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by

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OPTIMAL MONETARY AND FISCAL POLICY IN THE EMU: DOES FISCAL POLICY COORDINATION MATTER?*

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Abstract

I develop and analyze a DSGE model of a currency union to revise the question of how to conduct monetary and fiscal policy in countries that share the same currency. In contrast with the previous literature which assumes coordination, this paper analyzes the case where coordination lacks among fiscal authorities as well as between fiscal and monetary authorities. I show that the normative prescriptions emphasized by former analyses are not valid any more once policymakers are not coordinated. Indeed, in that case the common central bank does not stabilize the average union inflation as if it were in a closed economy because it has to take into account the distortions caused by the lack of coordination among fiscal policymakers. At the same time, if there is not a common agreement to coordinate fiscal policies, autonomous governments should use government expenditure as a stabilization tool even if shocks are symmetric.

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1 Introduction

The birth of the European Monetary Union (EMU) has sparkled interest in the question of how to conduct monetary and fiscal policy for a group of countries that share the same currency. There is a growing body of research that has tried to assess this issue within a fully micro-founded dynamic general equilibrium framework. However literature relies on the existence of a supra-national authority to which all monetary and fiscal policy decisions have been delegated. Yet, as matter of fact, in the EMU only the monetary policy is under the control of a common authority, the European Central Bank (ECB), whereas, even if bound by the Stability and Growth Pact (SGP), fiscal policies are still decided at national level. Consequently the following questions arise: How should monetary and fiscal policies be conducted in a monetary union where there is a common central bank but autonomous fiscal policies? Does this institutional arrangement lead to different normative prescriptions with respect to those highlighted by the previous literature?

In order to answer such questions, in this chapter I uses a generalized version of the DSGE model laid out by Galí and Monacelli (2005b)[1] and compares two different policy regimes: the regime of fiscal policy coordination considered as a benchmark, already analyzed by Galí and Monacelli (2005b) themselves, Beetsma and Jensen (2004) and (2005)[2] and the regime of fiscal policy no-coordination.

In our basic setup, the world is framed as a continuum of small open economies. Each country government chooses the optimal provision of a public consumption good and sets a time-invariant labour subsidy. The presence of lump sum taxes ensures compliance with SGP limits and rules out the additional problem of choosing how to finance optimally the public expenditure. Within this framework, under fiscal policy coordination, monetary and fiscal policies are chosen by a common policymaker in order to maximize the average union welfare. Conversely, under fiscal policy no-coordination[3], there is a multiplicity of policy authorities each of them taking as given other policymakers' decisions: governments that are concerned only about the welfare of their own country and the central bank of the Monetary Union that has the maximization of the average union welfare as objective.

According to my results, the no-coordination among fiscal authorities matters for the design of both optimal monetary and fiscal policies. The driving force of this

[1] See also Galí and Monacelli (2005a). Differently from Galí and Monacelli (2005b), not only final private goods but even public goods and intermediate inputs are traded, while the elasticity of substitution between home and foreign goods is not restricted to be equal to one. Moreover in the preference specification the intertemporal elasticities of substitution of public and private consumption are not necessarily equal. As it will be clarified below, the first two generalizations strengthen the incentive of uncoordinated fiscal policymakers to generate aggregate distortions. Conversely the last assumption is crucial to explain the results in the case of shocks to technology.

[2] Even Ferrero (2005) contributed to this debate. He analyzed the case of coordination in which, however, the exogenous government expenditure is financed through distortionary taxes and riskless bonds.

[3] There are some old contributions that consider the case of no-coordination (for instance Lambertini and Dixit (2003)). However, in general these papers do not assume fully-micro-founded welfare criteria. An exception in this respect is the work by Lombardo and Sutherland (2004). Yet they treat only marginally the case of a monetary union and reach results opposite to those of this paper by assuming an efficient steady state and considering only the case of optimal simple rules.

finding stems from countries monopoly power on their terms of trade. Indeed, given the imperfect substitutability between bundles produced in different countries, uncoordinated policymakers have an incentive to try to influence the terms of trade in their favour. This incentive works both at the steady state and over the business cycle[4]. At the steady state, independent fiscal authorities act as a monopolist. They try to increase the demand of the home produced goods and to decrease their supply by over-expanding government expenditure and reducing the labour subsidy. In this way they seek to render domestic goods relatively more expensive in order to reduce their production. In fact given that there is consumption risk sharing across countries, the increase in leisure associated with a terms of trade improvement more than compensates the corresponding fall in consumption. In other words through a terms of trade improvement governments seek to externalize labour effort to other countries consumers.

Over the business cycle instead they use government expenditure to restrain the terms of trade volatility and hence reduce the cost of the volatility of output or private consumption at other countries' expense.

This mechanism explains the differences in policy prescriptions under coordination and no-coordination. Under the benchmark case of fiscal policy coordination, Galí and Monacelli (2005b), Beetsma and Jensen (2004) and (2005) have pointed out two main findings. Firstly, under the optimal policy mix, the common monetary policy should seek to stabilize the average union inflation following the same normative prescriptions valid in a closed economy. Therefore, under technology shocks, it should pursue the stability of the average union price level; under mark up shocks, it has to trade off between stabilizing the average inflation and the average output gap. Secondly, in a monetary union fiscal policy is a useful tool for macroeconomic stabilization of single country economies. Indeed, at single country level fiscal policy should be employed to stabilize the effects of idiosyncratic shocks given that, because of the adoption of the common currency, the central bank is able only to stabilize the aggregate economy. However, at the aggregate level fiscal policy should only ensure on average the efficient provision of the public goods.

Under fiscal policy no-coordination, the previous results no longer hold. With regard to monetary policy, the common central bank should cope with the aggregate distortions generated both at the steady state and over the business cycle by independent governments and not stabilize the average union economy as if it were a closed economy. Therefore in the presence of productivity shocks strict inflation targeting is in general not optimal. In fact, under flexible prices output volatility is inefficiently high for the at least two reasons. On the one hand national authorities have an incentive to manipulate the terms of trade to their own advantage even over the business cycle. On the other hand the steady state government expenditure share in output is inefficiently high and thus amplifies the effects of government expenditure shocks on output fluctuations[5]. Moreover, in the response to mark up shocks, the monetary

[4]...as pointed out by the previous literature: see, among others, Corsetti and Pesenti (2001), Benigno and Benigno (2003) and Epifani and Gancia (2005).

[5]...at least under the baseline calibration. Given the inefficiently high steady state government expenditure share in output, one percentage increase in the government expenditure expands more output under no-coordination than under coordination. **?** has already emphasized that the government's size may have an

authority should be much more aggressive in fighting inflation under no-coordination than under coordination. This finding is explained by the inefficiently low steady state level of output. Given that distortion, an increase in output volatility in response to mark up shocks has some beneficial effects because it makes consumers willing to work more on average driving the economy, by so doing, towards the efficient allocation.

With regard to fiscal policies independent governments do not ensure on average, given their incentives, the efficient provision of the public goods. And, in the case of mark up shocks they use government expenditure for stabilization purposes even if shocks are symmetric. Indeed, by taking as given what other policymakers are doing, they do not realize that the common central bank is already stabilizing the aggregate economy and they go on seeking to stabilize, on their own, the undesirable effects of mark up shocks.

The chapter is organized as follows. Section 2 describes the basic framework. Section 3 introduces the equilibrium conditions. Section 4 examines the case of full coordination. Section 5 the case of no-coordination. Section 6 concludes.

2 The model

The currency union consists of a continuum of small open economies[6]. In each country there are two sectors: a competitive sector that produces one final good by using both home and foreign country intermediate inputs; a monopolistic competitive sector that produces a continuum of intermediate differentiated goods by using as input labour which is assumed immobile across countries.

2.1 Preferences

Preferences of a generic country representative household are defined over a private consumption bundle, C_t, a public consumption bundle, G_t and hours of labour $N_t(h)$[7]:

$$\sum_{t=0}^{\infty}\beta^t E_0\left[\frac{C_t^{1-\sigma}}{1-\sigma}+\chi\frac{G_t^{1-\gamma}}{1-\gamma}-\frac{N_t(h)^{\varphi+1}}{\varphi+1}\right] \quad 0<\beta<1 \tag{1}$$

where, as usual, β stands for the intertemporal preferences discount factor and χ is the weight attached to public consumption. Agents consume all the goods produced in the world economy. However preferences exhibit home bias. The private consumption index is, in fact, a CES aggregation of the following type:

$$C_t \equiv \left[(1-\alpha)^{\frac{1}{\eta}}C_{H,t}^{\frac{\eta-1}{\eta}}+\alpha^{\frac{1}{\eta}}C_{F,t}^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}} \qquad \eta>0 \tag{2}$$

with $1-\alpha$ being the degree of home bias in the private consumption and η denoting the elasticity of substitution between $C_{H,t}$, and $C_{F,t}$. $C_{H,t}$ represents the home household's

effect on output volatility.

[6]The general framework draws on Galí and Monacelli (2005a) and Galí and Monacelli (2005b).

[7]In this and in the following subsections we abstract from indexing the small open economy of reference.

consumption of the single home final good while $C_{F,t}$ is a CES aggregation of the goods produced in foreign countries namely:

$$C_{F,t} \equiv \left[\int_0^1 C_{j,t}^{\frac{\eta-1}{\eta}} dj \right]^{\frac{\eta}{\eta-1}} \tag{3}$$

η then represents even the elasticity of substitution between different foreign goods.

The public bundle is defined similarly to the private bundle, that is:

$$G_t \equiv \left[(1-\nu)^{\frac{1}{\eta}} G_{H,t}^{\frac{\eta-1}{\eta}} + \nu^{\frac{1}{\eta}} G_{F,t}^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} \qquad \eta > 0 \tag{4}$$

with

$$G_{F,t} \equiv \left[\int_0^1 G_{j,t}^{\frac{\eta-1}{\eta}} dj \right]^{\frac{\eta}{\eta-1}} \tag{5}$$

where $1-\nu$ indicates the degree of home bias in the public consumption which, in general, is allowed to be different from $1-\alpha$[8].

Public and private consumption index definitions (2), (4), (3) and (5) allow to determine consistent definitions of price indexes[9]. In particular, $P_{C,t}$ and $P_{G,t}$, the private and the public consumers' price indexes[10] are given by:

$$P_{C,t} \equiv \left[(1-\alpha) P_t^{1-\eta} + \alpha P_t^{*\,1-\eta} \right]^{\frac{1}{1-\eta}} \tag{6}$$

$$P_{G,t} \equiv \left[(1-\nu) P_t^{1-\eta} + \nu P_t^{*\,1-\eta} \right]^{\frac{1}{1-\eta}} \tag{7}$$

with P_t^* being specified as:

$$P_t^* \equiv \left[\int_0^1 P_t^{j\,1-\eta} dj \right]^{\frac{1}{1-\eta}} \tag{8}$$

Thus P_t and P_t^j are producers' price indexes [11]. There are no trading frictions being the law of one price assumed to hold in all single good markets. However, given the home biased preferences, the purchasing power parity does not hold for indexes $P_{C,t}$ and $P_{G,t}$.

2.2 Consumption demand, portfolio choices and labour supply

The consumption and price index definitions allow to solve the consumer problem in three stages. In the first two stages, agents decide how much real net income to allocate

[8] In fact Brülhart and Trionfetti (2004) point out that the home bias of public goods is higher than home bias of private goods.

[9] Namely price and consumption indexes are such that at the optimum expenditures for total consumption of both private and public goods, $P_t C_{H,t} + \int_0^1 P_t^j C_{j,t} dj$ and $P_t G_{H,t} + \int_0^1 P_t^j G_{j,t} dj$ are equal respectively to $P_{C,t} C_t$ and $P_{G,t} G_t$.

[10] In what follows, CPI stands for consumers' price index.

[11] Again in what follows, PPI stands for producers' price index.

to buy goods produced at home and abroad. According to the set of optimal conditions, it is possible to determine agent demands for $C_{H,t}$, $C_{F,t}$ and $C_{j,t}$, as:

$$C_{H,t} = (1-\alpha)\left(\frac{P_t}{P_{C,t}}\right)^{-\eta} C_t \qquad C_{F,t} = \alpha\left(\frac{P_t^*}{P_{C,t}}\right)^{-\eta} C_t \qquad C_{j,t} = \left(\frac{P_t^j}{P_t^*}\right)^{-\eta} C_{F,t} \quad (9)$$

for all j. The third stage coincides with the standard consumer problem. Agents are monopolistic competitive labour suppliers. Thus they maximize (1) with respect to C_t, D_{t+1} and $N_t(h)$ subject to the following sequence of constraints:

$$E_t\{Q_{t,t+1}D_{t+1}\} = D_t + W_t(h)N_t(h) - P_{C,t}C_t + T_t \quad (10)$$

$$N_t(h) = \left(\frac{W_t(h)}{W_t}\right)^{-\upsilon_t} N_t \quad (11)$$

where:

$$W_t \equiv \left[\int_0^1 W_t(h)^{1-\upsilon_t} dh\right]^{\frac{1}{1-\upsilon_t}} \quad (12)$$

Constraint (10) is the budget constraints which states that nominal saving, net of lump sum transfers, has to equalize the nominal value of a state contingent portfolio. In fact $W_t(h)$ stands for the per hour nominal wage, $Q_{t,t+1}$ denotes what is usually called the stochastic discount factor and D_{t+1} is the payoff of one maturity portfolio that includes firm shares.

Constraint (11) is a consequence of a CES aggregation of labour inputs which will be specified in the next sub-section and implicitly assumes that the elasticity of demand of labour, υ_t, is time-varying but equal across agents as in Clarida, Galí and Gertler (2002). Finally (12) is simply the aggregate wage index. Domestic and international markets are assumed to be complete.

By the optimality conditions of the household problem:

$$(1+\mu_t)N_t(h)^{\varphi}C_t^{\sigma} = \frac{W_t}{P_{C,t}} \quad (13)$$

$$\beta\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma}\left(\frac{P_{C,t}}{P_{C,t+1}}\right) = Q_{t,t+1} \quad (14)$$

which hold in all states of nature and at all periods and where $\mu_t \equiv \frac{1}{\upsilon_t - 1}$.

According to (13), workers set the real wage as mark up over the marginal rate of substitution between consumption and leisure, while the value of the intertemporal marginal rate of substitution of consumption should equalize the stochastic discount factor. Notice that since wages are perfectly flexible $N_t(h)$=N_t and $W_t(h)$=W_t for all h and t.

2.3 Final good aggregate demand

In each country the demand for the final good is the sum of four components: the demands of domestic and foreign households and governments namely:

$$Y_t = C_{H,t} + \int_0^1 C^j_{H,t} dj + G_{H,t} + \int_0^1 G^j_{H,t} dj \tag{15}$$

Condition (15) can be rewritten as:

$$Y_t = (1-\alpha)\left(\frac{P_t}{P_{C,t}}\right)^{-\eta} C_t + \alpha \int_0^1 \left(\frac{P_t}{P^j_{C,t}}\right)^{-\eta} C^j_t dj + (1-\nu)\left(\frac{P_t}{P_{G,t}}\right)^{-\eta} G_t + \nu \int_0^1 \left(\frac{P_t}{P^j_{G,t}}\right)^{-\eta} G^j_t dj \tag{16}$$

which follows from equation (3)[12] and the fact that:

$$G_{H,t} = (1-\nu)\left(\frac{P_t}{P_{G,t}}\right)^{-\eta} G_t \qquad G_{F,t} = \nu \left(\frac{P^*_t}{P_{G,t}}\right)^{-\eta} G_t \qquad G_{j,t} = \left(\frac{P^j_t}{P^*_t}\right)^{-\eta} G_{F,t} \tag{17}$$

for all j. According to (17) independently of the aggregate level of G_t, governments choose good demands by minimizing the total expenditure $P_t G_{H,t} + \int_0^1 P^j_t G_{j,t} dj$.

2.4 Firms and technology in the final good sector

Each final good is produced by using both home and foreign inputs according to the following CES technology:

$$Y_t = \left[(1-\psi)^{\frac{1}{\eta}} \left(Y^I_{H,t}\right)^{\frac{\eta-1}{\eta}} + \psi^{\frac{1}{\eta}} \left(Y^I_{F,t}\right)^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}} \qquad \eta > 0 \tag{18}$$

where $1-\psi$ is the degree of home bias in intermediate goods. Y^I_H and Y^I_F are defined as:

$$Y^I_{H,t} \equiv \left[\int_0^1 \left(y^I_{H,t}(k)\right)^{\frac{\varepsilon-1}{\varepsilon}} dk\right]^{\frac{\varepsilon}{\varepsilon-1}} \qquad Y^I_{F,t} \equiv \left[\int_0^1 \left(Y^I_{j,t}\right)^{\frac{\eta-1}{\eta}} dj\right]^{\frac{\eta}{\eta-1}} \tag{19}$$

with $Y^I_{j,t} \equiv \left[\int_0^1 \left(y^I_{j,t}(k)\right)^{\frac{\varepsilon-1}{\varepsilon}} dk\right]^{\frac{\varepsilon}{\varepsilon-1}}$ for all j and $y^I_{H,t}(k)$ and $y^I_{j,t}(k)$ being the demands for the k type of intermediate good produced in the home country and in country j respectively.

The final sector is perfectly competitive. Therefore firms maximize profits taking P_t, the price of the final good, as given. The optimality conditions of this problem lead to the following single and aggregate input demands:

$$y^I_{H,t}(k) = \left(\frac{p_{H,t}(k)}{P_{H,t}}\right)^{-\varepsilon} Y^I_{H,t} \qquad y^I_{j,t}(k) = \left(\frac{p_{j,t}(k)}{P_{j,t}}\right)^{-\varepsilon} Y^I_{j,t} \tag{20}$$

$$Y^I_{H,t} = (1-\psi)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} Y_t \qquad Y^I_{F,t} = \psi \left(\frac{P_{F,t}}{P_t}\right)^{-\eta} Y_t \qquad Y^I_{j,t} = \left(\frac{P_{j,t}}{P_{F,t}}\right)^{-\eta} Y^I_{F,t} \tag{21}$$

[12]... with the symmetric equations for foreign countries.

which allow to determine consistently the price indexes for final and intermediate goods as:

$$P_t = \left[(1-\psi)\left(P_{H,t}\right)^{1-\eta} + \psi\left(P_{F,t}\right)^{1-\eta}\right]^{\frac{1}{1-\eta}} \tag{22}$$

$$P_{H,t} = \left[\int_0^1 p_{H,t}(k)^{1-\varepsilon}dk\right]^{\frac{1}{1-\varepsilon}} \qquad P_{F,t} = \left[\int_0^1 \left(P_{j,t}\right)^{1-\eta}dj\right]^{\frac{1}{1-\eta}} \qquad P_{j,t} = \left[\int_0^1 p_{j,t}(k)^{1-\varepsilon}dk\right]^{\frac{1}{1-\varepsilon}} \tag{23}$$

where $p_{j,t}(k)$ is the price of intermediate input k produced in country j.

2.5 Intermediate good aggregate demand

The demand for home intermediate goods is generated by the demands of both home and foreign final good producers, namely:

$$y_{H,t}(k) \equiv y^I_{H,t}(k) + \int_0^1 y^{I,j}_{H,t}(k)dj \tag{24}$$

Condition (24) can be rewritten as:

$$y_{H,t}(k) = \left(\frac{p_{H,t}(k)}{P_{H,t}}\right)^{-\varepsilon}\left[(1-\psi)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta}Y_t + \psi\int_0^1\left(\frac{P_{H,t}}{P_t^j}\right)^{-\eta}Y_t^j dj\right] \tag{25}$$

which follows from equations (20) and (21)[13]. Given (25) it is possible to recover the aggregate demand $Y_{H,t} \equiv \left(\int_0^1 (y_{H,t}(k))^{\frac{\varepsilon-1}{\varepsilon}}dk\right)^{\frac{\varepsilon}{\varepsilon-1}}$. In fact by properly integrating (25) we obtain:

$$Y_{H,t} = \left[(1-\psi)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta}Y_t + \psi\int_0^1\left(\frac{P_{H,t}}{P_t^j}\right)^{-\eta}Y_t^j dj\right] \tag{26}$$

2.6 Firm technology and price setting in the intermediate good sector

In the intermediate sector each firm produces a single differentiated good with a constant return to scale technology of the type:

$$y_{H,t}(k) = A_t N_t(k) \tag{27}$$

with $N_t(k) = \left[\int_0^1 N_t(h)^{\frac{\upsilon_t-1}{\upsilon_t}}dh\right]^{\frac{\upsilon_t}{\upsilon_t-1}}$ and being the labour input and A_t the specific country technology shock. Given (27) and the fact that $N_t = N_t(h)$ for all h, the aggregate relationship between output and labour can be read as:

$$N_t = \frac{Y_{H,t}}{A_t}Z_t \tag{28}$$

[13]... with the symmetric equations for foreign countries.

where $Z_t \equiv \int_0^1 \frac{y_{H,t}(k)}{Y_{H,t}} dk$, and $N_t \equiv \int_0^1 N_t(k)dk$. Given (24) and (26) then $Z_t \equiv \int_0^1 \left(\frac{p_{H,t}(k)}{P_{H,t}}\right)^{-\varepsilon} dk$; thus Z_t can be interpreted as an index of the relative price dispersion across firms. We assume that good prices adjust according to a staggered mechanism *à la* Calvo. Therefore in each period a given firm can reoptimize its price only with probability $1-\theta$. As a result the fraction of firms that set a new price is fixed and the aggregate producer price index of the intermediate goods evolves accordingly to:

$$P_{H,t}^{(1-\varepsilon)} = \theta P_{H,t-1}^{(1-\varepsilon)} + (1-\theta)\tilde{p}_{H,t}(k)^{(1-\varepsilon)} \tag{29}$$

with $\tilde{p}_{H,t}(k)$ being the optimal price. Firms maximize the discounted expected sum of the future profits that would be collected if the optimal price could not be changed namely:

$$\sum_{s=0}^{\infty} (\theta)^s E_t \left\{ Q_{t,t+s} y_{H,t+s}(k) \left[\tilde{p}_{H,t}(k) - MC_{t+s}^n\right] \right\} \tag{30}$$

where $y_{H,t}(k)$ is given by (25) and $MC_t^n = \frac{(1-\tau)W_t}{A_t}$ is the nominal marginal cost with τ indicating a labour subsidy distributed to firms by the fiscal authority which is not supposed to vary over the business cycle. Taking into account (14) and that $MC_t \equiv \frac{MC_t^n}{P_{H,t}}$, the optimality condition of the firm problem can be written as:

$$\sum_{s=0}^{\infty} (\beta\theta)^s E_t \left\{ C_{t+s}^{-\sigma} \left(\frac{\tilde{p}_{H,t}(k)}{P_{H,t+s}}\right)^{-\varepsilon} Y_{H,t+s} \frac{P_{H,t}}{P_{C,t+s}} \left[\frac{\tilde{p}_{H,t}(k)}{P_{H,t}} - \frac{\varepsilon}{\varepsilon-1} \frac{P_{H,t+s}}{P_{H,t}} MC_{t+s}\right] \right\} = 0 \tag{31}$$

Condition (31) states implicitly that firms reset their prices as a mark up over a weighted average of the current and expected marginal costs, where the weight of the expected marginal cost at some date $t+s$ depends on the probability that the price is still effective at that date.

3 Equilibrium

The purpose of this section is twofold: on the one hand to recover the full set of conditions necessary and sufficient to determine the equilibrium of the monetary union; on the other hand to rewrite the single country equilibrium conditions in terms only of single country and average union variables. Indeed in this way, it is possible to simplify the fiscal policy problem under no-coordination. Being infinitesimally small, single country behaviour does not affect the average union performance. As a consequence, under no-coordination, the fiscal policy problem can be formulated just considering single country (and not the full set of the monetary union) equilibrium conditions.

3.1 International risk sharing

Under complete markets[14], condition (14) and the corresponding conditions for other countries imply:

$$\frac{P^j_{C,t}}{P_{C,t}} = \left(\frac{C^j_t}{C_t}\right)^{-\sigma} \tag{32}$$

for all j and where $P^j_{C,t}$ denotes the consumer price index of country j.

Notice that $P^*_t = \left[\int_0^1 (P^j_{C,t})^{1-\eta} dj\right]^{\frac{1}{1-\eta}}$ and let:

$$C^*_t \equiv \left[\int_0^1 (C^j_t)^{\sigma(\eta-1)} dj\right]^{\frac{1}{\sigma(\eta-1)}} \tag{33}$$

By properly integrating (32) we obtain:

$$\frac{P^*_t}{P_{C,t}} = \left(\frac{C^*_t}{C_t}\right)^{-\sigma} \tag{34}$$

Equation (32) and its aggregate version (34) state that, under complete markets, the marginal rate of substitution between home and other country consumption (or the average union consumption) has to be equal to the corresponding relative price. As a result, in equilibrium, any increase in the home relative to foreign CPI goes with a decrease of home relative to foreign consumption. Indeed a terms of trade improvement in the home country[15] induces private agents to reallocate the consumption between home and foreign goods. Then, because of the home bias, the home country consumers would decrease the *total* private consumption more than foreigners [16] .

By combining (34) with (6), (7) and (22) and considering that $P^*_t = P_{F,t}$, it follows that:

$$\frac{P_t}{P_{C,t}} = \left[\frac{1}{1-\alpha} - \frac{\alpha}{1-\alpha}\left(\frac{C^*_t}{C_t}\right)^{-\sigma(1-\eta)}\right]^{\frac{1}{1-\eta}} \tag{35}$$

$$\frac{P_{G,t}}{P_{C,t}} = \left[(1-\nu)\frac{P_t}{P_{C,t}}^{1-\eta} + \nu\left(\frac{C^*_t}{C_t}\right)^{-\sigma(1-\eta)}\right]^{\frac{1}{1-\eta}} \tag{36}$$

$$\frac{P_{H,t}}{P_t} = \left[\frac{1}{1-\psi} - \frac{\psi}{1-\psi}\left(\frac{C^*_t}{C_t}\right)^{-\sigma(1-\eta)}\left(\frac{P_t}{P_{C,t}}\right)^{\eta-1}\right]^{\frac{1}{1-\eta}} \tag{37}$$

which say that all the single country relative prices $P_t/P_{C,t}$, $P_{G,t}/P_{C,t}$ and $P_{H,t}/P_t$

[14]...and the assumption that the state contingent wealth at time zero is such that the lifetime discounted budget constraints are identical across agents.

[15]Namely the prices of the foreign goods in terms of home goods.

[16]In fact because of the home bias, even if there are complete markets, private agents consumption is not equal across countries.

and the terms of trade P^*/P_t and $P_t^*/P_{H,t}$ [17] are function exclusively of the difference between single country and average union private consumption.

In addition given (14) and (34):

$$\left(\frac{C_{t+1}^*}{C_t^*}\right)^{-\sigma} {\Pi_{t+1}^*}^{-1} = \left(\frac{C_{t+1}}{C_t}\right)^{-\sigma} \Pi_{C,t+1}^{-1} \tag{38}$$

with $\Pi_t^* \equiv P_t^*/P_{t-1}^*$ and $\Pi_{C,t} \equiv P_{C,t}/P_{C,t-1}$.

Thus in equilibrium the value of intertemporal marginal rate of substitution of private consumption should be equal across countries. This last condition combined with (35) and (37) can be log-linearized as:

$$\pi_{H,t} - \pi_t^* = -\omega_4 \left(\Delta\hat{c}_t - \Delta\hat{c}_t^*\right) \tag{39}$$

where $\omega_4 \equiv \frac{\sigma}{(1-\alpha)(1-\psi)}$ [18] . (38) (39) relate consumption variations differential from the union average to the *domestic* inflation differential. Moreover under complete markets, from conditions (14) and (34) it follows:

$$\frac{1}{1+r_t^*} = \beta E_t \left\{ \left(\frac{C_{t+1}^*}{C_t^*}\right)^{-\sigma} \left(\frac{P_t^*}{P_{t+1}^*}\right) \right\} \tag{40}$$

where $\frac{1}{1+r_t^*} = E_t\{Q_{t,t+1}\}$. When markets are complete, the expected value of the intertemporal marginal rate of substitution of private consumption, namely the price of a riskless portfolio, should be equal to the price of the riskless bond, being r_t^* the nominal interest rate.

The log-linear approximation of condition (40) leads to:

$$\hat{c}_t^* = E_t\{\hat{c}_{t+1}^*\} - \frac{1}{\sigma}(r_t^* - E_t\{\pi_{t+1}^*\} - \varrho) \tag{41}$$

where $\varrho \equiv -log\beta$. Condition (41) is the so called IS curve that relates the average union intertemporal marginal rate of substitution of private consumption with the real interest rate.

By(38), (40) can be read as:

$$\frac{1}{1+r_t^*} = \beta E_t \left\{ \left(\frac{C_{t+1}}{C_t}\right)^{-\sigma} \left(\frac{P_{C,t}}{P_{C,t+1}}\right) \right\} \tag{42}$$

In other words by (38) (41) is satisfied even at the single country level[19]. For this reason we can interpret (38) as a constraint imposed by the adoption of a common currency according to which in response to asymmetric shocks the terms of trade cannot adjust instantaneously because of the sluggish prices adjustment and the fix exchange

[17] In fact by (6) and (22), $P_{C,t}/P_t = \left[(1-\alpha) + \alpha \left(P_t^*/P_t\right)^{(1-\eta)}\right]^{\frac{1}{1-\eta}}$ and $P_t/P_{H,t} = \left[(1-\psi) + \psi \left(P_t^*/P_{H,t}\right)^{(1-\eta)}\right]^{\frac{1}{1-\eta}}$. P_t^*/P_t and $P_t^*/P_{H,t}$ are the so called *effective* terms of trade. In what follows, unless specified differently, we will refer only to the effective terms of trade.

[18] From now on the following conventions are used: $\hat{x}_t$ stands for the log deviation of X_t from the symmetric zero inflation steady state while $\Delta\hat{x}_t \equiv \hat{x}_t - \hat{x}_{t-1}$ and $\hat{x}_t^* \equiv \int_0^1 \hat{x}_t^i di$.

[19] However (38) is not only a sufficient but also a necessary condition for (42) to be satisfied given (40).

rates. Actually outside a monetary union, under flexible exchange rates (38) and (41) do not necessarily hold because the fluctuations of nominal exchange rates themselves assure the equality of the value of intertemporal marginal rate of substitution of private consumption and give reason of differences in the nominal interest rates across countries.

3.2 Good market clearing conditions

To rewrite the resource constraints as function of only aggregate variables, note that $P_t/P^j_{C,t} = (P_t/P_{C,t})(P_{C,t}/P^j_{C,t})$. Similarly $P_t/P^j_{G,t} = (P_t/P_{C,t})\,(P_{C,t}/P^j_{C,t})(P^j_{C,t}/P^j_{G,t})$ and $P_{H,t}P^j_t = (P_{H,t}/P_t)(P_t/P_{C,t})(P_{C,t}/P^j_{C,t})(P^j_{C,t}/P_{j,t})$. Then by substituting (32) in (**??**) and (26) we can express the good market clearing conditions as:

$$Y_t = \left(\frac{P_t}{P_{C,t}}\right)^{-\eta}\left[(1-\alpha)C_t + \alpha C_t^{\sigma\eta}\Upsilon_{C,t}^{1-\sigma\eta} + (1-\nu)\left(\frac{P_{C,t}}{P_{G,t}}\right)^{-\eta} G_t + \nu C_t^{\sigma\eta}\Upsilon_{G,t}^{1-\sigma\eta}\right] \tag{43}$$

$$Y_{H,t} = \left(\frac{P_{H,t}}{P_t}\right)^{-\eta}\left[(1-\psi)Y_t + \psi\left(\frac{P_t}{P_{C,t}}\right)^{-\eta} C_t^{\sigma\eta}\Upsilon_{Y,t}^{1-\sigma\eta}\right] \tag{44}$$

where:

$$\Upsilon_{C,t} \equiv \left[\int_0^1 {C_t^j}^{1-\sigma\eta} dj\right]^{\frac{1}{1-\sigma\eta}}$$

$$\Upsilon_{G,t} \equiv \left[\int_0^1 {C_t^j}^{-\sigma\eta}\left(\frac{P^j_{C,t}}{P^j_{G,t}}\right)^{-\eta} G_t^j dj\right]^{\frac{1}{1-\sigma\eta}} \qquad \Upsilon_{Y,t} \equiv \left[\int_0^1 {C_t^j}^{-\sigma\eta}\left(\frac{P^j_{C,t}}{P_{j,t}}\right)^{-\eta} Y_t^j dj\right]^{\frac{1}{1-\sigma\eta}} \tag{45}$$

Rewriting the good market clearing conditions in this way lead to the following considerations. If isolated, any change in the home terms of trade[20] does not affect the demands for final and intermediate goods of other countries as long the aggregate variables are given. This is because countries are assumed to be small[21]. At the same time any improvement in the home terms of trade makes private agents willing to switch expenditure from home to foreign goo ds[22].

The log-linear approximations of the resource constraints (43) and (44) and of conditions (35), (36) and (37) allow to retrieve the following condition:

$$\hat{y}_{H,t} + \frac{\psi}{1-\psi}(\hat{y}_{H,t} - \hat{y}_t^*) = \rho\hat{c}_t + \rho(\delta-1)(\hat{c}_t - \hat{c}_t^*) + (1-\rho)\hat{g}_t - (1-\rho)\nu(\hat{g}_t - \hat{g}_t^*) \tag{46}$$

where $\delta \equiv (1-\psi)\left[(1-\alpha) + \delta_1 + \delta_2(1-\rho)/(\rho) + \delta_3(1)/(\delta)\right] \rho \equiv \frac{C}{Y}$, $\delta_1 \equiv [(1-\alpha) + \xi\alpha(2-\alpha)]$, $\delta_2 \equiv [\xi\nu(2-\nu)]$, $\delta_3 \equiv [\xi\psi(2-\psi)]/(1-\psi)$ and $\xi \equiv \eta\sigma/(1-\alpha)$

[20] namely P_t^*/P_t and $P_t^*/P_{H,t}$

[21] To see why this is true it is sufficient to consider the analogous of condition (43) for a given foreign country.

[22] what in the literature is called the *switching expenditure* effect.

3.3 The Phillips curve

Given condition (**??**) the optimal price is determined as:

$$\frac{\tilde{p}_{H,t}(k)}{P_{H,t}} = \frac{K_t}{F_t} \tag{47}$$

with:

$$K_t \equiv \sum_{s=0}^{\infty} (\beta\theta)^s E_t \left[C_{t+s}^{-\sigma} Y_{H,t+s} \left(\frac{P_{H,t+s}}{P_{H,t}} \right)^{\varepsilon} \frac{P_{H,t+s}}{P_{C,t+s}} \frac{\varepsilon}{\varepsilon-1} MC_{t+s} \right] \tag{48}$$

$$F_t \equiv \sum_{s=0}^{\infty} (\beta\theta)^s E_t \left[C_{t+s}^{-\sigma} Y_{H,t+s} \left(\frac{P_{H,t+s}}{P_{H,t}} \right)^{\varepsilon-1} \frac{P_{H,t+s}}{P_{C,t+s}} \right] \tag{49}$$

which can be read as:

$$K_t = C_t^{-\sigma} Y_{H,t} \frac{P_{H,t}}{P_{C,t}} \frac{\varepsilon}{\varepsilon-1} MC_t + \beta\theta E_t \left\{ \Pi_{H,t+s}^{\varepsilon} K_{t+1} \right\} \tag{50}$$

$$F_t = C_t^{-\sigma} Y_{H,t} \frac{P_{H,t}}{P_{C,t}} + \beta\theta E_t \left\{ \Pi_{H,t+1}^{\varepsilon-1} F_{t+1} \right\} \tag{51}$$

where $\Pi_{H,t} \equiv \frac{P_{H,t}}{P_{H,t-1}}$. Following Benigno and Woodford (2004), from (47) and (29) we can retrieve the next conditions:

$$\frac{1-\theta\Pi_{H,t}^{\varepsilon-1}}{1-\theta} = \left(\frac{F_t}{K_t} \right)^{\varepsilon-1} \tag{52}$$

$$Z_t = \theta Z_{t-1} \Pi_{H,t}^{\varepsilon} + (1-\theta) \left(\frac{1-\theta\Pi_{H,t}^{\varepsilon-1}}{1-\theta} \right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{53}$$

which determines the law of motion of firms price dispersion. From the log linear approximation of (**??**) (48) (49) and (53):

$$\pi_{H,t} = \lambda \widehat{mc}_t + \beta E_t\{\pi_{H,t+1}\} \tag{54}$$

where:

$$\begin{aligned} \widehat{mc}_t &= (\widehat{w}_t - \hat{p}_{c,t}) - (\widehat{p}_t - \hat{p}_{c,t}) - (\widehat{p}_{H,t} - \hat{p}_t) - \hat{a}_t \\ &= \varphi \hat{y}_{H,t} + \sigma \hat{c}_t + \omega_4((1-\psi)\alpha + \psi)(\hat{c}_t - \hat{c}_t^*) - (1+\varphi)\hat{a}_t + \hat{\mu}_t \end{aligned} \tag{55}$$

Condition (54) is the New Keynesian Phillips Curve direct consequence of the Calvo mechanism. As usual, current *domestic* inflation depends on the expectation on future *domestic* inflation and the current real marginal cost of producing intermediate goods. Being the economy open, in equilibrium that cost is determined by the real wage (which is equal to the marginal rate of substitution between consumption and leisure), the labour productivity and the relative prices of intermediate and final goods. These prices are determined as made clear by (35) and (37) by the differences of private consumption from the average.

The rational expectation stochastic equilibrium of the monetary union is then defined as the sequence of $\{C_t^i,\, Y_t^i,\, Y_{H,t}^i,\, \Pi_{H,t}^i,\, Z_t^i,\, F_t^i,\, K_t^i\,\}_{t=0}^{\infty}$ for all i which, given $\{G_t^i$

, $r_t^* \}_{t=0}^{\infty}$ for all i, τ and the initial condition Z_{-1}, satisfies (**??**), (50), (51),(52), (53), (43), (44) (38) and (40) for all i where $P_t^i/P_{C,t}^i$, $P_{G,t}^i/P_{C,t}^i$, $P_{H,t}^i/P_t^i$ are determined according to (35), (36), (37).

What it is still missing is to determine the optimal monetary and fiscal policies. This is the objective of the next paragraphs.

4 The optimal policies

As mentioned in the introduction, the optimal monetary and fiscal policy mix is analysed under two different policy regimes: the regimes of coordination and no-coordination. Under coordination there is a common authority responsible for both monetary and fiscal policies which has the maximization of the average union welfare as objective. Under no-coordination there is a plurality of independent policymakers each of those takes other policy authorities' decisions as given. The central bank on the one hand which seeks to minimize the average losses of union households and the governments on the other hand which, conversely, which conversely are concerned about the average losses of the single country households. The solutions to the optimal policy problems under both regimes are derived by using the linear quadratic approach proposed by Benigno and Woodford (2004). This methodology requires to assume that policies are optimal from *timeless* perspective[23] and can be implemented as follows. First the zero-inflation deterministic steady state is retrieved; then a purely quadratic approximation to the single country and monetary union welfare around the deterministic steady state is obtained. Being the economies open and in the case of no-coordination the deterministic steady state distorted, these approximations are derived by using the second order approximations of the structural equations[24]. Finally, given the purely quadratic approximations of policymakers' objectives, the optimal policies[25] are recovered by using as constraints the equilibrium conditions approximated up to the first order.

4.1 The case of coordination

Under coordination the optimal policy problem of the common authority can be formulated as the maximization of:

$$\sum_{t=0}^{\infty} \beta^t E_0 \int_0^1 \left[\frac{C_t^{i\,1-\sigma}}{1-\sigma} + \chi \frac{G_t^{i\,1-\gamma}}{1-\gamma} - \frac{1}{\varphi+1} \left(\frac{Y_{H,t}^i}{A_t^i} \right)^{\varphi+1} \right] di \tag{56}$$

with respect to C_t^i, G_t^i, Y_t^i, $Y_{H,t}^i$, $\Pi_{H,t}^i$, Z_t^i, F_t^i and K_t^i for all i, subject to equilibrium conditions (**??**), (38), (43), (44), (50), (51), (52), and (53) for all i where $P_t^i/P_{C,t}^i$,

[23] See also Benigno and Benigno (2006), Benigno and De Paoli (2005) and Paoli (2004).

[24] If second order approximation of a policymaker objective contain a linear term, then the constraint to the policy problem should be approximated up to the second order.

[25] In the case of no-coordination, the Nash equilibrium policies are determined by the solutions to both the monetary and fiscal policy problems.

$P^i_{G,t}/P^i_{C,t}$, $P^i_{H,t}/P^i_t$ are determined according to (35), (36), (37). It is easy to show that the symmetric zero inflation deterministic steady state[26] reduces to the following system of equations:

$$C^{-\sigma} = Y^{\varphi} \tag{57}$$

$$\chi G^{-\gamma} = Y^{\varphi} \tag{58}$$

$$Y = C + G \tag{59}$$

where $A = 1$. Under coordination and in absence of shocks, the allocation chosen by the common policymaker is determined by three conditions: the first two that equate the marginal rates of substitution (MRS) between both private consumption and leisure and public consumption and leisure to the corresponding rates of transformation(MRT) and the third one, the resource constraint, that equates the private and public demands for final goods to the relative supply. In order to implement this allocation, the policy maker should provide an efficient level of public goods as embodied in (57)-(59), and use the steady state subsidy on labor in order to completely offset the distortion due to monopolistic competition. This ensures that in the case of coordination the steady state allocation is Pareto efficient.

As shown in the appendix, under coordination the average welfare of union households can be approximated as follows:

$$-\frac{1}{2}Y^{\varphi+1}\sum_{t=0}^{\infty}\beta^t E_0 \int_0^1 \Big[\frac{\varepsilon}{\lambda}(\pi^i_{H,t})^2 + \varphi(\tilde{y}^{c,i}_{H,t})^2 + \gamma(1-\rho)(\tilde{g}^{c,i}_t)^2 + \sigma\rho(\tilde{c}^{c,i}_t)^2 \tag{60}$$

$$+2(1-\rho)(1-\nu)\varsigma_1(\tilde{g}^{c,i}_t - \tilde{g}^{c,*}_t)(\tilde{c}^{c,i}_t - \tilde{c}^{c,*}_t) + \varsigma_3(\tilde{c}^{c,i}_t - \tilde{c}^{c,*}_t)^2\Big] di + s.o.t.i.p. \tag{61}$$

where $\varsigma_1 \equiv \xi(\nu+\psi)$, $\xi \equiv \frac{\eta\sigma}{1-\alpha}$, ς_3 is properly defined in appendix and $\tilde{x}_t{}^{c,i} \equiv \hat{x}^i_t - \hat{x}^{c,i}_t$ where $\hat{x}^{c,i}_t$ indicates the target of the common authority under coordination. Notice that by (35), (36) and (37), $\hat{c}_t{}^i - \hat{c}_t{}^*$ is perfectly negative correlated with the terms of trade.

According to (60), under coordination, welfare losses are increasing in inflation, output, private consumption and public expenditure gaps. At the same time, these losses are affected by the gaps of terms of trade and the consequent mis-allocation in private consumption, public expenditure and production which crucially depends on the different degrees of openness and the elasticity of substitution among bundles produced in different country.

Under coordination, the target of the union policymaker is the first best and corresponds to the flexible price allocation floating exchange rates and technology shocks:

[26] See the appendix.

$$-\gamma\hat{g}_t^{c,i} - (\varphi\hat{y}_{H,t}^{c,i} - (\varphi+1)a_t^i) = (1-\rho)^{-1}\varsigma_1(1-\nu)(\hat{c}_t^{c,i} - \hat{c}_t^{c,*}) \quad (62)$$

$$-\sigma\hat{c}_t^{c,i} + \gamma\hat{g}_t^{c,i} = \quad (63)$$
$$-(1-\rho)^{-1}\varsigma_1(1-\nu)(\hat{c}_t^{c,i} - \hat{c}_t^{c,*}) + \rho^{-1}\left[\varsigma_1(1-\rho)(1-\nu)(\hat{g}_t^{c,i} - \hat{g}_t^{c,*}) + \varsigma_3(\hat{c}_t^{c,i} - \hat{c}_t^{c,*})\right]$$

$$\hat{y}_{H,t}^{c,i} + \frac{\psi}{1-\psi}(\hat{y}_{H,t}^{c,i} - \hat{y}_t^{c,*}) = \quad (64)$$
$$\rho\hat{c_t^{c,i}} + (\delta-1)\rho(\hat{c}_t^{c,i} - \hat{c}_t^{c,*}) + (1-\rho)\hat{g}_t^{c,i} - \nu(1-\rho)(\hat{g}_t^{c,i} - \hat{g}_t^{c,*})$$

The difference across countries embodied in (62)- (64) are explained as efficient response to asymmetric technological shocks. In fact, if, for instance, its technological shock is above the average union shock, then a single country economy experiences a terms of trade worsening that efficiently increases home demand labor relative to those of other countries. However, once the system (62) - (64) is integrated:

$$\varphi\hat{y}_t^{c,*} + \sigma\hat{c}_t^{c,*} = (1+\varphi)a_t^* \quad \gamma\hat{g}_t^{c,*} = \sigma\hat{c}_t^{c,*} \quad \hat{y}_t^{c,*} = \rho\hat{c}_t^{c,*} + (1-\rho)\hat{g}_t^{c,*} \quad (65)$$

Thus, under coordination, the target of the common authority on average corresponds exactly to that in a closed economy [27]. Moreover, given (62) - (64), the set of constraints relevant for the optimal policy problem can be rewritten as:

$$\tilde{y}_{H,t}^{c,i} + \frac{\psi}{1-\psi}(\tilde{y}_{H,t}^{c,i} - \tilde{y}_t^{c,*}) = \rho\tilde{c}_t^{c,i} + (\delta-1)\rho(\tilde{c}_t^{c,i} - \tilde{c}_t^{c,*}) + (1-\rho)\tilde{g}_t^{i,c} - \nu(1-\rho)(\tilde{g}_t^{c,i} - \tilde{g}_t^{c,*}) \quad (66)$$

$$\pi_{H,t}^i = \lambda\left[\varphi\tilde{y}_{H,t}^{c,i} + \sigma\tilde{c}_t^{c,i} + \omega_4((1-\psi)\alpha + \psi)(\tilde{c}_t^{c,i} - \tilde{c}_t^{c,*})\right] + \beta E_t\{\pi_{H,t+1}^i\} + \lambda\hat{\mu}_t^i \quad (67)$$

$$\pi_{H,t}^i - \pi_t^* = -\omega_4(\Delta\tilde{c}_t^{c,i} - \Delta\tilde{c}_t^{c,*}) - \omega_4(\Delta\upsilon_{1,t}^i - \Delta\upsilon_{1,t}^*) \quad (68)$$

for all t and all i, where $\upsilon_{1,t}^i \equiv \hat{c}_t^{c,i}$. This system of equations makes clear which are the tradeoffs of the common policymaker under coordination. At the union level the only tradeoff is generated by the presence of aggregate markup shocks. To see why this is true, it is sufficient to integrate the constrains (66)-(68) and consider that if the average union inflation is completely stabilised then the average union marginal costs is as well stabilised at the desired level. Thus , not surprisingly, according to the first order conditions of the optimal policy problem[28] in absence of markup shocks:

$$\pi_t^* = \tilde{y}_t^{c,*} = \tilde{c}_t^{c,*} = \tilde{g}_t^{c,*} = 0 \quad (69)$$

for all t which implies that $\varphi\hat{y}_t^* + \sigma\hat{c}_t^* = (1+\varphi)a_t$ and $\gamma\hat{g}_t^* = \sigma\hat{c}_t^*$. In other words under coordination, the optimal policy entails a strict targeting of the average union inflation in order to replicate on average the flexible price allocation.

[27] where the only existing distortion is due to price stickiness.
[28] See the appendix.

However, at the single country level, there is another tradeoff, direct consequence of adopting a common currency. As emphasized by (68), if the nominal exchange rates are fix and prices are sticky , the terms of trade cannot adjust instantaneously in response to asymmetric shocks and the flexible prices allocation cannot be implemented. Therefore, as long as shocks are asymmetric, in general:

$$\pi^i_{H,t} \neq 0 \tag{70}$$

for all i for some t and the government expenditure can be used as a stabilizatio tool[29].

5 The case of no-coordination

Under no-coordination, fiscal authorities are not coordinated neither among each other nor with the common central bank. The monetary and fiscal policy problems are then formulated as follows.

Single country governments maximize the welfare of the small open economy representative agent:

$$\sum_{t=0}^{\infty} \beta^t E_0 \left[\frac{C_t^{1-\sigma}}{1-\sigma} + \chi \frac{G_t^{1-\gamma}}{1-\gamma} - \frac{1}{\varphi+1} \left(\frac{Y_{H,t}}{A_t} \right)^{\varphi+1} \right] \tag{71}$$

with respect to C_t, Y_t, $Y_{H,t}$, $\Pi_{H,t}$, Z_t, F_t and K_t, subject to the single country equilibrium conditions (**??**), (38), (43), (44), (50), (51), (52), and (53) where $P_t/P_{C,t}$, $P_{G,t}/P_{C,t}$, $P_{H,t}/P_t$ are determined according to (35), (36), (37). [30] and taking as given the union average variables including the nominal interest rate chosen by the common central bank[31].

Conversely, the monetary authority maximizes the average union welfare namely:

$$\sum_{t=0}^{\infty} \beta^t E_0 \int_0^1 \left[\frac{{C_t^i}^{1-\sigma}}{1-\sigma} + \chi \frac{{G_t^i}^{1-\gamma}}{1-\gamma} - \frac{1}{\varphi+1} \left(\frac{Y^i_{H,t}}{A^i_t} \right)^{\varphi+1} \right] di \tag{72}$$

with respect to C^i_t, Y^i_t, $Y^i_{H,t}$, $\Pi^i_{H,t}$, Z^i_t, F^i_t and K^i_t for all i, subject to equilibrium conditions (**??**), (38), (43), (44), (50), (51), (52), and (53) for all i, where $P^i_t/P^i_{C,t}$, $P^i_{G,t}/P^i_{C,t}$, $P^i_{H,t}/P^i_t$ are determined according to (35), (36), (37) and taken as given the fiscal policies[32].

Given the formulation of the monetary and fiscal policy problems, it can be shown that at the symmetric deterministic steady state, zero inflation is a Nash equilibrium policy[33]. In particular, the optimality conditions evaluated at the zero inflation steady

[29]These results have been highlighted by Galí and Monacelli (2005b).

[30]...and the constraints that render the problem *timeless* as in Benigno and Woodford (2004).

[31]Namely C^*_t,$\Upsilon_{C,t}$, $\Upsilon_{G,t}$ and $\Upsilon_{Y,t}$.

[32]Namely G^i_t and τ^i for all countries and in all periods. Notice that differently from governments, the common central bank takes into account the effects that a marginal change of C^i_t and Y^i_t produces on C^*_t $\Upsilon_{C,t}$, $\Upsilon_{G,t}$ and $\Upsilon_{Y,t}$.

[33]See the appendix.

state lead to:

$$C^{-\sigma} = (1-\psi)\left[\delta_1 + \delta_2\frac{G}{C} + \delta_3\frac{Y}{C}\right]Y^{\varphi} \tag{73}$$

$$\chi G^{-\gamma} = (1-\psi)(1-\nu)Y^{\varphi} \tag{74}$$

$$Y = C + G \tag{75}$$

where $A = 1$ and:

$$\delta_1 \equiv [\xi\alpha(2-\alpha) + (1-\alpha)] \qquad \delta_2 \equiv [\xi\nu(2-\nu)] \quad \delta_3 \equiv \frac{1}{(1-\psi)}[\xi\psi(2-\psi)].$$

The comparison between this systems of equations (73) - (75) and (57) - (59) makes clear that, when uncoordinated, fiscal policymakers have an incentive to generate *static* distortions. Indeed if at the symmetric steady state, in the case of coordination, the MRS between both leisure and private consumption and leisure and public expenditure are set equal to the correspondent MRT, in the case of no-coordination are respectively determined as $(1-\psi)\left[\delta_1 + \delta_2\frac{G}{C} + \delta_3\frac{Y}{C}\right]$ and $(1-\psi)(1-\nu)$.

The reason underlying this outcome is the following. As usual the optimal policy is chosen by equating the marginal benefit of an additional unit of private and public consumption with the marginal cost of labor effort necessary to produce that unit. Nevertheless, under no-coordination this cost includes only the marginal change in the labor effort of home country workers and not that of foreign countries workers[34] as the common authority under coordination does. Ignoring this external effect on other countries' welfare is what bring about the incentive for non-coordinated fiscal authorities to exert their monopoly power on its terms of trade. In fact, an autonomous government would consider that by decreasing the home private consumption, could generate in equilibrium a terms of trade improvement, reducing in this way the demands for home produced goods[35] and the labor effort necessary to match that demand[36]

For this reason non-coordinated fiscal policymaker prefers to over-expand the public expenditure and renders labor more expensive[37] in order to lower through the substitution effect private consumption. As long as the actions of the other fiscal authorities are taken as given, rendering home produced goods more expansive increases the profits revenue of households and makes up for the decrease in labor income and the increase in lump-sum taxes. Then households can consume more public goods and work less than under coordination. The decrease in private consumption is then more than compensated by the increase in the provision of public goods and the decrease of labor

[34] This is because countries are small and under no-coordination take the actions of the other policymakers as given.

[35] As emphasized before, this is a direct consequence of the complete market assumption: in equilibrium the marginal rate of substitution between the single private consumption and the average union private consumption should be equal to the corresponding relative price.

[36] Conversely the coordinated authority would also take into account how, by improving the home terms of trade, the same decrease in the home private consumption in equilibrium increase the demands for foreign goods and the labor effort of foreigners.

[37] ..with respect to the efficient levels and thank to the steady state labor subsidy.

effort. This is a way in which countries seek to externalize both the costs of production and taxation[38]. Obviously given that at the symmetric steady state everybody is doing the same, in equilibrium the prices of all goods are the same and everybody is worse off.

To get a sense of the magnitude of the inefficiency generated at the steady state by uncoordinated policies it is sufficient to look at the following table:

	Coordination	no-coordination
$\frac{C}{Y}$	0.97	0.73

Under the baseline calibration, according to which $\alpha = 0.4$, $\nu = 0.2$ and $\psi = 0.4$[39], if fiscal policies are not coordinated the steady state consumption output ratio is equal to the 73% (as in the European Monetary Union) whereas, if they are coordinated, it reaches 97%. In other words the steady state distribution of resources between private and public sector under no-coordination is highly inefficient. And as it will be clear in the next subsections this static distortion will be key even in determining the effects that that lack of coordination produces over the business cycle.

5.1 Fiscal policy

For the fiscal policymaker the single country welfare has been approximated as:

$$-\frac{1}{2}Y^{\varphi+1}\sum_{t=0}^{\infty}\beta^t E_0\Big[\frac{\varepsilon}{\lambda}(\pi_{H,t})^2 + \varphi(\tilde{y}^f_{H,t})^2 + (1-\rho)(1-\psi)(1-\nu)\gamma(\tilde{g_t}^f)^2 + (\rho(1-\psi)\delta\sigma + \varsigma_2)(\tilde{c_t}^f)^2$$
$$+2(1-\rho)(1-\nu)(\varsigma_1 - \xi\nu\psi)\tilde{g_t}^f\tilde{c_t}^f\Big] + t.i.f.p.$$

where ς_2 is properly determined in the appendix, $t.i.f.p$ stands for terms independent of fiscal policy, $\tilde{x}^f_t \equiv \hat{x}_t - \hat{x}^f_t$ and $\hat{x}^f_t$ indicates the target of the fiscal authority. Variables, target and weights that enter in this loss are different with respect to the case of coordination. Fiscal policymakers takes as given the average union allocation and weight more private variations in the private consumption gap and less that in the public expenditure. In the perspective of the independent governments , variations in the gaps of private consumption are more expansive because of their effects on the terms of trade, while those in public expenditure cost less because the external effects produced on the welfare of the other countries are not taken into account.

The target of the fiscal authority corresponds to the flexible price allocation in the hypotheses that fiscal policymakers are not coordinated and can use over the business

[38]In fact at the steady state the incentive to over-expand the government expenditure is present even when the labor supply is completely inelastic.

[39]and that will be discussed in details below.

cycle a labour subsidy as a policy instrument. This allocation is determined as:

$$-\gamma\hat{g}_t^f - (\varphi\hat{y}_{H,t}^f - (\varphi+1)a_t) = (1-\psi)^{-1}(\varsigma_1 - \psi\nu)(\hat{c}_t^f - \hat{c}_t^*) \tag{76}$$

$$-\sigma\hat{c}_t^f - (\varphi\hat{y}_{H,t}^f - (\varphi+1)a_t) = ((1-\psi)\rho\delta)^{-1}\Big[(1-\rho)(1-\nu)(\varsigma_1-\psi\nu)(\hat{g}_t^f - \hat{g}_t^*)$$
$$+\varsigma_2(\hat{c}_t^f - \hat{c}_t^*) + (1-\psi)\delta_3(\hat{y}_t^* - \hat{c}_t^*) + (1-\psi)(1-\rho)\delta_2(\hat{g}_t^* - \hat{c}_t^*)\Big] \tag{77}$$

$$\hat{y}_{H,t}^f + \frac{\psi}{1-\psi}(\hat{y}_{H,t}^f - \hat{y}_t^*) =$$
$$\rho\hat{c}_t^f + (\delta-1)\rho(\hat{c}_t^f - \hat{c}_t^*) + (1-\rho)\hat{g}_t^f - \nu(1-\rho)(\hat{g}_t^f - \hat{g}_t^*) \tag{78}$$

Notice that, in contrast with both the common authority under coordination and the central bank of the monetary union under no-coordination, uncoordinated governments take the average fluctuations of output, government expenditure and private consumption as exogenous[40].

The system (76) and (78) gives some insights on which are the incentives of the fiscal policymakers and the potential aggregate effects produced by their policy choices. To better understand the sources of these effects, let's restrict to symmetric technological shocks.

Under symmetric shocks, if *implemented*, the target of uncoordinated governments is determined by the following conditions:

$$\varphi\hat{y}_t^f + \gamma\hat{g}_t^f = (1+\varphi)a_t^* \tag{79}$$

$$\delta\rho(\gamma\hat{g}_t^f - \sigma\hat{c}_t^f) = (1-\psi)\delta_3(\hat{y}_t^f - \hat{c}_t^f) + (1-\psi)(1-\rho)\delta_2(\hat{g}_t^f - \hat{c}_t^f) \tag{80}$$

$$\hat{y}_t^f = \rho\hat{c}_t^f + (1-\rho)\hat{g}_t^f \tag{81}$$

According to both these conditions and condition (65), two are the channels through which the lack of coordination causes distortions at the average union level even over the business cycle.

The first channel works through the steady state public expediture output ratio namely $(1-\rho)$[41]. As already pointed out under no-coordination $(1-\rho)$ is inefficiently high because at the steady state both public expenditure is over-expanded and output under-produced. Under the baseline calibration, this static distortion carries a clear consequence: it inefficiently amplifies the impact of the government expenditure shocks over output fluctuations. Indeed if the government expenditure-output ratio is too high (meaning that private consumption-output ratio is too low), then one percent increase in the government expenditure (private consumption) would expand more (less) output under no-coordination than under coordination. But according to the baseline calibration, the intertemporal elasticity of substitution of the government expenditure, γ, is smaller than that of private consumption, σ. Hence over the cycles policymakers have an incentive to substitute private consumption with public expenditure in order to smooth the path of the more inelastic bundle. Thus in response to a positive

[40]For this reason even if shocks are symmetric it can happen that $\hat{c}_t^f - \hat{c}_t^* \neq 0$.

[41]In fact $\rho \equiv \frac{C}{Y}$

technological shock, they expand more public expenditure than private consumption producing an unconscious over-expansion of the average union output.

The second channel is given by the incentive of independent governments to influence the terms of trade in their favour. According to (80), $\widehat{mrs}_t$ between private consumption and public expenditure do not reflect that of the corresponding $\widehat{mrt}_t$ as long as $\gamma \neq \sigma$ and $\psi > 0$ or $\nu > 0$[42]. The reason of this difference is again explained by the evaluation of the cost of an additional unit home household's private consumption, which under no-coordination, internalizes only the effects on home demands[43] and not those on foreign demands. Because of that, even over the business cycle uncoordinated fiscal policymakers seek to exert their monopoly power on their terms of trade.

If, as a result, under a positive technology shock, there is either an over or under expansion of output with respect to the case of coordination depends on balance between these two incentives: namely the incentive to reduce labor effort and the incentive to increase the government expenditure which if $\gamma < \sigma$ [44] generates an additional motive for an over-expansion of output. According to the impulse responses to a technological shock, that would be analyzed in what follows, the second effect is prevailing.

Clearly, in general, fiscal policymakers would not be able to reach their target even under symmetric productivity shocks. Indeed they lack policy instruments and anyway the monetary authority would seek to correct the average distortionary effects due to their policy choices. Nevertheless it will turn out that, under symmetric technology shocks, there is special parametric restriction under which that target can be implemented namely when $\gamma = \sigma$. In fact, in that case there is no motive to substitute intertemporally between public and private consumption. Thus according to (79)-(81) $\hat{g}_t^f = \hat{c}_t^f = \hat{y}_t^f$. As consequence, there are no aggregate distortions over the cycles due the lack of coordination among fiscal policymakers because on the one hand the inefficiently high dimension of the public sector is not affecting the path of output and private consumption and on the other hand the the incentive of the fiscal policymakers to influence the terms of trade over the cycle does not generate aggregate effects [45]. And in fact when $\gamma = \sigma$, if implemented, the target of the uncoordinated governments coincides with the average target of the common authority under coordination.

Consider now the case of symmetric markup shocks. Symmetric markup shocks affect fiscal policymaker decisions in two ways. First, given an aggregate markup shock, both average union output and private consumption would contract lowering the demands of foreigners for home produced goods and potentially worsening the terms of trade of the single small open economy. And, as implicit in (76) and (78), the target of autonomous government would shift accordingly because the terms of trade worsening and the lower foreign goods increase the optimal level of leisure and decrease those of both private and public consumption. Second if shocks are symmetric even the private consumption and the output of the single small open economy would reduce and the home country inflation would increase in a way which the uncoordinated fiscal poli-

[42]i.e. even intermediate or public goods are traded. Note that if $\psi > 0$ or $\nu > 0$ then $\delta_3 > 0$ or $\delta_2 > 0$.

[43]In particular the average changes over the cycle of this cost is given by $(1-\psi)\delta_3(\hat{y}_t^f - \hat{c}_t^f)$ and $(1-\psi)(1-\rho)\delta_2(\hat{g}_t^f - \hat{c}_t^f)$ which embody the average shifts in the home demands of foreigners due to terms of trade fluctuations.

[44]Notice in fact that in equilibrium $(1-\psi)\delta_3(\hat{y}_t^f - \hat{c}_t^f) = (1-\psi)(1-\rho)\delta_3(\hat{g}_t^f - \hat{c}_t^f)$.

[45]In fact in that case $(1-\psi)\delta_3(\hat{y}_t^f - \hat{c}_t^f) + (1-\psi)(1-\rho)\delta_2(\hat{y}_t^f - \hat{c}_t^f) = 0$.

cymaker perceives as inefficient. Consequently autonomous fiscal policymakers should trade off between stabilizing inflation and keeping the fluctuations of the marginal rate of substitution between the private consumption and leisure and and between public and private consumption and leisure at the desired target. Notice, however, that they do so without internalizing neither the actions of the other governments nor that of the monetary authority who as well are trying to offset the negative contraction of average output and private consumption and the increase of the average inflation. Moreover they do not recognize as the monetary authority authorities does that the steady state distortion, which is in fact a distortion only from the monetary union point of view, generates a motive to stabilize more inflation than output[46].

It is can be shown that the set of the first order conditions of fiscal policy problem can be written as:

$$\pi_{H,t} = \frac{1}{\varphi\varepsilon}\left((1-L)+\lambda B(L)\right)\left[-\varphi\tilde{y}_t^f - \gamma\tilde{g}_t^f + (1-\psi)^{-1}(\varsigma_1 - \psi\nu)\tilde{c}_t^f\right]$$
$$-\frac{\lambda}{\omega_4\varepsilon}B(L)\left[(1-\psi)\delta\rho(-\varphi\tilde{y}_t^f - \sigma\tilde{c}_t^f) + (1-\rho)(1-\nu)(\varsigma_1-\psi\nu)\tilde{g}_t^f + \varsigma_2\hat{c}_t^f\right] \quad (82)$$

$$\tilde{y}_{H,t}^f = (1-\psi)\delta\rho\tilde{c}_t{}^f + (1-\psi)(1-\nu)(1-\rho)\tilde{g}_t^f \quad (83)$$

$$\pi_{H,t} = \lambda\left[\varphi\tilde{y}_{H,t}^f + \omega_4\tilde{c}_t^f\right] + \beta E_t\{\pi_{H,t+1}\} + \lambda(\hat{\mu}_t + \upsilon_{2,t}) \quad (84)$$

$$\pi_{H,t} = -\omega_4\Delta\tilde{c}_t^f + \upsilon_{3,t} \quad (85)$$

where $B(L) \equiv (1 - E_t L^{-1})$, $\upsilon_{2,t} \equiv \varphi\hat{y}_{H,t}^f + \sigma\hat{c}_t^f + \omega_4((1-\psi)\alpha+\psi)(\hat{c}_t^f - \hat{c}_t^*)$, $\upsilon_{3,t} \equiv \pi_t^* + \omega_4(\Delta\hat{c}_t^f - \Delta\hat{c}_t^*)$.

This system of equations determines the gaps of the small open economy under uncoordinated fiscal policies for a given path of the $\hat{c}_t^*$ and π_t^*[47]. Condition (83)-(85) are the constraints of the fiscal policy problem that corresponds to conditions resource constraint, the Phillips curve and the constraint due to the adoption of a common currency in terms of gap from the target of the fiscal policymaker. Conversely condition (82) is derived from the first order conditions with respect to $\tilde{y}_{H,t}^f$, $\tilde{g}_t^f$, $\tilde{c}_t^f$ and $\pi_{H,t}$. This last condition expresses the trade off among stabilizing at the desired level different objectives: the home country inflation, the marginal rate of substitution between private consumption and leisure and public consumption and leisure. A question arises: under which conditions the fiscal policymakers choose a policy consistent with home price stability?

Suppose that the monetary authority sets the nominal interest rate in order to completely stabilize the average union inflation. And at the same time suppose that shocks are to productivity and symmetric. Then if $\psi = 0$ and $\nu = 0$ or $\gamma = \sigma$ it can be shown $\pi_{H,t} = 0$ for all t is an optimal outcome. To see why this is true notice that

[46]Why this is true it will be clarified in the next section.

[47]To recover the average union allocation one has to find the optimal average level of provision of public expenditure and then determines the average union private consumption and output using the other equilibrium conditions: the average union resource constraint, the average union Phillips curve and the IS curve.

if $\psi = 0$ and $\nu = 0$ or $\gamma = \sigma$ and shocks are symmetric and to productivity then on the one hand stabilizing inflation allows to stabilize the marginal rate of substitution between consumption and leisure at the desired level because $\upsilon_{2,t} = \varphi\hat{y}^f_{H,t} + \sigma\hat{c}^f_t = 0$, on the on other hand there are no changes in the terms of trade so that there is no additional trade off generated by the adoption of a common currency because $\upsilon_{3,t} = 0$. Moreover under these conditions even the marginal rate of substitution between public consumption and leisure can be stabilize at the desired target.

But this result is conditional on the willingness of the monetary policymaker of completely stabilize inflation. Whether she finds it optimal or not it will be clarified in the next paragraph.

5.2 Monetary policy

Under fiscal policy no-coordination the objective of the common central bank can be approximated as:

$$-\frac{1}{2}Y^{\varphi+1}\sum_{t=0}^{\infty}\beta^t E_0 \int_0^1 \Big[\zeta_3\left(\frac{\varepsilon}{\lambda}(\pi^i_{H,t})^2 + \varphi(\tilde{y}^{m,i}_{H,t})^2\right) + \left(\rho\delta(\sigma-1) + \zeta_1\rho - \zeta_2\sigma^2\right)(\tilde{c}^{m,i}_t)^2$$
$$+\left(\zeta_1\varsigma_3 + \zeta_2(\omega_4 - \omega_5^2 - \sigma^2)\right)(\tilde{c}^{m,i}_t - \tilde{c}^{m,*}_t)^2 + 2\zeta_2\omega_4(\tilde{c}^{m,i}_t - \tilde{c}^{m,*}_t)(\tilde{y}^{m,i}_{H,t} - \tilde{y}^{m,*}_t)$$
$$+2\zeta_2\sigma\tilde{c}^{m,*}_t\tilde{y}^{m,*}_t\Big]di + s.o.t.i.m.p.$$

where $\tilde{x}^{m,i}_t \equiv \hat{x}^i_t - \hat{x}^{m,i}_t$ and $\hat{x}^{m,i}_t$ is the target for $\hat{x}^i_t$ chosen by the central bank.

Moreover $\zeta_1 \equiv \frac{((1-\psi)\delta\varphi\rho+\sigma)}{\varphi\rho+\sigma}$, $\zeta_2 \equiv \frac{((1-\psi)\delta-1)\rho}{\varphi\rho+\sigma}$, $\zeta_3 \equiv \frac{((\varphi+1)(1-\psi)\delta\rho+\sigma-\rho)}{\varphi\rho+\sigma}$ and ω_5 is properly defined in the appendix.

The objective approximation of the central bank under non-coordinated fiscal policies diverges from those of the uncoordinated fiscal authority and of the common policymaker under coordination. And this not only because the central bank does not choose the optimal provision of the public goods. Indeed even abstracting from this consideration, there are striking differences in target, weights and variables that enter in the approximation. The key determinant of these divergences is the steady state distortion as shown by the dependence of the weights and of the average target from ρ, ζ_1,ζ_2 and ζ_3.

The target of the central bank for the average union economy can be retrieved from:

$$\zeta_3(\varphi\hat{y}^{m,*}_t + \sigma\hat{c}^{m,*}_t - (1+\varphi)a^*_t) + \zeta_2(\frac{\sigma}{\rho}(\hat{y}^{m,*}_t - \hat{c}^{m,*}_t) + (1+\varphi)\hat{\mu}^*_t) = 0 \quad (86)$$

$$\hat{y}^{m,*}_t = \rho\hat{c}^{m,*}_t + (1-\rho)\hat{g}^*_t \quad (87)$$

Clearly according to (86) and (87) in general, the target of common central bank is not the first best and therefore does not coincide with the target of the common authority under coordination. Actually (86) and (87) help to understand the reasons why the steady state distortion, generated by uncoordinated fiscal policies, affects the common central bank policy target and her policy decisions.

The first is that, as pointed out by Benigno and Woodford (2004), in general, a non-efficient steady generates additional cyclical distortions that the central bank should try to correct. The source of these distortions has been partially already emphasized in the previous paragraph. If at the steady state the wedge in the marginal rate of substitution between leisure and private consumption is bigger than one, then the steady state consumption output ratio is sub-optimally low and the dimension of the public sector is sub-optimally high. Thus, differently from what happens in the case coordination, as long as $\hat{c}_t^{m,*} \neq \hat{y}_t^{m,*}$, the flexible price allocation not efficient. Indeed, when the percentage change in private consumption is lower (higher) than that of the government expenditure, *ceteris paribus* the output would over(under)- expand with the respect to what would be efficient. As a consequence, according to (86), the target of the monetary authority has to balance different purposes: on the one hand keeping the fluctuation of the marginal rate of substitution between private consumption and leisure at the efficient level; on the other hand cooping with the difference between $\hat{y}_t^{m,*}$ and $\hat{c}_t^{m,*}$; closing this gap would allow to eliminate at once the average dynamic effects of an inefficiently high dimension of the public sector. Indeed the closer $\hat{y}_t^{m,*}$ are $\hat{c}_t^{m,*}$ the less is the intratemporal substitution between private consumption and output and the less are the business cycle distortions generated by this substitution.

The second reason is that the cyclical welfare losses due to the steady distortion are increasing in the mark up shock. Namely differently from the case of coordination the second order approximation of the objective of the central bank does depend directly on the correlation between markup shocks and output. A negative markup shock decreases the inefficiently high wedge in the marginal rate of substitution between consumption and leisure generating welfare benefits. This explains why, according to (86), the central bank considers optimal to increase on average labor effort in response to a negative markup shocks. In other words, she tries to generate a negative correlation between the fluctuations of average output and those of the average markup because this reduces the average union welfare losses. This implies an important consequence: given a markup shock, under uncoordinated fiscal policy, the central bank finds optimal to stabilize on average more inflation than output with respect to what the common authority is willing to do under coordination.

The constraints to the monetary policy problem are:

$$\tilde{y}_{H,t}^{m,i} + \frac{\psi}{1-\psi}(\tilde{y}_{H,t}^{m,i} - \tilde{y}_t^{m,*}) = \rho\tilde{c}_t^{m,i} + (\delta-1)\rho(\tilde{c}_t^{m,i} - \tilde{c}_t^{m,*}) \tag{88}$$

$$\pi_{H,t}^i = \lambda\left[\varphi\tilde{y}_{H,t}^{m,i} + \sigma\tilde{c}_t^{m,i} + \omega_4((1-\psi)\alpha+\psi)(\tilde{c}_t^{m,i} - \tilde{c}_t^{m,*})\right] + \beta E_t\{\pi_{H,t+1}^i\}$$

$$+\lambda(\hat{\mu}_t^i + \upsilon_{5,t}) \tag{89}$$

$$\pi_{H,t}^i - \pi_t^* = -\omega_4(\Delta\tilde{c}_t^{m,i} - \Delta\tilde{c}_t^{m,*}) - \omega_4(\Delta\upsilon_{4,t}^i - \Delta\upsilon_{4,t}^*) \tag{90}$$

where $\upsilon_{4,t}^i \equiv \hat{c}_t^{m,i}$ and $\upsilon_{5,t} \equiv \varphi\hat{y}_{H,t}^{m,i} + \sigma\hat{c}_t^{m,i} + \omega_4((1-\psi)\alpha+\psi)(\hat{c}_t^{m,i} - \hat{c}_t^{m,*})$. Notice that given (86) $\upsilon_{5,t}^* \equiv -(\zeta_2\sigma)/(\zeta_3\rho)(y_t^{m,*} - \hat{c}_t^{m,*} + (1+\varphi)\hat{\mu}_t^*)$.

According to (89) and (90) the central bank has to balance the costs of the inflation with the other two objectives: first keeping the private consumption at its target; second correcting the dynamic distortion generated by fiscal authorities. This last objective explains the differences of between the optimal policy decisions under fiscal policy coordination and no-coordination. Indeed, the simple inspection of the (89) gives reason for two results that there will be stressed later on. First, in general, even if shocks are only to technology, completely stabilizing the average union inflation is not optimal. A finding this that differs from the findings of Galí and Monacelli (2005b). Second, for the reason just discussed, when the average union inflation is completely stabilized, a mark up shock produces a gap between the actual and the desired marginal rate of substitution between consumption and leisure that is smaller than under fiscal policy coordination.

Once integrated the system of optimality conditions of the monetary policymaker can be rewritten as:

$$\pi_t^* = -\frac{\rho(1-L)}{\varepsilon(\sigma+\varphi\rho)}\left[\varphi\tilde{y}_t^{m,*} + \sigma\tilde{c}_t^{m,*} + \frac{\zeta_2}{\zeta_3}\frac{\sigma}{\rho}(\tilde{y}_t^{m,*} - \tilde{c}_t^{m,*})\right] \tag{91}$$

$$\tilde{y}_t^{m,*} = \rho\tilde{c}_t^{m,*} \tag{92}$$

$$\pi_t^* = \lambda\left[\varphi\tilde{y}_t^{m,*} + \sigma\tilde{c}_t^{m,*}\right] + \beta E_t\{\pi_{t+1}^*\} + \lambda(\hat{\mu}_t^* + \upsilon_{5,t}^*) \tag{93}$$

Thanks to this set of equations it is possible to recover the average union allocation determined by the optimal reaction of the common central bank to given fiscal policies. Now it is possible to the question posed at the end of last paragraph. In presence of productivity shocks when does the central bank find optimal to completely stabilize the average union inflation?

Suppose that according to a policy rule $\hat{g}_t^* = \hat{c}_t^{m,*}$ for all t and there are only technological shocks. Then $\pi_t^* = 0$ for all t satisfies conditions (91),(92)and (93). Namely when the average union private and public consumptions are perfectly correlated, strict inflation targeting at the average union level is an optimal monetary policy.

However when $\hat{g}_t^* \neq \hat{c}_t^{m,*}$ for some t, then $\pi_t^* = 0$ for all t cannot be optimal. In that case a policy that completely stabilizes the average inflation does not allow to keep the average union output and private consumption at the constrained efficient level. Thus even when there is no trade in public and intermediate goods, namely $\nu = \psi = 0$ the monetary policymaker would not stabilize the average union inflation even under symmetric productivity shocks, while in that the fiscal policy maker would be willing to do that.

5.3 The case for average price stability

The analysis of the previous sections allows to formulate the following proposition:

Proposition 1 *If $\sigma = \gamma$ and shocks are to technology, then $\pi_t^* = 0$ for all t is an Nash equilibrium outcome of the monetary and fiscal policy game under fiscal policy no-coordination. Conversely $\sigma \neq \gamma$ and shocks are to technology, then $\pi_t^* = 0$ for all t cannot be a Nash equilibrium outcome of the monetary and fiscal policy game under fiscal policy no-coordination.*

Proof.1 See the appendix ■

Proposition 1 can be interpreted as follows: when $\gamma = \sigma$ and shocks are to technology, the lack of coordination among fiscal policymakers yields on *average* only *static* distortions[48] namely the steady state distortions. Indeed under this parametric restriction and the average union inflation is completely stabilized, two conditions are simultaneously satisfied. On the one hand the average marginal rates of substitution between private and public consumption and private consumption and leisure fluctuate as the marginal rates of transformation between the same variables, i.e. $\sigma\hat{c}_t^* - \gamma\hat{g}_t^* = 0$ and $\varphi(\hat{y}_t^* - \hat{a}_t^*) + \sigma\hat{c}_t^* = \hat{a}_t^*$; on the other hand the average union output co-move with the private and public consumption, i.e. $\hat{y}_t^* = \hat{g}_t^* = \hat{c}_t^*$. These two conditions ensure that, even if fiscal policies are uncoordinated, under flexible prices, the average union fluctuations of output, and public and private consumption replicate the fluctuations that would be achieved if the fiscal policies were coordinated. As a consequence the monetary authority seeks to remove the only remaining distortion that can be corrected: the average price stickiness. In fact stabilizing completely the average inflation is optimal: it allows at the same time to eliminate on average the inefficiencies produced by price rigidities and to keep the average allocation at the constrained-efficient level.

5.4 The general case

This section analyzes the general case allowing for different intertemporal elasticities of substitution of private and public consumption and different kinds of shocks. These differences generates an incentive for the fiscal authorities to seek to substitute intratemporally the public and private consumption. In the case of different elasticities in order to smooth intertemporally the path of more inelastic goods. In the case of markup shocks in order to reduce the home country private consumption and output gap. As a result because of this intratemporal substitution between private and public consumption, it is no more true that, under technological shocks, the symmetric allocation is proportional to the efficient one. And both monetary and fiscal policies at the average union level do not correspond to the ones that are optimal under coordination. In fact neither under technological shocks the common central bank should seek to pursue price stability nor fiscal policies ensure on average the efficient provision of public goods.

5.5 Calibration

Impulse responses to a one percent rise in technology and markup are recovered using the calibration indicated in the appendix which is close to those of Galí and Monacelli (2005b) and Galí and Monacelli (2005a). In particular γ^{-1} and φ^{-1} the intertemporal elasticities of substitution of public consumption and labor, α the degree of openness in private consumption, ε, the elasticity of substitution among goods produced in the same country, β the preferences discount factor, θ the parameter that governs the level of price stickiness in the economy and ac the first order autocorrelation of shocks[49] are set according to their calibration. Conversely σ^{-1} the intertemporal elasticity of substitution of private consumption and η the elasticity of substitution between bundles produced in different countries is set according to Benigno and Benigno (2006), ψ the

[48]...at least up to a first order approximation of the optimal policies.
[49]Both markup and productivity shocks are suppose to be AR(1).

degree of openness in the intermediate goods is equal to α and $\nu = 0.2$ as partially suggested by Brülhart and Trionfetti (2004). Finally χ the parameter that regulates the relative weight of the public good in the preferences is calibrated to match the average consumption output ratio of European Monetary Union.

The baseline calibration is described in the appendix an it is close to those of Galí and Monacelli (2005b), Galí and Monacelli (2005a). In particular α_s the degree of openness of the small open economies, ε, the elasticity of substitution among goods produced in the same country, β the preferences discount factor, θ the parameter that governs the level of price stickiness in the economy and ac the first order autocorrelation of shocks[50] are set according to their calibration. Conversely σ the intertemporal elasticity of substitution of private consumption and η the elasticity of substitution between bundles produced in different countries and α_b, the degree of openness of the areas are set according to Pappa (2004).

5.6 Dynamic Simulations

The appendix shows the impulse responses to a one percent increase in aggregate technology and markup that can be interpreted as follows.

When shocks are to technology and fiscal policies are coordinated, the optimal policy mix embodies two clear prescriptions for the average union economy: the nominal interest rate should be set to completely stabilize the average inflation while on average the fluctuations in the government expenditure should ensure an efficient provision of the public goods. These policies allow to close on average all the gaps and reach the efficient fluctuations. However under fiscal policy no-coordination none of these prescriptions is still valid. The first is not valid because of dynamic effects produced by fiscal policy no-coordination. In particular, given that $\gamma < \sigma$ the technology shock leads to an expansion of the provision of the public goods greater than that of consumption which, because of the inefficiently high dimension of the public sector and the incentive of uncoordinated fiscal policymakers to influence the terms of trade in their favor, implies an overexpansion of output. Thus, the central bank has to trade off between stabilizing the average union inflation and reducing the output. This explain why the monetary policy generates average union deflation, being more restrictive under no-coordination that under coordination as emphasized by the different path of the nominal interest rates. Obviously in this situation not even the average public good provision is efficient. Even over the cycle fiscal policymakers choose their policy disregarding the aggregate distortions that their joint action produces. Thus, they take their policy decisions without internalizing the fact that by expanding the government expenditure more than private consumption they are overexpanding output with respect to what would be efficient from the monetary union perspective. And in fact they choose the provision of the public goods trading off between stabilizing the home inflation and keeping marginal the rate of substitution between private consumption and leisure and between public consumption and leisure at the desired target. A target that embodies the incentive to try to influence the terms of trade in their favor. Indeed the fact that according to the impulse responses the government expenditure expansion is greater than σ/γ that of private consumption can be explained as a consequence

[50] Both markup and productivity shocks are suppose to be AR(1).

of this incentive for which public goods provision is used to generate a term of trade improvement through a reduction in private consumption.

When shocks are to the markup, the policy prescriptions under coordination are twofold. Fiscal policy is not a useful tool to stabilize the average effects of the markup shocks: for this purpose it is more efficient to use the nominal interest rate which is a costless instrument. Therefore under markup shocks, the average union government expenditure should be kept at the steady state level. At the same time, the monetary authority should trade off between stabilizing inflation and closing the output gap given the consequences of an inefficient shock to the mark up. The policy prescriptions under no-coordination are quite different. First because in response to a positive markup shock the optimal monetary policy is more restrictive under no-coordination than under coordination. Indeed from the average union perspective positive a markup shock does not only lead to an inefficient reduction of output and consumption and an increase in inflation by affecting firms decisions. As made clear by (86), it even increases the relevant welfare costs of the fluctuations due to the steady state distortion because it rises the already inefficient wedge in the marginal rate of substitution between leisure and private consumption. As a result, the common central bank wants to generate a negative correlation between average union output fluctuations and the markup shocks in order to decrease the welfare losses. Thus she has an incentive to stabilize more the average inflation rather than the average output. This explains why, according to the impulse responses, the nominal interest rate under-coordination is higher under no-coordination than under coordination, while the average inflation and output are lower under no-coordination. Second, because autonomous governments lower the provision of public goods. This is the result of the balance between different objectives. On the one hand the given that the markup shock is aggregate, the fall of the average union private consumption and output decrease the demands of foreign for home produced goods and potentially worsens the terms of trade. In response to these external shocks the non-coordinated policymaker would like to allow for an increase in the home leisure and a decrease in home private and public consumption[51]. However she has to trade off between this purpose and stabilizing the rise of home inflation and the decrease of home output due to the internal markup shock that she perceives as inefficient. Thus the provision of public good falls but not much more than the private consumption in order to diminish the reduction of the private consumption itself that actually after the first periods is higher than under coordination. Thus, while under coordination, the common authority recognizes that only the monetary policy should be used to stabilize the average effects of markup shocks, under no-coordination the single country government takes as given the actions of the other policymakers and tries on her own to stabilize the effects of the markup shock in her own country.

6 Conclusion

According to this paper the lack of coordination among policymakers has relevant normative implications in a monetary union. In fact, only under a special parametric restriction and when shocks are to technology, fiscal policy no-coordination does not

[51]This is made clear by the (76)-(78).

matter for the optimal monetary policy design. However, in general, this result in not verified and as opposed to the case of coordination under no-coordination it is possible to reach the following conclusions: first when shocks are to technology, stabilizing the average union prices is not optimal; second under markup shocks, the monetary authority is mainly focused on the stabilization of the average union inflation. Finally even if shocks are symmetric, fiscal policies are used as stabilization tool.

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A Proof of proposition 1

First part of the proof. If $\gamma = \sigma$ for all t then it can be shown that $\tilde{g}_t^{f,*} = \tilde{c}_t^{f,*} = \tilde{y}_t^{f,*}$ $\pi_t^* = 0$ satisfies the average of conditions (82)-(85). Then $\hat{g}_t^* = \hat{c}_t^* = \hat{y}_t^*$ which implies that $\tilde{g}_t^{m,*} = \tilde{c}_t^{m,*} = \tilde{y}_t^{m,*} = 0$.

The second part of the proof can be obtained by contradiction. If $\pi_t^* = 0$ for all t, then by (91) and (93) $\hat{c}_t^{m,*} = \hat{y}_t^{m,*}$ which implies that $\hat{c}_t^* = \hat{g}_t^*$. However $\hat{c}_t^* = \hat{g}_t^*$ is consistent with the average of conditions (82)-(85) only if only if $\gamma = \sigma$ which contradicts our initial hypothesis.

B The zero inflation deterministic steady states

B.1 The policy problem under coordination

Under coordination, the policy maker maximizes the following lagragian with respect to C_t^i, G_t^i, Y_t^i, $Y_{H,t}^i$, Z_t^i, K_t^i, F_t^i and $\Pi_{H,t}^i$ for all i and t:

$$
\begin{aligned}
L = \sum_{t=0}^{\infty} \beta^t E_0 \int_0^1 \Bigg\{ & \frac{{C_t^i}^{1-\sigma}}{1-\sigma} + \chi \frac{{G_t^i}^{1-\gamma}}{1-\gamma} - \frac{1}{\varphi+1}\left(\frac{Y_{H,t}^i Z_t^i}{A_t^i}\right)^{\varphi+1} \\
& + \lambda_{1,t}^{i,c}\left[Y_t^i - \left(\frac{P_t^i}{P_{C,t}^i}\right)^{-\eta}\left((1-\alpha)C_t^i + \alpha {C_t^i}^{\sigma\eta}\Upsilon_{C,t}^{1-\sigma\eta} + (1-\nu)\left(\frac{P_{C,t}^i}{P_{G,t}^i}\right)^{-\eta} G_t^i + \nu {C_t^i}^{\sigma\eta}\Upsilon_{G,t}^{1-\sigma\eta}\right)\right] \\
& + \lambda_{2,t}^{i,c}\left[Y_{H,t}^i - \left(\frac{P_{H,t}^i}{P_t^i}\right)^{-\eta}\left((1-\psi)Y_t^i + \psi\left(\frac{P_t^i}{P_{C,t}^i}\right)^{-\eta} {C_t^i}^{\sigma\eta}\Upsilon_{Y,t}^{1-\sigma\eta}\right)\right] \\
& + \lambda_{3,t}^{i,c}\left[K_t^i - \left(\frac{Y_{H,t}^i}{A_t^i}\right)^{\varphi+1} {Z_t^i}^{\varphi}(1-\tau)(1+\mu_t^i)\frac{\varepsilon}{\varepsilon-1}\right] - \lambda_{3,t-1}^{i,c}\theta {\Pi_{H,t}^i}^{\varepsilon} K_t^i \\
& + \lambda_{4,t}^{i,c}\left[F_t^i - Y_{H,t}^i {C_t^i}^{-\sigma}\frac{P_t^i}{P_{C,t}^i}\frac{P_{H,t}^i}{P_t^i}\right] - \lambda_{4,t-1}^{i,c}\theta {\Pi_{H,t}^i}^{(\varepsilon-1)} F_t^i \\
& + \lambda_{5,t}^{i,c}\left[F_t^i - K_t^i\left(\frac{1-\theta {\Pi_{H,t}^i}^{\varepsilon-1}}{1-\theta}\right)^{\frac{1}{\varepsilon-1}}\right] \\
& + \lambda_{6,t}^{i,c}\left[Z_t^i - \theta Z_{t-1}^i {\Pi^i}_{H,t}^{\varepsilon} - (1-\theta)\left(\frac{1-\theta {\Pi_{H,t}^i}^{\varepsilon-1}}{1-\theta}\right)^{\frac{\varepsilon}{\varepsilon-1}}\right] \\
& + \lambda_{7,t}^{i,c}\left[\left(\frac{C_t^i}{C_{t-1}^i}\right)^{-\sigma}\left(\frac{C_{t-1}^*}{C_t^*}\right)^{-\sigma}\Pi_t^* - \frac{P_{C,t}^i}{P_t^i}\frac{P_t^i}{P_{H,t}^i}\Pi_{H,t}^i \frac{P_{H,t-1}^i}{P_{t-1}^i}\frac{P_{t-1}^i}{P_{C,t-1}^i}\right]\Bigg\} di
\end{aligned}
$$

where $P_t^i/P_{C,t}^i$, $P_{G,t}^i/P_{C,t}^i$, $P_{H,t}^i/P_t^i$, C_t^*, $\Upsilon_{C,t}$, $\Upsilon_{G,t}$ and $\Upsilon_{Y,t}$ are determined according (35), (36), (37), (33) and (45) and $Z_{-1} = 1$According to the first order conditions evaluated at the zero inflation symmetric non-stochastic steady state:

$$
\begin{aligned}
&C^{-\sigma} = \lambda_1^c - \lambda_4^c \sigma Y C^{-\sigma-1} \\
&\chi G^{-\gamma} = \lambda_1^c \\
&\lambda_1^c = \lambda_2^c \\
&Y^{\varphi} = \lambda_2^c - \lambda_3^c(\varphi+1)Y^{\varphi}(1-\tau)(1+\mu)\frac{\varepsilon}{\varepsilon-1} - \lambda_4^c C^{-\sigma} \\
&Y^{\varphi+1} = -\lambda_3^c \varphi Y^{\varphi+1} + \lambda_6^c(1-\theta) \\
&\lambda_3^c(1-\theta) = \lambda_5^c \\
&\lambda_4^c(1-\theta) = -\lambda_5^c \\
&\lambda_3^c \theta \varepsilon K = -\lambda_4^c \theta(\varepsilon-1)F + \lambda_5^c \frac{\theta}{1-\theta}K
\end{aligned}
$$

If $(1-\tau) = (1/(1+\mu))(\varepsilon-1)/\varepsilon$[52], this system of equations jointly with (38), (43), (44), (50), (51), (52), and (53) can be satisfied by the following solution:

$$
\begin{aligned}
&C^{-\sigma} = Y^{\varphi} \\
&\chi G^{-\gamma} = Y^{\varphi} \\
&Y = C + G \\
&F = K = \frac{YC^{-\sigma}}{1-\theta} = \frac{Y^{\varphi+1}}{1-\theta}(1-\tau)(1+\mu)\frac{\varepsilon}{\varepsilon-1} \\
&Y_H = Y \qquad \Pi_H = 1 \qquad Z = 1 \\
&\lambda_1^c = Y^{\varphi} \quad \lambda_2^c = \lambda_1^c \quad \lambda_3^c = -\lambda_4^c = \frac{\lambda_5^c}{1-\theta} = 0 \quad \lambda_6^c = \frac{Y^{\varphi}}{1-\theta} \quad \lambda_7^c = 0
\end{aligned}
$$

B.2 The fiscal policy problem under no-coordination

The fiscal policy makers maximize the following lagrangian with respect to C_t, G_t, Y_t, $Y_{H,t}$, Z_t, K_t, F_t and $\Pi_{H,t}$:

[52] Namely if even τ is chosen optimally in such a way $\lambda_3 = -\lambda_4 = 0$

$$
\begin{aligned}
L = \sum_{t=0}^{\infty}\beta^t E_0\Big\{ & \frac{C_t^{1-\sigma}}{1-\sigma} + \chi\frac{G_t^{1-\gamma}}{1-\gamma} - \frac{1}{\varphi+1}\left(\frac{Y_{H,t}Z_t}{A_t}\right)^{\varphi+1} \\
& + \lambda_{1,t}^f\left[Y_t - \left(\frac{P_t}{P_{C,t}}\right)^{-\eta}\left((1-\alpha)C_t + \alpha C_t^{\sigma\eta}\Upsilon_{C,t}^{1-\sigma\eta} + (1-\nu)\left(\frac{P_{C,t}}{P_{G,t}}\right)^{-\eta}G_t + \nu C_t^{\sigma\eta}\Upsilon_{G,t}^{1-\sigma\eta}\right)\right] \\
& + \lambda_{2,t}^f\left[Y_{H,t} - \left(\frac{P_{H,t}}{P_t}\right)^{-\eta}\left((1-\psi)Y_t + \psi\left(\frac{P_t}{P_{C,t}}\right)^{-\eta}C_t^{\sigma\eta}\Upsilon_{Y,t}^{1-\sigma\eta}\right)\right] \\
& + \lambda_{3,t}^f\left[K_t - \left(\frac{Y_{H,t}}{A_t}\right)^{\varphi+1}Z_t^{\varphi}(1-\tau)(1+\mu_t)\frac{\varepsilon}{\varepsilon-1}\right] - \lambda_{3,t-1}^f\theta\Pi_{H,t}^{\varepsilon}K_t \\
& + \lambda_{4,t}^f\left[F_t - Y_{H,t}C_t^{-\sigma}\frac{P_t}{P_{C,t}}\frac{P_{H,t}}{P_t}\right] - \lambda_{4,t-1}^f\theta\Pi_{H,t}^{(\varepsilon-1)}F_t \\
& + \lambda_{5,t}^f\left[F_t - K_t\left(\frac{1-\theta\Pi_{H,t}^{\varepsilon-1}}{1-\theta}\right)^{\frac{1}{\varepsilon-1}}\right] \\
& + \lambda_{6,t}^f\left[Z_t - \theta Z_{t-1}\Pi_{H,t}^{\varepsilon} - (1-\theta)\left(\frac{1-\theta\Pi_{H,t}^{\varepsilon-1}}{1-\theta}\right)^{\frac{\varepsilon}{\varepsilon-1}}\right] \\
& + \lambda_{7,t}^f\left[\left(\frac{C_t}{C_{t-1}}\right)^{-\sigma}\left(\frac{C_{t-1}^*}{C_t^*}\right)^{-\sigma}\Pi_t^* - \frac{P_{C,t}}{P_t}\frac{P_t}{P_{H,t}}\Pi_{H,t}\frac{P_{H,t-1}}{P_{t-1}}\frac{P_{t-1}}{P_{C,t-1}}\right]\Big\}
\end{aligned}
$$

where $P_t/P_{C,t}$, $P_{G,t}/P_{C,t}$ and $P_{H,t}/P_t$ are determined according (35), (36) and (37) and C_t^*, $\Upsilon_{C,t}$, $\Upsilon_{G,t}$ and $\Upsilon_{Y,t}$ are taken as given. According to first order conditions evaluated at the zero inflation symmetric non-stochastic steady state:

$$C^{-\sigma} = \lambda_1^f(\delta_1 + \delta_2\frac{\rho}{1-\rho}) + \lambda_2^f(1-\psi)\delta_3\frac{1}{\rho} - \lambda_4^f YC^{-\sigma}[C^{-1}\sigma + (\omega_4 - 1)] + \lambda_7^f\frac{(1-\beta)}{C}[\sigma - (\omega_4 - 1)]$$

$$\chi G^{-\gamma} = \lambda_1^f$$

$$\lambda_1^f = \lambda_2^f(1-\psi)$$

$$Y^{\varphi} = \lambda_2^f - \lambda_3^f(\varphi+1)Y^{\varphi}(1-\tau)(1+\mu)\frac{\varepsilon}{\varepsilon-1} - \lambda_4^f C^{-\sigma}$$

$$Y^{\varphi+1} = -\lambda_3^f\varphi Y^{\varphi+1} + \lambda_6^f(1-\theta)$$

$$\lambda_3^f(1-\theta) = \lambda_5^f$$

$$\lambda_4^f(1-\theta) = -\lambda_5^f$$

$$\lambda_3^f\theta\varepsilon K = -\lambda_4^f\theta(\varepsilon-1)F + \lambda_5^f\frac{\theta}{1-\theta}K - \lambda_7^f$$

If $(1-\tau) = ((1/(1+\mu))(\varepsilon-1)/\varepsilon)(1-\psi)\left[\delta_1 + \delta_2\frac{G}{C} + \delta_3\frac{Y}{C}\right]$[53] this system of equations jointly with (38), (43), (44), (50), (51), (52), and (53) can be satisfied by the following

[53]Namely if even τ is chosen to maximize the objective of the fiscal policy maker ensuring $\lambda_3^f = 0$.

solution:

$$
\begin{aligned}
&C^{-\sigma} = (1-\psi)\left[\delta_1 + \delta_2\frac{G}{C} + \delta_3\frac{Y}{C}\right]Y^{\varphi}\\
&\chi G^{-\gamma} = (1-\psi)(1-\nu)Y^{\varphi}\\
&Y = C + G\\
&F = K = \frac{YC^{-\sigma}}{1-\theta} = \frac{Y^{\varphi+1}}{1-\theta}(1+\mu)(1-\tau)\frac{\varepsilon}{\varepsilon-1}\\
\\
&Y_H = Y \qquad \Pi_H = 1 \qquad Z = 1\\
&\lambda_1^f = (1-\psi)Y^{\varphi} \quad \lambda_2^f = Y^{\varphi} \quad \lambda_3^f = -\lambda_4^f = \frac{\lambda_5^f}{1-\theta} = 0 \quad \lambda_6^f = \frac{Y^{\varphi+1}}{1-\theta} \quad \lambda_7^f = 0
\end{aligned}
$$

B.3 The monetary policy problem under no-coordination

The monetary policy maker maximizes with respect to C_t^i, Y_t^i, $Y_{H,t}^i$, Z_t^i, K_t^i, F_t^i and $\Pi_{H,t}^i$ for all i and t the following lagragian:

$$
\begin{aligned}
L = &\sum_{t=0}^{\infty}\beta^t E_0 \int_0^1 \left\{ \frac{{C_t^i}^{1-\sigma}}{1-\sigma} + \chi\frac{{G_t^i}^{1-\gamma}}{1-\gamma} - \frac{1}{\varphi+1}\left(\frac{Y_{H,t}^i Z_t^i}{A_t^i}\right)^{\varphi+1}\right.\\
&+\lambda_{1,t}^{i,m}\left[Y_t^i - \left(\frac{P_t^i}{P_{C,t}^i}\right)^{-\eta}\left((1-\alpha)C_t^i + \alpha {C_t^i}^{\sigma\eta}\Upsilon_{C,t}^{1-\sigma\eta} + (1-\nu)\left(\frac{P_{C,t}^i}{P_{G,t}^i}\right)^{-\eta} G_t^i + \nu {C_t^i}^{\sigma\eta}\Upsilon_{G,t}^{1-\sigma\eta}\right)\right]\\
&+\lambda_{2,t}^{i,m}\left[Y_{H,t}^i - \left(\frac{P_{H,t}^i}{P_t^i}\right)^{-\eta}\left((1-\psi)Y_t^i + \psi\left(\frac{P_t^i}{P_{C,t}^i}\right)^{-\eta} {C_t^i}^{\sigma\eta}\Upsilon_{Y,t}^{1-\sigma\eta}\right)\right]\\
&+\lambda_{3,t}^{i,m}\left[K_t^i - \left(\frac{Y_{H,t}^i}{A_t^i}\right)^{\varphi+1} {Z_t^i}^{\varphi}(1-\tau)(1+\mu_t^i)\frac{\varepsilon}{\varepsilon-1}\right] - \lambda_{3,t-1}^{i,m}\theta {\Pi_{H,t}^i}^{\varepsilon} K_t^i\\
&+\lambda_{4,t}^{i,m}\left[F_t^i - Y_{H,t}^i {C_t^i}^{-\sigma}\frac{P_t^i}{P_{C,t}^i}\frac{P_{H,t}^i}{P_t^i}\right] - \lambda_{4,t-1}^{i,m}\theta {\Pi_{H,t}^i}^{(\varepsilon-1)} F_t^i\\
&+\lambda_{5,t}^{i,m}\left[F_t^i - K_t^i\left(\frac{1-\theta {\Pi_{H,t}^i}^{\varepsilon-1}}{1-\theta}\right)^{\frac{1}{\varepsilon-1}}\right]\\
&+\lambda_{6,t}^{i,m}\left[Z_t^i - \theta Z_{t-1}^i {\Pi^i}^{\varepsilon}_{H,t} - (1-\theta)\left(\frac{1-\theta {\Pi_{H,t}^i}^{\varepsilon-1}}{1-\theta}\right)^{\frac{\varepsilon}{\varepsilon-1}}\right]\\
&\left.+\lambda_{7,t}^{i,m}\left[\left(\frac{C_t^i}{C_{t-1}^i}\right)^{-\sigma}\left(\frac{C_{t-1}^*}{C_t^*}\right)^{-\sigma}\Pi_t^* - \frac{P_{C,t}^i}{P_t^i}\frac{P_t^i}{P_{H,t}^i}\Pi_{H,t}^i\frac{P_{H,t-1}^i}{P_{t-1}^i}\frac{P_{t-1}^i}{P_{C,t-1}^i}\right]\right\}di
\end{aligned}
$$

where $P_t^i/P_{C,t}^i$, $P_{G,t}^i/P_{C,t}^i$, $P_{H,t}^i/P_t^i$, C_t^*, $\Upsilon_{C,t}$, $\Upsilon_{G,t}$ and $\Upsilon_{Y,t}$ are determined according (35), (36), (37), (33) and (45) G_t^i is taken as given for all i and t and $Z_{-1} = 1$

. According to the first order conditions evaluated at the zero inflation symmetric non-stochastic steady state:

$$
\begin{aligned}
&C^{-\sigma} = \lambda_1^m - \lambda_4^m \sigma Y C^{-\sigma-1} \\
&\lambda_1^m = \lambda_2^m \\
&Y^{\varphi} = \lambda_2^m - \lambda_3^m(\varphi+1)Y^{\varphi}(1-\tau)(1+\mu)\frac{\varepsilon}{\varepsilon-1} - \lambda_4^m C^{-\sigma} \\
&Y^{\varphi+1} = -\lambda_3^m \varphi Y^{\varphi+1} + \lambda_6^m(1-\theta) \\
&\lambda_3^m(1-\theta) = \lambda_5^m \\
&\lambda_4^m(1-\theta) = -\lambda_5^m \\
&\lambda_3^m \theta\varepsilon K = -\lambda_4^m \theta(\varepsilon-1)F + \lambda_5^m \frac{\theta}{1-\theta}K
\end{aligned}
$$

It easy to show that if $(1-\tau) = ((1/(1+\mu))(\varepsilon-1)/\varepsilon)(1-\psi)\left[\delta_1 + \delta_2\frac{G}{C} + \delta_3\frac{Y}{C}\right]$ this system of equations jointly with (38), (43), (44), (50), (51), (52), and (53) can be satisfied by the following solution:

$$
\begin{aligned}
&C^{-\sigma} = (1-\psi)\left[\delta_1 + \delta_2\frac{G}{C} + \delta_3\frac{Y}{C}\right]Y^{\varphi} \\
&Y = C + G \\
&F = K = \frac{YC^{-\sigma}}{1-\theta} = \frac{Y^{\varphi+1}}{1-\theta}(1+\mu)(1-\tau)\frac{\varepsilon}{\varepsilon-1} \\
\\
&Y_H = Y \qquad \Pi_H = 1 \qquad Z = 1 \\
&\lambda_1^m = Y^{\varphi}\left[\frac{C}{Y}\delta\varphi + \sigma\right] \Big/ \left[\frac{C}{Y}\varphi + \sigma\right] \qquad \lambda_2^m = \lambda_1^m \\
&\lambda_3^m = -\lambda_4^m = \frac{\lambda_5^m}{1-\theta} = \frac{C}{Y}\frac{(\delta-1)}{\delta} \Big/ \left[\frac{C}{Y}\varphi + \sigma\right] \qquad \lambda_6^m = \frac{Y^{\varphi+1}(1-\varphi\lambda_4^m)}{1-\theta} \qquad \lambda_7^m = 0
\end{aligned}
$$

B.4 A purely quadratic approximation of policy makers' objectives

In order to recover the optimal policies we need to approximate up to the second order single country representative agent utility given by (**??**) in the following way.

First we can approximate the utility derived from private consumption as:

$$
\frac{C_t^{1-\sigma}}{1-\sigma} \simeq \frac{C^{1-\sigma}}{1-\sigma} + C^{1-\sigma}(\hat{c}_t + \frac{1}{2}\hat{c}_t^2) - \frac{\sigma}{2}C^{1-\sigma}\hat{c}_t^2 + t.i.p. \tag{94}
$$

where $\hat{c}_t$ stands for the log-deviations of private consumption from the steady state[54].

[54]From now this convention will be used: $\hat{x}_t$ represents the log-deviation of X_t from the steady state.

Similarly the utility derived from the consumption of public goods can be approximated:

$$\frac{G_t^{1-\gamma}}{1-\gamma} \simeq \frac{G^{1-\gamma}}{1-\gamma} + G^{1-\gamma}(\hat{g}_t + \frac{1}{2}\hat{g}_t^2) - \frac{\gamma}{2}G^{1-\gamma}\hat{g}_t^2 + t.i.p. \tag{95}$$

The labor disutility can be approximated by taking into account that $N_t = \frac{Y_{H,t}Z_t}{A_t}$ and, as showed by Galí and Monacelli (2005b), being $Z_t = \int_0^1 \left(\frac{p_{H,t}(k)}{P_{H,t}}\right)^{-\varepsilon} dk$:

$$\hat{z}_t \simeq \frac{\varepsilon}{2} Var_k(p_{H,t}(k)) \tag{96}$$

In words the approximation of Z_t around the symmetric steady state is purely quadratic. Moreover following Woodford (2001, NBER WP8071) it is possible to show that $\sum_{t=0}^{\infty} \beta^t Var_k(p_{H,t}(k)) = \frac{1}{\lambda}\sum_{t=0}^{\infty}\beta^t \pi_{H,t}^2$ with $\lambda \equiv \frac{(1-\theta)(1-\beta\theta)}{\theta}$. Thus:

$$\frac{1}{\varphi+1}\left(\frac{Y_t Z_t}{A_t}\right)^{\varphi+1} \simeq \frac{1}{\varphi+1}Y^{\varphi+1} + Y^{\varphi+1}(\hat{y}_{H,t} + \frac{1}{2}\hat{y}_{H,t}^2) + Y^{\varphi+1}\frac{\varepsilon}{2\lambda}(\pi_{H,t})^2 + \frac{\varphi}{2}Y^{\varphi+1}\hat{y}_{H,t}^2$$
$$-(\varphi+1)Y^{\varphi+1}\hat{y}_{H,t}a_t + t.i.p. \tag{97}$$

The welfare approximation under coordination

Under coordination, at the steady state, the fiscal authority chooses to produce the efficient level of public goods. Therefore $C^{-\sigma} = \chi G^{-\gamma} = Y^{\varphi}$ which implies that the second order approximation of the average union welfare can be rewritten as:

$$\sum_{t=0}^{\infty}\beta^t Y^{\varphi+1} E_0 \int_0^1 \left[\hat{s}_t^{i'} z_s - \frac{1}{2}\hat{s}_t^{i'} Z_{s,s}\hat{s}_t^i + \hat{s}_t^{i'} Z_{s,a}\hat{a}_t^i\right] + t.i.p. \tag{98}$$

where

$$\hat{s}_t' \equiv [\hat{y}_{H,t}^i,\ \hat{g}_t^i,\ \hat{c}_t^i,\ \pi_{H,t}^i] \qquad z_s' \equiv [-1,\ \rho,\ (1-\rho),\ 0]$$

$$Z_{s,s} \equiv \begin{bmatrix} (\varphi+1) & 0 & 0 & 0 \\ 0 & (\gamma-1)(1-\rho) & 0 & 0 \\ 0 & 0 & (\sigma-1)\rho & 0 \\ 0 & 0 & 0 & \frac{\varepsilon}{\lambda} \end{bmatrix}$$

$$Z_{s,a} \equiv \begin{bmatrix} (1+\varphi) \\ 0 \\ 0 \end{bmatrix}$$

Again it is possible to substitute the linear quadratic terms of (94) by using the second order approximation of the resource constraints namely:

$$0 \simeq -\int_0^1 \hat{y}_t^i di - \frac{1}{2}\int_0^1 \hat{y}_t^{i2} di + \int_0^1 \hat{s}_t^i di' \ h_s + \frac{1}{2}\int_0^1 \hat{s}_t^{i'} H_{s,s} \hat{s}_t^i di + \frac{1}{2}\int_0^1 \hat{s}_t^i di' H_{S,S} \int_0^1 \hat{s}_t^i di + t.i.p. \quad (99)$$

$$0 \simeq \int_0^1 \hat{s}_t^i di' \ p_s + \int_0^1 \hat{y}_t^i di + \frac{1}{2}\int_0^1 \hat{y}_t^{i2} di + \frac{1}{2}\int_0^1 \hat{s}_t^{i'} P_{s,s} \hat{s}_t^i di + \frac{1}{2}\int_0^1 \hat{s}_t^i di P_{S,S} \int_0^1 \hat{s}_t^i di + \int_0^1 \hat{y}_t^i P_{y,s} \hat{s}_t^i di$$

$$+ \int_0^1 \hat{y}_t^i di P_{Y,S} \int_0^1 \hat{s}_t^i di + t.i.p. \quad (100)$$

where

$$h_s' \equiv [0, \ (1-\rho), \ \rho, \ 0]$$

$$H_{s,s} \equiv \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & (1-\rho) & \xi\nu(1-\nu)(1-\rho) & 0 \\ 0 & \xi\nu(1-\nu)(1-\rho) & \rho + \omega_1\rho + \omega_2(1-\rho) & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$H_{S,S} \equiv \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & -\xi\nu(1-\nu)(1-\rho) & 0 \\ 0 & -\xi\nu(1-\nu)(1-\rho) & -\omega_1\rho - \omega_2(1-\rho) & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$p_s^{'} \equiv [-1, \ 0, \ 0, \ 0] \qquad P_{y,s} \equiv \begin{bmatrix} 0 & 0 & \xi\psi & 0 \end{bmatrix} \qquad P_{Y,S} \equiv \begin{bmatrix} 0 & 0 & -\xi\psi & 0 \end{bmatrix}$$

$$P_{s,s} \equiv \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \omega_3 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad P_{S,S} \equiv \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -\omega_3 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

where

$$\xi \equiv \frac{\eta\sigma}{1-\alpha}$$

$$\omega_1 \equiv \frac{\alpha\,\eta\,\sigma\,(\sigma - (1-\alpha)\,\alpha\,(1-\eta\,\sigma))}{(1-\alpha)^2}$$

$$\omega_2 \equiv \frac{\eta\,\nu\,\sigma\,((\nu-1) + (\sigma-1) - (1-2\,\eta)\,(1-\nu)\,\nu\,\sigma - \alpha\,(\nu-2)\,(1+(1-\eta)\,\sigma) - (1-\eta)\,\sigma\nu)}{(1-\alpha)^2}$$

$$\omega_3 \equiv -\left(\frac{\eta\,\sigma\,\psi\,((1-\alpha-\alpha\,(1-\eta)\,\sigma)\,(1-\psi)\,(2-\psi) - \sigma\,(1+\eta\,(1-\psi)\,\psi))}{(1-\alpha)^2\,(1-\psi)^2}\right)$$

Given (115) and (116) it is easy to show that:

$$0 \simeq \int_0^1 \hat{s}_t^i di' \ r_s + \frac{1}{2}\int_0^1 \hat{s}_t^{i'} R_{s,s} \hat{s}_t^i di + \frac{1}{2}\int_0^1 \hat{s}_t^i di' R_{S,S} \int_0^1 \hat{s}_t^i di + t.i.p. \quad (101)$$

where

$$r_s \equiv p_s + h_s \qquad R_{s,s} \equiv P_{s,s} + H_{s,s} + h_y P_{y,s} + P'_{y,s} h'_y$$

$$R_{S,S} \equiv \; P_{S,S} + H_{S,S} + h_Y P_{y,s} + P'_{y,s} h'_Y + h_s P_{Y,S} + P'_{Y,S} h'_s \tag{102}$$

and

$h_y^{'} \equiv [0,\ (1-\nu)(1-\rho),\ \delta_1\rho + \delta_2(1-\rho),\ 0] \qquad h_Y^{'} \equiv [0,\ \nu(1-\rho), \rho - \delta_1\rho - \delta_2(1-\rho),\ 0]$

where $\varsigma_1 \equiv \xi(\psi+\nu)$ $\varsigma_3 \equiv \rho\omega_1 + (1-\rho)\omega_2 + \omega_3 + 2\xi\psi(\rho\delta_1 + (1-\rho)\delta_2)$

Given that

$$z_s = r_s \tag{103}$$

under coordination, the second order approximation to the average union welfare can be rewritten as:

$$Y^{\varphi+1}\sum_{t=0}^{\infty}\beta^t E_0\Big[-\frac{1}{2}\int_0^1 \hat{s}_t^{i'}\Omega_{s,s}\hat{s}_t^i di - \frac{1}{2}\int_0^1 \hat{s}_t^i di'\Omega_{S,S}\int_0^1 \hat{s}_t^i di + \int_0^1 \hat{s}_t^{i'}\Omega_{s,a}\hat{a}_t^i di + \int_0^1 \hat{s}_t^i di'\Omega_{S,A}\int_0^1 \hat{a}_t^i di\Big]$$
$$+t.i.p. \tag{104}$$

where

$$\Omega_{s,s} \equiv Z_{s,s} + R_{s,s} \qquad \Omega_{S,S} \equiv R_{S,S}$$

$$\Omega_{s,a} \equiv Z_{s,a}$$

are equal to:

$$\Omega_{s,s} = \begin{bmatrix} \varphi & 0 & 0 & 0 \\ 0 & \gamma(1-\rho) & (1-\rho)(1-\nu)\varsigma_1 & 0 \\ 0 & (1-\rho)(1-\nu)\varsigma_1 & \sigma\rho + \varsigma_3 & 0 \\ 0 & 0 & 0 & \frac{\varepsilon}{\lambda} \end{bmatrix}$$

$$\Omega_{S,S} = \begin{bmatrix} 0 & -(1-\rho)(1-\nu)\varsigma_1 & 0 \\ -(1-\rho)(1-\nu)\varsigma_1 & -\varsigma_3 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

The welfare for the fiscal authority under no-coordination

By combining (94),(95) and (97) and considering that at the steady state $C^{-\sigma} = (1-\psi)\delta Y^{\varphi}$and $\chi G^{-\gamma} = (1-\psi)(1-\nu)Y^{\varphi}$ the second order approximation of single country representative agent welfare can be written in matrix notation as:

$$\sum_{t=0}^{\infty}\beta^t Y^{\varphi+1} E_0\left[\hat{s}_t' w_s - \frac{1}{2}\hat{s}_t' W_{s,s}\hat{s}_t + \hat{s}_t' W_{s,e}\hat{e}_t\right] + t.i.f.p. \tag{105}$$

where

$$\hat{s}_t' \equiv [\hat{y}_{H,t},\ \hat{g}_t,\ \hat{c}_t,\ \pi_{H,t}] \qquad w_s^{'} \equiv [-1,\ (1-\psi)(1-\nu)(1-\rho),\ (1-\psi)\delta\rho,\ 0] \qquad \hat{e}_t^{'} \equiv [\hat{y}_t^*,\ \hat{g}_t^*,\ \hat{c}_t^*,\ a_t]$$

$$W_{s,s} \equiv \begin{bmatrix} (\varphi+1) & 0 & 0 & 0 \\ 0 & (\gamma-1)(1-\psi)(1-\nu)(1-\rho) & 0 & 0 \\ 0 & 0 & (\sigma-1)(1-\psi)\delta\rho & 0 \\ 0 & 0 & 0 & \frac{\varepsilon}{\lambda} \end{bmatrix}$$

$$W_{s,e} \equiv \begin{bmatrix} 0 & 0 & 0 & (1+\varphi) \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

and with $\hat{y}_t^* \equiv \int_0^1 \hat{y}_t^j dj$, $\hat{g}_t^* \equiv \int_0^1 \hat{g}_t^j dj$ and $\hat{c}_t^* \equiv \int_0^1 \hat{c}_t^j dj$. This approximation can be written in purely quadratic way by using the second order approximation of the single country market clearing conditions (43) and (44). In particular notice that the second order approximation of these constraints can be read as:

$$0 \simeq \left[-\hat{y}_t - \tfrac{1}{2}\hat{y}_t^2 + \hat{s}_t' f_s - \hat{e}_t' f_e + \tfrac{1}{2}\hat{s}_t' F_{s,s}\hat{s}_t - \hat{s}_t' F_{s,e} e_t\right] + s.o.t.i.f.p. \tag{106}$$

$$0 \simeq \left[\hat{s}_t'\iota_s - \hat{e}_t'\iota_e + \hat{y}_t\iota_y + \tfrac{1}{2}\hat{y}_t^2\iota_y + \tfrac{1}{2}\hat{s}_t^{'}I_{s,s}\hat{s}_t - \hat{s}_t^{'}I_{s,e}\hat{e}_t + \hat{y}_t I_{y,s}\hat{s}_t - \hat{y}_t I_{y,e}\hat{e}_t\right] + s.o.t.i.f.p. \tag{107}$$

where

$$f_s^{'} \equiv [0,\ (1-\nu)(1-\rho),\ \delta_1\rho + \delta_2(1-\rho),\ 0] \qquad f_e^{'} \equiv [0,\ -\nu(1-\rho),\ -\rho + (\delta_1\rho + \delta_2(1-\rho)),\ 0]$$

$$F_{s,s} \equiv \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & (1-\nu)(1-\rho) & \xi\nu(1-\nu)(1-\rho) & 0 \\ 0 & \xi\nu(1-\nu)(1-\rho) & \delta_1\rho + \delta_2(1-\rho) + \omega_1\rho + \omega_2(1-\rho) & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$F_{s,e} \equiv \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & \xi\nu(1-\nu)(1-\rho) & 0 \\ 0 & -\xi\nu(1-\rho) & \omega_1\rho + \omega_2(1-\rho) + \xi\nu(2-\nu)(1-\rho) & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\iota_s^{'} \equiv [-1,\ 0,\ (1-\psi)\delta_3,\ 0] \qquad \iota_e^{'} \equiv [-\psi,\ 0,\ (1-\psi)\delta_3,\ 0] \qquad \iota_y \equiv [(1-\psi)]$$

$$I_{s,s} \equiv \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & (1-\psi)\delta_3 + \omega_3 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad I_{s,e} \equiv \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -\frac{\xi\psi}{(1-\psi)} & 0 & \omega_3 + \frac{\xi\psi(2-\psi)}{(1-\psi)} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$I_{y,s} \equiv \begin{bmatrix} 0 & 0 & \xi\psi & 0 \end{bmatrix} \qquad I_{y,e} \equiv \begin{bmatrix} 0 & 0 & \xi\psi & 0 \end{bmatrix}$$

Given (106), (107) an be rewritten as:

$$0 \simeq \hat{s}_t^{'}(\iota_s + (1-\psi)f_s) - \hat{e}_t^{'}(\iota_e + (1-\psi)f_e) + \frac{1}{2}\hat{s}_t^{'}\left(I_{s,s} + (1-\psi)F_{s,s} + f_s I_{y,s} + I_{y,s}^{'}f_s^{'}\right)\hat{s}_t$$

$$-\hat{s}_t^{'}\left[I_{s,e} + (1-\psi)F_{s,e} + f_s I_{y,e} + f_e I_{y,s}\right]\hat{e}_t + s.o.t.i.f.p. \tag{108}$$

Again thanks to conditions (73), (74) and (75) it follows that:

$$w_s = \iota_s + (1-\psi) f_s \tag{109}$$

Therefore by using (108), (105) can be approximated as:

$$Y^{\varphi+1} \sum_{t=0}^{\infty} \beta^t E_0 \Big[-\frac{1}{2} s_t^{'} \Omega_{s,s} s_t + s_t^{'} \Omega_{s,e} e_t \Big] + t.i.f.p. \tag{110}$$

which is purely quadratic and where $\Omega_{s,s} \equiv W_{s,s} + I_{s,s} + (1-\psi)F_{s,s} + f_s I_{y,s} + I_{y,s}' f_s'$ and $\Omega_{s,e} \equiv W_{s,e} + I_{s,e} + (1-\psi)F_{s,e} + f_s I_{y,e} + f_e I_{y,s}$ are respectively equal to:

$$\begin{bmatrix} \varphi & 0 & 0 & 0 \\ 0 & \gamma(1-\rho)(1-\nu)(1-\psi) & (1-\rho)(1-\nu)(\varsigma_1 - \xi\nu\psi) & 0 \\ 0 & (1-\rho)(1-\nu)(\varsigma_1 - \xi\nu\psi) & (1-\psi)\rho\sigma\delta + \varsigma_2 & 0 \\ 0 & 0 & 0 & \frac{\varepsilon}{\lambda} \end{bmatrix} \tag{111}$$

$$\begin{bmatrix} 0 & 0 & 0 & 1+\varphi \\ 0 & 0 & (1-\rho)(1-\nu)(\varsigma_1 - \xi\nu\psi) & 0 \\ -(1-\psi)\delta_3 & -(1-\rho)(1-\psi)\delta_2 + (1-\rho)(1-\nu)(\varsigma_1 - \xi\nu\psi) & (1-\psi)((1-\rho)\delta_2 + \delta_3) + \varsigma_2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{112}$$

with $\delta \equiv \delta_1 + \frac{(1-\rho)}{\rho}\delta_2 + \frac{1}{\rho}\delta_3$, $\varsigma_1 \equiv \xi(\nu+\psi)$ $\varsigma_2 \equiv (1-\psi)(\omega_1\rho + \omega_2(1-\rho)) + \omega_3 + 2\xi\psi(\rho\delta_1 + (1-\rho)\delta_2)$.

B.5 The welfare approximation for the monetary authority

The central bank of the monetary union maximizes:

$$\sum_{t=0}^{\infty} \beta^t E_0 \int_0^1 \left[\frac{C_t^{i\,1-\sigma}}{1-\sigma} + \chi \frac{G_t^{i\,1-\gamma}}{1-\gamma} - \frac{N_t^{i\,\varphi+1}}{\varphi+1} \right] di \quad 0 < \beta < 1 \tag{113}$$

By combining (94) and (97) and given that $C^{-\sigma} = (1-\psi)\delta Y^{\varphi}$, the second order approximation of (113) can be written as:

$$\sum_{t=0}^{\infty} \beta^t Y^{\varphi+1} E_0 \int_0^1 \left[\hat{l}_t^{i'} w_l - \frac{1}{2} \hat{l}_t^{i'} W_{l,l} \hat{l}_t^i + \hat{l}_t^{i'} W_{l,u} \hat{u}_t^i \right] di + t.i.m.p. \tag{114}$$

where

$$\hat{l}_t^{i'} \equiv \left[\hat{y}_{H,t}^i,\ \hat{c}_t^i,\ \hat{\pi}_{H,t}^i \right] \qquad \hat{u}_t^{i'} \equiv \left[\hat{g}_t^i,\ a_t^i, \mu_t^i \right] \qquad w_l' \equiv [-1,\ (1-\psi)\delta\rho,\ 0]$$

$$W_{l,l} \equiv \begin{bmatrix} (\varphi+1) & 0 & 0 \\ 0 & (\sigma-1)(1-\psi)\delta\rho & 0 \\ 0 & 0 & \frac{\varepsilon}{\lambda} \end{bmatrix} \qquad W_{l,u} \equiv \begin{bmatrix} 0 & (\varphi+1) & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

and $t.i.m.p.$ stands for terms independent of monetary policy inclusive of the government expenditure. In order to express that approximation in a purely quadratic way, it is necessary to recover the second order approximations of (43), (44), (52), (48) and (49). By integrating the first two approximation we obtain:

$$0 \simeq -\int_0^1 \hat{y}_t^i di - \frac{1}{2}\int_0^1 \hat{y}_t^{i2} di + \int_0^1 \hat{l}_t^i di'\ f_l - \int_0^1 \hat{u}_t^i di'\ f_u + \frac{1}{2}\int_0^1 \hat{l}_t^{i'} F_{l,l}\hat{l}_t^i di + \frac{1}{2}\int_0^1 \hat{l}_t^i dj' F_{L,L}\int_0^1 \hat{l}_t^i di$$
$$-\int_0^1 \hat{l}_t^{i'} F_{l,u}\hat{u}_t^i di - \int_0^1 \hat{l}_t^{i'} di F_{L,U}\int_0^1 \hat{u}_t^i di + s.o.t.i.m.p. \tag{115}$$

$$0 \simeq \int_0^1 \hat{l}_t^i di'\ \iota_l + \int_0^1 \hat{y}_t^i di + \frac{1}{2}\int_0^1 \hat{y}_t^{i2} di + \frac{1}{2}\int_0^1 \hat{l}_t^{i'} I_{l,l}\hat{l}_t^i di + \frac{1}{2}\int_0^1 \hat{l}_t^i di' I_{L,L}\int_0^1 \hat{l}_t^i di + \int_0^1 \hat{y}_t^i I_{y,l}\hat{l}_t^i di$$
$$+\int_0^1 \hat{y}_t^i di I_{Y,L}\int_0^1 \hat{l}_t^i di + s.o.t.i.m.p. \tag{116}$$

where

$$f_l' \equiv [0,\ \rho,\ 0] \qquad f_u' \equiv [-(1-\rho),\ 0, 0]$$

$$F_{l,l} \equiv \begin{bmatrix} 0 & 0 & 0 \\ 0 & \rho+\omega_1\rho+\omega_2(1-\rho) & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad F_{l,u} \equiv \begin{bmatrix} 0 & 0 & 0 \\ -\xi\nu(1-\nu)(1-\rho) & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$F_{L,L} \equiv \begin{bmatrix} 0 & 0 & 0 \\ 0 & -\omega_1\rho-\omega_2(1-\rho) & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad F_{L,U} \equiv \begin{bmatrix} 0 & 0 & 0 \\ \xi\nu(1-\nu)(1-\rho) & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\iota_l' \equiv [-1,\ 0,\ 0] \qquad I_{y,l} \equiv \begin{bmatrix} 0 & \xi\psi & 0 \end{bmatrix} \qquad I_{Y,L} \equiv \begin{bmatrix} 0 & -\xi\psi & 0 \end{bmatrix}$$

$$I_{l,l} \equiv \begin{bmatrix} -1 & 0 & 0 \\ 0 & \omega_3 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad I_{L,L} \equiv \begin{bmatrix} 0 & 0 & 0 \\ 0 & -\omega_3 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Given (115) and (116) it is easy to show that:

$$0 \simeq \int_0^1 \hat{l}_t^i di'\ r_l - \int_0^1 \hat{u}_t^i di'\ r_u + \frac{1}{2}\int_0^1 \hat{l}_t^{i'} R_{l,l}\hat{l}_t^i di + \frac{1}{2}\int_0^1 \hat{l}_t^i di' R_{L,L}\int_0^1 \hat{l}_t^i di$$
$$-\int_0^1 \hat{l}_t^{i'} R_{l,u} u_t^i di - \int_0^1 \hat{l}_t^{i'} di R_{L,U}\int_0^1 u_t^i di + s.o.t.i.m.p. \tag{117}$$

where

$$r_l \equiv \iota_l + f_l \qquad r_u \equiv f_u$$

$$R_{l,l} \equiv I_{l,l} + F_{l,l} + f_y I_{y,l} + I_{y,l}' f_y' \qquad R_{L,L} \equiv I_{L,L} + F_{L,L} + f_Y I_{y,l} + I_{y,l}' f_Y + f_l I_{Y,L} + I_{Y,L}' f_l'$$
$$R_{l,u} \equiv F_{l,u} + I_{y,l}' f_g' \qquad R_{L,U} \equiv F_{L,U} + f_G I_{y,l} + I_{Y,L}' f_u' \tag{118}$$

and

$$f_y' \equiv [0,\ \delta_1\rho+\delta_2(1-\rho),\ 0] \qquad f_Y' \equiv [0,\ \rho-\delta_1\rho-\delta_2(1-\rho),\ 0]$$

$$f_g' \equiv [-(1-\nu)(1-\rho),\ 0] \qquad f_G' \equiv [-\nu(1-\rho),\ 0]$$

By combining the second order approximation of the (52), (50) and (51) as in Benigno and Woodford (2004), we obtain the following condition:

$$V_0 = \frac{1-\theta}{\theta}(1-\beta\theta)\sum_{t=0}^{\infty}\beta^t E_0\Big[\int_0^1 \hat{l}_t^i di' v_l - \int_0^1 \hat{u}_t^{i'} v_u + \frac{1}{2}\int_0^1 \hat{l}_t^{i'} V_{l,l}\hat{l}_t^i di + \frac{1}{2}\int_0^1 \hat{l}_t^i di' V_{L,L}\int_0^1 \hat{l}_t^i di - \int_0^1 \hat{l}_t^{i'} V_{l,u}\hat{u}_t^i di\Big] + s.o.t.i.m.p. \tag{119}$$

where

$$v_l' \equiv [\varphi,\ \sigma,\ 0] \qquad v_u' \equiv [0,\ (\varphi+1),\ -1]$$

$$V_{l,l} \equiv \begin{bmatrix} \varphi(\varphi+2) & \omega_4 & 0 \\ \omega_4 & -\omega_4^2+\omega_5 & 0 \\ 0 & 0 & \frac{\varepsilon(\varphi+1)}{\lambda} \end{bmatrix} \qquad V_{l,u} \equiv \begin{bmatrix} 0 & (\varphi+1)^2 & -(\varphi+1) \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$V_{L,L} \equiv \begin{bmatrix} 0 & \sigma-\omega_4 & 0 \\ \sigma-\omega_4 & -\sigma^2+\omega_4^2-\omega_5 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

with

$$\omega_5 \equiv -\frac{\sigma\,\psi\,(-1+(1-\eta)\,\sigma\,(1+\alpha\,(1-\psi))+\alpha\,(1-\psi)+(1-\sigma)\,\psi)}{(1-\alpha)^2\,(1-\psi)^2} + \frac{\alpha\,\sigma\,(1-\alpha\,(1-\sigma)-(1-\eta)\,\sigma)}{(1-\alpha)^2}$$

$$\omega_4 \equiv \frac{\sigma}{(1-\alpha)\,(1-\psi)} \qquad \lambda \equiv \frac{(1-\theta)(1-\beta\theta)}{\theta}$$

Conditions (117) and (120) allow to substitute the linear term of the union welfare approximation with purely quadratic terms. In fact given these conditions:

$$0 \simeq Y^{\varphi+1}\sum_{t=0}^{\infty}\beta^t E_0\Big[\int_0^1 \hat{l}_t^i di'(\zeta_1 r_l+\zeta_2 v_l) + \frac{1}{2}\int_0^1 \hat{l}_t^{i'}(\zeta_1 R_{l,l}+\zeta_2 V_{l,l})\hat{l}_t^i di - \int_0^1 \hat{l}_t^{i'}(\zeta_1 R_{l,u}+\zeta_2 V_{l,u})\hat{u}_t^i di + \frac{1}{2}\int_0^1 \hat{l}_t^i di'(\zeta_1 R_{L,L}+\zeta_2 V_{L,L})\int_0^1 \hat{l}_t^i di - \int_0^1 \hat{l}_t^i di'(\zeta_1 R_{L,U})\int_0^1 \hat{u}_t^i di\Big] + t.i.m.p. \tag{120}$$

where $\zeta_1 \equiv \frac{(1-\psi)\delta\varphi\rho+\sigma}{\varphi\rho+\sigma}$ and $\zeta_2 \equiv \frac{((1-\psi)\delta-1)\rho}{\varphi\rho+\sigma}$. It is easy to show that:

$$w_l = \zeta_1 r_l + \zeta_2 v_l \tag{121}$$

Hence we can write the second order approximation of union welfare as:

$$Y^{\varphi+1}\sum_{t=0}^{\infty}\beta^t E_0\Big[-\frac{1}{2}\int_0^1 \hat{l}_t^{i'}\Omega_{l,l}\hat{l}_t^i di - \frac{1}{2}\int_0^1 \hat{l}_t^i di'\Omega_{L,L}\int_0^1 \hat{l}_t^i di + \int_0^1 \hat{l}_t^{i'}\Omega_{l,u}\hat{u}_t^i di + \int_0^1 \hat{l}_t^i di'\Omega_{L,U}\int_0^1 \hat{u}_t^i di\Big] + t.i.m.p. \tag{122}$$

where

$$\Omega_{l,l} \equiv W_{l,l} + \zeta_1 R_{l,l} + \zeta_2 V_{l,l} \qquad \Omega_{L,L} \equiv \zeta_1 R_{L,L} + \zeta_2 V_{L,L}$$
$$\Omega_{l,u} \equiv W_{l,u} + \zeta_1 R_{l,u} + \zeta_2 V_{l,u} \qquad \Omega_{L,U} \equiv \zeta_1 R_{L,U} \tag{123}$$

are equal to:

$$\Omega_{l,l} = \begin{bmatrix} \varphi\zeta_3 & \zeta_2\omega_4 & 0 \\ \zeta_2\omega_4 & \delta(\sigma-1)(1-\psi)\rho + \zeta_1(\rho+\varsigma_3) + \zeta_2(\omega_5-\omega_4^2) & 0 \\ 0 & 0 & \frac{\varepsilon\zeta_3}{\lambda} \end{bmatrix}$$

$$\Omega_{L,L} = \begin{bmatrix} 0 & \zeta_2(\sigma-\omega_4) & 0 \\ \zeta_2(\sigma-\omega_4) & -\zeta_1\varsigma_3 - \zeta_2(\sigma^2+\omega_5-\omega_4^2) & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\Omega_{l,u} = \begin{bmatrix} 0 & (\varphi+1)\zeta_3 & -(\varphi+1)\zeta_2 \\ -\zeta_1(1-\nu)(1-\rho)\varsigma_1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad \Omega_{L,U} = \begin{bmatrix} 0 & 0 & 0 \\ \zeta_1(1-\nu)(1-\rho)\varsigma_1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

with $\zeta_3 \equiv 1 + (\varphi+1)\zeta_2$ and $\varsigma_3 \equiv \rho\omega_1 + (1-\rho)\omega_2 + \omega_3 + 2\xi\psi(\rho\delta_1 + (1-\rho)\delta_2)$.