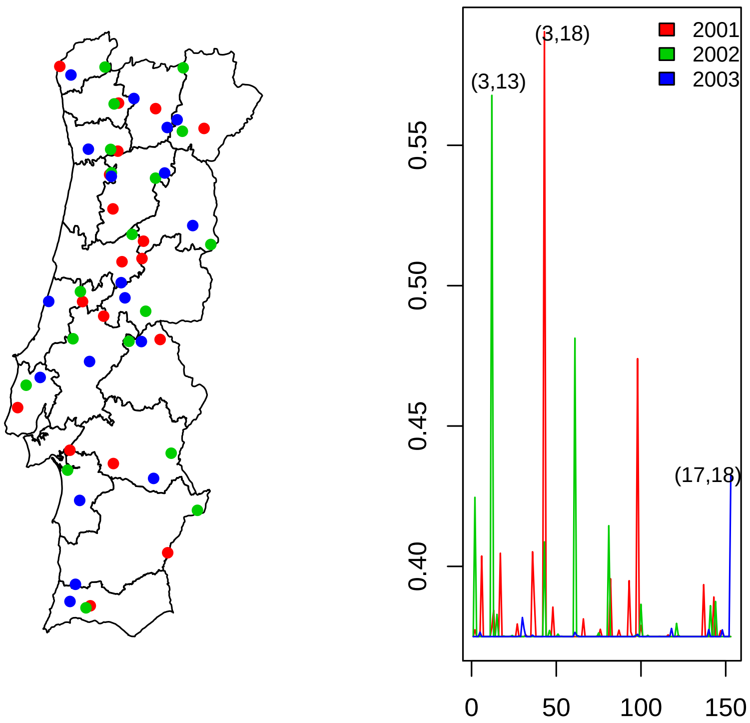


Figure 7: Traces

extremal coefficients

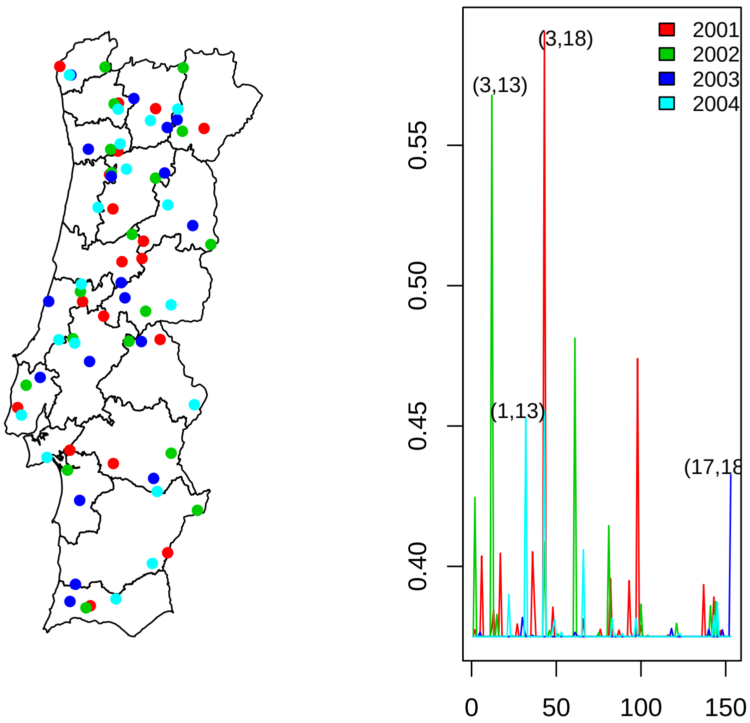
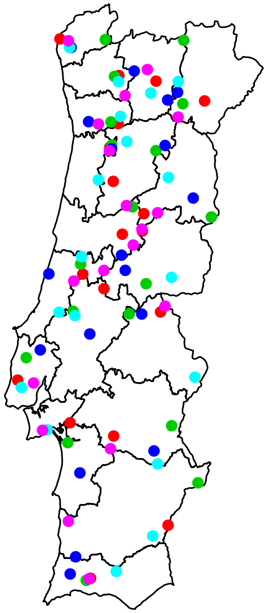


Figure 8: Traces



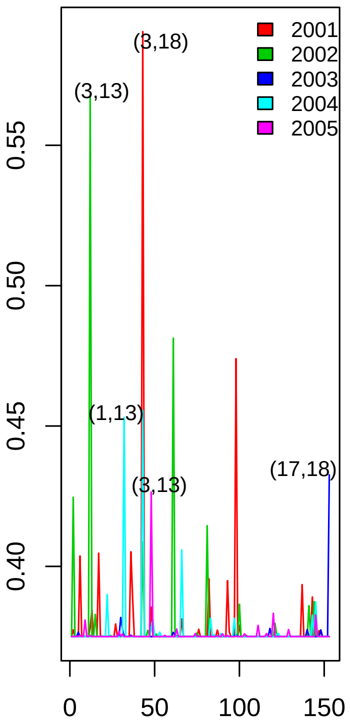
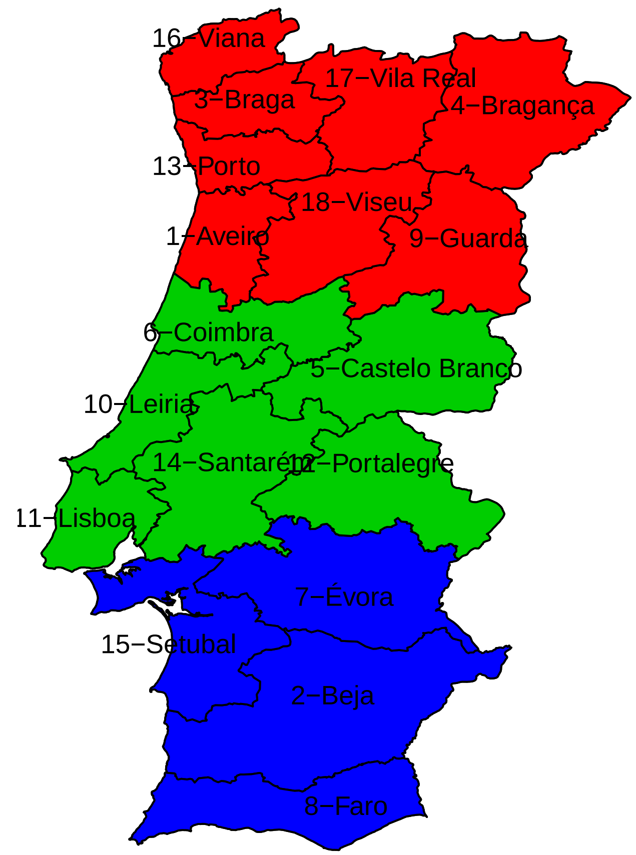


Figure 9: Traces

Max-stable model

- Simplest model possible, but not realistic.
- Extreme fires behave differently over the territory.
- Parameters change in space as well as in time.
- Dependence due to meteorological and local conditions confound in this model.
- How to introduce separate long range and short range dependence structures in the model?
- 18 Administrative regions are divided into 3 main Geographical regions: North, Center and South.

Geographical regions



3 models

- Data is re-organized: $Y(s_{j_k,t}) =$ maximum (log) burned area in year $t = 1, .., 31$, region $j_k = 1, ..., n_k$, belonging to the geographical region $k = 1, 2, 3$, located at spatial point $s_{j_k,t}$ (centroid of fire scar).
- General strategy:Fit 3 max-stable processes for each geographical regions and link these processes through the model parameters.
- Dependence structure introduced through the parameters will represent the meteorological conditions acting over large distances, whereas the dependence structure of each the Brown-Resnick processes will represent the local dependence structures.

Hierarchy

- $Y(s_{j_k,t}) = \sigma(k,t)Z_k(s_{j_k,t}) + \mu(k,t)$,
- $Z_k(s)$ are 3 independent Brown- Resnick max-stable processes fitted to each of the 3 Geographical regions
- $Z_k(s_{j_k},t)$ are $N = 31$ independent replicates of the Brown-Resnick process $Z_k(s)$, observed at spatial locations $s_{j_k,t}$ (different locations at each year)
- $\log(\sigma(k,t)) = \sigma_k + \eta_1(k) + \zeta_{1k}(t)$
- $\mu(k,t) = \mu_k + \eta_2(k) + \zeta_{2k}(t)$
- $\eta_1(k)$, $\eta_2(k)$ are co-regionalized CAR,
- $\zeta_1(t)$,$\zeta_2(t)$ are independent, first order autoregressive processes to capture time dependence, specific for each region.

Results

- 3 max-stable processes have different behaviour. Extremes in North region seem to be more dependent than other two regions.
- Significant temporal effect capturing the expected cycle of vegetation growth.
- Statistically insignificant spatial effect connecting the 3 regions
- How to compare conditionally independent model and this max-stable model?
- Conditionally specified max-stable models of Fougères-Nolan-Rootzén can give better results.