Discussion of "Application of a Nonstandard Explicit Integration to Solve Green and Ampt Infiltration Equation" by Damodhara R. Mailapalli, Wesley W. Wallender, Rajendra Singh, and Narendra S. Raghuwanshi

D. A. Barry[1]; J.-Y. Parlange[2]; R. Bakhtyar[3]

[1] Professor, Ecole polytechnique fédérale de Lausanne, Faculté de l'environnement naturel, architectural et construit, Institut des sciences et technologies de l'environnement, Laboratoire de technologie écologique, Station no. 2, CH-1015 Lausanne, Switzerland. Ph. +41 (21) 693-5576; Fax. +41 (21) 693-5670; Email: andrew.barry@epfl.ch.

[2] Professor, Department of Biological and Environmental Engineering, Cornell University, Ithaca, New York 14853-5701 USA. Ph. +1 (607) 255-2476; Fax. +1 (607) 255-4080; Email: jp58@cornell.edu.

[3] Postdoctoral Researcher, Ecole polytechnique fédérale de Lausanne, Faculté de l'environnement naturel, architectural et construit, Institut des sciences et technologies de l'environnement, Laboratoire de technologie écologique, Station no. 2, CH-1015 Lausanne, Switzerland. Ph. +41 (21) 693-8087; Fax. +41 (21) 693-5670; Email: roham.bakhtyar@epfl.ch.

Mailapalli et al. (2009) presented an explicit numerical scheme for calculation of solutions to the (implicit) 1D Green and Ampt (1911) infiltration equation. They compared their numerical solution to a numerical solution obtained using the Newton-Raphson method (e.g., Rao et al., 2006) and the "approximate explicit solutions" of Salvucci and Entekhabi (1994) and Barry et al. (1995e). Below we wish to point out (i) that they could also have employed a simple formula to approximate very accurately solutions to the Green-Ampt model, (ii) that the effective hydraulic conductivity mentioned by the authors is consistent with another infiltration model and (iii) that, using a different starting value, their numerical scheme would also agree with the other results.

Barry et al. (1995e) used the approximations presented by Barry et al. (1993), who provided an exact explicit solution to the Green-Ampt infiltration equation in terms of the Lambert W function (Corless et al. 1996). This function can be calculated to arbitrary precision using public domain software (Barry et al., 1995a,b). Besides the approximation to the Lambert W function provided by Barry et al. (1993), Parlange et al. (2002) and Barry et al. (2005) provide several compact approximations, with the lowest relative error presented by Barry et al. (2005) being about 5×10^{-5}%, an error that is orders of magnitude less than errors that might be present an any experimental data set.

Mailapalli et al. (2009) compared the results of their numerical scheme and the other solutions mentioned above with experimental data from Fok and Chiang (1984), which considered infiltration from a rectangular furrow into a 2D vertical soil slice. Figures 1 and 2 of Mailapalli et al. (2009) show comparisons for, respectively, dry and wet soil. Mailapalli et al. (2009) found that, when compared with the experimental data, the three solutions all

gave virtually the same range of relative error, and that all three over-predicted the infiltration measured by Fok and Chiang (1984). The numerical scheme of Mailapalli et al. (2009) mainly under-predicted the experimental data, but gave a lower relative error.

For a given soil, the Green-Ampt model is considered as being the upper limit of infiltration at a given time. The other limit (Parlange, 1980) is given by the model of Talsma and Parlange (1972). Both the Green-Ampt and Talsma-Parlange models rely on two soil properties, these being the sorptivity, S, and the saturated hydraulic conductivity, K_s. Parlange et al. (1982) provided a three-parameter infiltration equation that reduces to the Green-Ampt and Talsma-Parlange models as limiting cases. It has been shown that infiltration into soil is usually closer to the Talsma-Parlange model than the Green-Ampt model (e.g., Fuentes et al., 1992; Barry et al., 1995c; Ross et al., 1996; Clausnitzer et al., 1998).

The sorptivity, S, for the Green-Ampt model is given by (e.g., Parlange et al., 1992; Hsu et al., 2002):

$$S = (2K_s \Delta\psi \Delta\theta)^{1/2},\tag{1}$$

where K_s is the saturated hydraulic conductivity, $\Delta\psi$ is the difference between the head at the surface (where water is ponded) and that at the wetting front, and $\Delta\theta$ is the difference between the saturated and initial moisture contents.

The implicit Green-Ampt equation can be written as an infinite series in powers of $t^{1/2}$, where t is time. The first two terms of this series gives the approximation (e.g., Haverkamp et al., 1990; Barry et al., 1995d):

$$I_{GA}(t) \approx St^{1/2} + 2K_s t/3, \tag{2}$$

where I is the cumulative infiltration and the subscript GA refers to Green-Ampt. For the parameter values used by Mailapalli et al. (2009), we find that the magnitude of the second term on the right side of Eq. (2) is much less than first for both the dry and wet soils so long as the condition

$$t \ll 1.5 \times 10^4 \text{ min} \tag{3}$$

holds. The experimental data sets analyzed by Mailapalli et al. (2009) range up to about 10^2 min, and so for both data sets the approximation in Eq. (2) is reasonable. For the Talsma-Parlange infiltration equation, the short-time expansion corresponding to Eq. (2) is:

$$I_{TP}(t) \approx St^{1/2} + K_s t/3, \tag{4}$$

where the subscript TP refers to Talsma-Parlange. The factor ½ difference in the second term on the right hand sides of Eqs. (2) and (4) provides support for the statement of Mailapalli et al. (2009) that the "value of K_e can be taken as 0.5 times the value of the saturated hydraulic conductivity, K_s", where K_e is the soil's effective conductivity. That is, replacing K_s by $K_s/2$ in Eq. (2) effectively reduces it to the more accurate Eq. (4).

The initial condition for the Green-Ampt model is $I_{GA}(0) = 0$. The condition cannot be used directly in the numerical scheme of Mailapalli et al. (2009) as this would result in division by zero (see their Eqs. 6 and 7). Mailapalli et al. (2009) thus took a non-zero value for $I_M(0)$, i.e., $I_M(0) = 0.0001$ m, where the subscript M refers to the numerical solution given by the authors. They found that this value gave better agreement with the experiments than the other results. However had they started their numerical scheme with values con-

sistent with Eqs. (2)or (4), their results would have been essentially the same as the others. In applying their scheme, it would be useful, if possible, to determine a priori the starting value giving the best agreement with experiments.

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