Dimensional Metrology
Part -2

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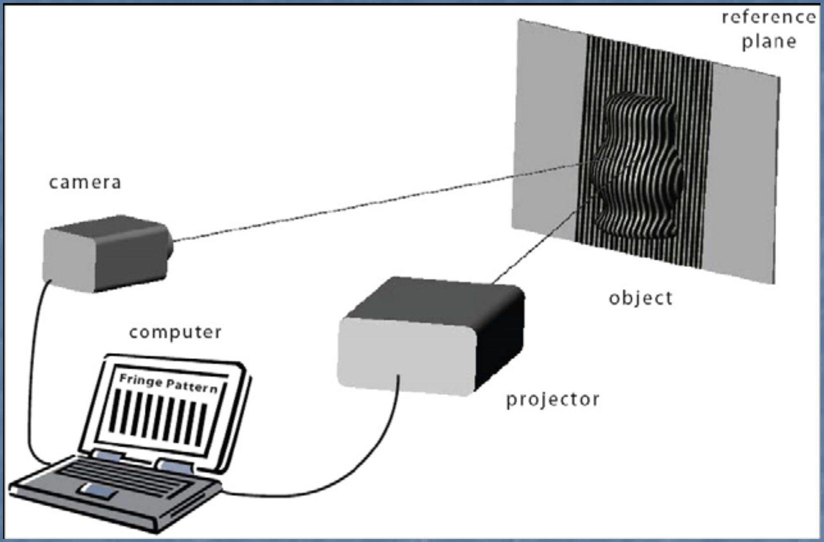
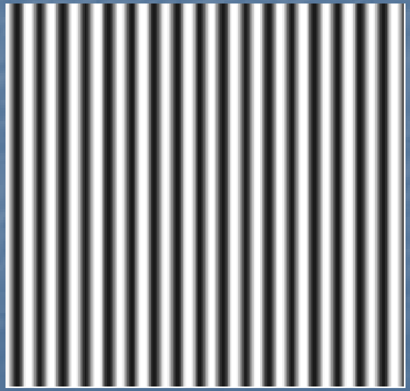
Lecture Notes: Photomechanics for Engineers

IMAC, EPFL

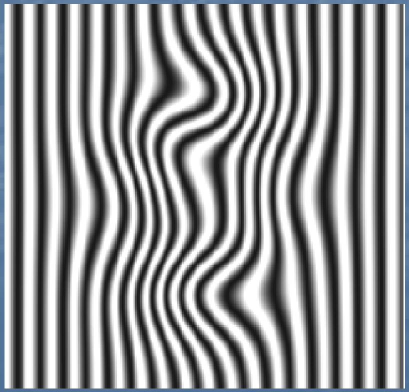
6th Oct. 2009

Fringe Projection Technique





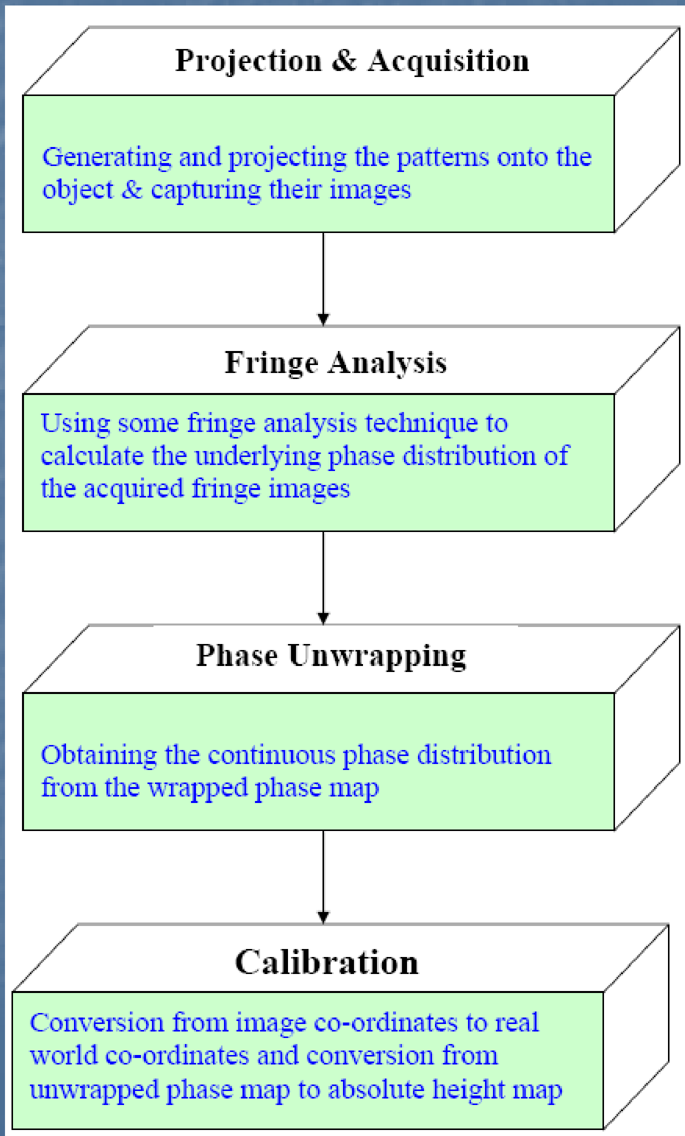
$$h(x, y) = \frac{l_0 * \Delta\varphi(x, y)}{\Delta\varphi(x, y) - 2\pi f_0 d}$$

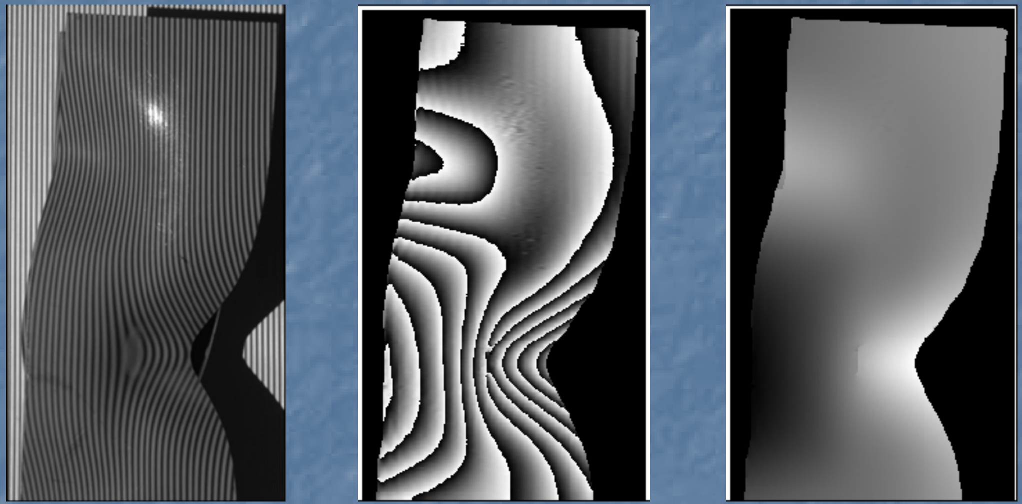


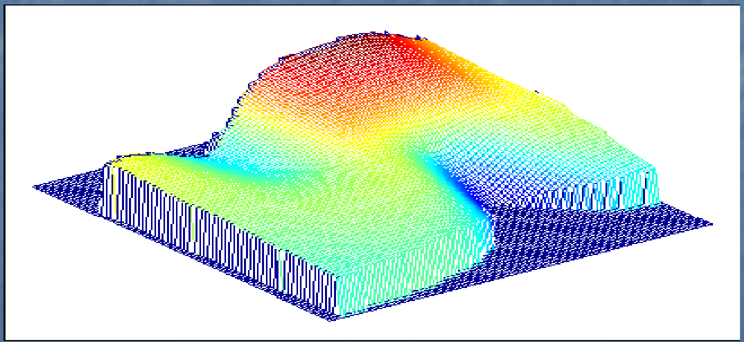
$g_0(x, y) = a(x, y)+b(x, y) \cos(2\pi f_0 x+\phi_0(x, y))$

$g(x, y) = a(x, y) + b(x, y) \cos(2\pi f_0 x+\phi(x, y))$

3D shape measurement of objects using fringe projection technique: Overview of measurement methodology







Fringe Analysis

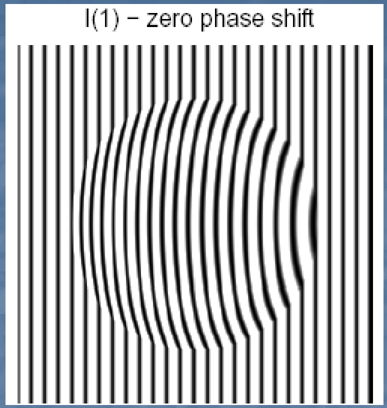
$$g(x, y) = a(x, y) + b(x, y) \cos[2\pi f_0 x + \phi(x, y)]$$

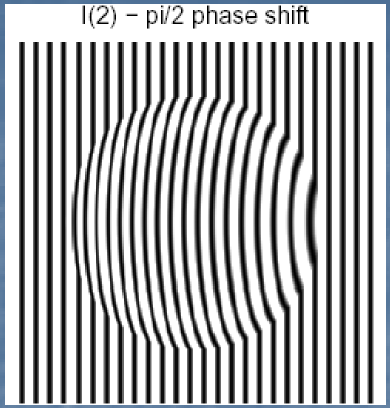
- *a(x, y)* represents the intensity variations of the background (related to the object's texture)
- *b(x, y)* represents non-uniform reflectivities of the object surface (fringe amplitude modulation term)
- 2πf0x represents the spatial carrier
- $\phi(x, y)$ is the phase term which contains the information of the object's shape
- Fringe analysis methods aim at extracting $\phi(x, y)$ from g(x, y)

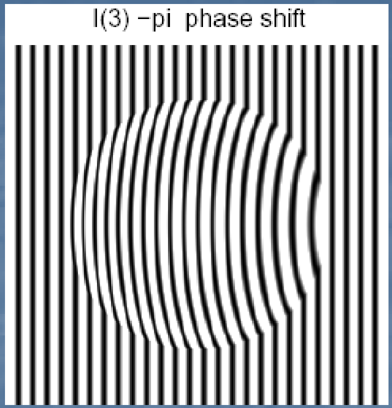
Fringe Analysis Methods

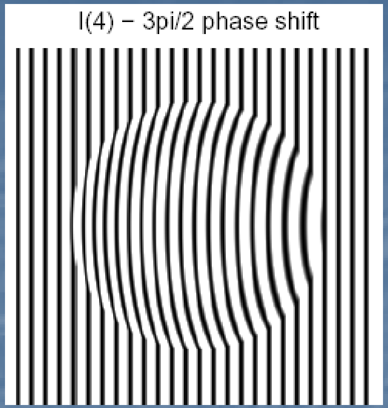
- Phase shifting method
- Multi-channel based phase shifting method
- Spatial filtering method
- Fourier transform method
- Wavelet transform method
- Windowed Fourier transform method

Phase shifting profilometry









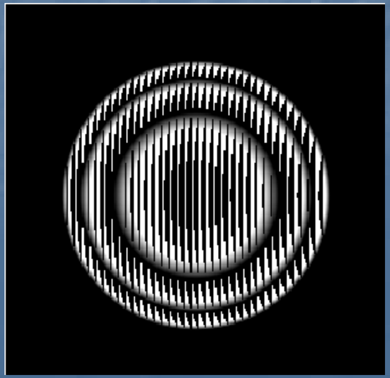
$I_1(x, y) = a(x, y) + b(x, y)\cos[2\pi f_0 x + \phi(x, y)]$

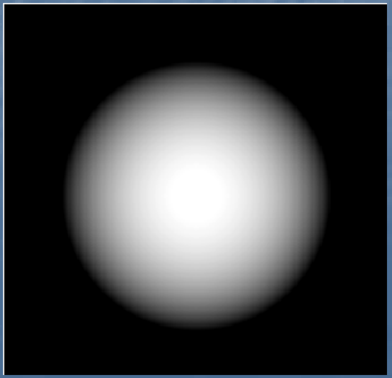
$I_2(x, y) = a(x, y) + b(x, y)\cos[2\pi f_0 x + \phi(x, y) + (\pi/2)]$

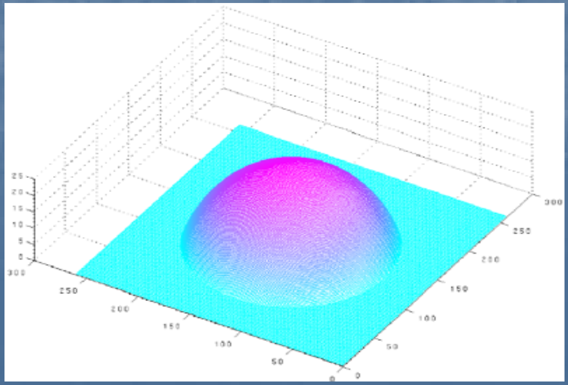
$I_3(x, y) = a(x, y) + b(x, y)\cos[2\pi f_0 x + \phi(x, y) + (\pi)]$

$I_4(x, y) = a(x, y) + b(x, y)\cos[2\pi f_0 x + \phi(x, y) + (3\pi/2)]$

$$\phi(x, y) = \tan^{-1}\left(\frac{I_4 - I_2}{I_1 - I_3}\right)$$







Multi-channel Approach

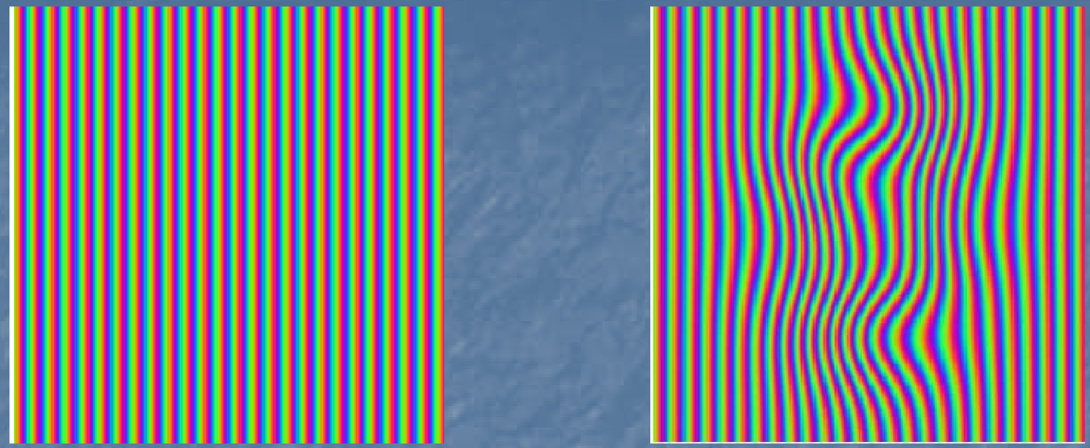
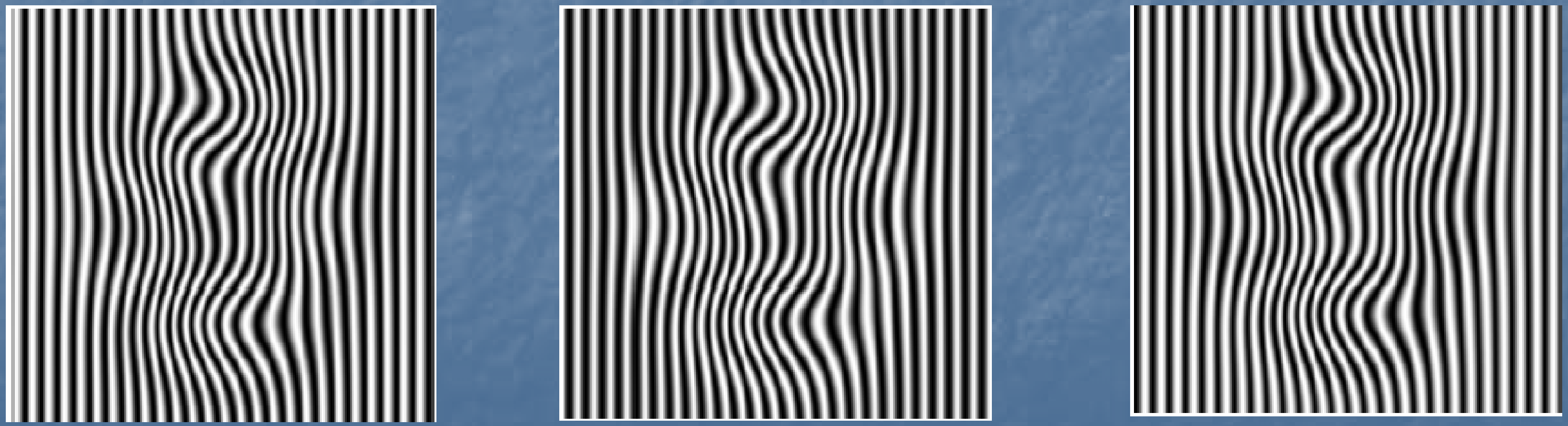


Figure (a) RGB color encoded fringe pattern on reference plane (b) the deformed fringe pattern when projected on to the object surface.



Three phase shifted fringe patterns extracted by using individual R, G, B filters

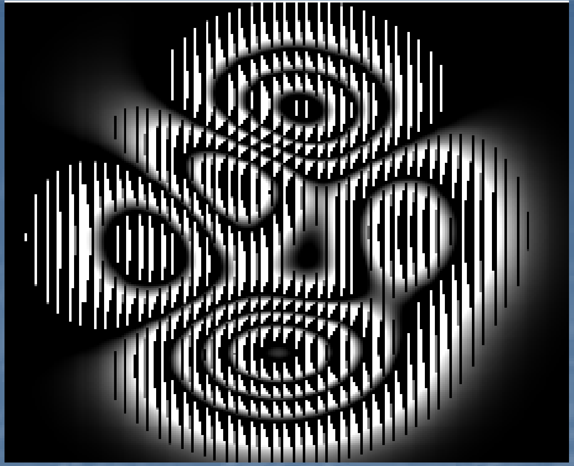
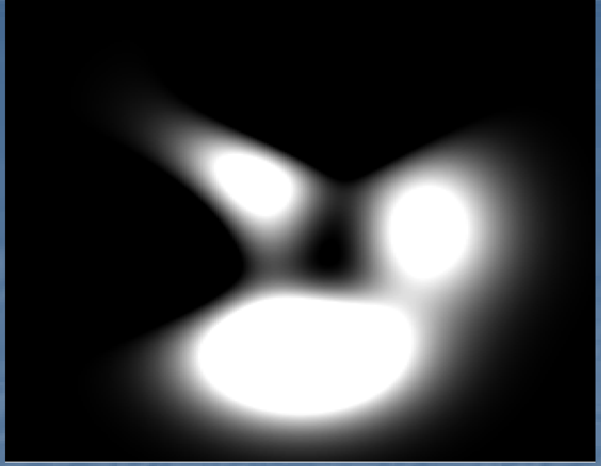


Fig. (a) Wrapped phase map



(b) Unwrapped phase map

$$\varphi = \tan^{-1}\left(\sqrt{3}\,\frac{I_1 - I_3}{2I_2 - I_1 - I_3}\right)$$

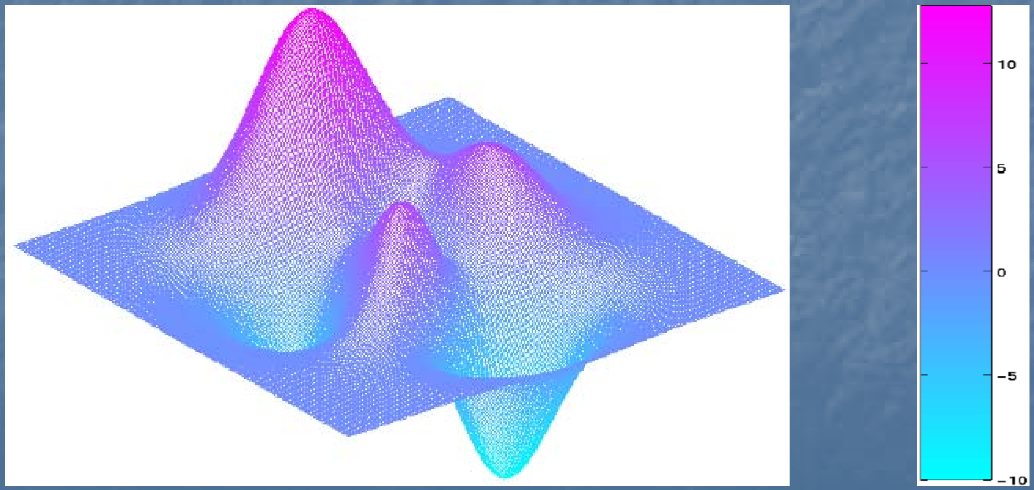


Figure. Three-dimensional surface plot of the reconstructed profile.

Fourier transform profilometry

$$g(x, y) = a(x, y) + b(x, y) \cos(2\pi f_0 x + \phi(x, y))$$

above equation can be written as :

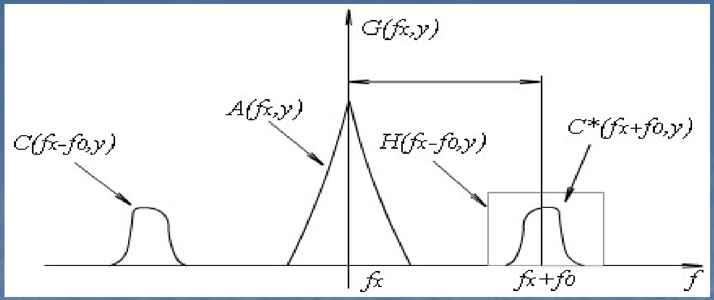
$$g(x, y) = a(x, y) + \tfrac{1}{2} b(x, y) \exp j(2\pi f_0 x + \varphi(x, y)) + \tfrac{1}{2} b(x, y) \exp -j(2\pi f_0 x + \varphi(x, y))$$

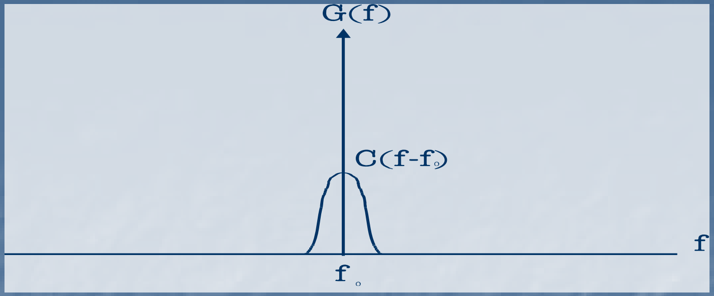
$$g(x, y) = a(x, y) + c(x, y) \exp(j 2\pi f_0 x) + c^*(x, y) \exp(-j 2\pi f_0 x)$$

where $c(x, y) = \tfrac{1}{2} b(x, y) \exp[j\, \varphi(x, y)]$

Fourier Transform of the above equation is of the form:

$$\mathbf{G(f, y) = A(x, y) + C(f - f_0, y) + C^*(f - f_{0,}\ y)}$$





Filtering the spectrum centered around f0 and translating to the origin results in

$G_f(f, y) = C(f, y)$

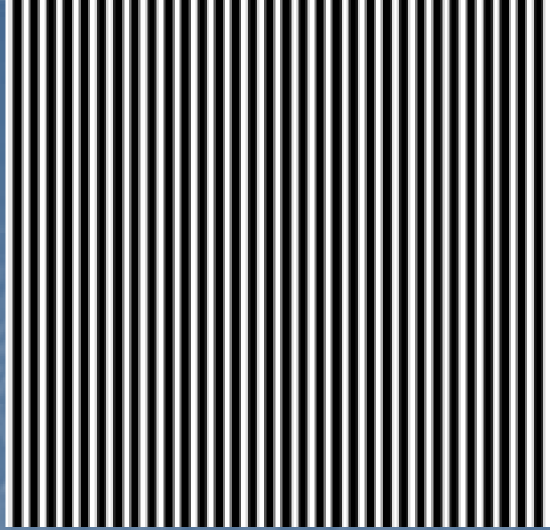
Inverse Transformation to the image shown in the second figure and calculating phase using

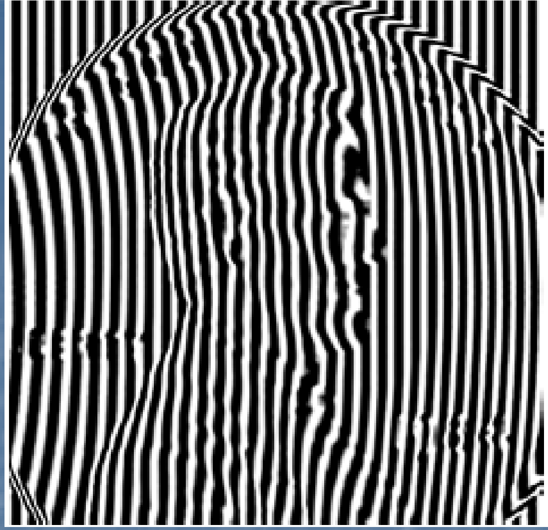
$\varphi(x, y) = \text{atan}\{\text{Im}[c(x, y)] / \text{Re}[c(x, y)]\}$ results in wrapped phase map.

Unwrapping algorithm is subsequently used to obtain continuous phase map.

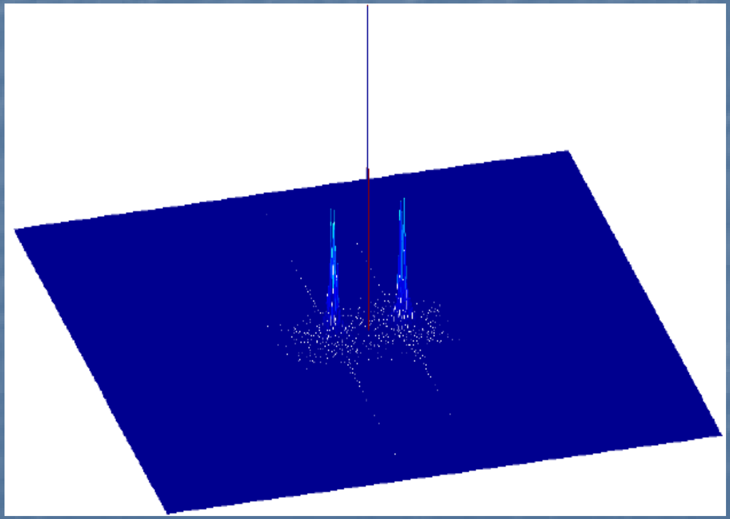
Calculating the height of all points from phase using the equation below results surface profile

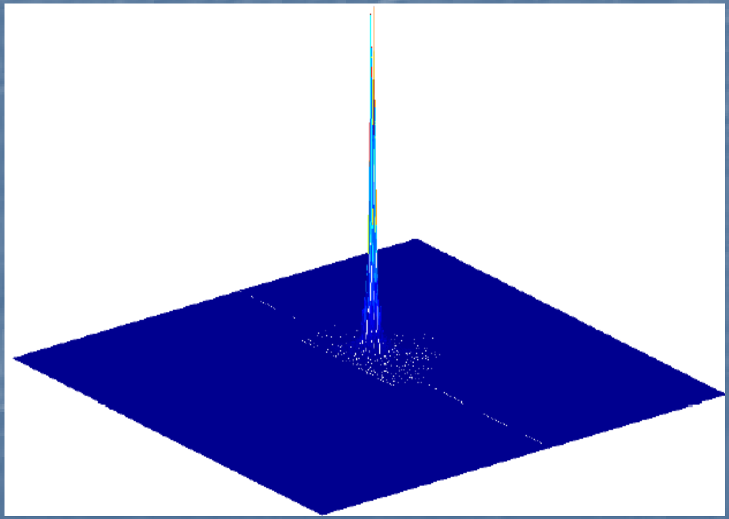
$$h(x, y) = \frac{l_0 * \Delta\varphi(x, y)}{\Delta\varphi(x, y) - 2\pi f_0 d}$$





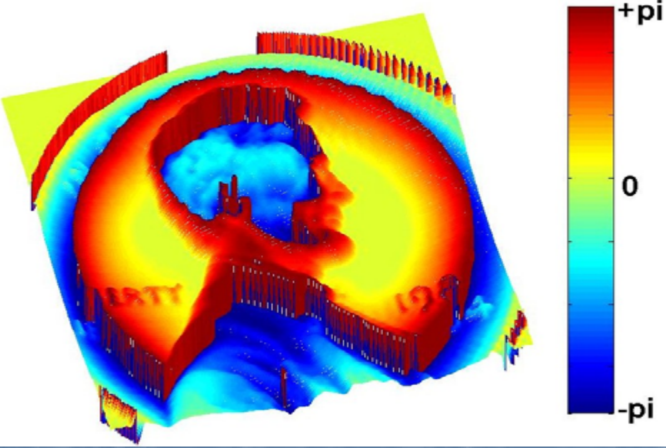
Fringe pattern on reference plane and on object surface

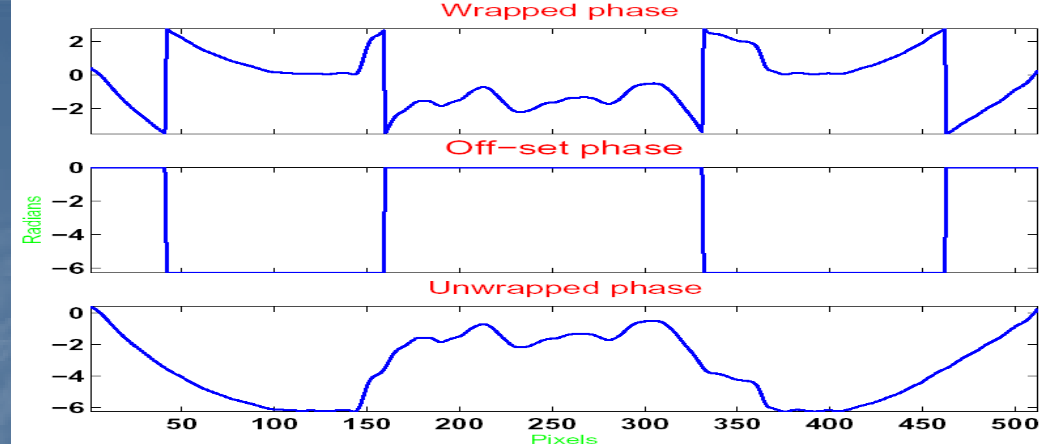




2-D FFT of the fringe pattern

Filtered and shifted version of the spectrum centered around f_0





Surface plot of wrapped phase map

Gray scale representation of unwrapped phase map

3-D surface plot of the reconstructed surface

Spatial phase detection

$$I_S(x,y) = I(x,y) \times \sin(2\pi f_0 x)$$

$$= a(x,y)\sin(2\pi f_0 x) + b(x,y)\cos\left[2\pi f_0 x + \varphi(x,y)\right]\sin(2\pi f_0 x)$$

$$= a(x,y)\sin(2\pi f_0 x) + \frac{1}{2}b(x,y)\sin\left[4\pi f_0 x + \varphi(x,y)\right] - \frac{1}{2}b(x,y)\sin\left[\varphi(x,y)\right]$$

$$r(x,y) * I_S(x,y) = -\frac{1}{2}b(x,y)\sin\left[\varphi(x,y)\right]$$

$$I_C(x,y) = I(x,y) \times \cos(2\pi f_0 x)$$

$$= a(x,y)\cos(2\pi f_0 x) + b(x,y)\cos\left[2\pi f_0 x + \varphi(x,y)\right]\cos(2\pi f_0 x)$$

$$= a(x,y)\cos(2\pi f_0 x) + \frac{1}{2}b(x,y)\cos\left[4\pi f_0 x + \varphi(x,y)\right] + \frac{1}{2}b(x,y)\cos\left[\varphi(x,y)\right]$$

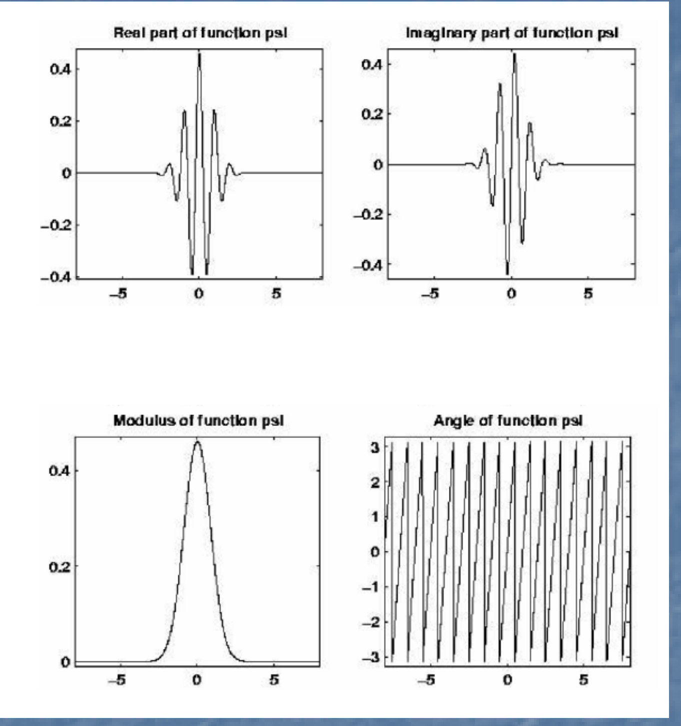
$$r(x,y) * I_C(x,y) = \frac{1}{2}b(x,y)\cos\left[\varphi(x,y)\right]$$

$$\varphi(x,y) = \arctan\left[-\frac{r(x,y) * I_S(x,y)}{r(x,y) * I_C(x,y)}\right]$$

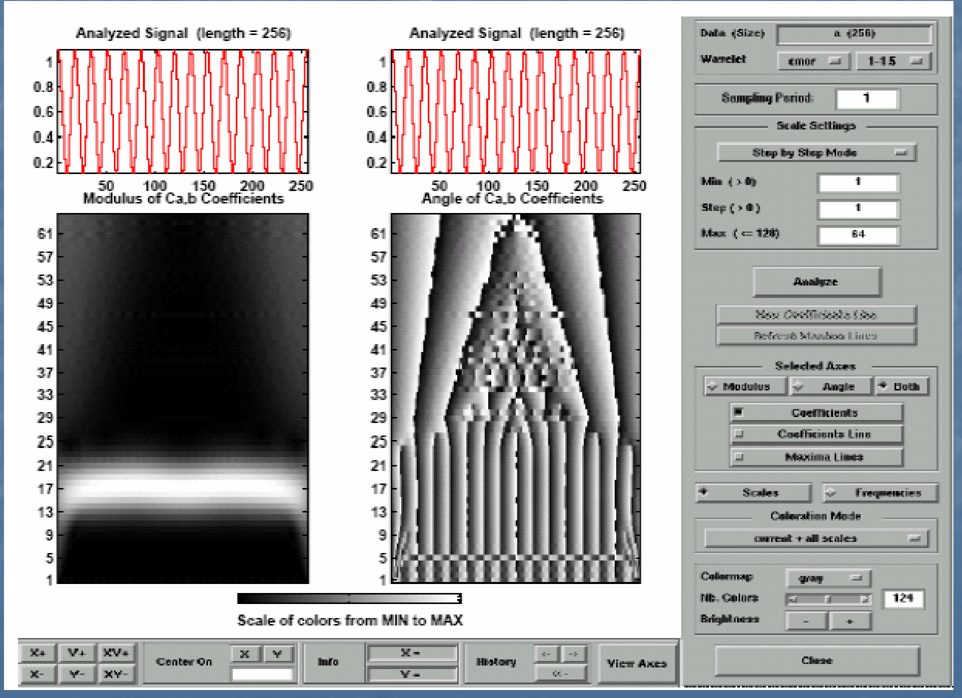
Wavelet transform profilometry

$$\psi_{a,b}(x) = (1/a)\,\psi\left(\frac{x-b}{a}\right)$$

$$\psi(x) = \pi^{1/4} e^{jcx} e^{-\frac{x^2}{2}}$$



$$W(a,b) = \int g(x,y)\,\psi^{*}_{a,b}(x)\,dx$$

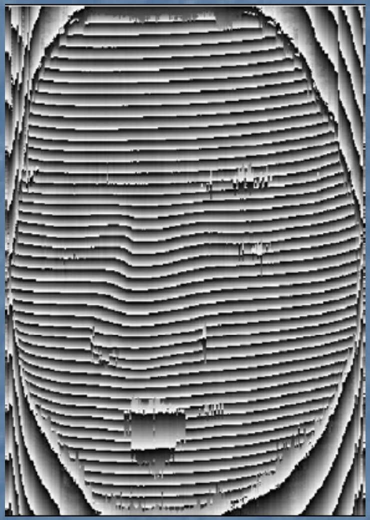


$$A(a,b) = \left[\left(\mathrm{imag}[w(a,b)]^2\right)^2 + \left(\mathrm{real}[w(a,b)]^2\right)^2\right]^{1/2}$$

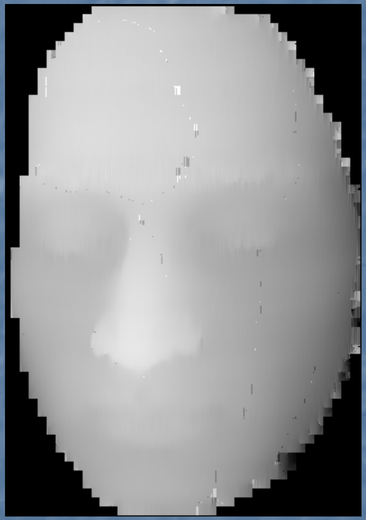
$$\phi(a,b) = \tan^{-1}\left(\frac{\mathrm{imag}[W(a,b)]}{\mathrm{real}[W(a,b)]}\right)$$

Wavelet transform profilometry

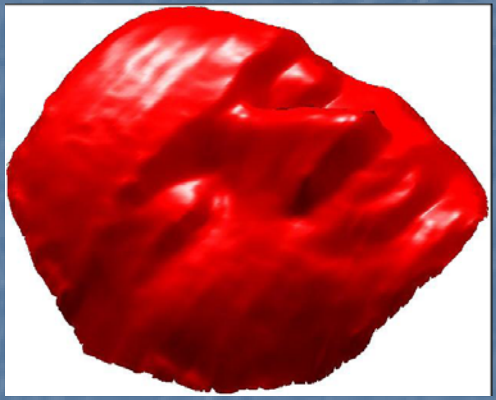
Deformed fringe pattern



Wrapped phase map



Unwrapped phase map



3D surface plot

Experiments

- Fourier transform profilometry
- Simple 2D unwrapping algorithm
- Phase shifting profilometry
- Wavelet transform profilometry
- Texture extraction and mapping

You are welcome to bring the objcts of your choice for measuring 3D shapes

Three-dimensional shape measurement using fringe projection

Experiments as part of the course on ***Photo-mechanics for engineers***

October 2009

Names	**7th**	**8th**	**9th**	**13th**	**14th**	**15th**	**16th**	**19th**
Marc Tinguely **Vlad Hasmatuchi**	**14:00-19:00**							
Epely Gael **Lindqvist Maria**			**08:00-12:30**					
Amirreza Zobeiri **Francisco Botero**		**10-12 and 14-16:30**						
Martin Calmon **Steven Roth**	**08:00-12:30**							
Georgios Violakis **Nandita Aggarwal**							**8:00-12:30**	
Strässle Rahel **Canonica Michael** **Jutzi Fabio**				**11:00-16:00**				
Nuttapol Pootrakulchote and Sami Goekce								**14:00-18:30**
Lubrano Emanuele **Claire Sauthier**						**8:00-12:30**		
Rajshekhar G **Roshan Ghias Alireza**					**14:00-18:30**			

Experiments on 3D shape measurement using fringe projection technique: Plan of action

1. Simulation of reference and deformed fringe patterns for a known height distribution and estimating the underlying phase distribution of the patterns using
   (a) Phase shifting method
   (b) One-dimensional Fourier transform analysis method
   (c) Two-dimensional Fourier transform analysis method

2. Writing simple 2D unwrapping algorithm to unwrap the wrapped phase maps obtained in the earlier step.

3. Conducting experiments to reconstruct the 3D shape of a real object using the above methods

4. Understanding the MATLAB routines given to you which automate the 3D shape measurement process. The routines automates the process of generating and projecting patterns; acquiring images and saving them on the hard disk; allows pre-processing of images by cropping and generating mask; analysis of the images to obtain wrapped phase distribution; and unwrapping them and plotting the recovered 3D height distribution.