# Strain estimation in digital holographic interferometry using piecewise polynomial phase approximation based method

**Sai Siva Gorthi, G Rajshekhar and Pramod Rastogi**

*Applied Computing and Mechanics Laboratory, Ecole Polytechnique Fédérale de Lausanne, 1015-Lausanne, Switzerland.*

*pramod.rastogi@epfl.ch*

**Abstract:** Measurement of strain is an important application of digital holographic interferometry. As strain relates to the displacement derivative, it depends on the derivative of the interference phase corresponding to the reconstructed interference field. The paper proposes an elegant method for direct measurement of unwrapped phase derivative. The proposed method relies on approximating the interference phase as a piecewise cubic polynomial and subsequently evaluating the polynomial coefficients using cubic phase function algorithm. The phase derivative is constructed using the evaluated polynomial coefficients. The method's performance is demonstrated using simulation and experimental results.



**OCIS codes:** (120.2880) Holographic interferometry; (090.1995) Digital holography; (120.4290) Non-destructive testing

## References and links

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## 1. Introduction

Digital holographic interferometry (DHI) is a popular non-invasive and whole field measurement technique which has emerged as an important tool for deformation analysis in areas like non-destructive testing, experimental mechanics etc. In DHI, the information about the deformation or displacement is encoded in the interference phase of the complex reconstructed interference field whose real part constitutes a fringe pattern [1] . For many applications, the displacement derivative or equivalently the interference phase derivative is of particular interest since it gives information about the strain distribution. A popular approach for phase derivative estimation in DHI is digital shearing where a superposition between pixel shifted i.e. sheared complex amplitude of reconstructed wave and the original complex amplitude is performed to approximate the phase differentiation operation [2, 3, 4]. The sensitivity of this method depends on the amount of the shearing introduced. The phase derivative obtained using the digital shearing approach is usually susceptible to noise and hence methods involving various filtering operations have been proposed [5]. The iterative filtering operation in [5] is time-consuming and has to be implemented with caution so as not to smear the dense fringes. It needs to be emphasized that the phase derivative obtained by the above methods is wrapped and hence requires an unwrapping algorithm. Some of the other methods developed for phase derivative estimation are [6, 7]. Recently, methods [8, 9] based on the high-order ambiguity function (HAF)[10] were proposed with potential benefits for fringe analysis in DHI. However the performance of HAF based methods is adversely affected in the presence of severe noise [11].

In this paper, we propose an elegant method to directly estimate the unwrapped phase derivative in DHI even in the presence of severe noise using the cubic phase function (CPF) algorithm [11]. The proposed method works by modelling the complex reconstructed interference field obtained in DHI as a piecewise polynomial phase signal. In other words, the reconstructed interference field is divided in many segments and the interference phase is assumed to behave like a polynomial in each segment. The major benefit of using piecewise polynomial approximation is that even phase distribution with rapid variations can be modelled as a low order polynomial with sufficient accuracy in a small segment. In the proposed method, the phase is modelled as a cubic polynomial or equivalently the phase derivative as a quadratic, and the quadratic coefficients are evaluated using the CPF algorithm in each segment. The phase derivative is then constructed using the evaluated coefficients. The theory of the proposed method is outlined in the next section and simulation and experimental results are presented in section 3 followed by conclusions and acknowledgements.

## 2. Theory

The reconstructed interference field in DHI is given as

$$I(x,y) = A(x,y)\,\exp\left[j\phi(x,y)\right] + \eta(x,y) \tag{1}$$

where $A(x,y)$ is the amplitude term; $\phi(x,y)$ is the interference phase and $\eta(x,y)$ represents the noise assumed to be zero mean additive white gaussian noise (AWGN). Here $x$ and $y$ refer to the pixels or equivalently columns and rows along the $N \times N$ fringe pattern. To implement the piecewise polynomial phase approximation, we divide an arbitrary column $x$ into say $N_w$ segments such that each segment is of size $N_s = N/N_w$. So for the $k^{th}$ segment such that $k \in [1, N_w]$, Eq. (1) can be written as

$$I_k(y) = A_k(y)\,\exp\left[j\phi_k(y)\right] + \eta_k(y) \tag{2}$$

Assuming a cubic phase approximation with the cubic coefficients $[a_{0k}, a_{1k}, a_{2k}, a_{3k}]$ for the $k^{th}$ segment, we have

$$\phi_k(y) = a_{0k} + a_{1k}y + a_{2k}y^2 + a_{3k}y^3 \tag{3}$$

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$$\frac{\partial \phi_k(y)}{\partial y} = a_{1k} + 2a_{2k}y + 3a_{3k}y^2 \tag{4}$$

$$\frac{\partial^2 \phi_k(y)}{\partial y^2} = 2(a_{2k} + 3a_{3k}y) \tag{5}$$

From Eq. (4), it is clear that the phase derivative for the $k^{th}$ segment can be evaluated by determining the coefficients $[a_{1k}, a_{2k}, a_{3k}]$. They are estimated using the CPF algorithm [11]. The CPF of $I_k(y)$ is given as

$$CPF_k(y,\Omega) = \sum_{\tau=0}^{(N_s-1)/2} I_k(y+\tau)I_k(y-\tau)exp(-j\Omega\tau^2) \tag{6}$$

The peak of the CPF's magnitude corresponds to the second order derivative of phase also known as the instantaneous frequency rate in signal processing. Hence we have

$$U_k(y) = \arg\max_{\Omega} |CPF_k(y,\Omega)| \tag{7}$$

where '$\arg\max$' indicates the value of the argument $\Omega$ at which $|CPF_k|$ attains the maximum value. So using Eq. (5), we have

$$U_k(y) = 2(a_{2k} + 3a_{3k}y) \tag{8}$$

Equation (8) which involves the coefficients $a_{2k}$ and $a_{3k}$ gives the second order phase derivative as a function of $y$ for the $k^{th}$ segment. In order to estimate these coefficients, $U_k(y)$ is evaluated for two positions of $y$ i.e. $y_1$ and $y_2$ to generate two equations in the two variables $[a_{2k}, a_{3k}]$. Hence, we have

$$U_k(y_1) = 2(a_{2k} + 3a_{3k}y_1) \tag{9}$$

$$U_k(y_2) = 2(a_{2k} + 3a_{3k}y_2) \tag{10}$$

Equation (9) and Eq. (10) are solved to get the estimates $[\hat{a}_{2k}, \hat{a}_{3k}]$. The recommended values of $y_1$ and $y_2$ are 0 and $0.11N_s$ to keep the estimation error minimum [11]. The remaining coefficient $a_{1k}$ is then estimated using a dechirping operation which is carried out as

$$I'_k(y) = I_k(y)exp[-j(\hat{a}_{2k}y^2 + \hat{a}_{3k}y^3)] \tag{11}$$

Equation (11) is equivalent to peeling off the contribution of the polynomial coefficients $a_{2k}$ and $a_{3k}$ from the phase of $I_k(y)$ which effectively yields $I'_k(y)$ as a single tone signal with frequency $a_{1k}$. Hence estimation of $a_{1k}$ boils down to single tone frequency estimation from $I'_k(y)$ which can be implemented using a Fourier transform (FT). In other words,

$$G_k(\omega) = FT[I'_k(y)] \tag{12}$$

$$\hat{a}_{1k} = \arg\max_{\omega} |G_k(\omega)| \tag{13}$$

Equation (12) is efficiently implemented using fast Fourier transform (FFT). The estimation accuracy for $\hat{a}_{1k}$ is further improved using iterative frequency estimation by interpolation on Fourier coefficients (IFEIF) technique [12]. IFEIF is a computationally efficient technique with enhanced accuracy for single tone frequency estimation. With the coefficient estimates $[\hat{a}_{1k}, \hat{a}_{2k}, \hat{a}_{3k}]$ known, the phase derivative for the $k^{th}$ segment can be constructed using Eq. (4).

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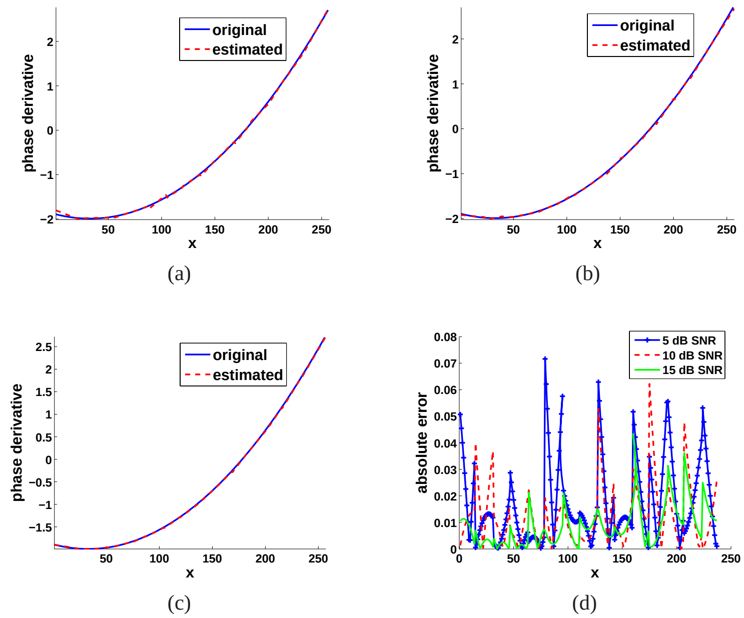


Fig. 1. Original vs estimated phase derivative in radians/pixel at SNR of (a) 5 dB, (b) 10 dB, (c) 15 dB. (d) Absolute error for phase derivative estimation

By repeating the above procedure for all $N_w$ segments, the phase derivative for the column $x$ is determined. A 50 % overlapping segment strategy is used to minimize the error at the boundaries of adjacent segments. The phase derivative is determined in a similar fashion for all columns $x \in [1,N]$ to give the overall phase derivative for the entire fringe pattern. It needs to be emphasized that the phase derivative obtained by the proposed method is unwrapped and hence no further unwrapping algorithm is required.

The major advantage of the proposed method is the inherent robustness of the CPF algorithm to severe noise [11]. To show the applicability of the proposed method for phase derivative estimation, we simulated a one dimensional signal at signal to noise ratios (SNR) of 5 dB, 10 dB and 15 dB. The performance of the proposed method is shown in Fig. 1(a)-1(c). The absolute errors in phase derivative estimation for different SNRs are shown in Fig. 1(d). It is clear from Fig. 1 that even for SNR as low as 5 dB, the proposed method works reasonably well for phase derivative estimation.

## 3. Simulation and experimental results

The fringe pattern corresponding to the real part of the reconstructed interference field in DHI simulated at SNR of 5 dB is shown in Fig. 2(a). The original phase derivative along $y$ direction in radians/pixel is shown in Fig. 2(b). The phase derivative estimate $\omega_1(x,y)$ in radians/pixel obtained by applying the proposed method is shown in Fig. 2(c). We used $N_w = 8$ for analysis throughout the paper. Though the phase derivative obtained from the proposed method is unwrapped, the corresponding wrapped form is shown for illustration purpose only in Fig.2(d). The wrapped form was evaluated using $\arctan\{\mathrm{Im}(exp[j\omega_1(x,y)])/\mathrm{Re}(exp[j\omega_1(x,y)])\}$ where 'Im' and 'Re' denote the imaginary and real parts of a complex number. The root mean square error (RMSE) for phase derivative estimation was 0.0166 radians/pixel. Note that the pixels near the borders were neglected for the RMSE calculation to ignore the errors at the bound-

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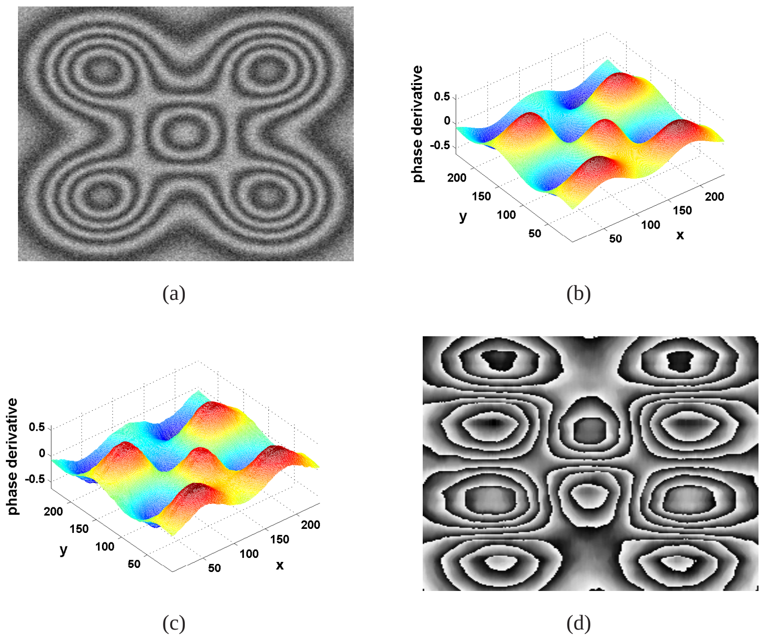


Fig. 2. (a) Simulated fringe pattern. (b) Original phase derivative in radians/pixel. (c) Estimated phase derivative in radians/pixel. (d) Wrapped form of the estimated phase derivative

aries.

The practical applicability of the proposed method is tested for a DHI experiment. A circularly clamped object was subjected to central loading and two holograms were recorded before and after deformation using a Coherent Verdi laser (532 nm). Numerical reconstruction was performed using Discrete Fresnel transform [1] which gave the complex amplitudes of the object wave before and after deformation. The complex amplitude of the post-deformation object wave was multiplied with the conjugate of the complex amplitude of the object wave prior to deformation to obtain the reconstructed interference field. The corresponding fringe pattern is shown in Fig. 3(a). The estimated phase derivative along $y$ direction after applying the proposed method and the corresponding wrapped form are shown in Fig. 3(b) and Fig. 3(c). For the sake of comparison, the phase derivative was also estimated using the digital shearing method [4] where the sheared complex amplitude of the reconstructed wave was superimposed on the original complex amplitude to approximate the phase differentiation operation. The wrapped phase derivative estimate thus obtained is shown in Fig. 3(d). It is clear from Fig. 3(d) that the digital shearing method is susceptible to noise besides requiring an unwrapping algorithm. Compared to the digital shearing method, the proposed method offers better ability to handle fringe patterns with severe noise.

## 4. Conclusions

The paper proposes an elegant cubic phase function algorithm based method for phase derivative estimation in DHI. The major advantages of the proposed method are its ability to directly provide the unwrapped phase derivative thereby eliminating the requirement of unwrapping algorithms and its robustness to noise. The proposed method's performance is verified by the simulations whereas its practical applicability is validated by the experimental results presented in the paper. The results indicate that the method has the potential to be established as an im-

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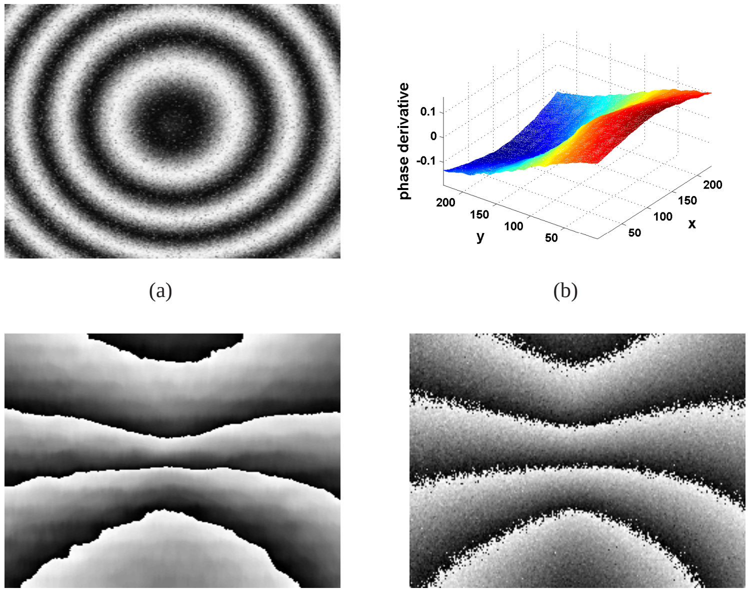


Fig. 3. (a) Fringe pattern obtained in a DHI experiment. (b) Estimated phase derivative in radians/pixel. (c) Wrapped estimated phase derivative. (d) Wrapped phase derivative estimate using digital shearing method.

portant technique for phase derivative estimation in DHI.


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