Comparing two design strategies for tensegrity structures

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Abstract. Tensegrity structures are cable-strut configurations that employ compressive struts to maintain topology of a surrounding tensile cable net. They are closely coupled structures that often display geometrically nonlinear behaviour. There is no generally agreed systematic method for configuration and sizing for a range of design situations. Several configurations have been proposed over the past fifty years; most are too flexible and difficult to erect. Once a configuration is selected, elements are usually sized using local search within the scope of a parametric analysis. Since this is equivalent to gradient search, such a procedure implicitly includes the assumption that there are no local minima in the objective function used for design. This paper compares the results of parametric design with design using stochastic search for a tensegrity-ring pedestrian bridge. Stochastic search revealed a design solution having a fabricated cost that is 26% less than the cost of the solution identified using traditional parametric design.

1 Introduction

Tensegrity structures are spatial structural systems composed of struts and cables with reticulated connections. Thus, they can be considered as a subclass of cable structures with the important property that tensile forces are not anchored. Their stability is based on a self-equilibrated self-stress state. Due to the initial self-stress, their two sets of compressed and tensioned components combine to form a stable system. Tensegrity systems present many interesting structural properties. Due to their composition, they are relatively lightweight systems offering high resistance for a small amount of building material. Modules can be combined together to form larger systems. Therefore, tensegrity systems have potential to be good structural systems for bridges.

Designing tensegrity structures is a challenge for engineers since no generally agreed guidelines exist. The topology of a tensegrity structure is affected by the initial self-stress state. Self-stress also increases load-bearing capacity. The initial equilibrium position can be found either by analytical or experimental methods. A review of form-finding methods can be found in Tibert and Pellegrino (2003). Tensegrity action involves large displacements and therefore analysis should include geometrical nonlinearity. Additionally, the behavior of the structure cannot be predicted from analyzing the behavior of individual components. Studies on simple tensegrity structures have revealed the importance of parameters such as the level of self-stress and the rigidity ratio between struts and cables (Kebiche et al. 1999). Due to these peculiarities, design of tensegrity structures can be a complex task (Quirant et al. 2003) .

In practice, an iterative approach similar to a gradient-based search is employed for design. The idea is to start with a trial solution and then modify design parameters depending upon constraint violations to arrive at a feasible solution. For example, engineers designing tensegrity systems may gradually increment the areas of struts and cables to meet stability and serviceability requirements. The adjustments to design parameters are based upon engineering experience with respect to the influence of parameters on objectives and constraints. The assumption is that the search space has a single trough corresponding to the minimum cost solution and individually adjusting the design parameters would lead to this minimum.

However, this assumption is seldom valid as design spaces have multiple local minima. Gradient-based search often results in solutions that are only locally optimal. Also, it is often of interest to generate a number of good designs. So that designers can select preferred solutions using design criteria that are not modelled explicitly in the objective function.

Researchers have extensively studied the use of optimization methods (Arora et al. 1995) such as genetic algorithms and simulated annealing for structural design optimization (Camp and Bichon 2004; Degertekin et al. 2008; Griffiths and Miles 2003; Kicinger et al. 2005a; Kicinger et al. 2005b; Miles et al. 2001; Shea and Smith 2006; Svenerudh et al. 2002). Tensegrity structures involve a higher level of complexity compared to other structural systems due to their nonlinear behavior and can benefit from the use of stochastic optimization methods (Domer et al. 2003a). Search using stochastic sampling techniques explore the search space by generating and testing many solutions to find good ones (Domer et al. 2003b).

In spite of extensive research on design optimization, general acceptance is slow. This paper aims to provide further evidence by providing a systematic comparison of two methods on the same design task of a tensegrity pedestrian bridge. The first method simulates traditional design through parametric analyses, while the second uses a direct stochastic search called PGSL (Probabilistic Global Search Lausanne). PGSL is a stochastic sampling method for global optimization that has been shown to give better performance than other optimization techniques such as genetic algorithms (Domer et al. 2003b; Raphael and Smith 2003). Results and performance of the two design strategies are compared and discussed.

2 Design Specifications

Tensegrity ring modules are elementary tensegrity systems that were first describe more than thirty years ago (Pugh 1976). Their conception includes the idea of a circuit of compressed components which enhances bending stiffness (Tibert 2003). Motro et al (2006) showed that ring modules are easy to construct based on straight prisms with n-sided polygonal bases. Additionally, tensegrity ring modules are deployable and can be assembled together to form a hollow rope. In this study, a pentagonal hollow rope is used as a structural system for a pedestrian bridge. Four identical modules are connected base to base to span a 20 m pedestrian bridge (Figure 1). The bridge geometry is chosen such that it has the minimum internal space required for two pedestrians to walk side-by-side. This space can be represented by a rectangle with a height of 2.5 m and a width of 2 m. Symmetry about midspan is obtained by mirroring two modules.

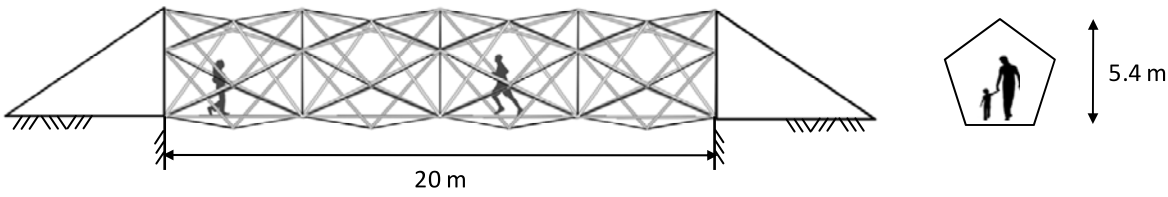

Figure 1: The tensegrity pedestrian bridge

A pentagonal module contains 15 struts and 35 cables. Struts have a length of 678 cm. Cables can be separated in two classes based on their topology and length: layer cables and x-cables. Layer cables form a pentagon on each base of the module with a side length of 457 cm. On the other hand, x-cables form a web around the longitudinal axis of the module. X-cables have a length of 347 cm. This geometry is found to satisfy internal space requirements. The above member dimensions are constant for the whole study. The cables and the struts of the bridge are also assumed to be made of steel. The nodes of the structure at both extremities are fixed in all three directions.

This tensegrity bridge is designed to meet the norms for safety and serviceability specified by the Swiss codes SIA 260 and 263 (SIA 2003a; SIA 2003b). Safety criteria ensure that there is sufficient resistance to avoid failure and instabilities. Therefore, strut design is governed by buckling resistance and slenderness limits. Another constraint is the ratio between the diameter and thickness for tubular struts used for struts. Tension members are only governed by tensile strength. Vertical displacements at midspan are limited to span/700 by serviceability criteria. Two independent live load models are employed for a pedestrian bridge: one with a uniform load and another with a concentrated midspan load. Loads are applied on the walking plate, which transmits the loads to four base nodes of each module. The walking path is 2 m wide and 20 m long. Considering loads and partial factors the uniform load model presents the largest displacements at midspan and the largest forces in the members. Thus, it is taken to be the critical load model for the design of the structure.

The structure is analyzed using dynamic relaxation with kinematic damping. It is an iterative method that traces the motion of the structure until it converges to an equilibrium state (Barnes 1999). Dynamic relaxation is an attractive analysis method for tensegrity structures since it includes geometrical nonlinearities efficiently and without matrix inversion.

3 Cost Model

In this study, a cost model is used that reflects the total cost C of fabricating the structure. This model includes two parts: the cost of elements c_s and the cost of joints c_j.

$$C = c_s(d,t,l,A) + c_j \tag{1}$$

The cost of element depends on the outer diameter d, the thickness t, the length l and the cross-section area A. The cost can be further separated into cost of struts ($c_{s,s}$) and cost of cables ($c_{s,c}$). Struts are made out of steel hollow tubes. Data obtained from local steel construction companies indicate that the price of hollow tubes varies according to cross-section area. The cost of hollow tubes is calculated using Equation 2 obtained using linear regression on commercial data. In Equation 2, $c_{s,c}$ is the cost per unit length in CHF/m and A_s is the area in cm^2.

$$c_{s,s} = 74.8 \cdot A_s \tag{2}$$

For cables, the cost varies with cross-section area and length. Equation 3 relating cost per unit length to the area has been obtained using linear regression. In this equation, $c_{s,c}$ is the cost in CHF/m and A_c is the area in cm^2.

$$c_{s,c} = 66.3 \cdot A_c^{0.72} \tag{3}$$

The second component in Equation 1, and potentially the most important factor affecting the total cost of the structure, is the cost of fabricating the joints. In steel construction, joints are very expensive details that may determine other aspects of the design of a structure. In this study, only a single topology is analyzed. Therefore, the number of joints, and hence their cost, is assumed to remain constant for all design solutions.

4 Design Using Parametric Analysis and Traditional Design

Engineers generally adopt an iterative approach similar to a gradient-based search to design structures. The initial design often violates design constraints. Depending upon the nature of constraint violations, larger member sizes are used and then verified for other criteria. This approach is simulated using parametric studies. Parametric studies are conducted to reveal the individual influence of each variable on the responses related to the design constraints. The goal is to identify the optimal direction, similar to the steepest slope in a gradient-based search. For this case, the effects of parameters on vertical displacements at midspan and the maximum compressive force are required. Vertical displacements at midspan reflect serviceability criteria, while the maximum compressive force is related to failure due to buckling.

Deflection dependencies related to the following five parameters are examined: cross-section area of x-cables, layer cables, and struts as well as the rigidity ratio between struts and cables (varying the Young's modulus of struts) and self-stress. For all parametric studies, a reference design configuration is assumed, including struts with a section of 5 cm^2 and cables of 0.5 cm^2. The parameter of interest is alone varied, while the values for other parameters are left unchanged from the reference configuration.

The parametric analysis reveals that the cross-sectional area of x-cables is the parameter with the most influence on the vertical displacement (Figure 2). Increasing the cross-section area of these cables significantly increases the total rigidity of the system, and thus decreases vertical displacements. However, when the areas of struts are increased displacements initially decrease and then gradually increase, contrary to engineering intuition. Figure 2 shows that areas of layer cables have negligible influence on vertical displacements.

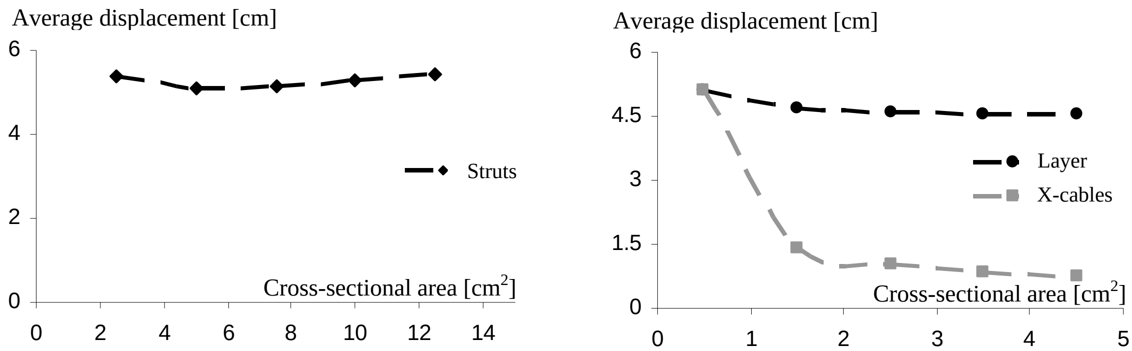

Figure 2: Influence of the cross-sectional area of struts, layer cables and x-cables on the average vertical displacement at midspan

Figure 3 shows that increasing the rigidity ratio results in a decrease in the average vertical displacement at mid-span. The reduction in displacement becomes very small beyond a certain value for the rigidity ratio. Consequently, an optimal rigidity ratio between tensile and compressive elements can be identified for each configuration. This optimal value can be used to guide the design of the structure. The self-stress state is responsible for the stability and the high resistance of tensegrity structures. In this study, self-stress is specified in terms of cable elongation. Figure 3 shows also that increasing self-stress decreases vertical displacements contributing by the way to the overall rigidity of the structure. Figure 4 shows that increasing the cross-section area of x-cables or struts results in a decrease in the compressive force normalized to the buckling capacity. The reduction in the normalized compressive force becomes negligible beyond a certain value for the cross-section area.

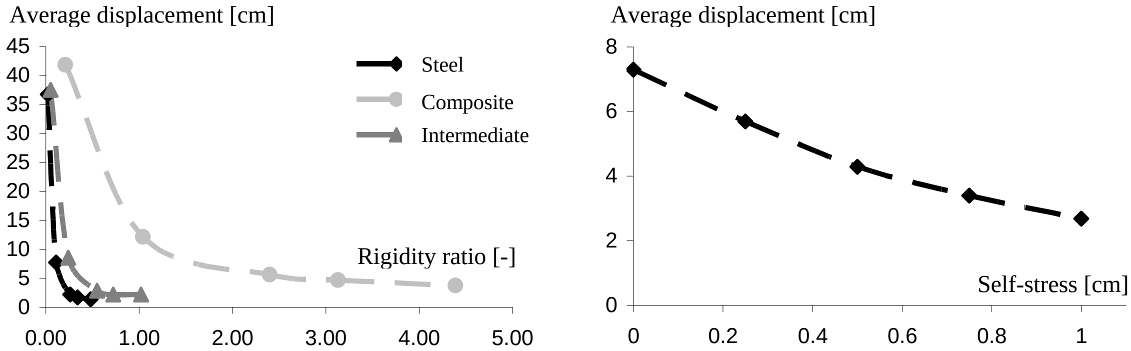

Figure 3: Influence of the rigidity ratio between bars and cables and self-stress on the average vertical displacement at midspan

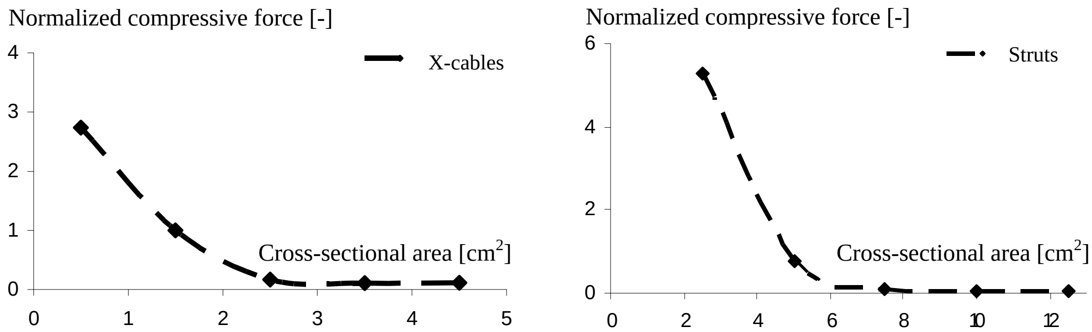

Figure 4: Influence of the cross-sectional area of x-cables and struts on the compressive force normalized to the buckling capacity

A feasible configuration is found based on the trends from the parametric analyses. The first parameter that is modified is the area of the x-cables. A value is found for which both vertical displacements and tensile strength constraints are satisfied. The next variable considered is the area of the struts. A tube with sufficient cross-section area and moment of inertia is chosen for the struts such that it avoids the instability due to buckling. Finally, self-stress is introduced in order to increase the rigidity of the structure and avoid slack cables. At every step of the procedure, the dynamic relaxation method is used to analyze the bridge. The details of a feasible solution are given in Table 1. The dead load of the structure is 44.4 kN. Based on the cost model, the fabricated cost for this bridge configuration (joints not included) is estimated to be 54’900 CHF.

Table 1: Design found using parametric analyses

Characteristic	Struts	Layer cables	X cables
L [cm]	677.8	457.5	346.8
D [cm]	10.1	1.0	1.0
A [cm^2]	11.1	3.0	3.0

4 Design through Stochastic Search

Structural design is an abductive task for which engineers search for solutions, given required functionality and behavior. The design solution obtained through parametric studies satisfies all design constraints. However, this solution may simply be a local minimum in a very large and complex solution space. Plots from parametric studies show the influence of a single parameter only. Implicitly, the assumption in parametric studies is that the general trends are valid, even when the initial design configuration is altered. This assumption is often false.

Global optimization techniques, such as stochastic search, are powerful techniques for complex engineering tasks. They find solutions that have a greater chance of being the global minimum than solutions provided by parametric analysis. Design optimization of the tensegrity bridge was addressed using PGSL (Probabilistic Global Search Lausanne). The PGSL technique is based on the assumption that sets of near-optimal solutions are more likely to be found near sets of good solutions. Search is driven by a probability density function that is iteratively modified so that more exhaustive searches are made in regions of good solutions (Raphael and Smith 2003).

In this study, the element topology and the span of the bridge are assumed to be fixed. The optimization method modifies only design parameters related to member sizing. The following six parameters are considered as design variables:

1. Area of layer cables [0.05; 10 cm^2]
2. Area of x-cables [0.05; 10 cm^2]
3. Outer diameter of tubular struts [2; 15 cm]
4. Diameter to thickness ratio of tubular struts [5; 50]
5. Self-stress in layer cables [0; 1 cm]
6. Self-stress in x-cables [0; 1 cm]

The numbers within rectangular brackets beside each parameter indicate lower and upper bounds of possible values. For self-stress, the numbers in brackets correspond to elongations in the respective cables.

The objective function consists of two components: the cost (*C*) of the structure including joints as given in Equation 4, and penalty costs (*P*) that account for each constraint violation.

$$OF = C + P \tag{4}$$

There is often a tradeoff between the two components of the objective function. For example, decrease in the cost *C* may result in constraint violations and increase the value of the objective function through the penalty function *P*. The penalties for the violation of constraints are calculated as the additional costs that are likely to be incurred to force the solution to satisfy the constraints.

$$\sum P = P_d + P_t + P_c + P_{sc} \tag{5}$$

There are four penalty components as described in Equation 5. The penalty cost P_d corresponds to the cost that is estimated to reduce displacements so that they satisfy displacement criteria. P_t and P_c correspond to the penalty costs that are estimated to make the design solution satisfy the tensile and compressive stress limits. An additional penalty P_{sc} is considered in order to eliminate slack cables. Parametric studies have revealed that the x-cables are the most important load bearing component in the structure. Thus, P_d is calculated as the cost corresponding to the additional x-cable area required to reduce vertical displacements to the allowable limit.

$$A = 5.16 \cdot \delta^{-1.23} \tag{6}$$

$$P_d = \begin{cases} 0, \text{ if } \delta < \delta_0 = 2.85 \text{ cm} \\ C(A_0) - C(A), \text{ if } \delta > \delta_0 \end{cases} \quad (7)$$

For a given solution with vertical displacement δ, P_d is calculated according to Equation 7, where A_0 is the estimated minimum cross-sectional area that is required to keep vertical displacements under the allowable limit δ_0. A is the cross-section area corresponding to the evaluated vertical displacement δ as given in Table 1. A and A_0 are calculated using Equation 6. The penalty costs for excessive stress are also estimated. If tensile stresses exceed the tensile capacity of the cables, a penalty cost is estimated for the additional cable area required to take the calculated force. The additional area is calculated according to Figure 2. For the struts, if compressive stresses exceed their buckling strength, a penalty cost is estimated for the additional area necessary to prevent buckling.

Table 2: Solution generated by PGSL

Characteristic	Struts	Layer cables	X cables
L [cm]	677.8	457.3	346.5
D [cm]	10.2	0.7	0.8
A [cm^2]	6.2	1.6	2.1

PGSL provides consistently good results for a number of evaluations greater than 4000. The details of a solution obtained by the stochastic search algorithm are given in Table 2. The dead load of the structure is 25.7 kN. Although values for design parameters may vary between different PGSL runs, the cost of PGSL solutions remains consistently close to CHF 40’170. This is 26% lower than a solution found by parametric analysis. PGSL can be used to generate a range of good solutions so that criteria that are hard to specify in an objective function, such as aesthetics and buildability, can be used to make the best design decision.

5 Conclusions

This paper focuses on the use of stochastic search for the design with minimum cost of a pedestrian bridge made of hollow-rope tensegrity ring modules. Two design methods aiming to find the minimal fabricated cost solution are compared. The first method attempts to simulate the practice in design offices using parametric analyses. The second method uses PGSL, a stochastic search algorithm. Results from parametric analyses show that certain cables (x-cables) are the fundamental load-bearing elements in these tensegrity bridges. Their stiffness has the largest effect on vertical displacements and a large impact on internal compressive forces. Both parametric analysis and stochastic search generate designs that satisfy safety and serviceability criteria. However, the best solution using stochastic search has a cost that is 26% lower than the cost of the solution identified using traditional parametric analysis. Parametric analyses are useful to obtain a broad understanding of the influence of each parameter and can also help in defining effective penalty costs for enforcing constraints during stochastic search. These results underline the complexity of tensegrity structure design and the efficiency of advanced computing methods. Work in progress includes studies of more elaborate cost models and a representation that models design parameters as discrete variables. Additionally, a prototype of the tensegrity bridge will be built and studied experimentally.

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