CENTER FOR FISCAL POLICY

Optimal Exchange Rate Targeting with Large Labor Unions

by

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May 2009

Center for Fiscal Policy Working Paper Series

Working Paper 01-2009

Optimal Exchange-Rate Targeting with Large Labor Unions*

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Abstract

We study whether monetary policy should target the exchange rate in a two-country model with non-atomistic wage setters, non-traded goods and different degrees of exchange-rate pass through. Commitment to an exchange rate target reduces the labor market distortion. Large labor unions anticipate that higher wages depreciate the exchange rate, which triggers an increase in the interest rate and restrain wage demands. However, reduced exchange rate flexibility worsens the distortion stemming from preset pricing. Targeting the nominal exchange rate will be optimal when the labor market distortion is larger than the preset-pricing one. This result arises with cooperation both under producer and local currency pricing, even though the optimal degree of exchange-rate targeting is higher under local currency pricing. In the Nash equilibrium, the terms-of-trade effect raises optimal wage mark-ups thereby reducing the optimal weight on the exchange rate target. The terms-of-trade effect is stronger as openness and substitutability among Home and Foreign goods increase.

JEL Classification: F3, F41, E52

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*We thank Philippe Bacchetta, Chiara Forlati, Stefano Gnocchi, Stephanie Schmitt-Grohé and seminar participants at the University of Lausanne and at the Workshop on Macroeconomics policies and labour market institutions in Milan for very helpful comments on earlier drafts. Vincenzo Cuciniello thanks the CFI for financial support and for the stimulating research environment. The usual disclaimer applies.

1 Introduction

Inflation targeting is becoming the norm for monetary policy and interest rate rules are simplified descriptions of how central banks operate. The question we ask is whether monetary policy should explicitly target the nominal exchange rate in addition to targeting producer price inflation (PPI) and a measure of resource utilization such as the output gap.

We address this question in an open-economy two-country model with non-traded goods, productivity shocks, preset pricing and different degrees of exchange-rate pass through: producer currency pricing (PCP) and local currency pricing (LCP). Monetary policy is committed and it follows an interest rate rule that allows for exchange-rate targeting. The novelty of our work is to introduce collective wage bargaining through non-atomistic unions, i.e. unions that are large enough to internalize the effect of their own wages on the aggregate wage level.[1]

We find that the optimal interest rate rule may require exchange-rate targeting. On one hand, exchange-rate targeting restrains wages and reduces labor market distortions in our model. On the other hand, it reduces the flexibility of the exchange rate, which is needed in the presence of non-traded goods. If labor market distortions are large – and larger than the distortion stemming from preset pricing – exchange-rate targeting is optimal within the interest-rate rules analyzed here. We also find that the optimal degree of exchange-rate targeting, namely the weight assigned to the exchange rate target in the interest rate rule, is larger under LCP than PCP.

Since targeting the exchange rate implies reducing its flexibility, our paper is closely related to the literature on the optimality of exchange rate regimes. According to this literature, the case for exchange rate flexibility is based on the need to cushion national economies from real idiosyncratic shocks in a world where nominal prices adjust slowly while pass-through is rapid. Intuitively, freely floating exchange rates permit a rapid adjustment of relative prices even though the nominal prices have not changed much. A large body of empirical evidence, however, suggests that the degree of exchange rate pass-through is far from complete in the short run and departures from the law of one price are large and persistent (see, for example, Engel and Rogers, 1996; Goldberg and Knetter, 1997; Goldberg and Campa, 2005).

One reason why complete pass-through and PCP fail to describe reality well is that firms preset prices in the currency of the markets where they sell their goods (e.g. Betts and Devereux, 2000; Devereux and Engel, 2003). In such case, exchange rate flexibility may not be desirable after all. Devereux and Engel (2003) study the optimal degree of exchange rate flexibility under PCP and LCP. They consider a two-country model where all goods are traded internationally, with country-specific productivity shocks, preset prices and a central bank that can commit to a monetary rule. They find that the optimal monetary rule is consistent with fully flexible exchange rates under PCP but with fixed exchange rates

[1] Wage determination in most OECD countries is structured in collective bargaining between employers and trade unions at the plant, firm, industry or aggregate level (see Nickell, Nunziata, and Ochel, 2005; International Labour Office 2008).

under LCP. Under PCP, domestic monetary policy can be used to achieve the flexible price equilibrium for the goods produced at home while foreign monetary policy does the same for foreign-produced goods. The exchange rate must be flexible to ensure that the flexible price equilibrium can emerge in such setting.

Under LCP, on the other hand, prices are set in consumers' currencies so that the CPI is unaffected by the exchange rate – as a matter of fact, the CPI is completely preset. This implies that the terms of trade is constant and it does not influence demand or employment. Another way of understanding this result is to think of the Backus-Smith condition stating that the ratio of the marginal utilities of consumption at home and abroad is equal to the inverse of the real exchange rate in equilibrium. When consumer price indices are constant, the marginal utility ratio is constant when the exchange rate is also constant. Hence, the optimal degree of exchange rate flexibility depends on the degree of exchange rate pass-through.

Duarte and Obstfeld (2008) find that flexible exchange rates are optimal with LCP. Non-traded goods are introduced in the same setting of Devereux and Engel (2003) and this brings back the need for exchange rate flexibility. With non-traded goods, an idiosyncratic technological shock generates divergent consumption movements that need to be supported by exchange rate movements. At the same time, exchange rate movements are necessary to guarantee risk sharing with nominally complete asset markets.

Our paper reconsiders the optimal degree of exchange rate flexibility in the setting of Duarte and Obstfeld (2008) augmented by distortions in the labor market. In our model, the need for exchange rate flexibility stemming from the presence of non-traded goods must be balanced against the need to target exchange rates to reduce labor market distortions.

The literature on optimal monetary policy in New Open Economy Macroeconomics is large. Some of these works focus on environments with PCP and show that a policy pursuing domestic price stability can be optimal or close to the first best.[2] A number of papers have analyzed monetary policy in the presence of imperfect exchange rate pass-through. For example Corsetti and Pesenti (2005) and Sutherland (2005) introduce a variable degree of exchange rate pass-through and demonstrate that optimal monetary rules are no longer inward-looking. We depart from their work by introducing non-atomistic wage setters. Smets and Wouters (2002) and Monacelli (2005) analyze optimal monetary policy with intermediate pass-through, but they focus on a small open economy.

Engel (2009) studies optimal discretionary, cooperative monetary policy in a two-country economy à la Clarida, Galí and Gertler (2002) with LCP. He finds that optimal monetary policy should target currency misalignments. More precisely, the targeting rule for monetary policy involves CPI rather than PPI inflation. Our work shares with Engel (2009) the environment with LCP and price rigidities,[3] while it differs by introducing large wage setters and by focusing on optimal committed interest rate rules. Like Engel, we find that monetary

[2]See Clarida, Galí and Gertler (1999), Galí and Monacelli (2005), Obstfeld and Rogoff (2002) and Benigno and Benigno (2003), to cite a few.

[3]Engel (2009) features sticky prices à la Calvo while we assume price setting is synchronized with prices set one period in advance.

policy should respond to the exchange rate, but the channel is different. In our model, the reason to target the exchange rate is to discipline wage setters and reduce labor market distortions; in Engel, the reason is to eliminate the inefficient allocations stemming from currency misalignment.

Benigno and Benigno (2008) analyze the determinacy and dynamics of the nominal exchange rate under alternative interest rate rules in an environment with PCP. More precisely, they consider three alternative regimes – fixed, floating and managed exchange rate – and design interest rate rules that are consistent with these regimes. They find that the nominal exchange rate displays a unit root behavior under the floating exchange rate regime where the interest rate responds to the inflation rate. Benigno (2004) focuses on a model with LCP to study how interest rate rules for monetary policy affect the persistence of the real exchange rate. Woodford (2007) proposes a simplified version of the model in Benigno (2004), based on the framework in Clarida, Galí and Gertler's (2002), to assess the effect of domestic monetary policy on domestic aggregate demand and domestic inflation under different degree of openness. All these works, do not ask whether monetary policy is optimal.

Our paper is also related to a large literature that considers the role of large unions and their impact on optimal monetary policy.[4] This literature considers simple strategic games between the government and organized labor unions and it is based on one of two alternative assumptions: (i) unions care about inflation, following Cubitt (1992) and Skott (1997); (ii) unions care about employment and real wages, following Soskice and Iversen (1998, 2000) and Bratsiotis and Martin (1999). Moreover, these models are deterministic and/or static.

Gnocchi (2006) studies optimal interest rate rules in a New Keynesian model in the presence of non-atomistic unions. He finds that it is indeed optimal to pursue more aggressive inflation stabilization policies when labor markets are concentrated, as tougher stabilization policies induce more wage restraint and bring the level output closer to its efficient level. We depart from his work in two ways. First, by considering an open economy. Second, by allowing the interest rate rule to be an explicit function of the exchange rate.[5]

The remainder of the paper is organized as follows. Section 2 presents the model and Section 3 solves for the equilibrium with flexible prices. Section 4 solves for the equilibrium with preset prices and Section 5 analyzes optimal monetary policy when prices are preset. Section 6 concludes.

2 The Model

For simplicity, we follow the notation of Duarte and Obstfeld (2008). The economy consists of two ex-ante equally-sized countries, Home (H) and Foreign (F), inhabited by a continuum of agents (with population size normalized to 1) and a finite number of unions. In monopolistic competitive markets, firms produce tradable goods ($H \in [0,1]$ and $F \in [1,2]$) and a

[4] See Calmfors (2001) and Cukierman (2002) for extensive surveys of this literature.

[5] Cuciniello (2008, 2009) focuses on the time-consistent monetary policy in presence of non-atomistic wage setters by introducing an open-economy dimension in Lippi's (2003) framework.

continuum of non-tradable goods indexed by $N \in [0,1]$ in the Home country and $N^* \in [0,1]$ in the Foreign country. Production of the Home (Foreign) goods requires a continuum of differentiated labor inputs indexed by $[0,1]$. Each worker acts as a monopolistic supplier of a particular labor service to both non tradable and tradable sectors.

Under the PCP specification the Home central bank sets the nominal interest rate according to the following rule

$$\iota_t = \bar{\iota} + \psi p_{Ht} + \chi(e_t - \mathrm{E}_{t-1}e_t), \tag{1}$$

where $\iota_t \equiv \log(1+i_t)$ is the log of nominal interest earned between dates t and $t+1$, $p_H \equiv \log P_H$ is the log[6] of Home PPI in domestic currency, e_t is the log exchange rate (Home currency price of Foreign currency) and $\mathrm{E}_{t-1}e_t$ is its expectation based on information at time $t-1$. u is a normally distributed shocks with mean of zero and variance σ_u^2.

Equation (1) is a Taylor-type rule that targets the PPI and the nominal exchange rate. We assume that the weight on the exchange rate target is positive and finite, namely $0 \leq \chi < \infty$. When $\chi = 0$ monetary policy does not target the exchange rate, which is freely floating. As χ increases, the interest rate responds more aggressively to changes in the exchange rate. We refer to this regime as"managed floating," as exchange rate flexibility is reduced in this case. We allow for χ to become arbitrarily large and therefore for exchange rate to approach a fixed exchange rate regime, but do not consider the case where χ is infinite. In fact, the interest rate rules (1) and (2) do not constitute a fixed exchange rate regime as $\chi \to \infty$.

The Foreign central bank has similar rule,

$$\iota_t^* = \bar{\iota} + \psi^* p_{Ft}^* - \chi^*(e_t - \mathrm{E}_{t-1}e_t), \tag{2}$$

where $\iota_t^* \equiv \log(1+i_t^*)$ and p_F^* expresses Foreign-currency PPI.

Under the LCP specification the interest rate rules are identical to (1) and (2), except they target CPI rather than PPI:

$$\iota_t = \bar{\iota} + \psi p_t + \chi(e_t - \mathrm{E}_{t-1}e_t) \qquad \iota_t^* = \bar{\iota} + \psi^* p_t^* - \chi^*(e_t - \mathrm{E}_{t-1}e_t), \tag{3}$$

where p and p^* respectively express Home- and Foreign-currency CPI, respectively.

2.1 Households

Preferences of the representative Home agent $z \in [0,1]$ are defined over consumption C and labor supplied L:

$$U_0(z) = \mathrm{E}_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[\frac{(C_t(z))^{1-\rho}}{1-\rho} - kL_t(z) \right] \right\}, \tag{4}$$

where $\rho > 0$ and $\beta \in (0,1)$. For any person z the overall consumption index C is a Cobb-Douglas aggregate of the tradable and non-tradable composite goods, given by

$$C = \frac{C_T^{\gamma} C_N^{1-\gamma}}{\gamma^{\gamma}(1-\gamma)^{1-\gamma}},$$

[6]Henceforth lower-case letters, except for interest rates, will denote natural logarithms.

where the tradable goods subindex C_T is given by

$$C_T = 2C_H^{\frac{1}{2}} C_F^{\frac{1}{2}}, \tag{5}$$

and C_H, C_F and C_N are CES aggregators of the available varieties,

$$C_j = \left[\int_0^1 C_j(i)^{\frac{\theta-1}{\theta}} \mathrm{d}i \right]^{\frac{\theta}{\theta-1}} \qquad \theta > 1, \quad j \in [H, F, N].^7$$

The consumption-based price index expressed in domestic currency is defined as

$$P = P_T^{\gamma} P_N^{1-\gamma}, \tag{6}$$

with

$$P_T = P_H^{\frac{1}{2}} P_F^{\frac{1}{2}}$$

and

$$P_j = \begin{cases} \left[\int_0^1 P_j(i)^{1-\theta} \mathrm{d}i \right]^{\frac{1}{1-\theta}}, & j = H, N \\ \left[\int_1^2 P_j(i)^{1-\theta} \mathrm{d}i \right]^{\frac{1}{1-\theta}}, & j = F. \end{cases}$$

Each z-th individual trades state-contingent nominal bonds denominated in the Home currency. We denote the price at date t when the state of the world is s_t of a bond paying one unit of Home currency at date $t+1$ if the state of the world is s_{t+1} by $Q_{s_{t+1}|s_t}$. The quantity of these bonds purchased by the Home agent z at date t is $B_{s_{t+1}}$, while revenues received at date t when the state of the world is s_t are denoted by B_{s_t}. Firm's profits are entirely redistributed as dividends among domestic agents.

A typical Home agent z faces the following budget constraint in nominal terms

$$\sum_{s_{t+1}} Q_{s_{t+1}|s_t} B_{s_{t+1}}(z) + P_t C_t(z) = B_{s_t}(z) + P_t T_t(z) + W_t(z) L_t(z) + \int_0^1 [\Pi_{H,t}(i,z) + \Pi_{N,t}(i,z)] \mathrm{d}i, \tag{7}$$

where

$$L(z) \equiv \int_0^1 [L_H(i,z) + L_N(i,z)] \mathrm{d}i,$$

$\Pi_j(i)$ denotes nominal dividends received from firm i operating in sector $j \in [H, N]$ and $T(z)$ are per capita lump-sum real transfers from the Home government. Individuals take firm behavior and lump-sum transfers as given. Foreign individuals face isomorphic budget constraints (with * denoting Foreign variables).

The optimal intra-temporal allocation of consumption of Home and Foreign tradable varieties and the non-traded varieties yields the following demands for the i-th domestic

[7] For $j = F$, the integration is over the interval [1,2].

firm:

$$C_H(i) = \frac{\gamma}{2}\left(\frac{P_H(i)}{P_H}\right)^{-\theta}\left(\frac{P_H}{P_T}\right)^{-1}\left(\frac{P_T}{P}\right)^{-1}C, \tag{8}$$

$$C_H^*(i) = \frac{\gamma}{2}\left(\frac{P_H^*(i)}{P_H^*}\right)^{-\theta}\left(\frac{P_H^*}{P_T^*}\right)^{-1}\left(\frac{P_T^*}{P^*}\right)^{-1}C^*, \tag{9}$$

$$C_N(i) = (1-\gamma)\left(\frac{P_N(i)}{P_N}\right)^{-\theta}\left(\frac{P_N}{P}\right)^{-1}C. \tag{10}$$

The inter-temporal Euler equation is given by

$$\frac{C_t^{-\rho}}{P_t} = (1+i_t)\beta \mathrm{E}_t\left\{\frac{C_{t+1}^{-\rho}}{P_{t+1}}\right\} \tag{11}$$

in the Home country. A symmetric equation holds in the Foreign country.

Since markets are complete in claims on future money payments, the ex-post allocation satisfies the Backus and Smith's (1993) condition

$$\frac{C_t^{-\rho}}{P_t} = \frac{(C_t^*)^{-\rho}}{\mathcal{E}_t P_t^*}, \tag{12}$$

where $\mathcal{E}$ is the exchange rate (domestic price of foreign currency). Note the purchasing-power-parity condition need not hold in the model ex post because of non-traded goods. As a result, the marginal utilities of consumption are not necessarily equated among countries.

2.2 Firms

Let $Y_j(i)$ denote the level of output produced by the monopolistically competitive firm i and supplied to the Home tradable market ($j = H$) or to the Home non-tradable market ($j = N$). Technology is described by the following production functions:

$$Y_{j,t}(i) = A_t L_{j,t}(i) \qquad j \in [N, H], \tag{13}$$

where A is an economy-wide productivity shock whose stochastic process in log-terms is given by:

$$a_t = \lambda a_{t-1} + u_t, \tag{14}$$

with $\lambda \in [0,1]$, and $L_j(i)$ is a labor index defined as a Dixit-Stiglitz aggregate of the quantities hired of each differentiated labor type z:

$$L_{j,t}(i) = \left[\int_0^1 L_{j,t}(i,z)^{\frac{\sigma-1}{\sigma}}\mathrm{d}z\right]^{\frac{\sigma}{\sigma-1}} \qquad j \in [N, H], \quad \sigma > 1.^8 \tag{15}$$

[8] A symmetric production function holds in the Foreign country with the productivity shock $a_t^* = \lambda a_{t-1}^* + u_t^*$.

For a given level of production, demands of labor type z by producer i solve the dual problem of minimizing total cost, $\int_0^1 W(z)L(i,z)\mathrm{d}z$, subject to the employment index (15):

$$L_{j,t}(i,z) = \left[\frac{W_t(z)}{W_t}\right]^{-\sigma} L_{j,t}(i), \tag{16}$$

where $W(z)$ denotes the nominal wage of labor type z and W is the nominal wage index defined as

$$W_t = \left[\int_0^1 W_t(z)^{1-\sigma}\mathrm{d}z\right]^{\frac{1}{1-\sigma}}. \tag{17}$$

Firms set nominal prices one period in advance. As in Devereux and Engel (2003), we allow two different price-setting specifications. Under the PCP specification, Home firms producing traded goods choose a single price in domestic currency so as to maximize the state-contingent value of profits:

$$\max_{P_{Ht}(i)} \mathrm{E}_{t-1}\left[d_t\left((P_{Ht}(i) - W_t/A_t)(C_{Ht}(i) + C^*_{Ht}(i))\right)\right], \tag{18}$$

subject to the demand functions (8) and (9). In equilibrium, the pricing kernel d_t is given by $\beta\frac{C_t^{-\rho}P_{t-1}}{C_{t-1}^{-\rho}P_t}$.

Under the LCP specification, firms can choose two different prices: one to charge Home residents, and one to charge Foreign residents. Thus, the Home producer of $Y^*_H(i)$, $Y_H(i)$ sells the former to the Foreign market at the Foreign-currency price $P^*_H(i)$ and the latter to the Home market at the Home-currency price $P_H(i)$. Specifically, Home firms producing traded goods solve the following problem:

$$\max_{P_{Ht}(i),P^*_{Ht}(i)} \mathrm{E}_{t-1}\left[d_t\left(P_{Ht}(i)C_{Ht}(i) + \mathcal{E}_t P^*_{Ht}(i)C^*_{Ht}(i) - \frac{W_t}{A_t}(C_{Ht}(i) + C^*_{Ht}(i))\right)\right], \tag{19}$$

subject to the demand functions (8) and (9).

The Home firms producing non-traded goods choose a single price in domestic currency and their maximization problem is similar to (18) both under PCP and LCP.

Consider first domestic price setting. Under either specification, the optimal price set by Home firms selling in the Home market is given by

$$P^{PCP}_{Ht}(i) = P^{PCP}_{Ht} = P^{LCP}_{Ht} = P_{Nt} = \frac{\theta}{\theta-1}\frac{\mathrm{E}_{t-1}\left[C_t^{1-\rho}\frac{W_t}{A_t}\right]}{\mathrm{E}_{t-1}\left[C_t^{1-\rho}\right]}. \tag{20}$$

Conversely, the optimal price set in the Foreign market is different under the two specifications. With PCP the law of one price holds and the price set by Home firms is

$$P^{*PCP}_{Ht}(i) = P^{*PCP}_{Ht} = P^{PCP}_{Ht}/\mathcal{E}_t. \tag{21}$$

With LCP the solution to (19) is given by

$$P_{Ht}^{*LCP}(i) = P_{Ht}^{*LCP} = \frac{\theta}{\theta-1}\frac{\mathrm{E}_{t-1}\left[C_t^{*1-\rho}\frac{W_t}{A_t\mathcal{E}_t}\right]}{\mathrm{E}_{t-1}\left[C_t^{*1-\rho}\right]}. \tag{22}$$

Similarly, Foreign firms selling in the Foreign market choose

$$P_{Ft}^{*PCP}(i^*) = P_{Ft}^{*PCP} = P_{Ft}^{*LCP} = P_{Nt}^{*} = \frac{\theta}{\theta-1}\frac{\mathrm{E}_{t-1}\left[C_t^{*1-\rho}\frac{W_t^*}{A_t^*}\right]}{\mathrm{E}_{t-1}\left[C_t^{*1-\rho}\right]}, \tag{23}$$

while Foreign firms selling in the Home market choose

$$P_{Ft}^{PCP}(i^*) = P_{Ft}^{PCP} = \mathcal{E}_t P_{Ft}^{*PCP} \tag{24}$$

under the PCP specification, and

$$P_{Ft}^{LCP}(i^*) = P_{Ft}^{LCP} = \frac{\theta}{\theta-1}\frac{\mathrm{E}_{t-1}\left[\mathcal{E}_t C_t^{1-\rho}\frac{W_t^*}{A_t^*}\right]}{\mathrm{E}_{t-1}\left[C_t^{1-\rho}\right]}, \tag{25}$$

under the LCP specification.

3 The Equilibrium with Flexible Prices

In this section we consider the flexible-price case. We assume that Home and Foreign workers are organized in $n = n^* > 1$ labor unions. All labor types are unionized and equally distributed among unions. Therefore, each union x has mass $1/n = \int_{z\in x} \mathrm{d}z$ and each union's ability to internalize the consequences of its own actions is proportional to union size: the larger the union, the more it internalizes the impact of its wage settlement on aggregate wage. In choosing the nominal wage $W(x)$, union x anticipates that (see Appendix A)

$$\frac{\partial W}{\partial W(x)} = \frac{1}{n}\left(\frac{W(x)}{W}\right)^{-\sigma}, \tag{26}$$

taking other unions' wages both at Home and abroad as given.

When price are flexible, (20), (23), (22), and (25) simplify to

$$P_{Ht} = P_{Ht}^*\mathcal{E}_t = P_{Nt} = \frac{\theta}{\theta-1}\frac{W_t}{A_t}, \tag{27}$$

$$P_{Ft}^* = \frac{P_{Ft}}{\mathcal{E}_t} = P_{Nt}^* = \frac{\theta}{\theta-1}\frac{W_t^*}{A_t^*}, \tag{28}$$

and the law of one price holds for traded goods. Combining (27) and (28) yields an expression for the exchange rate

$$\mathcal{E}_t = W_t/W_t^*. \tag{29}$$

As long as n is finite, an increase in union's wage is perceived to affect aggregate wage that, in turn, affects the nominal exchange rate. In a symmetric equilibrium ($W(x) = W$), the elasticity of domestic demand for labor type x and the elasticity of CPI to wage x are respectively given by (see Appendix B)

$$Z_l^{FL} \equiv -\frac{\partial \log L(x)}{\partial \log W(x)} = \sigma\left(1-\frac{1}{n}\right) + Z_L^{FL}\frac{1}{n}, \quad \text{where} \quad Z_L^{FL} \equiv -\frac{\partial \log L}{\partial \log W} = \frac{1+\psi+\chi}{\rho}, \tag{30}$$

$$Z_p^{FL} \equiv \frac{\partial \log P}{\partial \log W(x)} = \frac{1}{n}, \tag{31}$$

from which we derive the labor-type elasticity to real wage

$$\delta^{FL} \equiv -\frac{\partial \log L(z)}{\partial \log(W(x)/P)} = \sigma + \frac{1+\psi+\chi}{\left(1-\frac{1}{n}\right)\rho\, n} > 1. \tag{32}$$

With atomistic wage setters ($n \to \infty$), only the elasticity of substitution between labor types σ matters because unions do not internalize the impact on CPI and on aggregate employment of their wage demands – $Z_p^{FL} \to 0$ and $Z_L^{FL}/n \to 0$ as $n \to \infty$. It is worth noticing that Z_L^{FL} is increasing in χ and ψ, the two parameters governing the interest rate response to exchange rate and price level, respectively. A non-atomistic union anticipates that its wage claims will trigger a higher interest rate, thereby reducing current consumption and aggregate employment. The larger the weights χ and ψ, the stronger such effect on employment.

Each union aims at maximizing the utility (4) of its members[9] subject to the budget constraint (7) and labor demand (16). By symmetry in labor markets, the solution to unions' problem implies the following condition (Appendix C)

$$\frac{W_t}{P_t} = k\frac{\delta^{FL}}{\delta^{FL}-1}\, C_t^{\rho}. \tag{33}$$

Substituting the price level in the Home country (B.1) into the expression above, consumption under flexible prices is given by:

$$C_t = \left[\left(\frac{\theta-1}{\theta k}\frac{\delta^{FL}-1}{\delta^{FL}}\right) A_t^{1-\frac{\gamma}{2}} (A_t^*)^{\frac{\gamma}{2}}\right]^{\frac{1}{\rho}}. \tag{34}$$

4 The Equilibrium with Predetermined Prices

In Appendix B.2 we show that the elasticity of labor demand and CPI to Home wage under preset pricing are given by:

$$Z_L^{PR} \equiv -\frac{\partial \log L_t}{\partial \log W_t} = \frac{Z_E^{PR}(2\chi + \gamma(1+\eta(1+\gamma(\rho-1)-2\rho)))}{2\rho}, \tag{35}$$

[9] The benevolent union hypothesis is consistent with the traditional labor union theory (see e.g. Oswald, 1985).

$$Z_p^{PR} \equiv \frac{\partial \log P_t}{\partial \log W_t(x)} = \frac{\gamma\eta}{2n} Z_E^{PR}, \tag{36}$$

where Z_E^{PR} is the elasticity of the exchange rate to the domestic nominal wage, which is equation (B.13) in Appendix B.2, and $\eta \in \{0, 1\}$ is the degree of exchange rate pass-through. Specifically, if $\eta = 1$, exchange rate pass-through is complete and we are in the standard case of PCP. If $\eta = 0$, exports are invoiced in the importer's currency and exchange rate pass-through is nil. This is LCP.

Aggregate labor demand elasticity Z_L^{PR} is increasing in χ, the weight on the exchange rate target in the interest rate rule. The more aggressive the interest rate response to a depreciation, the greater is aggregate labor demand elasticity. Non-atomistic unions know that aggregate employment depends on aggregate demand, which in turn depends on consumption and relative prices. They anticipate that an increase in their wage claims trigger an increase in the interest rate in response to an increase in prices (this is the closed-economy channel) *and* in response to a depreciation (this is the open-economy channel). But a higher interest rate reduces consumption and aggregate labor demand. Appendix B presents the mathematical derivations.

This mechanism is captured by the elasticity of demand for labor type x

$$Z_l^{PR} \equiv -\frac{\partial \log L(x)}{\partial \log W(x)} = \sigma\left(1 - \frac{1}{n}\right) + Z_L^{PR}\frac{1}{n}. \tag{37}$$

When $n \to \infty$ and wage setters are atomistic, unions do not internalize the impact of their wage demands on aggregate wage and Z_l^{PR} reduces to the elasticity of substitution between labor types, σ. When wage setters are non-atomistic, the aggregate labor demand elasticity Z_L^{PR} matters with a weight that is increasing in union's size. With a single all-encompassing union ($n = 1$), the labor demand elasticity coincides with the aggregate one.

Assuming symmetry in labor markets, the first order condition under preset prices is[10]

$$\frac{W_t}{P_t} = k\frac{\delta^{PR}}{\delta^{PR} - 1} C_t^{\rho}, \tag{38}$$

where

$$\begin{aligned}
\delta^{PR} &\equiv -\frac{\partial \log L_t(x)}{\partial \log(W_t(x)/P_t)} = Z_l^{PR}\frac{\partial \log W_t(x)}{\partial \log(W_t(x)/P_t)} \\
&= \frac{\left(1 - \frac{1}{n}\right)\sigma}{1 - \frac{\gamma Z_E^{PR}\eta}{2n}} + \frac{Z_E^{PR}(\chi + \gamma/2(1 + \eta(1 + \gamma(\rho - 1) - 2\rho)))}{n\left(1 - \frac{\gamma Z_E^{PR}\eta}{2n}\right)\rho}
\end{aligned}$$

is the elasticity of the demand for labor type x to real wage. The ratio $\delta^{PR}/(\delta^{PR} - 1)$ is the wage mark-up over the marginal rate of substitution between consumption and leisure. In models without differentiated labor inputs, such mark-up is absent and the ratio is equal

[10] Appendix C gives the details of this derivation.

to one. If labor inputs are differentiated but unions are atomistic, the mark-up boils down to σ. In our model with non-atomistic unions, the mark-up becomes a function of χ, the weight of the exchange rate in the interest rate rule.

It is worth noticing that the wage mark-up is higher, and therefore the labor market distortion larger, under LCP. Intuitively, the CPI is predetermined under LCP and unions directly control the real wage. Under PCP unions anticipate that real wage will increase less than one-to-one with nominal wage and the exchange rate will depreciate to a larger extent because Z_E^{PR} is larger. Non-atomistic unions anticipate this and restrain their wage demands.

4.1 LCP specification

Under the LCP specification, i.e. when $\eta = 0$, and assuming log-normally distributed consumption, we show in Appendix D that Home consumption innovation is given by

$$c_t - \mathrm{E}_{t-1}c_t = \frac{\lambda\psi}{\rho(1+\psi-\lambda)}\left[\left(1-\frac{\gamma}{2}\right)u_t + \frac{\gamma}{2}u_t^*\right] - \frac{1}{\rho}(\iota_t - \mathrm{E}_{t-1}\iota_t), \tag{39}$$

while innovations to Foreign consumption are

$$c_t^* - \mathrm{E}_{t-1}c_t^* = \frac{\lambda\psi}{\rho(1+\psi-\lambda)}\left[\left(1-\frac{\gamma}{2}\right)u_t^* + \frac{\gamma}{2}u_t\right] - \frac{1}{\rho}(\iota_t^* - \mathrm{E}_{t-1}\iota_t^*). \tag{40}$$

As noted in Duarte and Obstfeld (2008), the presence of non-traded goods makes Home and Foreign consumption depend on different combinations of Home and Foreign productivity shocks. As a result, exchange-rate innovations depend on productivity shocks. Using the risk-sharing condition $e_t = p_t - p_t^* + \rho(c_t - c_t^*)$ we obtain (see Appendix D)

$$e_t - \mathrm{E}_{t-1}e_t = -\left[(\iota_t - \iota_t^*) - \mathrm{E}_{t-1}(\iota_t - \iota_t^*)\right] + \frac{\lambda\psi(1-\gamma)}{1+\psi-\lambda}(u_t - u_t^*). \tag{41}$$

In the absence of non-traded goods ($\gamma = 1$), the exchange rate is not affected directly by country-specific shocks. Finally, one can easily verify that the responses of interest rates to technology shocks are given by

$$\iota_t - \mathrm{E}_{t-1}\iota_t = \frac{\chi(\gamma\Psi - (2-\gamma)\Psi^*)}{2(1+\chi+\chi^*)}u_t^* + \frac{\chi((2-\gamma)\Psi - \gamma\Psi^*)}{2(1+\chi+\chi^*)}u_t, \tag{42}$$

$$\iota_t^* - \mathrm{E}_{t-1}\iota_t^* = \frac{\chi^*(\gamma\Psi^* - (2-\gamma)\Psi)}{2(1+\chi+\chi^*)}u_t + \frac{\chi^*((2-\gamma)\Psi^* - \gamma\Psi)}{2(1+\chi+\chi^*)}u_t^*, \tag{43}$$

where $\Psi \equiv \frac{\lambda\psi}{1+\psi-\lambda}$ and $\Psi^* \equiv \frac{\lambda\psi^*}{1+\psi^*-\lambda}$.

4.2 PCP specification

With PCP pricing ($\eta = 1$), the law of one price holds for traded goods. Hence, from the risk-sharing condition:

$$\rho[c_t - \mathrm{E}_{t-1}c_t - (c_t^* - \mathrm{E}_{t-1}c_t^*)] = (1-\gamma)(\tau_t - \mathrm{E}_{t-1}\tau_t) = (1-\gamma)(e_t - \mathrm{E}_{t-1}e_t), \tag{44}$$

where $\tau_t \equiv e_t + p_{Ft}^* - p_{Ht} = p_{Ft} - p_{Ht}$ indicates the terms of trade. Unanticipated changes in the exchange rate affect the real exchange rate and hence relative consumption levels. The presence of non-traded goods implies that a change in the nominal exchange rate alters the relative price of non-traded goods, which in our model corresponds to the terms of trade. Exchange rate innovations are (see Appendix D):

$$e_t - \mathrm{E}_{t-1}e_t = \iota_t^* - \mathrm{E}_{t-1}\iota_t^* - (\iota_t - \mathrm{E}_{t-1}\iota_t) + \frac{\lambda\psi}{1+\psi-\lambda}u_t - \frac{\lambda\psi^*}{1+\psi^*-\lambda}u_t^*, \tag{45}$$

while Home and Foreign consumption innovations are

$$\rho(c_t - \mathrm{E}_{t-1}c_t) = \frac{\lambda\psi}{1+\psi-\lambda}u_t - (\iota_t - \mathrm{E}_{t-1}\iota_t) - \frac{\gamma}{2}(\tau_t - \mathrm{E}_{t-1}\tau_t), \tag{46}$$

$$\rho(c_t^* - \mathrm{E}_{t-1}c_t^*) = \frac{\lambda\psi^*}{1+\psi^*-\lambda}u_t^* - (\iota_t^* - \mathrm{E}_{t-1}\iota_t^*) + \frac{\gamma}{2}(\tau_t - \mathrm{E}_{t-1}\tau_t). \tag{47}$$

Home consumption innovations depend on domestic productivity shocks and the revision to current fundamentals, i.e. unanticipated movements in interest rates and the terms of trade.

From the above expressions we find that interest rate responses to productivity shocks are

$$\iota_t - \mathrm{E}_{t-1}\iota_t = \frac{\chi(u_t\Psi - u_t^*\Psi^*)}{1+\chi+\chi^*}, \tag{48}$$

$$\iota_t^* - \mathrm{E}_{t-1}\iota_t^* = \frac{\chi^*(u_t^*\Psi^* - u_t\Psi)}{1+\chi+\chi^*}. \tag{49}$$

5 Welfare and optimal monetary policy rules

Central banks can commit to interest rate rules. In this section we analyze optimal monetary policy from a welfare perspective, namely we find the parameters of the interest rate rule that maximize expected utility.

Expected utility is

$$\mathrm{E}_{t-1}U_t = \mathrm{E}_{t-1}\frac{C_t^{1-\rho}}{1-\rho} - k\mathrm{E}_{t-1}L_t. \tag{50}$$

In the following sections we illustrate the optimal cooperative and non-cooperative policies under LCP and PCP.

Appendix E shows that expected employment in the Home country as of date $t-1$ is

$$\mathrm{E}_{t-1}L_t = \frac{1-\gamma/2}{\mathcal{M}_p\mathcal{M}_w k}\mathrm{E}_{t-1}\left(C_t^{1-\rho}\right) + \frac{\gamma/2}{\mathcal{M}_p\mathcal{M}_w k}\mathrm{E}_{t-1}\left(C_t^{*1-\rho}\right). \tag{51}$$

where $\mathcal{M}_p \equiv \theta/(\theta-1)$ and $\mathcal{M}_w \equiv \delta^{PR}/(\delta^{PR}-1)$.

Substituting expected employment in the expected utility, the latter becomes a function of C_t, C_t^* and the parameters of the model

$$\mathrm{E}_{t-1}U_t = \left(\frac{\mathcal{M}_p\mathcal{M}_w - (1-\gamma/2)(1-\rho)}{(1-\rho)\mathcal{M}_p\mathcal{M}_w}\right)\mathrm{E}_{t-1}C_t^{1-\rho} - \frac{\gamma/2}{\mathcal{M}_p\mathcal{M}_w}\mathrm{E}_{t-1}\left(C_t^{*1-\rho}\right). \tag{52}$$

5.1 LCP: Optimal Policies

Coordination

Under coordination, the monetary authorities solve the following problem

$$\max_{\chi,\chi^*,\psi,\psi^*} 1/2\left(\mathrm{E}_{t-1}U_t + \mathrm{E}_{t-1}U_t^*\right). \tag{53}$$

The choice of ψ does not affect the wage mark-up in eq. (38). The central bank chooses ψ so as to reduce the distortion stemming from preset pricing. The larger ψ, the closer is the response of total consumption to technological shocks to the flexible-price equilibrium (see eq. (34)). Hence, for a given value of χ, the expected utility is always increasing in ψ.

The optimal value of χ when $\psi \to \infty$ is obtained (in a symmetric equilibrium) by solving the following expression

$$-\mathcal{M}_w'[\chi] = \frac{\mathcal{M}_w(1-\gamma)^2\lambda(\mathcal{M}_w\mathcal{M}_p + \rho - 1)\sigma_u^2(1-\lambda+2\chi)}{(\mathcal{M}_w\mathcal{M}_p - 1)\rho(1+2\chi)^3}, \tag{54}$$

where $\mathcal{M}_w'[\chi] \equiv \mathrm{d}\mathcal{M}_w/\mathrm{d}\chi$. As noted earlier, the union's mark-up is endogenous to the exchange-rate targeting policy. In particular, a higher weight on the exchange rate target reduces the monopolistic distortion in the labor market, i.e. $\mathcal{M}_w'[\chi] < 0$.

Intuitively, the central bank faces a trade-off between *dynamic* and *static* distortions. The former is due to the fact that prices are rigid in the short run while the latter is the labor market distortion. The monetary authority aims to equalize the marginal benefits from a reduction in the labor market distortion (L.H.S. of eq. (54)) to the marginal cost of a less efficient response of consumption to productivity shocks (R.H.S. of eq. (54)). When the labor market distortions at Home and Foreign are large – and larger than the dynamic distortion, it is optimal to put a large weight on the exchange rate target, i.e χ, χ^* large but less than infinity. Conversely, when dynamic distortions are relatively more important, it is optimal not to target the exchange rate, i.e. $\chi, \chi^* = 0$.

To analyze these trade-offs, we calibrate our simple model using the parameters in Table 1.[11] The elasticity of substitution among the same type of goods, θ, the elasticity of substitution among labor types, σ, the inverse of the rate of intertemporal substitution, ρ, and

[11] The value of ψ is set equal to 125 in all the calibration exercises.

Table 1: Calibrated Parameters

	Parameter	**Value**
θ	Elasticity of Substitution (Goods)	8
σ	Elasticity of Substitution (Labor Types)	4
λ	Autocorrelation	0.82
σ_u	Standard Deviation Innovation	0.0229
ρ	Inverse of Intertemporal Rate of Substitution	5
γ	Share of Traded Goods in Total Consuption	0.4
n	Number of unions	3

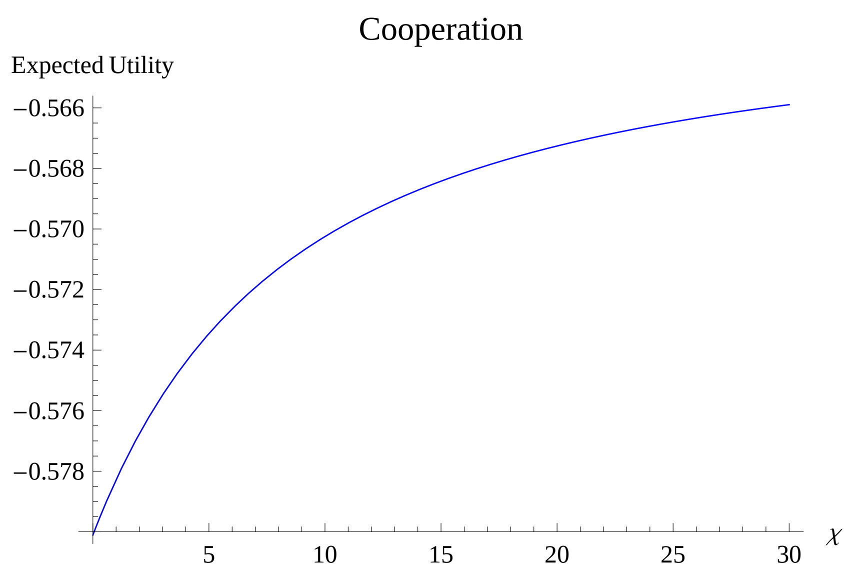


Figure 1: Benchmark calibration: LCP

the share of traded goods in total consumption, γ, are from Benigno and Thoenissen (2003). The standard deviation of productivity innovations is taken from Schmitt-Grohé and Uribe (2004). Foreign and Home are symmetric.

Figure 1 shows that, under the benchmark calibration, the cooperative equilibrium requires large weights on the exchange rate target, namely $\chi = \chi^*$ have large positive values. Intuitively, the labor market distortion is large in this calibration and the central banks are better off to commit to strong interest rate responses to exchange rate innovations in order to reduce wage mark-ups. A lower distortion in the labor market and/or a higher volatility of the productivity shocks reduce the importance of the labor market distortion relative to the dynamic one. Figure 2 depicts the case where the standard deviation of the productivity shocks is more than 60 times larger than under the benchmark parametrization, $\sigma_u = 1.4$. In this case, it is optimal *not* to target the exchange rate under cooperation, namely $\chi = \chi^* = 0$. Intuitively, idiosyncratic productivity shocks are extremely volatile and

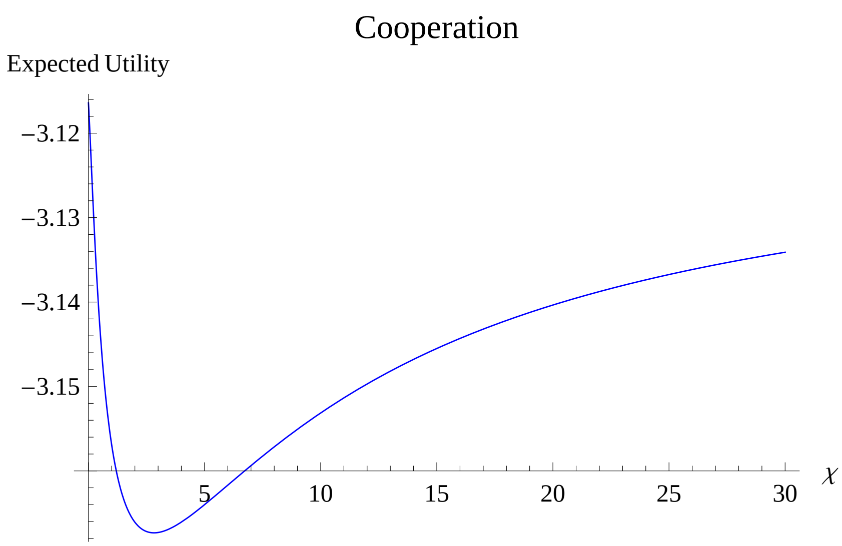
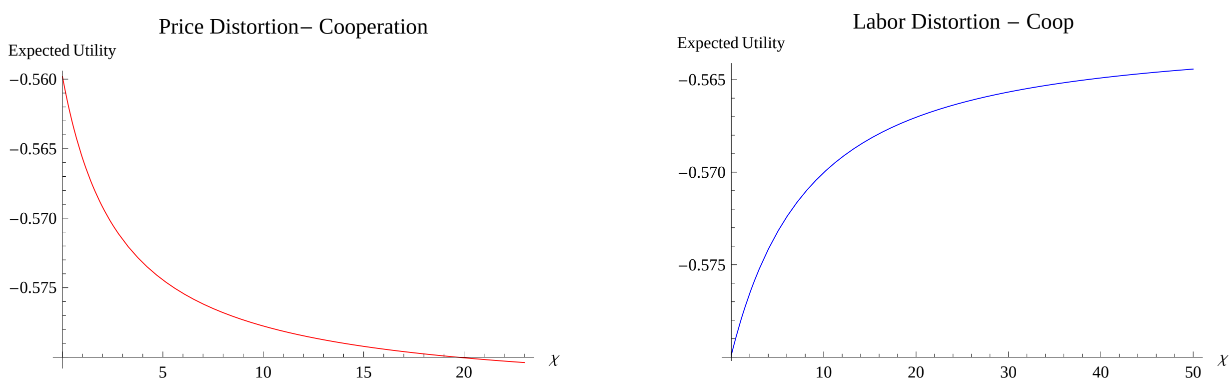


Figure 2: $\sigma_u = 1.4$: LCP



(a) Dynamic distortion

(b) Static distortion

Figure 3: Welfare decomposition

the need for full exchange rate flexibility is restored.

Under the benchmark calibration, it is optimal to target the exchange rate with a large weight. To understand this result, we evaluate welfare under two extreme assumptions. First, we assume that the wage mark-up is constant and monetary policy cannot affect it. This is Figure 3(a), showing that if we consider only the distortion stemming from preset pricing, optimal monetary policy would *not* target at all the exchange rate. Second, we assume that the variance of the technological shock is zero, namely $\sigma_u^2 = \sigma_{u^*}^2 = 0$. In this case, optimal monetary policy targets the exchange rate targeting with large weights. This is shown in Figure 3(a) respectively. Notice that the cooperative equilibrium is characterized by either *no* exchange rate targeting at all or by a very large weight on the exchange rate in the interest rate rule. This feature is going to disappear in the Nash equilibrium.

Nash

Under Nash the Home monetary authority chooses χ, ψ to solve

$$\max_{\chi,\psi} \mathrm{E}_{t-1} U_t. \tag{55}$$

taking χ^*, ψ^* as given. Expected utility is increasing in ψ and the optimal weight on the CPI target is large, specifically $\psi \to \infty$. As noted in the cooperative case, choosing a large ψ reduces the dynamic distortion without any impact on the static one.

In a symmetric equilibrium, the optimal value of χ solves the following expression:

$$\mathcal{M}'_w[\chi] = \frac{\mathcal{M}_w(1-\gamma)^2\sigma_u^2(2\mathcal{M}_w\mathcal{M}_p(1+\chi)+(\rho-1)(2-\gamma+2(1-\gamma)\chi))(1+2\chi-\lambda)\lambda}{(2-\mathcal{M}_w\mathcal{M}_p(2-\gamma)+(2-\gamma)\gamma(\rho-1))\rho(1+2\chi)^3}.$$

Three factors determine the optimal weight on the exchange rate target, χ. A higher value of χ reduces the labor market distortion; a lower value of χ makes price and consumption responses closer to the flexible price equilibrium. These two effects are the same we have seen under cooperation and they push χ to opposite extremes. Under Nash there is a third factor: the central bank wants to strategically increase the labor mark-up so as to increase domestic vis-à-vis foreign PPI. This is the terms-of-trade effect as in Corsetti and Pesenti (2001), where the gains from appreciating the terms of trade offset the efficiency loss stemming from lower output.

To assess the strategic incentive to raise the terms of trade, we eliminate the dynamic distortion and set $\sigma_u^2 = 0$, which is a good approximation of our benchmark model. The first-order condition relative to χ implies:

$$\mathcal{M}_w = \frac{2+(2-\gamma)\gamma(\rho-1)}{\mathcal{M}_p(2-\gamma)}, \qquad \mathcal{M}_w \geq 1.$$

Intuitively, an interior solution to the optimal value of χ satisfies the equation and the constraint above. In particular, the solution is increasing in γ and, for $\rho > 1$, in ρ as well. An increase in γ raises the fraction of traded goods in the consumption basket and makes the economy more open to Foreign goods. As the economy becomes more open, the terms-of-trade effect becomes stronger and optimal monetary policy achieves a *higher* wage mark-up in order to raise domestic PPI relative to the foreign one. Similarly, when $\rho > 1$ Home and Foreign goods are substitutes. An increase in ρ makes Home and Foreign goods more substitutable, thereby making the terms-of-trade effect stronger.

Figure 4 depicts expected utility for the Home country as a function of χ for the benchmark specification. In the Nash equilibrium, the optimal weight on the exchange rate target is zero, $\chi = \chi^* = 0$, and the exchange rate is freely floating. In our benchmark specification, the degree of openness is high. As a result, the terms-of-trade effect prevails over the labor market distortion. While it is optimal to target the exchange rate under cooperation, it is not so under Nash.

As γ falls and the economy becomes less open, the terms-of-trade effect weakens. Figure 5 shows the case where the economy is more closed relative to benchmark ($\gamma = 0.1$). Now

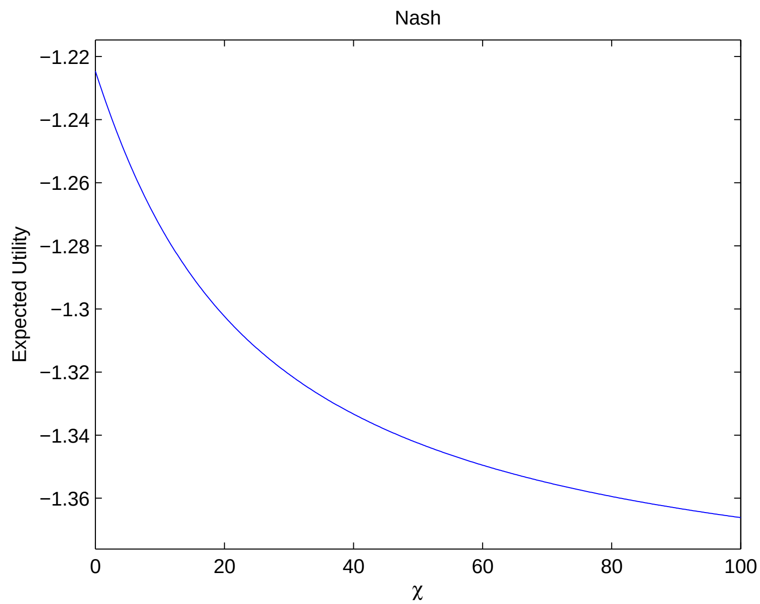


Figure 4: Benchmark calibration: LCP

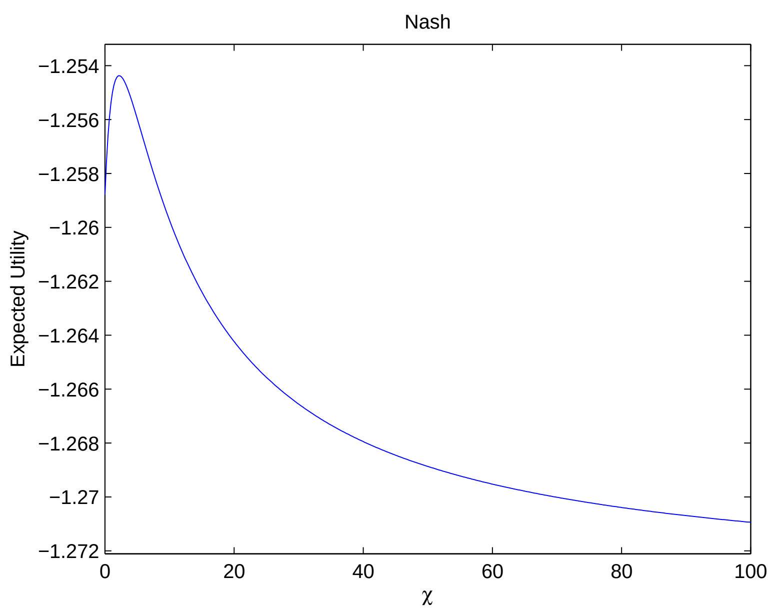


Figure 5: $\gamma = 0.1$: LCP

the optimal weight on the exchange rate target is positive and finite, $\chi = \chi^* = 4.20$, which implies an exchange rate regime of managed floating.

5.2 PCP: Optimal Policies

Coordination

This optimization problem is conceptually similar to the one under LCP. Appendix E.3 spells out the mathematical details. Once again, it is optimal to target the PPI with a large weight in order to bring the allocation closer to its flexible price counterpart, namely $\psi, \psi^* \to \infty$. The first-order condition relative to χ when $\psi, \psi^* \to \infty$ is

$$-\mathcal{M}_w'[\chi] = \frac{\mathcal{M}_w(1+(2-\gamma)\gamma(\rho-1))(\mathcal{M}_w\mathcal{M}_p - 1 + \rho)\sigma_u^2(1+2\chi-\lambda)\lambda}{(\mathcal{M}_w\mathcal{M}_p - 1)\rho(1+2\chi)^3}.$$

The trade-off in the choice of χ, χ^* is similar to the LCP case and we refer the reader to our earlier discussion. In the remainder of this section we focus on how this trade off is affected by the two price specifications.

For realistic parameter values, the labor-market distortion is smaller under PCP than LCP. Since prices are preset, the CPI is completely predetermined under LCP. Unions therefore set not only the nominal but also the real wage under LCP. Under PCP, on the other hand, only the PPI is predetermined while the CPI varies with the exchange rate. Unions anticipate that an increase in wages is going to depreciate the exchange rate and thereby raise the CPI. When the country is sufficiently open (γ large enough) and Home and Foreign goods are substitutes ($\rho > 1$), the anticipation of a CPI increase will restrain wage demands. As a result, the wage mark-up is lower under PCP than LCP.

Under the benchmark calibration, optimal monetary policy under PCP and coordination does *not* target the exchange rate, $\chi = \chi^* = 0$, and the exchange rate is flexible. Because the labor market distortion is lower under PCP than LCP, the optimal χ, χ^* are lower under PCP. This implies that the case for *not* targeting the exchange rate arises for smaller variances of the technological shocks under PCP than LCP. For our benchmark calibration, values of $\sigma_u^2 = \sigma_{u^*}^2 > 0.06$ are sufficient for $\chi = \chi^* = 0$ to be optimal under PCP, while in the LCP specification this threshold has to be larger, i.e. $\sigma_u^2 = \sigma_{u^*}^2 > 1.96$. Figure 6 displays expected utility as function of χ under the benchmark calibration of Table 1 but $\sigma_u^2 = \sigma_{u^*}^2 = 0.09$.

Nash

Under PCP, as well as LCP, it is optimal to choose a large value of ψ in the Nash equilibrium in order to reduce the welfare losses stemming from preset pricing (see Appendix E.3). The optimal choice of χ depends on the balance between the terms-of-trade effect, the dynamic, and the static distortion. For $\chi \to \infty$, the optimal value of χ solves

$$\begin{aligned}-\mathcal{M}_w'[\chi] \;=\; & \frac{\mathcal{M}_w(\rho-1)\sigma_u^2(-2(1+\chi)+(\gamma-2)\gamma(\rho-2(1+\chi)))(1+2\chi-\lambda)\lambda}{(2+\mathcal{M}_w\mathcal{M}_p(\gamma-2)+(2-\gamma)\gamma(\rho-1))\rho(1+2\chi)^3} + \\ & + \frac{\mathcal{M}_p\mathcal{M}_w^2\sigma_u^2((\gamma-2)(1+\gamma(\rho-1))-2(1-\gamma)\chi)(1+2\chi-\lambda)\lambda}{(2+\mathcal{M}_w\mathcal{M}_p(\gamma-2)+(2-\gamma)\gamma(\rho-1))\rho(1+2\chi)^3}.\end{aligned}$$

Under the benchmark calibration, it is optimal not to target the exchange rate and set $\chi = 0$. As for the case with LCP, the incentive to improve the terms of trade depends on the degree

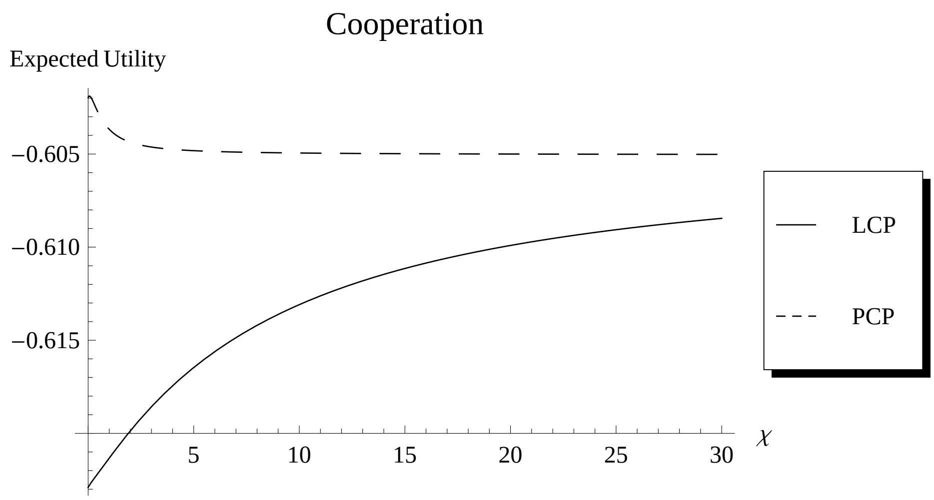


Figure 6: Expected utility with $\sigma_u^2 = 0.09$

of openness γ and the risk aversion coefficient ρ. In particular, as γ increases, Home becomes more open and therefore more affected by terms-of-trade changes. Similarly, a higher value of ρ makes Home and Foreign goods more substitutable and raises the incentive to strategically improve the terms of trade.

Figure 7 shows expected utility in the Nash equilibrium under PCP and LCP for $\gamma = 0.1$ – a lower degree of openness than for the benchmark calibration. Under both specifications, χ is positive and finite and the resulting exchange rate regime is a managed float. The optimal weight on the exchange rate target is lower under PCP than LCP, as argued earlier.

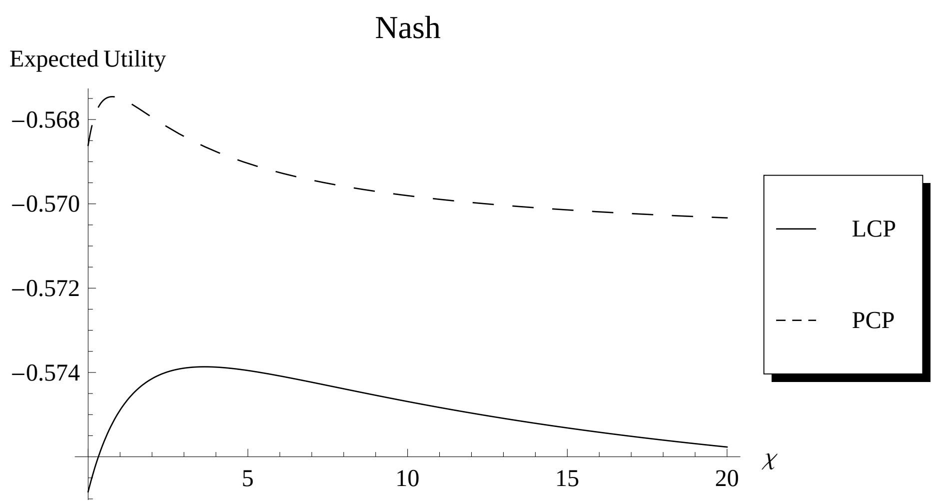


Figure 7: Expected utility with $\gamma = 0.1$

Notice that the results concerning the optimal degree of exchange-rate targeting under LCP and PCP are robust to the presence of permanent productivity shocks, $\lambda = 1$, as assumed by Duarte and Obstfeld (2008) and Devereux and Engel (2003).[12] We have repeated our calibration exercises with a standard deviation corresponding to permanent productivity innovations (e.g. Galí (2003)) and found qualitatively similar results. Moreover, when $\lambda = 1$ the weight on the PPI/CPI in the interest rate rule, ψ, drops out of expected utility and therefore becomes irrelevant for welfare – as long as it is strictly positive in order for prices to be determined.

6 Conclusions

This paper asks whether an optimal interest rate rule should explicitly target the exchange rate in an environment with non-traded goods, different degrees of pass-through and non-atomistic unions. The main finding is that exchange-rate targeting can be optimal when the labor market distortion is large. The reason is that non-atomistic labor unions take into account the interest rate response to the depreciation brought by their wage claims. Hence, monetary policy can reduce labor market distortions by committing to target the exchange rate.

The case for exchange-rate targeting is stronger under cooperation than Nash. In the absence of cooperation, each monetary authority tries to strategically improve the terms-of-trade. This is achieved by targeting less (or not targeting at all) the exchange rate so as to raise the wage mark up. The case of exchange-rate targeting is also stronger under LCP than PCP because the labor market distortion is itself smaller under PCP.

A number of extensions would be desirable. We have not analyzed the case of a fixed exchange rate regime because it requires an asymmetric setup. However, our model can be easily changed to allow for a fixed exchange rate and the methodology presented here can be easily applied. We have focused on preset pricing because they allow us to solve most of the model analytically. More realistic price setting mechanisms could be assumed and studied.

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Appendix

A Impact of union's wage on aggregate wage

From the wage index (17), we obtain

$$\begin{aligned}
\frac{\partial W_t}{\partial W_t(x)} &= \frac{\partial}{\partial W_t(x)}\left[\int_0^1 W_t(z)^{1-\sigma}\mathrm{d}z\right]^{\frac{1}{1-\sigma}} \\
&= \frac{\partial}{\partial W_t(x)}\left[\int_{z\in x} W_t(z)^{1-\sigma}\mathrm{d}z + \int_{z\notin x} W_t(z)^{1-\sigma}\mathrm{d}z\right]^{\frac{1}{1-\sigma}} \\
&= \frac{1}{n}\left[\frac{W_t(x)}{W_t}\right]^{-\sigma} = \frac{1}{n} \qquad \text{(A.1)}
\end{aligned}$$

where the last equality holds in a symmetric equilibrium, i.e. when $W(x) = W$.

B Labor type demand elasticity to union's wage

B.1 Flexible prices

Substituting eq. (29) into the definition of CPI, we may rewrite eq. (6) and the corresponding one in the Foreign country as follows:

$$P_t = \frac{\theta}{\theta - 1}\frac{W_t}{(A_t^*)^{\frac{\gamma}{2}} A_t^{1-\frac{\gamma}{2}}} \qquad \text{(B.1)}$$

$$P_t^* = \frac{\theta}{\theta-1}\frac{W_t^*}{A_t^{\frac{\gamma}{2}}(A_t^*)^{1-\frac{\gamma}{2}}} \tag{B.2}$$

Using eq. (A.1), the elasticity of CPI to wage x is directly obtained from eq. (B.1) in a symmetric equilibrium as follows:

$$Z_p^{FL} \equiv \frac{\partial \log P_t}{\partial \log W_t(x)} = \frac{1}{n}. \tag{B.3}$$

The aggregate labor demand can be written as function of domestic consumption as follows:

$$L_t = \frac{C_{Ht}+C_{Nt}+C_{Ht}^*}{A_t} = \frac{C_t}{2}\left(A_t A_t^*\right)^{-\gamma/2}\left(A_t^{-1+\gamma}(2-\gamma)+(A_t^*)^{\frac{(1-\gamma)(1-\rho)}{\rho}}A_t^{-\frac{1-\gamma}{\rho}}\gamma\right), \tag{B.4}$$

where we have used eqs. (8), (9), (10), (B.1), (B.2), and (12). Then, from the Euler equation (11) and the Home interest rate rule (1) or (3), it follows that

$$Z_C^{FL} \equiv= \frac{\partial \log C_t}{\partial \log W_t(x)} = -\frac{1+\psi+\chi}{\rho}Z_p^{FL} = -Z_L^{FL}\frac{1}{n}, \tag{B.5}$$

where Z_L^{FL} corresponds to the elasticity of aggregate labor demand to the aggregate union's wage $Z_L^{FL} \equiv \frac{1+\psi+\chi}{\rho}$ above. Finally, the elasticity of labor type x to the union's real wage x is given by

$$\begin{aligned}\delta^{FL} &\equiv -\frac{\partial \log L_t(z)}{\partial \log(W_t(x)/P_t)} = Z_l^{FL}\frac{\partial \log W_t(x)}{\partial \log(W_t(x)/P_t)} \\ &= \frac{\sigma}{1-Z_p^{FL}}\left(1-\frac{1}{n}\right)+\frac{Z_L^{FL}}{1-Z_p^{FL}}\frac{1}{n} = \sigma+\frac{1+\psi+\chi}{\left(1-\frac{1}{n}\right)\rho}\frac{1}{n} > 1.\end{aligned} \tag{B.6}$$

B.2 Predetermined prices

Using the optimal prices expressions (20), (25), (23), (22) and the definition of CPI (6), we may rewrite the Home CPI as follows:

$$P_t = \frac{\theta}{\theta-1}\mathcal{E}_t^{\frac{\gamma}{2}\eta}\left(\frac{\mathrm{E}_{t-1}\left(C_t^{1-\rho}\frac{W_t}{A_t}\right)}{\mathrm{E}_{t-1}\left(C_t^{1-\rho}\right)}\right)^{1-\frac{\gamma}{2}}\left(\frac{\mathrm{E}_{t-1}\left(C_t^{1-\rho}\mathcal{E}_t^{1-\eta}\frac{W_t^*}{A_t^*}\right)}{\mathrm{E}_{t-1}\left(C_t^{1-\rho}\right)}\right)^{\frac{\gamma}{2}}, \tag{B.7}$$

where $\eta \in \{0,1\}$ is the degree of exchange rate pass-through, i.e. $\eta = 0$ with LCP and $\eta = 1$ with PCP.

Wages are set under discretion. Therefore, trade unions cannot affect expectation at time t. From eq. (B.7), the only channel through which wage claims have an impact on prices is the exchange rate.

Now, in order to assess how trade unions internalize the impact of their wage settlement on the exchange rate, it is convenient to log-linearize some equations around a steady state with $A_0 = A_0^* = 1$. Henceforth, we will denote a log-linearized variable V around its steady state V_0 as follows: $\mathsf{v} \equiv (V - V_0)/V_0$; and its cross-country difference as $\mathsf{v}^R \equiv \mathsf{v} - \mathsf{v}^*$. Under the assumption that the initial net asset position of Home and Foreign is equal to zero, at the steady state

$$C_0 = C_0^* = L_0 = L_0^*, \qquad P_0 = \frac{\theta}{\theta - 1} W_0, \qquad P_0^* = \frac{\theta}{\theta - 1} W_0^*. \tag{B.8}$$

From the Back and Smith condition (12) we have that $\mathcal{E}_0 = W_0/W_0^*$ and

$$\mathsf{e}_t = \mathsf{p}_t^R + \rho \mathsf{c}_t^R. \tag{B.9}$$

Since unions take expectations on marginal costs as given, the log-linearized difference between the Home CPI (B.7) and the corresponding for the Foreign country turns out to be (disregarding constant additive terms)

$$\mathsf{p}_t^R = \gamma \eta \mathsf{e}_\mathsf{t}. \tag{B.10}$$

Similarly, from the aggregate demands ((8), (9), and (10)) and the household's budget constraint (7), we have

$$\mathsf{y}_t^R = (1-\gamma)(\mathsf{p}_t^R + \mathsf{c}_t^R) + \gamma \eta \mathsf{e}_t, \tag{B.11}$$

$$\frac{\mathsf{div}_t^R}{\theta} = (1-\gamma)\left(\mathsf{p}_t^R + \mathsf{c}_t^R\right) + \gamma \mathsf{e}_t - \frac{\theta - 1}{\theta}(\mathsf{w}_t^R + \mathsf{y}_t^R - \mathsf{a}_t^R). \tag{B.12}$$

Thus, combining eqs. (B.9)-(B.11), the elasticity of exchange rate to aggregate Home wage is given by

$$\frac{\partial \mathsf{e}_t}{\partial \mathsf{w}_t} = \frac{(\theta - 1)\rho}{1 - \gamma(1 - \theta\rho + \eta(1 - \gamma + (\gamma + \theta - 2)\rho))} \equiv Z_E^{PR} > 0, \tag{B.13}$$

where we have assumed that unions take the impact of their wage on dividends as given, i.e. $\partial \mathsf{div}_t^R / \partial \mathsf{w} = 0$.[13]

Now, making use of eq. (B.13), it is straightforward to calculate the elasticity of CPI (B.7) to wage x as follows:

$$Z_p^{PR} \equiv \frac{\partial \log P_t}{\partial \log W_t(x)} = \frac{\gamma \eta}{2n} Z_E^{PR}. \tag{B.14}$$

Under LCP (i.e. when $\eta = 0$) the above elasticity is zero and, for reasonable calibration of parameters, eq. (B.14) is greater than zero and less than one under the PCP specification (i.e. when $\eta = 1$).

Notice that the aggregate labor demand is given by

$$L_t = \frac{C_{Ht} + C_{Nt} + C_{Ht}^*}{A_t} = \frac{C_t}{A_t}\left[\left(1 - \frac{\gamma}{2}\right)\frac{P_t}{P_{Ht}} + \frac{\gamma}{2}\left(\frac{\mathcal{E}_t P_t^*}{P_t}\right)^{1-\frac{1}{\rho}} \frac{P_t}{\mathcal{E}_t P_{Ht}^*}\right] = \frac{C_t}{A_t} rel_t, \tag{B.15}$$

[13]This is consistent with the literature. See e.g. Lippi (2003) and Gnocchi (2006).

where we have used eqs. (8), (9), (10), and (12). It turns out that the above expression depends on two components: consumption and relative prices rel.

First, from the Euler equation (11) and the Home interest rate rule (3) or (1), it follows that

$$\frac{\partial \log C_t}{\partial \log W_t(x)} = -\left[\frac{\frac{\gamma\eta}{2}+\chi}{\rho}\right]\frac{Z_E^{PR}}{n}. \tag{B.16}$$

Next, we rewrite t in eq. (B.15) as follows:

$$\mathsf{rel}_t = \left(1-\frac{\gamma}{2}\right)(\mathsf{p}_t-\mathsf{p}_{Ht}) + \frac{\gamma}{2}\left[\left(1-\frac{1}{\rho}\right)\mathsf{p}_t^* + \frac{1}{\rho}(\mathsf{p}_t-\mathsf{e}_t) - \mathsf{p}_{Ht}^*\right], \tag{B.17}$$

which yields

$$\frac{\partial \mathsf{rel}_t}{\partial \mathsf{w}_t} = -[1+\gamma\eta(\rho-1)-2\eta\rho]\frac{\gamma Z_E^{PR}}{2\rho}. \tag{B.18}$$

The elasticity of aggregate labor demand to aggregate wage is hence given by

$$Z_L^{PR} \equiv -\frac{\partial \log L_t}{\partial \log W_t} = \frac{Z_E^{PR}(2\chi+\gamma(1+\eta(1+\gamma(\rho-1)-2\rho)))}{2\rho}. \tag{B.19}$$

Finally, the elasticity of labor type x to the union's real wage x is given by

$$\begin{aligned}
\delta^{PR} &\equiv -\frac{\partial \log L_t(z)}{\partial \log(W_t(x)/P_t)} = Z_l^{PR}\frac{\partial \log W_t(x)}{\partial \log(W_t(x)/P_t)} \\
&= \frac{\sigma}{1-Z_p^{PR}}\left(1-\frac{1}{n}\right) + \frac{Z_L^{PR}}{1-Z_p^{PR}}\frac{1}{n} \\
&= \frac{\left(1-\frac{1}{n}\right)\sigma}{1-\frac{\gamma Z_E^{PR}\eta}{2n}} + \frac{Z_E^{PR}(2\chi+\gamma(1+\eta(1+\gamma(\rho-1)-2\rho)))}{2n\left(1-\frac{\gamma Z_E^{PR}\eta}{2n}\right)\rho}.
\end{aligned} \tag{B.20}$$

C First order condition union x

In order to derive the x-th union first-order condition, it is convenient to reproduce the Lagrangian relevant to this purpose

$$\begin{aligned}
\mathcal{L}^{\mathcal{W}} &= \frac{C_t(x)^{1-\rho}}{1-\rho} - kL_t(x) + \lambda_t\Bigg[C_t(x) + \sum_{s_{t+1}} Q_{s_{t+1}|s_t}B_{s_{t+1}}(x)/P_t \\
&\quad + M_t(x)/P_t - B_{s_t}(x)/P_t - M_{t-1}(x)/P_t - T_t(x) - W_t(x)L_t(x)/P_t \\
&\quad -1/P_t\int_0^1[\Pi_{H,t}(i,x)+\Pi_{N,t}(i,x)]\mathrm{d}i\Bigg],
\end{aligned} \tag{C.1}$$

where the labor demand (16) has been plugged in. The first-order condition with respect to $W_t(x)$ is given by

$$
\begin{aligned}
-k\frac{\partial L_t(x)}{\partial W_t(x)} &= \lambda_t \left[\frac{L_t(x) + \frac{\partial L_t(x)}{\partial W_t(x)} W_t(x)}{P_t} - \frac{\partial P_t}{\partial W_t(x)} \frac{W_t(x) L_t(x)}{P_t^2} \right] \\
-k\frac{\partial L_t(x)}{\partial W_t(x)} &= -\frac{C_t^{-\rho}(x) L_t(x)}{P_t} \left[1 + \frac{\partial L_t(x)}{\partial W_t(x)} \frac{W_t(x)}{L_t(x)} - \frac{\partial P_t}{\partial W_t(x)} \frac{W_t(x)}{P_t} \right] \\
k Z_l^r &= -\frac{C_t^{-\rho}(x) W_t(x)}{P_t} \left[1 - Z_l^r - Z_p^r \right] \\
k\delta^r &= \frac{C_t^{-\rho} W_t}{P_t} \left[\delta^r - 1 \right], \qquad r \in [PR, FL],
\end{aligned}
$$

where in the last equation we have dropped the x index because of the symmetry between workers in equilibrium and used the definition of δ^r derived in Appendix B.

D Equilibrium with preset prices

Expected consumption.

Using (B.7), (12), and (38), taking expectations dated period $t-1$ yields

$$
1 = \frac{\theta k}{\theta - 1} \left(\frac{\delta^{PR}}{\delta^{PR} - 1} \right)^{1-\frac{\gamma}{2}} \left(\frac{\delta^{*PR}}{\delta^{*PR} - 1} \right)^{\frac{\gamma}{2}} \frac{\left[\mathrm{E}_{t-1} \frac{C_t \mathcal{E}_t^{\frac{\gamma\eta}{2}}}{A_t} \right]^{1-\frac{\gamma}{2}} \left[\mathrm{E}_{t-1} \frac{C_t \mathcal{E}_t^{\frac{\gamma\eta}{2} - \eta}}{A_t^*} \right]^{\frac{\gamma}{2}}}{\mathrm{E}_{t-1} C_t^{1-\rho}}. \tag{D.1}
$$

From the log-normality assumption, $c_t = \log C_t \sim \mathcal{N}(\mathrm{E}(c_t), \sigma_c^2)$ and $e_t = \log \mathcal{E}_t \sim \mathcal{N}(\mathrm{E}(e_t), \sigma_e^2)$, we can write eq. (D.1) as

$$
\begin{aligned}
& \exp\left\{ (1-\rho)\mathrm{E}_{t-1} c_t + \frac{(1-\rho)^2}{2} \sigma_c^2 \right\} = \frac{\theta k}{\theta - 1} \left(\frac{\delta^{PR}}{\delta^{PR} - 1} \right)^{1-\frac{\gamma}{2}} \left(\frac{\delta^{*PR}}{\delta^{*PR} - 1} \right)^{\frac{\gamma}{2}} \times \\
\times & \left\{ \exp\left[\mathrm{E}_{t-1} c_t - \mathrm{E}_{t-1} a_t + \frac{\gamma}{2} \eta \mathrm{E}_{t-1} e_t + \frac{\sigma_c^2 + \sigma_u^2 + \gamma^2 \eta^2 / 2\sigma_e^2}{2} + \frac{\gamma\eta\sigma_{ec} - 2\sigma_{cu} - \gamma\eta\sigma_{eu}}{2} \right] \right\}^{1-\frac{\gamma}{2}} \times \\
\times & \left\{ \exp\left[\mathrm{E}_{t-1} c_t - \mathrm{E}_{t-1} a_t^* - \eta \left(1 - \frac{\gamma}{2} \right) \mathrm{E}_{t-1} e_t + \frac{\sigma_c^2 + \sigma_{u^*}^2 + (1-\gamma/2)^2 \eta^2 \sigma_e^2}{2} + \right. \right. \\
- & \left. \left. \eta \left(1 - \frac{\gamma}{2} \right) \sigma_{ec} - \sigma_{cu^*} + \eta \left(1 - \frac{\gamma}{2} \right) \sigma_{eu^*} \right] \right\}^{\frac{\gamma}{2}}.
\end{aligned} \tag{D.2}
$$

Taking logs and simplifying, we may solve for the expected Home consumption (expected

Foreign consumption is obtained in an analogous way)

$$
\begin{aligned}
\mathrm{E}_{t-1}c_t &= -\frac{1}{\rho}\log\left[\frac{\theta k}{\theta-1}\left(\frac{\delta^{PR}}{\delta^{PR}-1}\right)^{1-\frac{\gamma}{2}}\left(\frac{\delta^{*PR}}{\delta^{*PR}-1}\right)^{\frac{\gamma}{2}}\right]+\frac{(\rho-2)}{2}\sigma_c^2 \\
&- \frac{1}{\rho}\left[\eta^2\frac{\gamma}{2}\left(1-\frac{\gamma}{2}\right)\right]\frac{\sigma_e^2}{2}+\frac{1}{\rho}\left[\left(1-\frac{\gamma}{2}\right)\sigma_{cu}+\frac{\gamma}{2}\sigma_{cu^*}\right]+\frac{1}{\rho}\frac{\gamma}{2}\eta\left(1-\frac{\gamma}{2}\right)(\sigma_{eu}-\sigma_{eu^*}) \\
&+ \frac{1}{\rho}\left[\left(1-\frac{\gamma}{2}\right)\mathrm{E}_{t-1}a_t+\frac{\gamma}{2}\mathrm{E}_{t-1}a_t^*\right]-\frac{1}{\rho}\left[\left(1-\frac{\gamma}{2}\right)\frac{\sigma_u^2}{2}+\frac{\gamma}{2}\frac{\sigma_{u^*}^2}{2}\right], \qquad \text{(D.3)}
\end{aligned}
$$

$$
\begin{aligned}
\mathrm{E}_{t-1}c_t^* &= -\frac{1}{\rho}\log\left[\frac{\theta k}{\theta-1}\left(\frac{\delta^{*PR}}{\delta^{*PR}-1}\right)^{1-\frac{\gamma}{2}}\left(\frac{\delta^{PR}}{\delta^{PR}-1}\right)^{\frac{\gamma}{2}}\right]+\frac{(\rho-2)}{2}\sigma_{c^*}^2 \\
&- \frac{1}{\rho}\left[\eta^2\frac{\gamma}{2}\left(1-\frac{\gamma}{2}\right)\right]\frac{\sigma_e^2}{2}+\frac{1}{\rho}\left[\left(1-\frac{\gamma}{2}\right)\sigma_{c^*u^*}+\frac{\gamma}{2}\sigma_{c^*u}\right]+\frac{1}{\rho}\frac{\gamma}{2}\eta\left(1-\frac{\gamma}{2}\right)(\sigma_{eu}-\sigma_{eu^*}) \\
&+ \frac{1}{\rho}\left[\left(1-\frac{\gamma}{2}\right)\mathrm{E}_{t-1}a_t^*+\frac{\gamma}{2}\mathrm{E}_{t-1}a_t\right]-\frac{1}{\rho}\left[\left(1-\frac{\gamma}{2}\right)\frac{\sigma_{u^*}^2}{2}+\frac{\gamma}{2}\frac{\sigma_u^2}{2}\right]. \qquad \text{(D.4)}
\end{aligned}
$$

Solution in the PCP specification ($\eta = 1$). We introduce some new notation. For any variable $v_{t+j} \geq 0$, define

$$\hat{\mathrm{E}}_t v_{t+j} = \mathrm{E}_t v_{t+j} - \mathrm{E}_{t-1} v_{t+j}.$$

Moreover, when $j = 0$, we will write simply $\hat{v}_t = v_t - \mathrm{E}_{t-1}v_t$

The interest rate rules in the PCP case are given by:

$$\iota_t = \bar{\iota} + \psi p_{Ht} + \chi \hat{e}_t$$

$$\iota_t^* = \bar{\iota}^* + \psi^* p_{Ft}^* - \chi^* \hat{e}_t,$$

so that

$$\hat{\iota}_t = \chi \hat{e}_t \qquad \text{(D.5)}$$

$$\hat{\iota}_t^* = -\chi^* \hat{e}_t \qquad \text{(D.6)}$$

Moreover, with PCP pricing, the law of one price holds for traded goods. Hence, from the risk-sharing condition:

$$\rho(\hat{c}_t - \hat{c}_t^*) = (1-\gamma)\hat{\tau}_t = (1-\gamma)\hat{e}_t, \qquad \text{(D.7)}$$

where $\tau_t \equiv e_t + p_{Ft}^* - p_{Ht} = p_{Ft} - p_{Ht}$ indicates the terms of trades. It turns out that

$$p_t = p_{Ht} + \frac{\gamma}{2}\tau_t \qquad ; \qquad p_t^* = p_{Ft}^* - \frac{\gamma}{2}\tau_t.$$

Using (D.3), (D.4), (D.7), taking expectations dated period $t-1$ of the Euler equation, we obtain (disregarding additive constants)

$$p_{Ht} = \frac{\lambda(\lambda-1)}{1+\psi-\lambda} a_{t-1},$$

$$p^*_{Ft} = \frac{\lambda(\lambda-1)}{1+\psi^*-\lambda} a^*_{t-1}.$$

Using the above expressions and $\hat{\mathrm{E}}_t \tau_{t+1} = \lambda(u_t - u^*_t)$ into the Euler equation we have that

$$\hat{\iota}_t = \frac{\lambda\psi}{1+\psi-\lambda} u_t - \rho\hat{c}_t - \frac{\gamma}{2}\hat{\tau}_t \tag{D.8}$$

$$\hat{\iota}^*_t = \frac{\lambda\psi^*}{1+\psi^*-\lambda} u^*_t - \rho\hat{c}^*_t + \frac{\gamma}{2}\hat{\tau}_t, \tag{D.9}$$

which correspond to eqs. (46) and (47) in the text.

Eqs. (D.5)-(D.9) yield

$$\sigma^2_c = \left(\frac{(2-\gamma+2\chi^*)\Psi}{2\rho(1+\chi+\chi^*)}\right)^2 \sigma^2_u + \left(\frac{(\gamma+2\chi)\Psi^*}{2\rho(1+\chi+\chi^*)}\right)^2 \sigma^2_{u^*}; \tag{D.10}$$

$$\sigma_{cu} = \frac{(2-\gamma+2\chi^*)\Psi}{2\rho(1+\chi+\chi^*)}\sigma^2_u \quad ; \quad \sigma_{cu^*} = \frac{(\gamma+2\chi)\Psi^*}{2\rho(1+\chi+\chi^*)}\sigma^2_{u^*}; \tag{D.11}$$

$$\sigma^2_{c^*} = \left(\frac{(\gamma+2\chi^*)\Psi}{2\rho(1+\chi+\chi^*)}\right)^2 \sigma^2_u + \left(\frac{(2-\gamma+2\chi)\Psi^*}{2\rho(1+\chi+\chi^*)}\right)^2 \sigma^2_{u^*}; \tag{D.12}$$

$$\sigma_{c^*u} = \frac{(\gamma+2\chi^*)\Psi}{2\rho(1+\chi+\chi^*)}\sigma^2_u \quad ; \quad \sigma_{c^*u^*} = \frac{(2-\gamma+2\chi)\Psi^*}{2\rho(1+\chi+\chi^*)}\sigma^2_{u^*}; \tag{D.13}$$

$$\sigma^2_e = \left(\frac{\Psi}{1+\chi+\chi^*}\right)^2 \sigma^2_u + \left(-\frac{\Psi^*}{1+\chi+\chi^*}\right)^2 \sigma^2_{u^*}; \tag{D.14}$$

$$\sigma_{eu} = \frac{\Psi}{1+\chi+\chi^*}\sigma^2_u \quad ; \quad \sigma_{eu^*} = -\frac{\Psi^*}{1+\chi+\chi^*}\sigma^2_{u^*}, \tag{D.15}$$

where $\Psi \equiv \frac{\lambda\psi}{1+\psi-\lambda}$ and $\Psi^* \equiv \frac{\lambda\psi^*}{1+\psi^*-\lambda}$.

Solution in the LCP specification ($\eta = 0$). For any variable $v_{t+j} \geq 0$, define

$$\hat{\mathrm{E}}_t v_{t+j} = \mathrm{E}_t v_{t+j} - \mathrm{E}_{t-1} v_{t+j}.$$

Moreover, when $j = 0$, we will write simply $\hat{v}_t = v_t - \mathrm{E}_{t-1} v_t$.

The interest rate rules in the LCP case are given by:

$$\iota_t = \bar{\iota} + \psi p_t + \chi \hat{e}_t$$

$$\iota_t^* = \bar{\iota}^* + \psi^* p_t^* - \chi^* \hat{e}_t,$$

so that

$$\hat{\iota}_t = \chi \hat{e}_t \tag{D.16}$$

$$\hat{\iota}_t^* = -\chi^* \hat{e}_t. \tag{D.17}$$

From the risk-sharing condition we obtain

$$\hat{e}_t = \rho(\hat{c}_t - \hat{c}_t^*). \tag{D.18}$$

The Euler equation in the Home country yields

$$\iota_t \quad = \quad \mathrm{E}_t\left(\rho c_{t+1} + p_{t+1}\right) - p_t - \rho c_t + \Gamma_i \tag{D.19}$$

where Γ_i is a constant. Using (D.3), taking expectations dated period $t-1$, and solving for p_t we have that

$$p_t = \frac{\lambda(\lambda-1)}{1+\psi-\lambda}\left[\left(1-\frac{\gamma}{2}\right)a_{t-1} + \frac{\gamma}{2}a_{t-1}^*\right] + \Gamma_i/\psi. \tag{D.20}$$

From the difference between (D.19) and its expected value at $t-1$, and using eqs. (D.3) and (D.20), we can write the following expressions for innovations to consumption:

$$\hat{c}_t = \Psi/\rho[(1-\gamma/2)u_t + \gamma/2u_t^*] - \hat{\iota}_t/\rho \tag{D.21}$$

$$\hat{c}_t^* = \Psi^*/\rho[(1-\gamma/2)u_t^* + \gamma/2u_t] - \hat{\iota}_t^*/\rho, \tag{D.22}$$

where $\Psi = \frac{\lambda\psi}{1+\psi-\lambda}$ and $\Psi^* = \frac{\lambda\psi^*}{1+\psi^*-\lambda}$.

Thus, combining eqs. (D.16)-(D.22) we obtain

$$\sigma_c^2 = \left(\frac{((2-\gamma)(1+\chi^*)\Psi + \gamma\chi\Psi^*)}{2\rho(1+\chi+\chi^*)}\right)^2 \sigma_u^2 + \left(\frac{(\gamma(1+\chi^*)\Psi + (2-\gamma)\chi\Psi^*)}{2\rho(1+\chi+\chi^*)}\right)^2 \sigma_{u^*}^2 \tag{D.23}$$

$$\sigma_{cu} = \frac{((2-\gamma)(1+\chi^*)\Psi + \gamma\chi\Psi^*)}{2\rho(1+\chi+\chi^*)}\sigma_u^2; \tag{D.24}$$

$$\sigma_{cu^*} = \frac{(\gamma(1+\chi^*)\Psi + (2-\gamma)\chi\Psi^*)}{2\rho(1+\chi+\chi^*)}\sigma_{u^*}^2; \tag{D.25}$$

$$\sigma_{c^*}^2 = \left(\frac{((2-\gamma)\chi^*\Psi + \gamma(1+\chi)\Psi^*)}{2\rho(1+\chi+\chi^*)}\right)^2 \sigma_u^2 + \left(\frac{(\gamma\chi^*\Psi + (2-\gamma)(1+\chi)\Psi^*)}{2\rho(1+\chi+\chi^*)}\right)^2 \sigma_{u^*}^2 \tag{D.26}$$

$$\sigma_{c^*u} = \left(\frac{((2-\gamma)\chi^*\Psi + \gamma(1+\chi)\Psi^*)}{2\rho(1+\chi+\chi^*)}\right)\sigma_u^2; \tag{D.27}$$

$$\sigma_{c^*u^*} = \left(\frac{(\gamma\chi^*\Psi + (2-\gamma)(1+\chi)\Psi^*)}{2\rho(1+\chi+\chi^*)}\right)\sigma_{u^*}^2. \tag{D.28}$$

E Optimal monetary policies

E.1 Expected Employment.

The goods-market equilibrium condition are

$$
\begin{aligned}
A_t L_{Tt} &= C_{Ht} + C_{Ht}^* = \frac{\gamma}{2}\frac{P_t}{P_{Ht}}C_t + \frac{\gamma}{2}\frac{P_t^*}{P_{Ht}^*}C_t^* \\
A_t L_{Nt} &= C_{Nt} = (1-\gamma)\frac{P_t}{P_{Nt}}C_t.
\end{aligned}
$$

Using the above equations with the optimal pricing equations (20) and (22) and the risk-sharing condition (12) yields

$$
\begin{aligned}
L_t &= \frac{1-\frac{\gamma}{2}}{\mathcal{M}_p\mathcal{M}_w k}\frac{\mathrm{E}_{t-1}\left(C_t^{1-\rho}\right)}{\mathrm{E}_{t-1}\left(C_t^{1-\rho}W_t/A_t\right)}\frac{C_t^{1-\rho}W_t}{A_t} \\
&+ \frac{\frac{\gamma}{2}}{\mathcal{M}_p\mathcal{M}_w k}\frac{\mathrm{E}_{t-1}\left(C_t^{*1-\rho}\right)}{\mathrm{E}_{t-1}\left(C_t^{*1-\rho}W_t^*/A_t\right)}\frac{C_t^{*1-\rho}W_t^*}{A_t},
\end{aligned}
$$

where $\mathcal{M}_p \equiv \theta/(\theta-1)$ and $\mathcal{M}_w \equiv \delta^{PR}/(\delta^{PR}-1)$. Therefore expected employment in the Home country at date $t-1$ is

$$
\mathrm{E}_{t-1}L_t = \frac{1-\gamma/2}{\mathcal{M}_p\mathcal{M}_w k}\mathrm{E}_{t-1}\left(C_t^{1-\rho}\right) + \frac{\gamma/2}{\mathcal{M}_p\mathcal{M}_w k}\mathrm{E}_{t-1}\left(C_t^{*1-\rho}\right). \tag{E.1}
$$

E.2 Optimal policies in the LCP specification.

Coordination:

In a Cooperative solution, the monetary authorities coordinate on rules so as to solve the following problem

$$
\max_{\chi,\chi^*,\psi,\psi^*} 1/2\left(\mathrm{E}_{t-1}U_t + \mathrm{E}_{t-1}U_t^*\right).
$$

The first-order condition in a symmetric equilibrium for $\chi = \chi^*$ is given by

$$
-\mathcal{M}_w'[\chi] = \frac{\mathcal{M}_w(1-\gamma)^2(\mathcal{M}_w\mathcal{M}_p+\rho-1)\sigma_u^2(1+2\chi-\Psi)\Psi}{(\mathcal{M}_w\mathcal{M}_p-1)\rho(1+2\chi)^3}, \tag{E.2}
$$

where $\mathcal{M}_w'[\chi] \equiv \mathrm{d}\mathcal{M}_w/\mathrm{d}\chi$. The optimal values of χ is obtained by eq. (E.2).

The (symmetrical) optimal value of $\psi = \psi^*$ is instead obtained by the following first-order condition

$$
\Psi \equiv \frac{\lambda\psi}{1+\psi-\lambda} = 1 + \frac{2(1-\gamma)^2\chi}{2-(2-\gamma)\gamma+4\chi(1+\chi)} \geq 1,
$$

which implies setting $\psi \to \infty$.

Nash solution:

$$\max_{\chi,\psi} \mathrm{E}_{t-1} U_t = D \exp S - D^* \exp S^*,$$

where

$$S = \left(\frac{1}{2}(1-\rho)^2 + \frac{1}{2}(1-\rho)(-2+\rho)\right)\sigma_c^2 + \frac{(1-\rho)\left(\left(1-\frac{\gamma}{2}\right)\sigma_{cu} + \frac{\gamma\sigma_{cu^*}}{2}\right)}{\rho},$$

$$S^* = \left(\frac{1}{2}(1-\rho)^2 + \frac{1}{2}(1-\rho)(-2+\rho)\right)\sigma_{c^*}^2 + \frac{(1-\rho)\left(\frac{\gamma\sigma_{c^*u}}{2} + \left(1-\frac{\gamma}{2}\right)\sigma_{c^*u^*}\right)}{\rho},$$

$$D = \mathcal{M}_p^{\frac{\rho-1}{\rho}} \mathcal{M}_w(\chi)^{\frac{(1-\frac{\gamma}{2})(-1+\rho)}{\rho}} \mathcal{M}_{w^*}^{\frac{\gamma(-1+\rho)}{2\rho}} \left(-\frac{1-\frac{\gamma}{2}}{\mathcal{M}_p\mathcal{M}_w(\chi)} + \frac{1}{1-\rho}\right),$$

$$D^* = \frac{\mathcal{M}_w(\chi)^{\frac{\gamma(\rho-1)-1}{2\rho}} \mathcal{M}_{w^*}^{\frac{(1-\frac{\gamma}{2})(-1+\rho)}{\rho}} \gamma}{2\mathcal{M}_p} \mathcal{M}_p^{\frac{\rho-1}{\rho}}.$$

The first-order condition for χ evaluated at the symmetric equilibrium $\sigma_u^2 = \sigma_{u^*}^2$ and $\chi^* = \chi$ is given by:[14]

$$\frac{\partial(D-D^*)}{\partial\mathcal{M}_w(\chi)}\mathcal{M}_w'(\chi) - D^*\frac{\partial S^*}{\partial\chi} + D\frac{\partial S}{\partial\chi} = 0.$$

$$\mathcal{M}_w'[\chi] = \frac{\mathcal{M}_w(1-\gamma)^2\sigma_u^2(2\mathcal{M}_w\mathcal{M}_p(1+\chi) + (\rho-1)(2-\gamma+2(1-\gamma)\chi))(1+2\chi-\Psi)\Psi}{(2+\mathcal{M}_w\mathcal{M}_p(\gamma-2)+(2-\gamma)\gamma(\rho-1))\rho(1+2\chi)^3}. \tag{E.3}$$

Again, the optimal value of χ has to satisfy eq. (E.3).

The optimal value of ψ instead solves the following first-order condition:

$$\begin{aligned}\Psi \;=\; & \frac{(1+2\chi)(2\mathcal{M}_w\mathcal{M}_p(2+(\gamma-2)\gamma)(1+\chi))}{2\mathcal{M}_w\mathcal{M}_p(1+\chi)\left(2-2\gamma+\gamma^2+2\chi\right)+(\rho-1)G} + \\ & + \frac{(1+2\chi)((2-\gamma)(\rho-1)(2(1+\chi)+\gamma(-2+\gamma+2(\gamma-1)\chi)))}{2\mathcal{M}_w\mathcal{M}_p(1+\chi)\left(2-2\gamma+\gamma^2+2\chi\right)+(\rho-1)G},\end{aligned}$$

where $G \equiv 4-\gamma(6-(4-\gamma)\gamma)+8\chi-2\gamma(4-(3-\gamma)\gamma)\chi+4\chi^2 > 0$. The above expression reduces to $\Psi = 1$ when $\gamma = 1$. Since the R.H.S. of the above expression is decreasing in γ, it turns out that the optimal value of ψ will be an extreme one, i.e. $\psi \to \infty$.

[14] In a symmetric equilibrium, it turns out that $S = S^*$.

E.3 Optimal policies in the PCP specification.

Coordination:

In a Cooperative solution, the monetary authorities coordinate on rules so as to solve the following problem

$$\max_{\chi,\chi^*,\psi,\psi^*} 1/2\left(\mathrm{E}_{t-1}U_t + \mathrm{E}_{t-1}U_t^*\right).$$

The first order condition with respect to χ in a symmetric equilibrium is given by

$$-\mathcal{M}_w'[\chi] = \frac{\mathcal{M}_w(1+(2-\gamma)\gamma(\rho-1))(\mathcal{M}_w\mathcal{M}_p - 1 + \rho)\sigma_u^2(1+2\chi-\Psi)\Psi}{(\mathcal{M}_w\mathcal{M}_p - 1)\rho(1+2\chi)^3}, \tag{E.4}$$

while the first-order with respect to ψ is given by

$$\Psi = 1 + \frac{2(1+(2-\gamma)\gamma(\rho-1))\chi}{(2-\gamma)\gamma(\rho-1) + 2(1+2\chi(1+\chi))} > 1,$$

which is solved by $\psi \to \infty$.

Nash solution:

Home monetary authority solves the following problem

$$\max_{\chi,\psi} \mathrm{E}_{t-1}U_t = D\exp S - D^*\exp S^*$$

taking χ^* and ψ^* as given, where

$$S = \frac{(\rho-1)(4\rho\sigma_c^2 - 8\sigma_{cu} + \gamma((2-\gamma)\sigma_e^2 + 4\sigma_{cu} - 4\sigma_{cu^*} - 2(2-\gamma)(\sigma_{eu} - \sigma_{eu^*})))}{8\rho},$$

$$S^* = \frac{(\rho-1)(4\rho\sigma_{c^*}^2 - 8\sigma_{c^*u^*} + \gamma((2-\gamma)\sigma_e^2 - 4\sigma_{c^*u} + 4\sigma_{c^*u^*} - 2(2-\gamma)(\sigma_{eu} - \sigma_{eu^*})))}{8\rho},$$

$$D = \mathcal{M}_w(\chi)^{\frac{(1-\frac{\gamma}{2})(\rho-1)}{\rho}}\mathcal{M}_{w^*}^{\frac{\gamma(\rho-1)}{2\rho}}\mathcal{M}_p^{\frac{\rho-1}{\rho}}\left(\frac{1}{1-\rho} - \frac{1-\frac{\gamma}{2}}{\mathcal{M}_p\mathcal{M}_w(\chi)}\right),$$

$$D^* = \frac{\mathcal{M}_w(\chi)^{\frac{\gamma(\rho-1)-1}{2\rho}}\mathcal{M}_{w^*}^{\frac{(1-\frac{\gamma}{2})(\rho-1)}{\rho}}\gamma}{2\mathcal{M}_p}\mathcal{M}_p^{\frac{\rho-1}{\rho}}.$$

The first-order condition for χ evaluated at the symmetric equilibrium $\sigma_u^2 = \sigma_{u^*}^2$ and $\chi^* = \chi$ is as follows:

$$\frac{\partial(D-D^*)}{\partial\mathcal{M}_w(\chi)}\mathcal{M}_w'(\chi) - D^*\frac{\partial S^*}{\partial\chi} + D\frac{\partial S}{\partial\chi} = 0.$$

$$
\begin{aligned}
-\mathcal{M}_w'[\chi] \quad = \quad & \frac{\mathcal{M}_w(\rho-1)\sigma_u^2(-2(1+\chi)+(-2+\gamma)\gamma(\rho-2(1+\chi)))(1+2\chi-\Psi)\Psi}{(2+\mathcal{M}_w\mathcal{M}_p(-2+\gamma)-(-2+\gamma)\gamma(-1+\rho))\rho(1+2\chi)^3}+ \\
& +\frac{\mathcal{M}_p\mathcal{M}_w^2\sigma_u^2((-2+\gamma)(1+\gamma(-1+\rho))+2(-1+\gamma)\chi)(1+2\chi-\Psi)\Psi}{(2+\mathcal{M}_w\mathcal{M}_p(-2+\gamma)-(-2+\gamma)\gamma(-1+\rho))\rho(1+2\chi)^3},
\end{aligned}
$$

while the first-order condition for ψ yields

$$
\begin{aligned}
\Psi \quad = \quad & \frac{(1+2\chi)\left((\rho-1)\left(\gamma^2(\rho-3-2\chi)-4(1+\chi)+\gamma(6-2\rho+4\chi)\right)\right)}{\mathcal{M}_w\mathcal{M}_p\left(\gamma^2(\rho-1)-4(1+\chi)^2+\gamma(4-2\rho+4\chi)\right)+(\rho-1)G'}+ \\
& +\frac{(1+2\chi)(\mathcal{M}_w\mathcal{M}_p(\gamma-2)(\gamma(\rho-1)+2(1+\chi)))}{\mathcal{M}_w\mathcal{M}_p\left(\gamma^2(\rho-1)-4(1+\chi)^2+\gamma(4-2\rho+4\chi)\right)+(\rho-1)G'},
\end{aligned}
$$

where $G' \equiv \gamma^2(\rho-3-4\chi)-4(1+\chi)^2+\gamma(6-2\rho+8\chi)<0$. The R.H.S. of the above expression is always larger than one for any calibration of the parameters. It turns out that the optimal value of ψ is chosen such that the L.H.S. be at its maximum value, i.e. λ. This is clearly obtained by setting $\psi \to \infty$.