NON-SYMMETRICAL PUNCHING OF FLAT SLABS AND SLAB BRIDGES WITHOUT TRANSVERSE REINFORCEMENT

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Abstract: *Punching of flat slabs without transverse reinforcement has mostly been investigated in the past for slabs with equal reinforcement ratios in the two directions and loaded under axis-symmetrical conditions. However, in practice, slab bridges as well as many flat slabs have different span lengths and reinforcement ratios along the two principal directions. For such cases, where punching shear is typically the governing design criterion for the ultimate state, two major differences with respect to axis-symmetrical slabs are found. Firstly, the shear forces developed in a flat slab with different span lengths might lead to concentrations of shear stresses near the column region, which in turn can reduce the punching shear strength. Secondly, the larger width of the flexural cracks along one direction of the slab with respect to the other direction can influence the punching shear strength, since the capacity to transmit shear forces is reduced as crack width increases.*

In this paper the phenomenon of non-symmetrical punching shear is revised according to the Critical Shear Crack Theory (CSCT) and compared with other design approaches suggested in codes such as EC2, BS8110 and ACI-318. The results of an experimental series of 7 tests (3×3×0.25m) carried out at Ecole Polytechnique Fédérale de Lausanne (EPFL) on non-symmetrical conditions and *with various concrete strengths and reinforcement ratios are presented. A comparison between the experimental results and the different theoretical models is finally introduced. Two approaches are presented with respect the CSCT, in order to estimate the required load-rotation response of the tests; firstly by means of approximate design formulas and secondly with a more refined non-linear finite element analysis using bending shell elements. Both approaches can provide accurate predictions of the ultimate strength and ductility when used in combination with the failure criterion of the Critical Shear Crack Theory.*

1. INTRODUCTION

1.1 Punching shear in slabs with no shear reinforcement

Punching shear is often the governing design criterion in structures such as flat slabs or slab bridges, as shown in Figure 1. Hence, punching shear has been a topic of research for the last 50 years and extensive experimental and analytical work has been performed. Experimental work focused mainly on individual slab elements with equal amount of flexural reinforcement in both orthogonal directions. Furthermore, the type of loading applied in most of the cases corresponded to an axis-symmetrical arrangement, which can represent the region of a flat slab near to a column, see Figure 1.a.

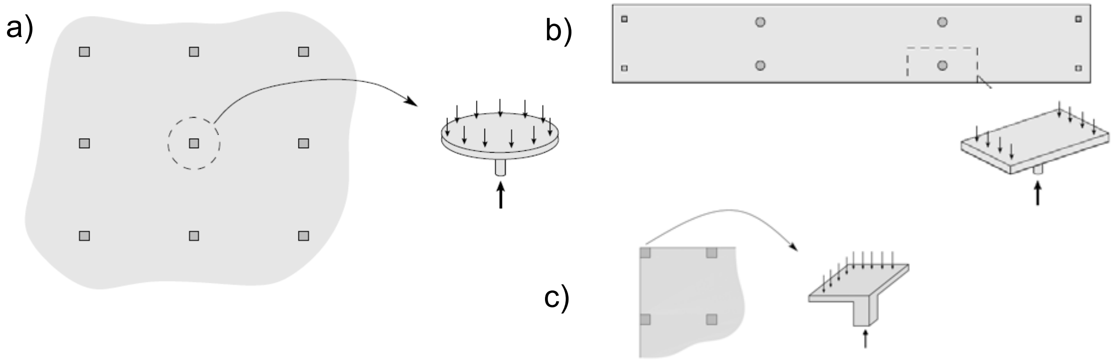


Figure 1: Punching shear in: (a) flat slabs with similar spans in both directions (axis-symmetrical behaviour); (b) slab bridge with non-axis-symmetrical behaviour; and (c) columns located at the perimeter of flat slabs

As presented by Muttoni and Fernández Ruiz[1,2], punching shear can be investigated using the Critical Shear Crack Theory (CSCT). This approach is based on the experimental observation that punching shear strength decreases with increasing the rotation of the slab (ψ), as shown in Figure 2. This is due to the presence of a critical shear crack developing through the theoretical compression strut (Muttoni and Schwartz[3]). According to the CSCT, the crack width (w) of the critical shear crack is assumed to be proportional to the rotation of the slab ψ. The failure criterion is written in terms of the concrete strength, crack width and crack roughness, which is given by the maximum aggregate size (D_{max}), as shown in Figure 2.

In practice, there are many instances of slabs supported on columns with a non-symmetrical behaviour, which can be due to either eccentricity in the loading or/and different amount of flexural reinforcement in both orthogonal directions. Typical examples include already mentioned slab bridges (refer to Figure 1.b) or columns located at the perimeter of flat slabs (Figure 1.c). In such cases, the rotation of the slab and the width of flexural cracks can be considerably larger in one direction compared to the other one.

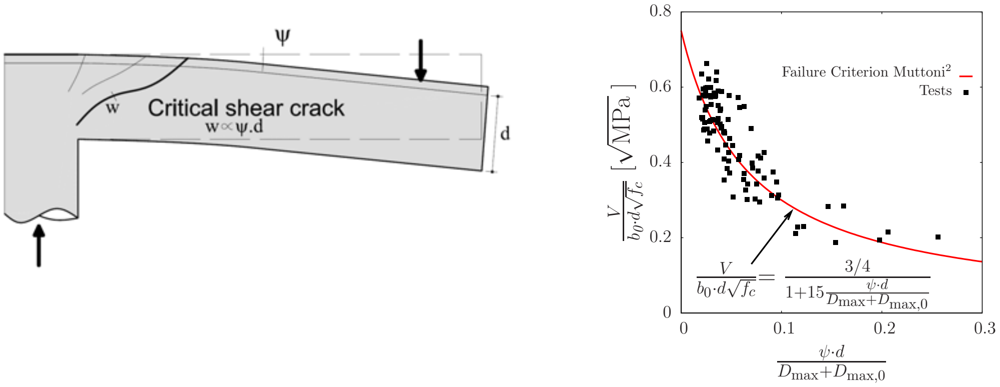


Figure 2: Failure criteria according to CSCT for two-way slabs

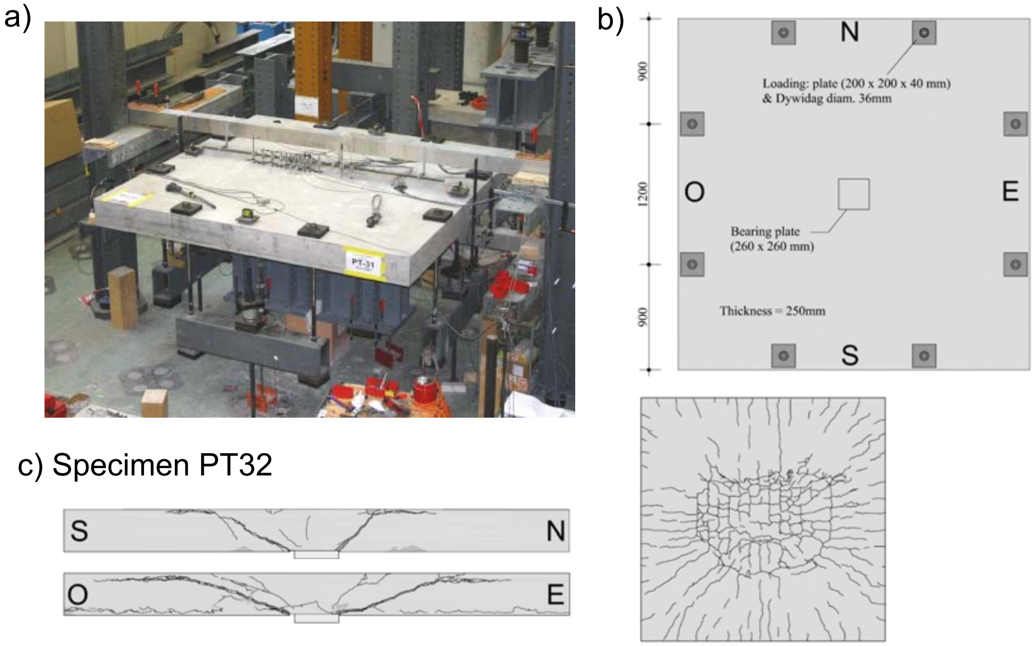
In order to assess the ultimate failure load using the CSCT, the load-rotation response must be estimated by means of approximate analytical solutions or more refined numerical analysis[4,5]. According to the CSCT, the perimeter at which the shear verification must be made corresponds to a distance of $d/2$ from the column face. The CSCT approach, from which SIA262[6] design formulas are grounded, has been shown to provide accurate predictions of the ultimate strength of test data (Muttoni[2]), as shown in Figure 2. Alternatively, punching shear strength in members without shear reinforcement can be estimated by using empirical formulae, as suggested in design codes such as EC2[7] or BS8110[8].

1.2 Eccentricity of loads

In order to account for load eccentricity, EC2 suggests an effective value of the shear stress $v_{eff}=\beta v$ in which β is estimated according to the geometry of the column and the eccentricity of the reaction (e). Similarly, Swiss code SIA262[7] applies a reduction factor k_e for the control perimeter given by simple formula $k_e=1/(1+e/b)$ where b is the radius of a circle with same surface as the column. Overall, both EC2 and BS8110 methods are very similar except for slight differences in the formulas used for $V_{Rd,c}$, control perimeter and parameter β.

2. EXPERIMENTAL PROGRAMME

An experimental programme is being carried out at Ecole Polytechnique Fédérale de Lausanne on punching shear slabs with different loading and reinforcement configurations. In order to investigate rotations and crack development in both orthogonal directions, a series of 7 punching shear tests (3×3×0.25m) supported on a 260mm square column (see Figure 3) were performed by Muttoni et al.[9]. Tests included cases with symmetrical and non-symmetrical reinforcement ratios, which were used to validate design formulas. All tests were loaded symmetrically with 8 point loads as shown in Figure 3, except for specimen PT34, in which only loads at north-south sides were applied.



c) Specimen PT32

Figure 3: Punching shear test with different reinforcement ratios in both orthogonal directions (Muttoni et al.[9]): (a) test arrangement; (b) loading and general dimensions; and (c) crack pattern

Table 1 summarizes the experimental results obtained for the two series tested. The cylinder concrete strength was between 40MPa and 67.5MPa. Rotations were measured in both orthogonal directions by means of vertical LVTDs and inclinometers. An additional inclinometer was placed at the diagonal N-E. The crack pattern observed in the tests (refer to Figure 3.c) clearly showed an asymmetric behaviour with considerably wider cracks in the weaker reinforced direction.

Test	Type	$d_{nominal}$ [mm]	f_c' [MPa]	f_{yx} [MPa]	f_{yy} [MPa]	ρ_x [%]	ρ_y [%]	V_R [kN]
PT21	*a.r*	214	67.5	597	552	1.64	0.84	959
PT22	*sym.*	214	67.0	552	552	0.82	0.82	989
PT23	*a.r*	220	66.0	552	568	0.85	0.36	591
PT31	*sym.*	210	66.3	540	540	1.48	1.48	1433
PT32	*a.r*	214	40.0	540	558	1.46	0.75	1157
PT33	*a.r*	220	40.2	558	533	0.76	0.32	602
PT34	*a.l*	214	47.0	533	533	0.74	0.74	879

Note: *sym* = symmetrical reinforcement & load; *a.r* = asymmetrical reinforcement
a.l = asymmetrical loading

Table 1: Summary of experimental results[8] (y-axis coincides with direction N-S)

3. STRENGTH PREDICTIONS

3.1 Theoretical approaches

In order to estimate the ultimate strength of slabs with $\rho_x f_{yx} \neq \rho_y f_{yy}$ using the CSCT, the failure criterion (Figure 2) is used in combination with the maximum rotation of the slab as shown in Figure 4.a. This approach is conservative but adequate as confirmed by Muttoni et al.[10] for the analysis of slab bridges. In order to estimate the load-rotation response a general FEA can be performed[4,5] or conversely a simple expression, which has been recently implemented in the Swiss code SIA262[6] (see equation 1) can be applied, even for non-axis-symmetric cases. The results obtained using equation (1) are referred to as "CSCT(simplified)" approach.

$$\psi = 1.5\frac{r_s}{d}\cdot\frac{f_y}{E_s}\left(\frac{V}{V_{flex}}\right)^{3/2} \qquad (1)$$

where r_s is related to the flexure failure mechanism and is generally taken as half distance of the yielding line (see Figure 4.b-c); E_s is the Young's modulus of steel. V_{flex} is the shear force when the flexural strength is reached, according to the flexural failure mechanism of the slab (see Figure 4.b-c).

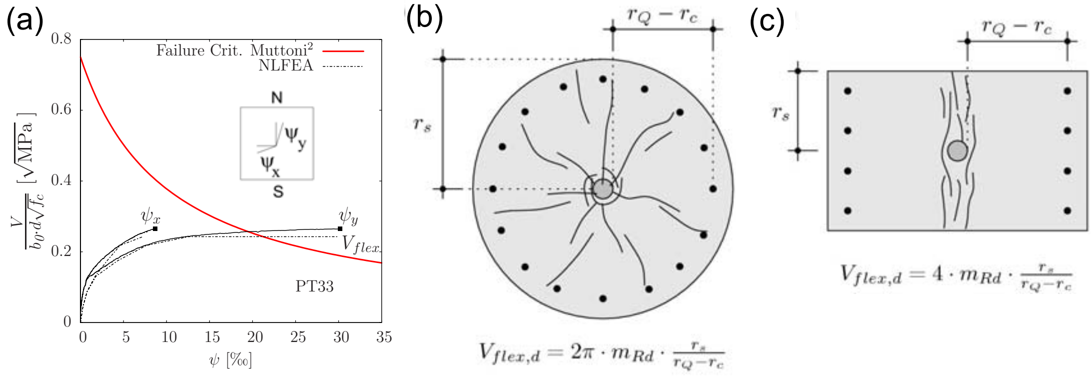


Figure 4: (a) Measured rotations (PT33) in x-y directions; (b) value of r_s for axis-symmetric case; and (c) punching shear in non-axis-symmetric case (slab bridges)

Figures 4.a and 5 show the comparison between the experimental and predicted behaviour from a non-linear finite element analysis (NLFEA), considering shell bending[5] together with the failure criterion shown in Figure 2. These results are denoted as "CSCT(refined)" in Table 2. As it can be observed from Figure 5 and Table 2, accurate predictions of the ultimate strength were obtained for most of the tests by considering the direction of maximum rotations only. However, ductility of specimens with lower reinforcement ratios was slightly underestimated. This underestimation of the ductility was probably due to a significant level of shear stress redistribution after cracking in these specimens.

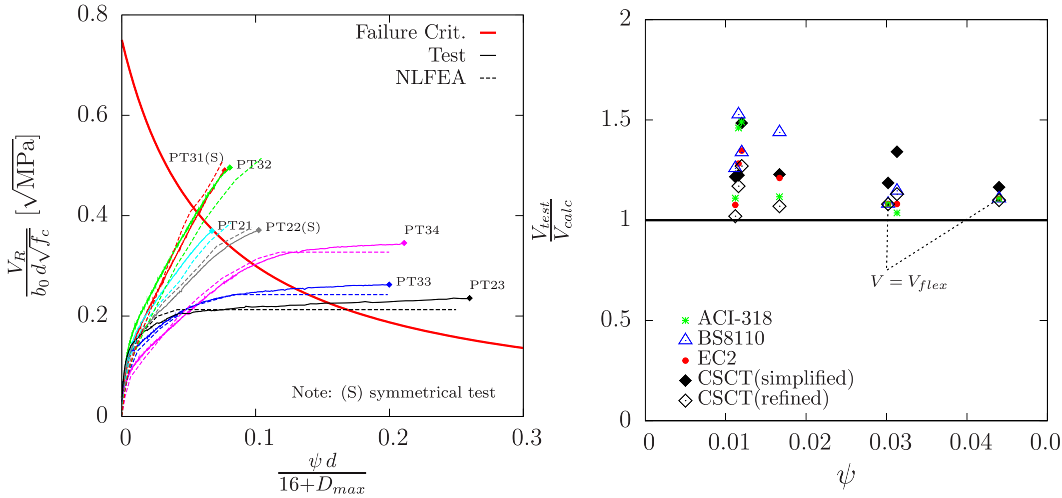


Figure 5: Experiments vs. predictions: (a) load-rotation curves; and (b) V_{test}/V_{calc} ratio using different design codes (ACI318-05, BS8110, EC2, CSCT)

3.2 Comparison to other design codes (EC2, BS8110 or ACI318)

The main difference between the CSCT method and other design formulas is that the former considers the kinematics of the system by means of the slab rotation. As shown in Figure 5.b, the other design methods investigated tend to be less conservative for larger values of ψ, showing the CSCT approaches the lowest dispersion in the predictions. In addition, punching failure after yielding of the main reinforcement can be taken into account by the CSCT thus ductility at failure can be estimated. As shown in Figure 5.b, ACI318-05 and EC2 methods provided a similar performance, although the scatter of ACI-318 was significantly larger. BS8110 predictions were slightly more conservative since $f_{c,cube}$ is limited to 40MPa (a conversion factor of 0.8 was assumed between cylinder and cube strengths). However, this underestimation is partially compensated in design by the relatively lower material factor used for concrete in BS8110 (γ_c=1.25).

V_{test}/V_{calc}	ACI318	BS8110	EC2	CSCT (simplified)	CSCT (refined)
PT21	1.11	1.26	1.08	1.22	1.02
PT22	1.12	1.44	1.21	1.23	1.07
PT23	1.10*	1.10*	1.10*	1.16	1.10*
PT31	1.46	1.53	1.28	1.22	1.17
PT32	1.49	1.34	1.35	1.48	1.27
PT33	1.08*	1.08*	1.08*	1.19	1.08*
PT34	1.04	1.15	1.08	1.34	1.13
Avg.-(COV)	1.20-(15%)	1.27-(13%)	1.17-(9%)	1.26-(9%)	1.12-(7%)

Note: *$V = V_{flex}$

Table 2: Ultimate strength predictions according to different design codes

4. CONCLUSIONS

Experimental and analytical work on punching shear focused in the past on specimens with axis-symmetrical behaviour. However, scarce experimental evidence of tests with different bending reinforcement ratios in both orthogonal directions and non-axis-symmetrical loading is available, although many practical cases correspond to this situation. Current analytical and experimental work is being carrying out at EPFL in order to investigate the load-rotation performance and stress redistribution capacity of punching shear with non-symmetrical arrangements. This paper presents a series of punching shear tests in which the bending reinforcement ratios were different in both orthogonal directions. The tests showed a clear reduction of the punching shear strength in specimens with lower reinforcement ratios due to larger slab rotations.

As shown in this paper, predictions based on the Critical Shear Crack Theory, can be applied to estimate punching shear strength for both symmetrical and non-symmetrical cases. More refined predictions of the load-rotation response can be obtained from a NLFEA with bending shell elements along with the CSCT.

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