

Dimensional Metrology

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Lecture Notes: Photomechanics for Engineers
IMAC, EPFL
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Outline

- One-dimensional (1D) and two-dimensional (2D) measurements
- Three-dimensional (3D) measurements
 - Fringe Projection Technique
 - Applications
 - Overview
 - Detailed Perspective

1D and 2D Measurement Tools

- Hand Tools:

- Yardstick



- Vernier Caliper



- Screw Gauge



1D and 2D Measurements Cont...

(Wish List)

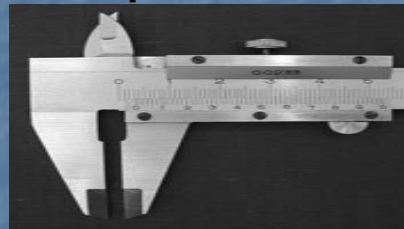
- Can we make measurements without touching the object?
- Can we have variable measurement range?
- Can we have variable measurement resolution?
- Can we automate the measurement process?
- ❖ Do we have a device which is having all of the above features???



Yes: It is a digital camera!!!

1D and 2D Measurements Cont...

- Digital Image (Grayscale Image): 2D Matrix
 - Elements of the matrix are referred to as pixels
 - Values of the elements represent intensity (0 to 255)



- Zoom-in, zoom-out and changing the distance from the object: enable to have variable resolution and variable range of measurement !
- Analysis of the acquired images to automate the measurement process !!
- We don't have to touch the object !!!

1D and 2D Measurements Cont...

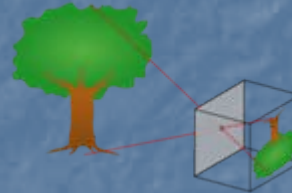


- Height of the Eiffel tower?
- There are many factors that govern the conversion between the number of pixels to real world dimensions.
- Some of them: Zoom (i.e. Focal length of the lens), position of the camera w.r.t the object, type of camera used etc.
- How do we know them in practice?

1D and 2D Measurements Cont...

- Is there a mathematical relation that governs this conversion?

- Pinhole camera model



- Projective mapping of world coordinates to pixel coordinates is described by:

$$z_c \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = A \begin{bmatrix} R & T \end{bmatrix} \begin{bmatrix} x_w \\ y_w \\ z_w \\ 1 \end{bmatrix}$$

$[u \ v \ 1]^T$ 2D positions in pixel coordinates
 $[x_w \ y_w \ z_w \ 1]^T$ 3D positions in real world coordinates

Intrinsic parameters

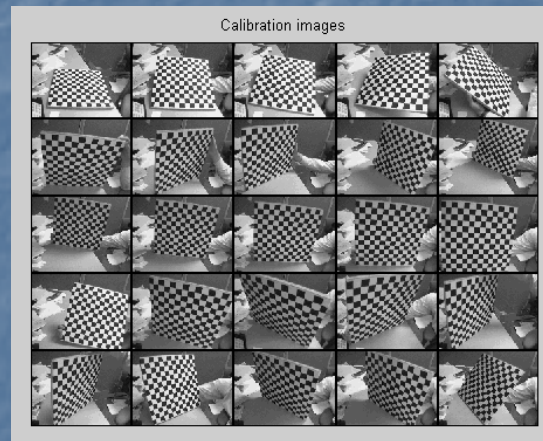
$$A = \begin{bmatrix} \alpha & \gamma & u_0 \\ 0 & \beta & v_0 \\ 0 & 0 & 1 \end{bmatrix}$$

$(u_0 \ v_0)$ coordinates of the principle point
 $(\alpha \ \beta)$ scale factors in image coordinates
 γ parameter describing the skewness of the two image axes.

R and T are the extrinsic parameters which denote the coordinates system transformations from 3D world coordinates to 3D camera coordinates. They define the position of the camera center and the camera heading in the world coordinates.

1D and 2D Measurements Cont...

- Practical Solution?
- Camera Calibration Toolbox for MATLAB

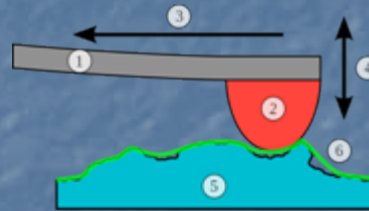


http://www.vision.caltech.edu/bouguetj/calib_doc/

- 'Comparative review of camera calibrating methods with accuracy evaluation' by Salvi et al., Pattern Recognition 2002; 35(7):1617-1635.

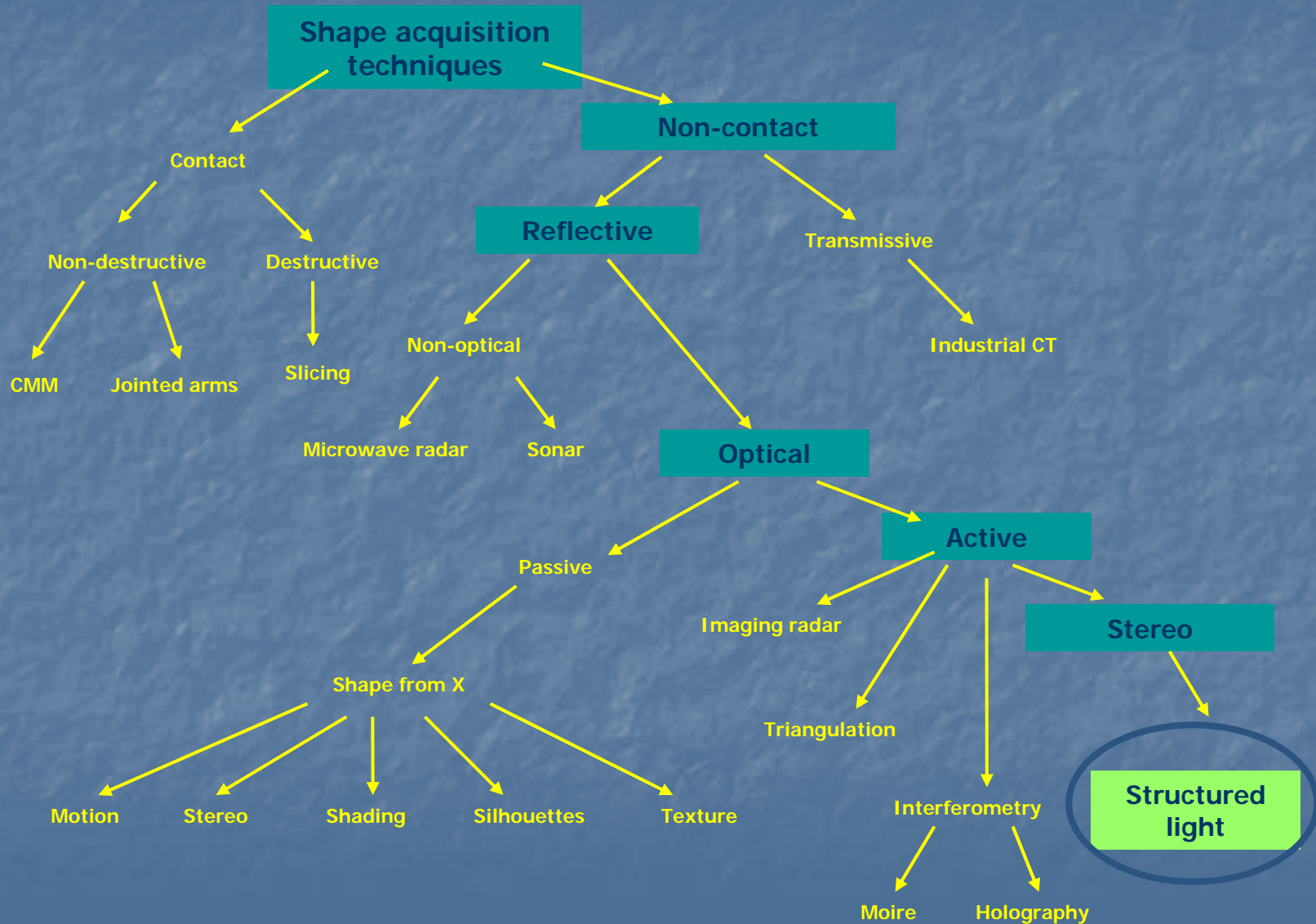
3D Shape Measurement

- Coordinate Measurement Machine (CMM)



- Measurements are defined by a probe attached to the third moving axis of this machine
- Can we inherit the benefits that we have in making 1D and 2D measurements with digital camera, while measuring 3D shapes?

3D Shape Measurement Techniques



Optical Triangulation

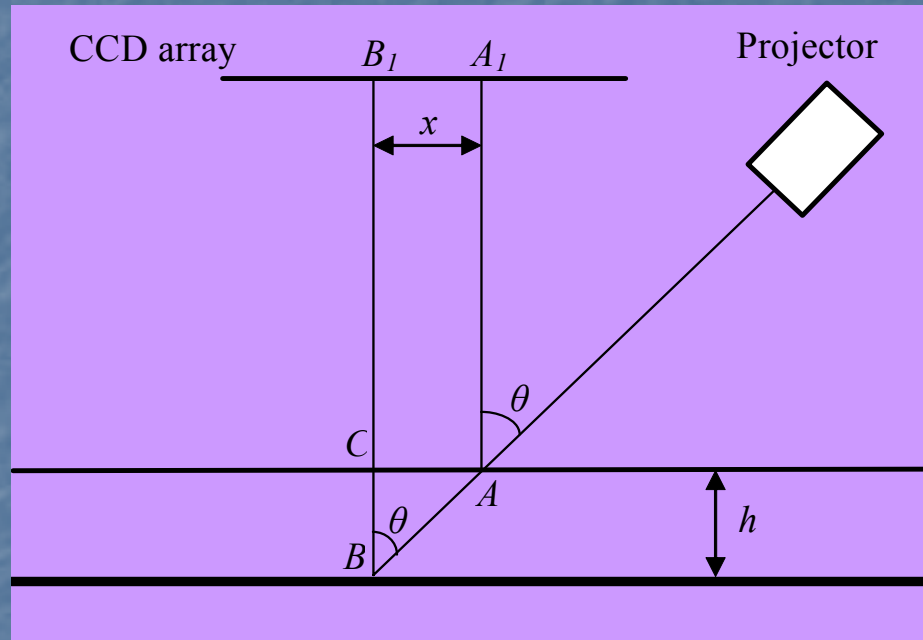
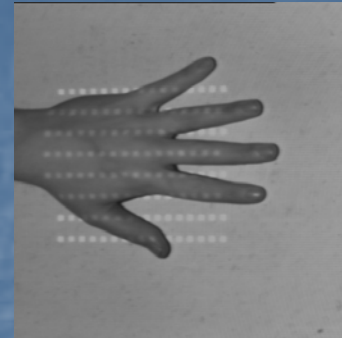


Figure: Optical triangulation geometry

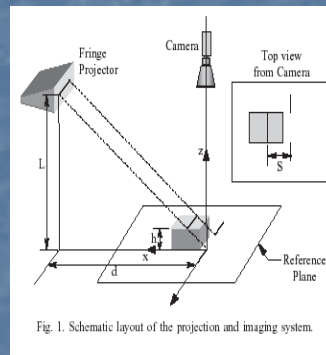
$$h = \frac{x}{\tan(\theta)}$$

3D Shape Measurement cont..

- Point raster – grid of dots

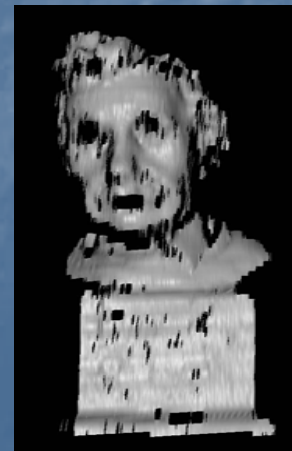
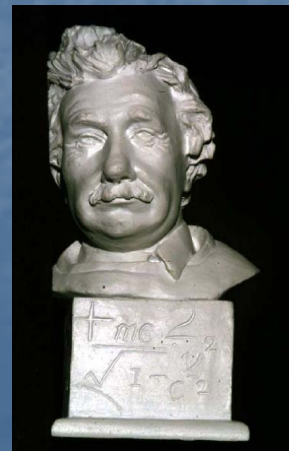
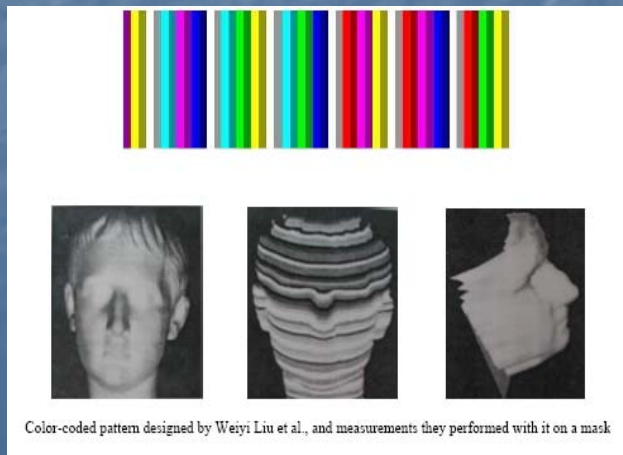


- Line raster

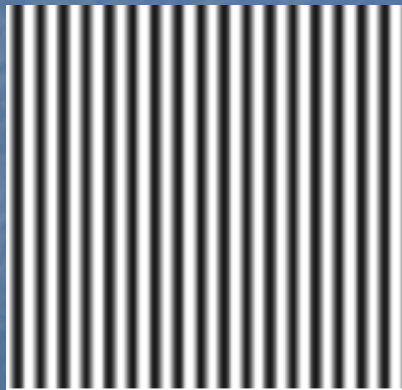
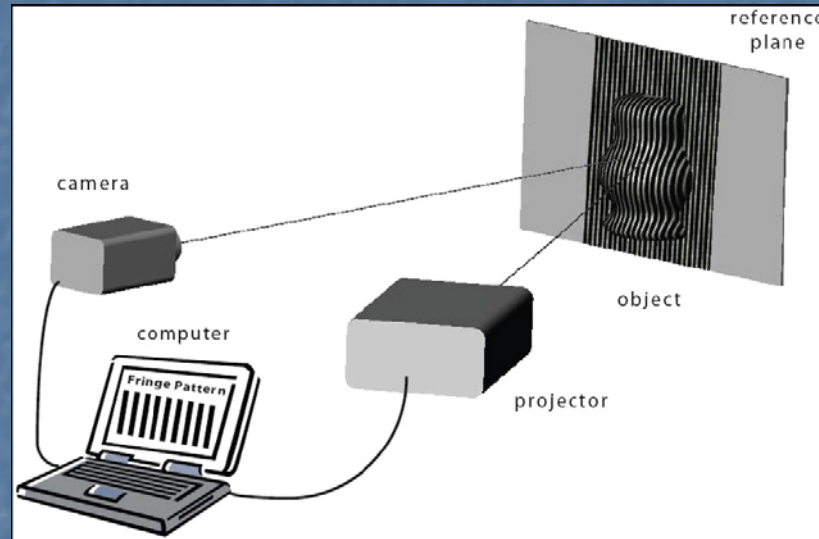


- Color-coded projection

Lie W et al., Applied optics 39 (20), 3504, 2000
Li Zhang et al., 3DPVT'02, 2000



Fringe Projection Technique



$$h(x, y) = \frac{I_0 * \Delta\varphi(x, y)}{\Delta\varphi(x, y) - 2\pi f_0 d}$$

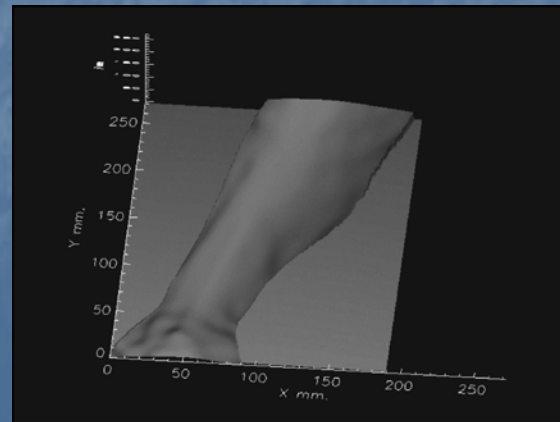
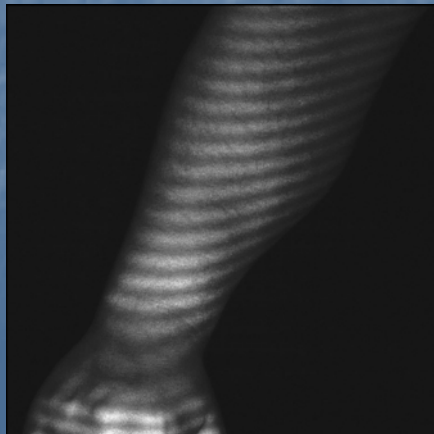
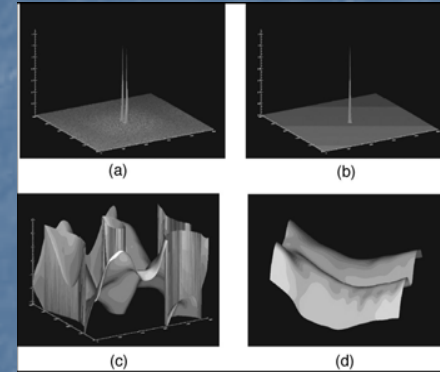
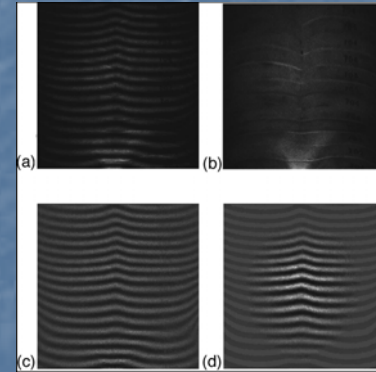
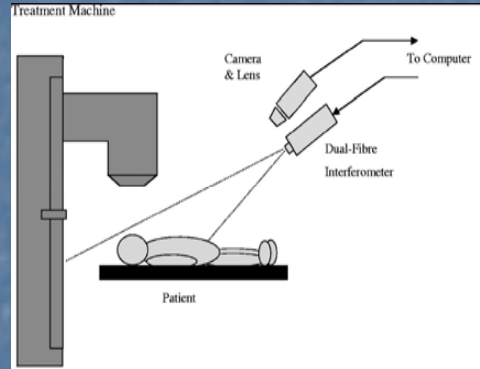
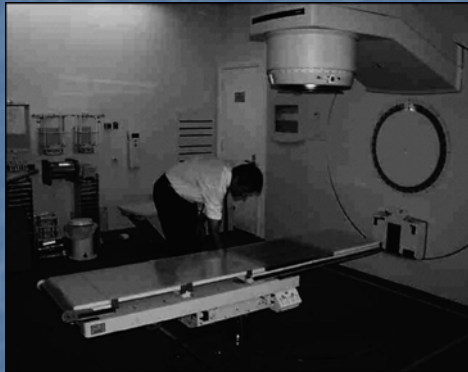


$$g_0(x, y) = a(x, y) + b(x, y) \cos(2\pi f_0 x + \phi_0(x, y))$$

$$g(x, y) = a(x, y) + b(x, y) \cos(2\pi f_0 x + \phi(x, y))$$

Applications of Fringe Projection Technique

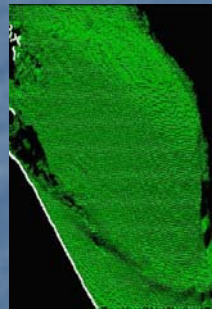
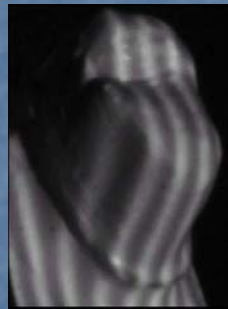
- Biomedical Applications
 - Shape guided radiotherapy treatment



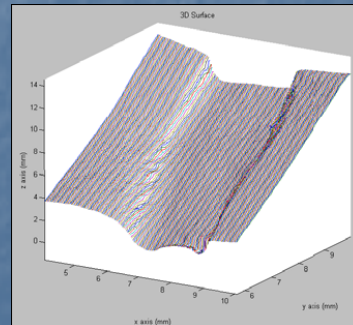
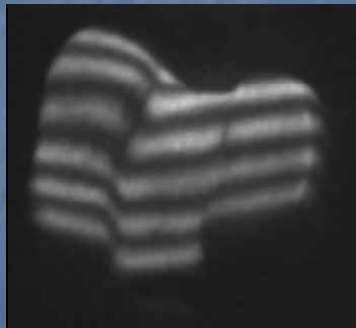
Lilley F et al., optical
Engineering 39(1), 187, 2000

Applications of Fringe Projection Technique cont..

- Biomedical Applications
 - 3D intra-oral dental measurements



Chen L et al., Measurement Science and Technology 16(5), 1061, 2005



Applications of Fringe Projection Technique cont..

- Biomedical Applications cont..
 - Non-invasive 3D monitoring of vascular wall deformations
 - Lower back deformation measurement
 - Detection and monitoring of scoliosis
 - Inspection of wounds
 - Skin topography measurement for use in cosmetology

Applications of Fringe Projection Technique cont..

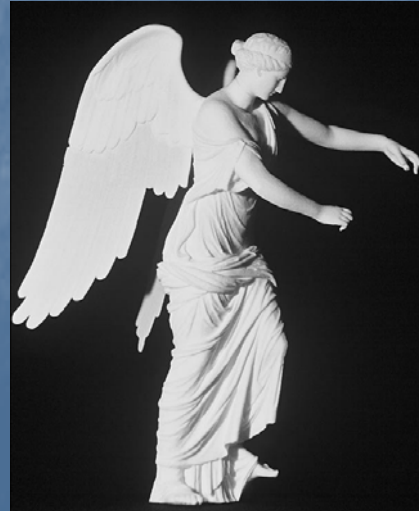
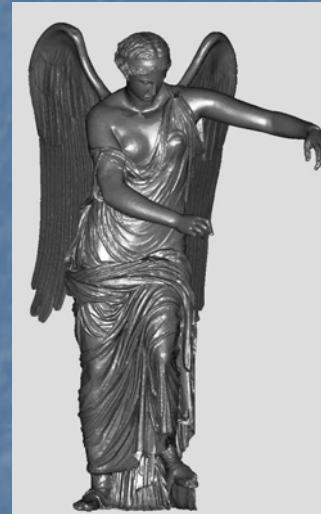
- Cultural heritage and preservation



(a)



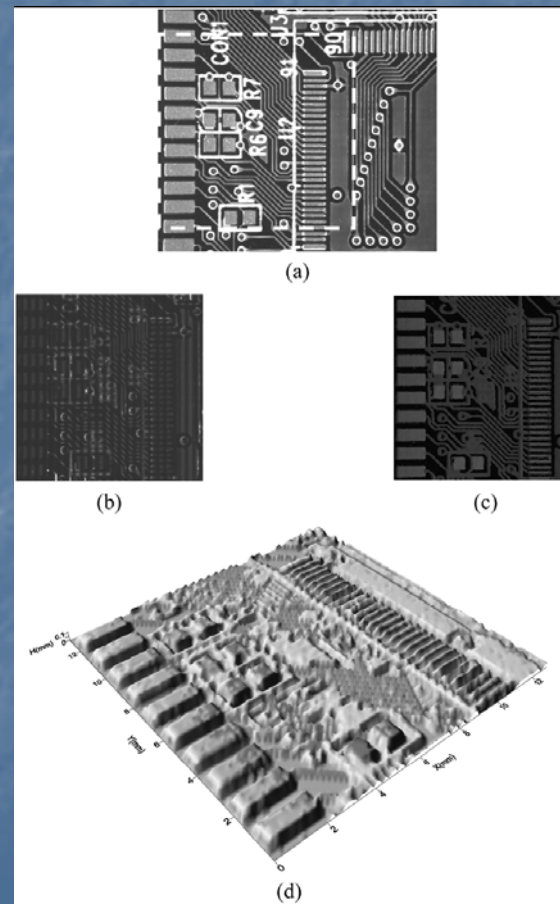
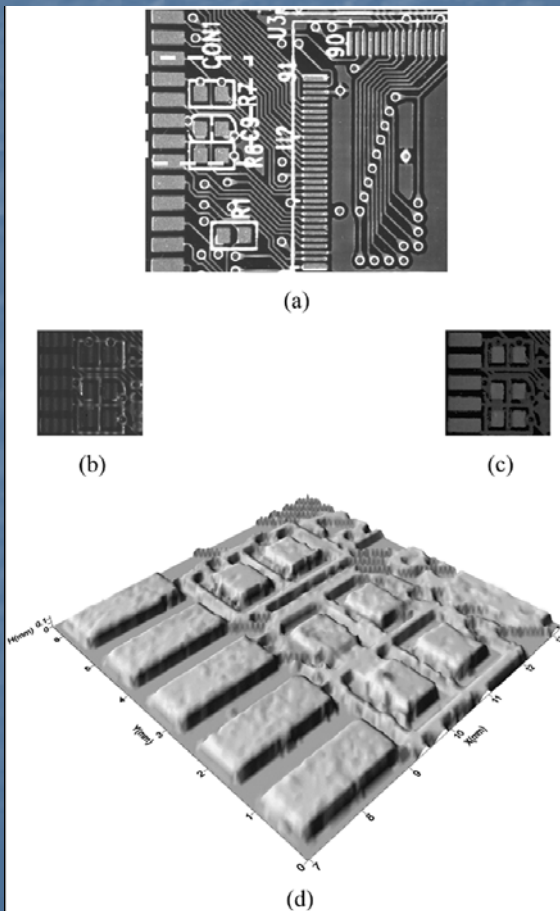
(b)



Sansoni G et al., IEEE Trans.
Instrumentation and
Measurement 54(1), 359,
2005

Applications of Fringe Projection Technique cont..

- Quality control of printed circuit board manufacturing



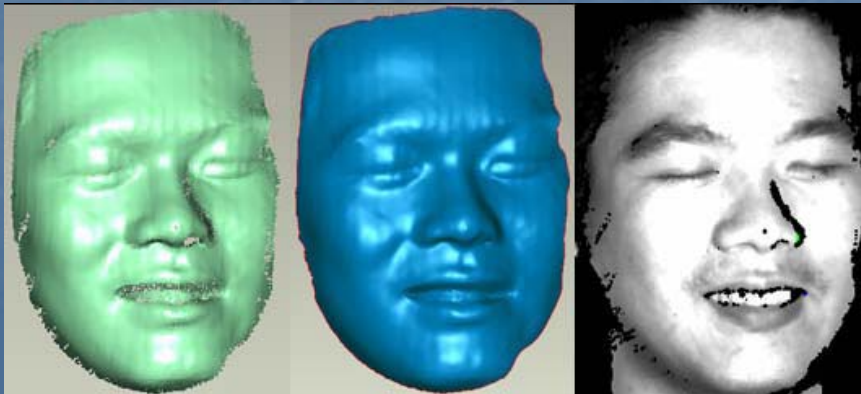
Yen H et al., IEEE Trans.
Electron Packaging Manuf
29(1), 50, 2006

Applications of Fringe Projection Technique cont..

- 3D face reconstruction: Applications in security systems (face recognition systems), gaming, virtual reality etc.

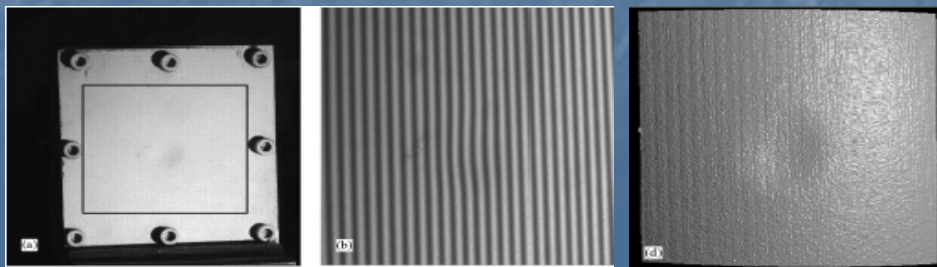
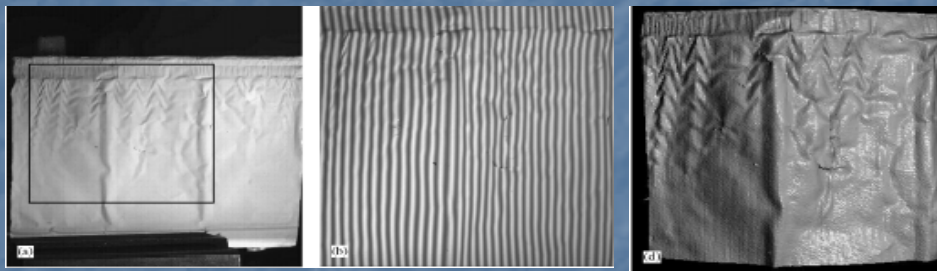
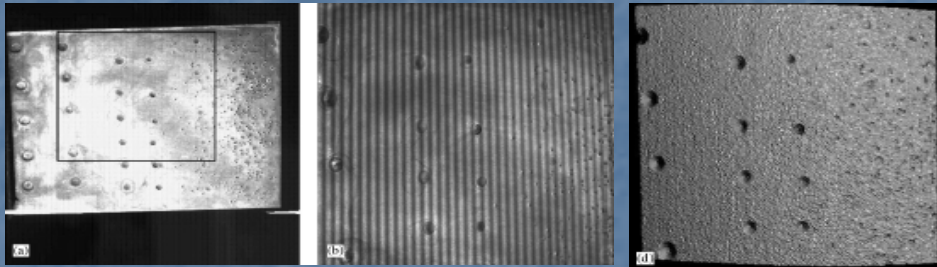


Zhou G et al., Tsinghua Sci
Technol 14, 62, 2009



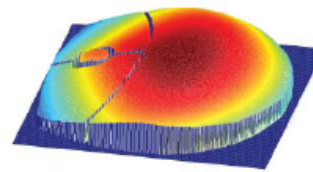
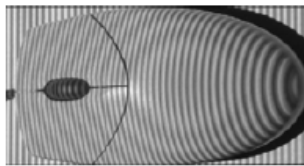
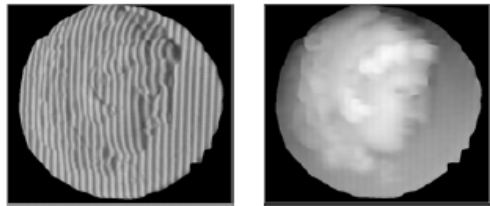
Applications of Fringe Projection Technique cont..

- Industrial and scientific applications
 - Corrosion analysis

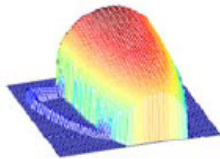
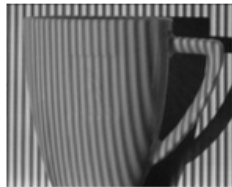


Huang PS et al., Optics and Lasers in Engineering 31(5), 371, 1999

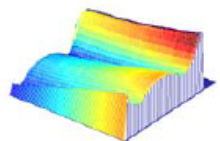
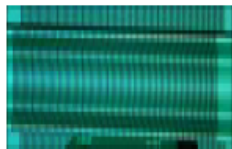
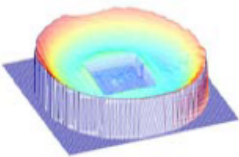
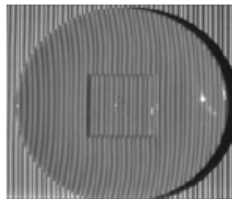
Experimental work at IMAC



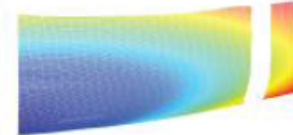
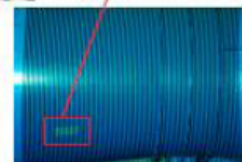
• real texture mapping



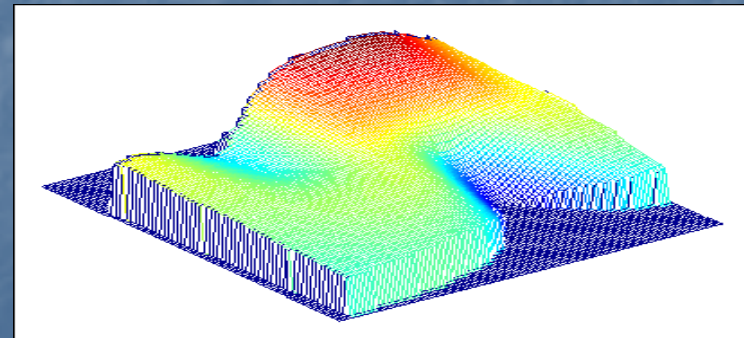
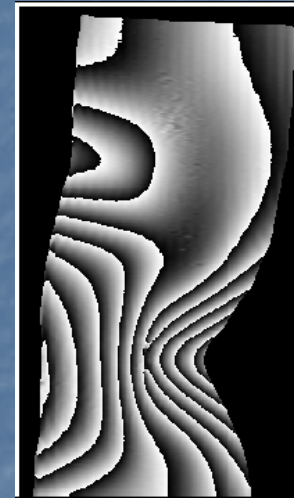
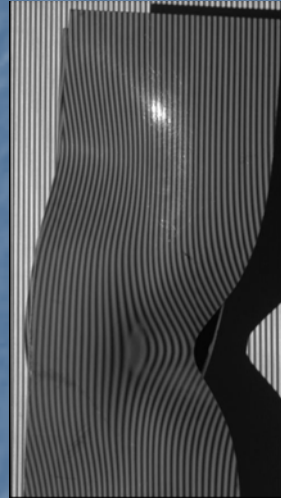
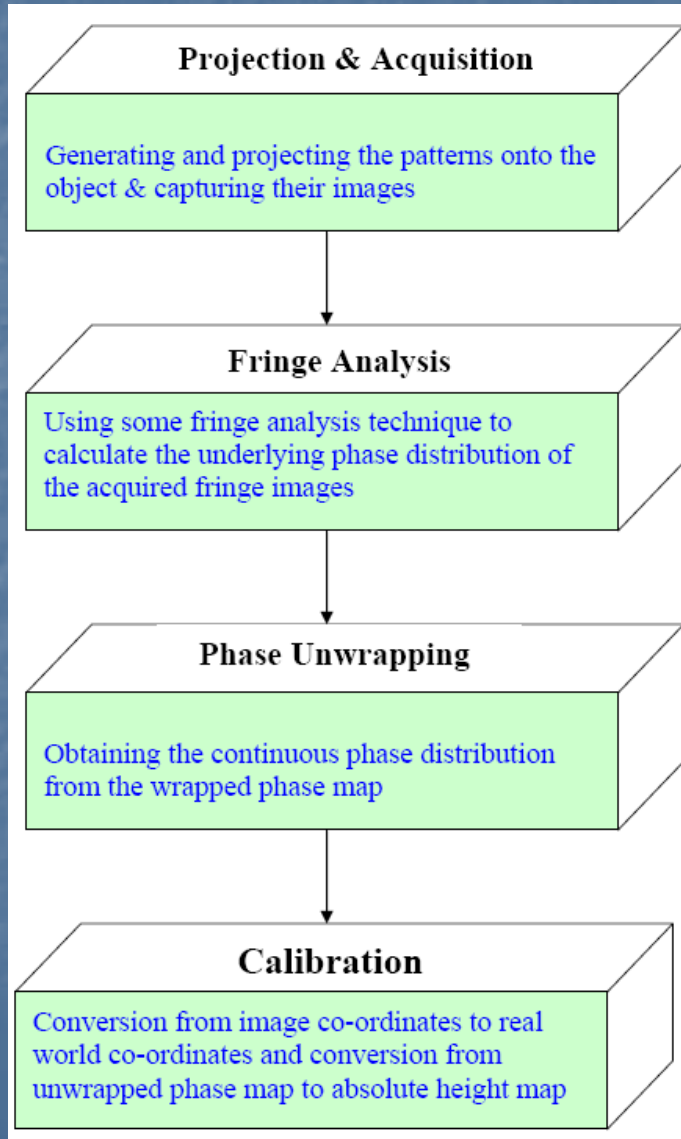
• extracted texture mapping



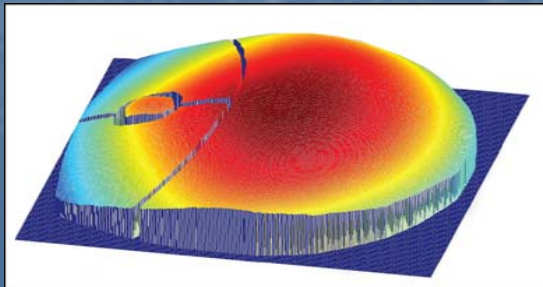
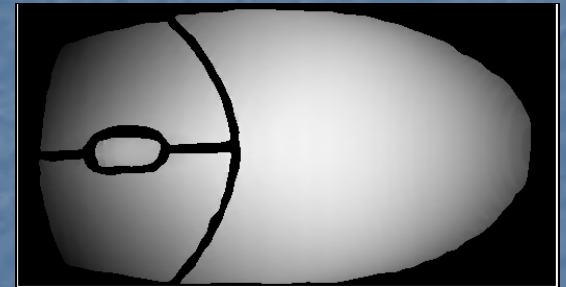
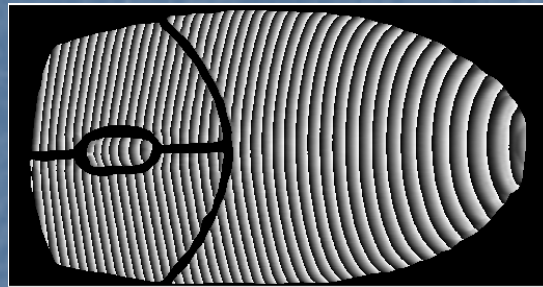
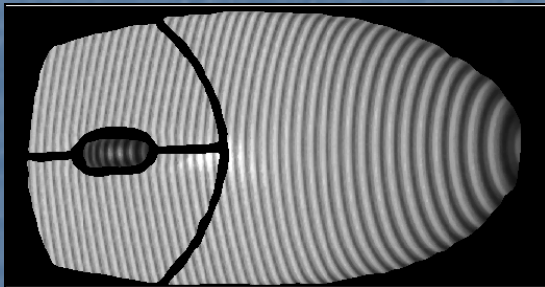
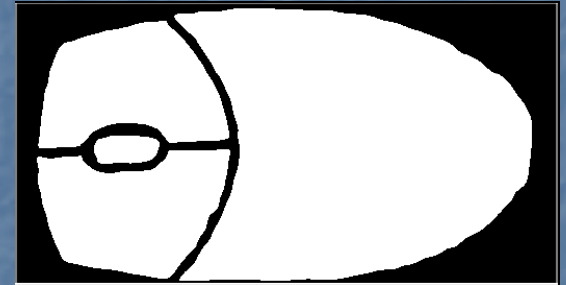
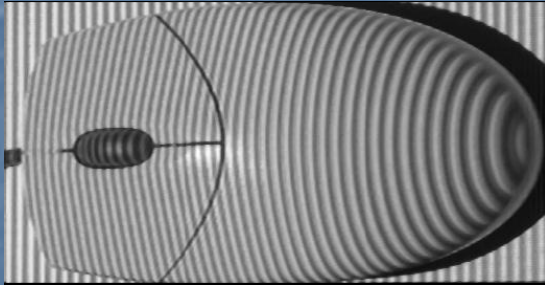
• non-accessible object



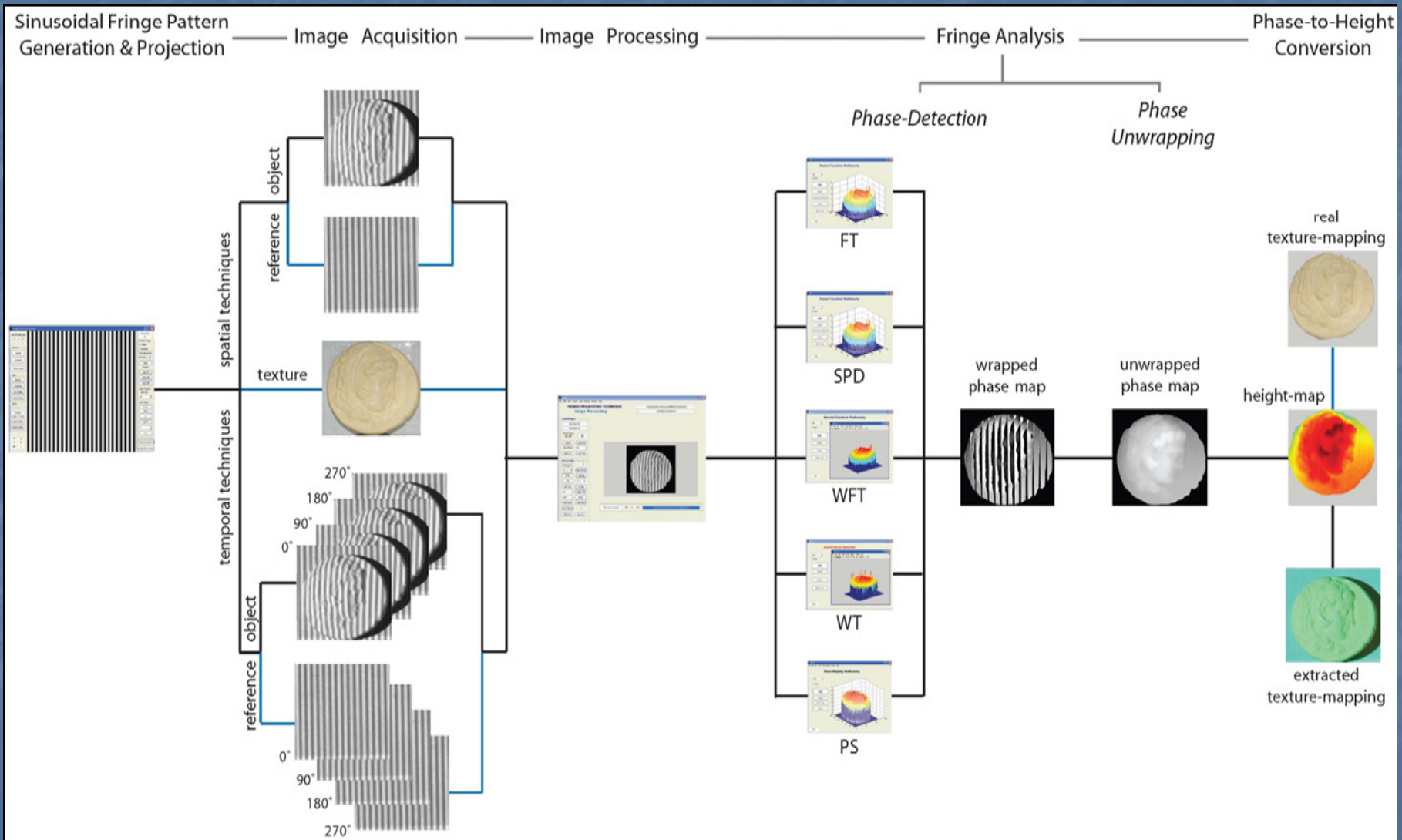
3D shape measurement of objects using fringe projection technique: Overview of measurement methodology



3D shape measurement of objects using fringe projection technique: Overview of measurement methodology



Work-flow in fringe projection profilometry

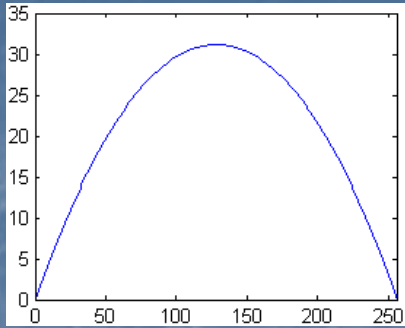


Fringe Analysis

$$g(x, y) = a(x, y) + b(x, y) \cos[2\pi f_0 x + \phi(x, y)]$$

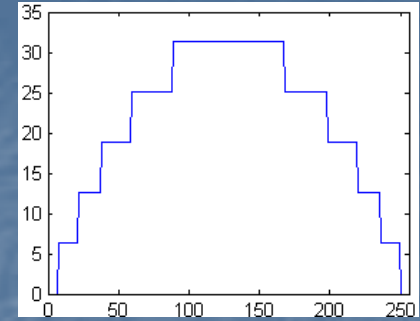
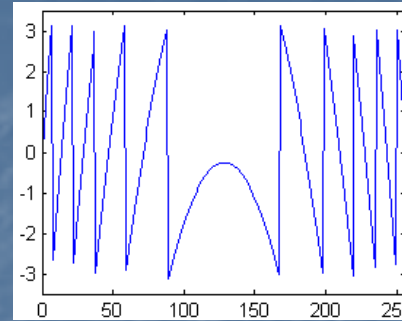
- $a(x, y)$ represents the intensity variations of the background (related to the object's texture)
- $b(x, y)$ represents non-uniform reflectivities of the object surface (fringe modulation term)
- $2\pi f_0 x$ represents the spatial carrier
- $\phi(x, y)$ is the phase term which contains the information of the object's shape
- Fringe analysis methods aim at extracting $\phi(x, y)$ from $g(x, y)$
- Most commonly used fringe analysis methods are:
 - Fourier transform method
 - Wavelet transform method
 - Windowed Fourier transform method
 - Phase shifting method

Phase Unwrapping



$$h(x) = ae^{j\phi(x)}$$

$$\hat{\phi}(x) = \tan^{-1} \left\{ \frac{\text{Im}[h(x)]}{\text{Re}[h(x)]} \right\}$$



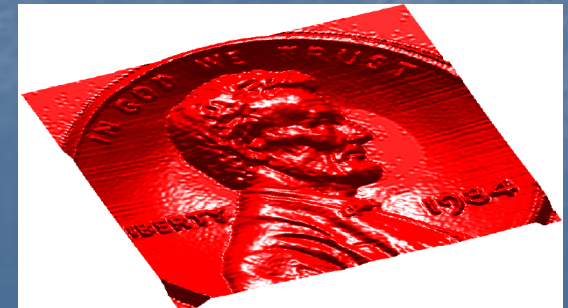
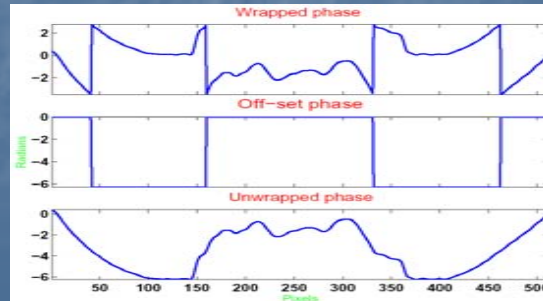
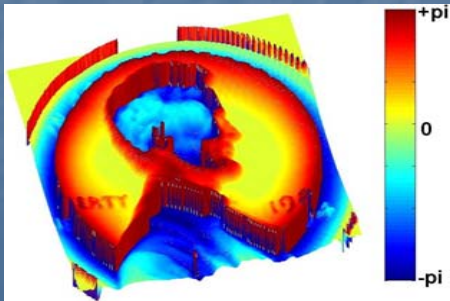
Phase $\phi(x)$

Wrapped Phase $\hat{\phi}(x)$

Offset

The process of determining the unknown integral multiple of 2π to be added at each pixel of the wrapped phase map to make it continuous by removing the artificial 2π discontinuities is referred to as phase unwrapping.

$$\phi_{unwrap}(x, y) = \phi_{wrap}(x, y) + 2\pi n$$



Wrapped phase map

Phase unwrapping

Unwrapped phase

Phase Unwrapping Cont..

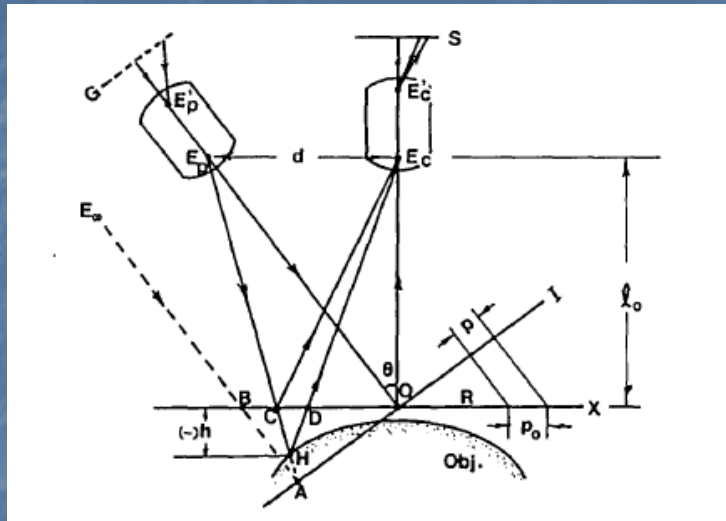
- Phase unwrapping is a trivial task if the wrapped phase map is ideal.
- In practice, the presence of the following makes unwrapping a difficult and path-dependent problem:
 - Shadows
 - Low fringe modulations
 - Non-uniform reflectivities of the object surface
 - Fringe discontinuities
 - Noise etc.
- Some of the most commonly used unwrapping algorithms are:
 - Goldstein's unwrapping algorithm
 - ZpiM unwrapping algorithm
 - Quality guided phase unwrapping algorithm

System Calibration

$$g_T(x,y) = \sum_{n=-\infty}^{\infty} A_n \exp(2\pi i n f_0 x)$$

$$g_0(x,y) = \sum_{n=-\infty}^{\infty} A_n \exp\{2\pi i n f_0 [x + s_0(x)]\}$$

$$g_0(x,y) = \sum_{n=-\infty}^{\infty} A_n \exp\{i[2\pi n f_0 x + n\phi_0(x)]\}$$



Takeda M and Kazuhiro M, Applied optics 22 (24), 3977, 1983

Crossed-optical-axes geometry

$$g(x,y) = r(x,y) \cdot \sum_{n=-\infty}^{\infty} A_n \exp\{2\pi i n f_0 [x + s(x,y)]\}$$

$$g(x,y) = r(x,y) \cdot \sum_{n=-\infty}^{\infty} A_n \exp\{i[2\pi n f_0 x + n\phi(x,y)]\}$$

$$\overline{CD} = -dh(x,y)/[l_0 - h(x,y)]$$

Linear calibration

Non-linear calibration

$$h(x,y) = \frac{l_0 * \Delta\phi(x,y)}{\Delta\phi(x,y) - 2\pi f_0 d}$$

$$h(x,y) = \frac{l_0 \Delta\phi(x,y)}{2\pi f_0 d} = K(x,y) \Delta\phi(x,y)$$

$$h(x,y) = \frac{\Delta\phi(x,y)}{C_1(x,y) + C_2(x,y) \Delta\phi(x,y)}$$