

# Design of Collision Avoidance System for a Chicken Robot Based on Fuzzy Relation Equations

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**Abstract**—Design and study of mixed animal-robot societies are the fields of scientific exploration that can bring new opportunities for research into the group behavior of social animals. Our goal is to develop a Chicken Robot – an autonomous mobile robot, socially acceptable by a group of chicks and able to interact with them using appropriate communication channels. One of the basic requirements to such a robot is the safety of its motion with respect to the chicks, so it has to be endowed with an efficient real-time collision avoidance system. In this paper we present a fuzzy obstacle avoidance system that was designed for the Chicken Robot using the theory of fuzzy relation equations. This approach allows to easily check a consistency of a used rule base and provides a more systematic approach to design of fuzzy control systems comparing with the classical techniques. The experimental results demonstrate that a mobile robot equipped with the presented system is able to successfully avoid obstacles and safely navigate on an experimental arena.

## I. INTRODUCTION

The goal of the Chicken Robot project is to develop a mobile robot able to interact with a group of domestic chicks using communication channels relevant for them, and to control specific group behaviors. A mobile robot capable to reply to an animal and to adapt to an animal's behavior offers a promising opportunity to ethology. It can be used in purely scientific experiments to study animal social behavior as well as in farming applications to improve breeding conditions of intensively produced animals.

Design of such a robot and its control system is also an interesting challenge for robotics. There are very few projects in the animal-robot interaction field [1]–[5]. In our research we follow a general methodology developed within the European project Leurre, where a mixed society of cockroaches and robots was created and successfully controlled [5]. We apply and modify this methodology for a case of more complex social animals such as chickens.

In order to overcome the problem of social recognition and acceptance of the robot by chicks we use the mechanism of the filial imprinting [6]. Once the robot is presented to very young chicks, they consider it as their hen and then will follow it, forming a stable group.

The young chicks have fragile bodies, hence one of the main requirements to the Chicken Robot is the safety of its

displacements with respect to chicks, i.e. a collision risk must be low and the eventual effects of collision be harmless.

A number of reactive navigation systems have been implemented in robotics. Among them the *potential field* method, where a robot follows to a superposed force field from obstacles and the goal [7]; the *dynamic window approach* that takes into account dynamics of a robot and deals with constraints imposed by robot's limited velocities and accelerations [8]; and the *vector field histogram* method [9], where a motion command is deduced from the polar histogram of obstacles density constructed from the current sensor readings. Finally, a number of mobile navigation approaches based on the principles of fuzzy control were proposed [10]–[12].

In this work we also take a fuzzy control based approach to the task of collision avoidance. In comparison with other techniques the fuzzy control has such advantages as use of expert's heuristic knowledge in a natural linguistic form and ability to provide a satisfactory performance in case of noisy input. This makes the fuzzy control a good choice for the needs of autonomous robotics, where a mathematical model of the environment is often not available, sensory data are noisy and real-time operation is necessary [11].

The conventional way to construct fuzzy control systems is to determine the relationship between input and output sets in form of fuzzy rules and then to use the resulted rule base directly to compute the output of the system. Being in the common use, this approach by default assumes that all of the rules create a consistent control algorithm that is not always the case; design of an efficient control system demands an intense tuning of the system parameters via trial-and-error adjustment. An alternative way is to represent the knowledge about the mapping of the input space to the output space contained in the rule base in the form of a single fuzzy relation by applying the *fuzzy relation equations* theory. Initially proposed by Sanchez [13] in 1976 these equations have been intensively studied since then [14]–[21] and found their applications in different fields [22]–[24]. Use of fuzzy relation equations allows to easily check the consistency of the rule base and provides a more systematic approach to design of the fuzzy control system comparing with other approaches [22].

In this paper we present a fuzzy collision avoidance system designed for the Chicken Robot using the theory of fuzzy relation equations. We represent a relationship between an input set of robot's sensor values and an output set of steering directions in the form of a fuzzy rule base. Then we prove the consistency of the rule base and find a mathematical

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representation of the knowledge that it contains in the form of a fuzzy relation  $R$  by solving a corresponding fuzzy relation equation. The fuzzy relation  $R$  is further used to compute a robot's steering angle allowing to avoid detected obstacles. To validate efficiency of the proposed obstacle avoidance system we conduct experiments with a real robot.

The paper is organized as follows. Section II presents an overview of the Chicken Robot and its control system. Section III explains how the obstacle avoidance system works and discusses efficiency issues, followed by the experimental results in Section IV.

## II. THE CHICKEN ROBOT AND ITS CONTROL SYSTEM

### A. The Chicken Robot

In animal-robot experiments we use a modification of a marXbot robot (Modular All-teRrain eXperimentation roBOT) developed in our group (Figure 1a). It is a modular mobile robot that can be expanded with different extensions supporting specific mechanical or electronic features. A base module of the marXbot (Figure 2) guarantees a stable contact with the ground and a remarkable maneuverability. It consists of two treels (tracks plus wheels) with two motors; 24 infrared proximity sensors around the robot used as bumpers; twelve infrared ground and close ground sensors; three dsPIC33 microcontrollers, one for each motor and one to manage the sensors; and space for a battery pack. The infrared bumpers have a distance range about 3 cm.

We extended the base module by adding a speaker module with a Sony Ericsson MBS-100 Bluetooth speaker used to improve quality of the imprinting and a top board with color markers (Figure 1b). The markers are used in the off-line analysis of recorded animal-robot experiments video data to determine the position and orientation of the robot.

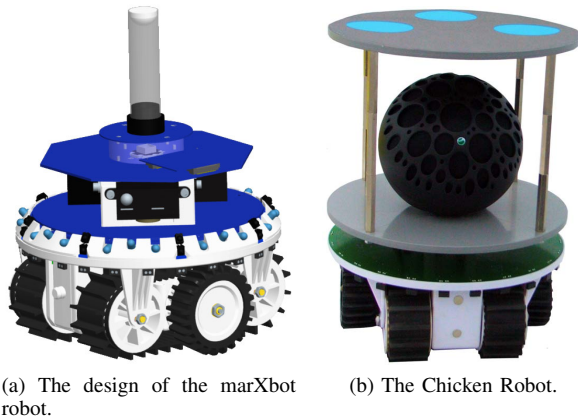


Fig. 1: The design of the marXbot robot and its Chicken Robot modification.

The Chicken Robot has a size of an adult chicken, energy autonomy up to several hours and Bluetooth connectivity.

### B. The robot's behavior

The control architecture of the Chicken Robot is a behavior-based controller [25]. It consists of a collection of

behaviors arranged in a hierarchy where behaviors on higher levels integrate behaviors on lower levels (Figure 3). The highest level behaviors are manually activated through the GUI on the user PC that has a Bluetooth connection with the robot; the behaviors of the second level work on the robot's microcontroller managing sensors.

When a *user control* behavior is activated, a user remote controls robot's displacements through the Bluetooth connection. If a *wandering* behavior is activated, the robot executes a forward move till sensors detect any obstacle. Based on the number of adjoining sensors detecting the obstacle the robot can distinguish two types of obstacles – walls and not walls. If the obstacle is a wall, the robot turns away. The walls avoidance algorithm is simple, the robot goes away from the wall with a turn-away angle equal to an angle of incident.

If the obstacle is not a wall, then the fuzzy collision avoidance system is activated. Based on the positions of obstacles it generates a new heading angle. When the obstacle is not detected any more, odometry data are used to return the robot to the original direction and it continues to move forward.

This behavioral model can be expanded by more complex behaviors depending on results of biological experiments. To combine behaviors we plan to apply the multivalued behavior control approach [10], [26].



Fig. 2: The base module of the marXbot robot.

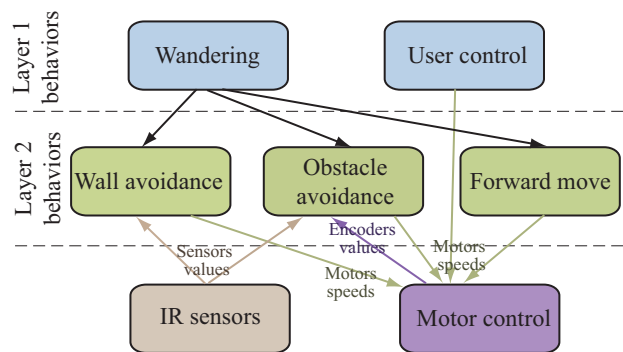


Fig. 3: An overview of the behavioral architecture of the Chicken Robot.

## III. THE OBSTACLE AVOIDANCE METHOD

### A. The method's description

To design the fuzzy control system responsible for obstacle avoidance we use the approximate reasoning method. We first

construct fuzzy partitions of an input and an output sets, and then determine a relationship between elements of the fuzzy partitions in the form of IF-THEN rules. This relationship is further used to determine the mapping of the input to the output space in the form of a single fuzzy relation.

An input set  $S$  is a finite set of infrared sensors of the Chicken Robot. To decrease computational load on the system we reduced the cardinality of  $S$  by grouping sensors in pairs (Figure 4a), thus  $|S| = 12$ . We chose an output set  $F$  to be a set of forbidden directions of travel, i.e. directions that lead to a collision with an obstacle. We also reduced its cardinality by partitioning a  $360^\circ$  range into sectors of  $30^\circ$  each (Figure 4b),  $|F| = 12$ .

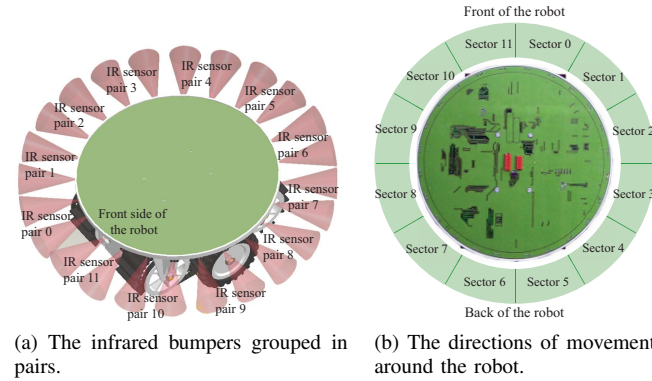


Fig. 4: The input and the output sets of the obstacle avoidance system.

A family of fuzzy subsets of  $X$   $\mathcal{A}^n = \{A_1, A_2, \dots, A_n\}$  is called a *fuzzy partition* of  $X$  iff

$$\forall i, j \in \{1, \dots, n\} (i \neq j \Rightarrow ((A_i \neq A_j) \wedge (0 < \sum_{k=1}^m A_i(x_k) < m)))$$

where  $m$  is a cardinality of  $X$  [14].

The Figures 5-6 represent the fuzzy partitions  $\mathcal{O}^4$  and  $\mathcal{D}^4$  of sets  $S$  and  $F$  correspondingly that we constructed for our fuzzy inference system. For simplicity reasons we restricted the number of elements in each of two partitions to four. We used triangular and trapezoid membership functions.

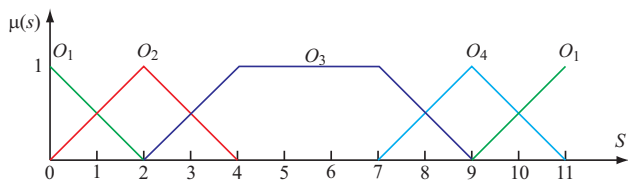


Fig. 5: The fuzzy sets combining the fuzzy partition  $\mathcal{O}^4$ .

Fuzzy sets of  $\mathcal{O}^4$  have the following linguistic representation:

- $O_1$  – an obstacle is detected in front of the robot;
- $O_2$  – an obstacle is detected in the direction of the sensor pair 2;
- $O_3$  – an obstacle is detected behind the robot;
- $O_4$  – an obstacle is detected in the direction of the sensor pair 10;

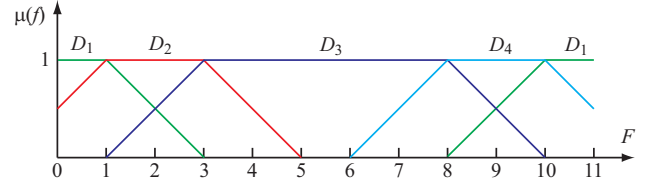


Fig. 6: The fuzzy sets combining the fuzzy partition  $\mathcal{D}^4$ .

while the fuzzy sets of  $\mathcal{D}^4$  have the following linguistic representation:

- $D_1$  – a movement in directions of sectors 0, 1, 10 and 11 is disallowed;
- $D_2$  – a movement in directions of sectors 1-3 is disallowed;
- $D_3$  – a movement in directions of sectors 3-8 is disallowed;
- $D_4$  – a movement in direction of sectors 8-10 is disallowed.

The wideness of the fuzzy set  $O_3$  and hence  $D_3$  is explained by the fact that  $O_3$  corresponds to obstacles behind the robot that are less important than obstacles close to the front of the robot.

After the fuzzy partitions on the input and the output sets are chosen, the next step is to establish a relationship between them. Since the elements of  $\mathcal{O}^4$  and  $\mathcal{D}^4$  have a clear linguistic meaning, empirical linguistic rules can be used in order to describe this relationship. Combining rules with the generalized modus ponens rule of inference, we obtain an approximate reasoning scheme for our obstacle avoidance system in the following form:

- |              |                 |                   |
|--------------|-----------------|-------------------|
| RULE 1:      | IF $o$ IS $O_1$ | THEN $d$ IS $D_1$ |
| RULE 2:      | IF $o$ IS $O_2$ | THEN $d$ IS $D_2$ |
| RULE 3:      | IF $o$ IS $O_3$ | THEN $d$ IS $D_3$ |
| RULE 4:      | IF $o$ IS $O_4$ | THEN $d$ IS $D_4$ |
| ANTECEDENT:  | $o$ IS $O$      |                   |
| CONSEQUENCE: | $d$ IS $D$ .    |                   |
- (1)

A mathematical representation of the procedure (1) can be obtained by applying the compositional rule of inference. If we define knowledge contained by the rule base as a fuzzy relation  $R : S \times F \rightarrow [0, 1]$  and  $\mathbf{R}_{12 \times 12}$  is a matrix form of  $R$ , then any input  $o$  (a fuzzy subset on  $S$ ) can be represented by a vector  $\mathbf{o} = [O(s_1), O(s_2), \dots, O(s_{12})]$  and an output  $\mathbf{d} = [D(f_1), D(f_2), \dots, D(f_{12})]$  is given by

$$\mathbf{d} = \mathbf{o} \circ \mathbf{R},$$

where  $\circ$  denotes a *max-T* composition with  $T$  being a continuous  $T$ -norm. In this work we use  $\min(a, b)$   $T$ -norm.

The main problem here is to find the fuzzy relation  $R$ , i.e. the matrix  $\mathbf{R}$ . The conventional approach suggests to use any well-known fuzzy implication to represent each rule [27]. An alternative way is to solve a system of fuzzy relation equations, where each equation represents one rule  $O_i \rightarrow D_i, i \in \{1, 2, 3, 4\}$ :

$$\begin{cases} \mathbf{d}_1 = \mathbf{o}_1 \circ \mathbf{R} \\ \mathbf{d}_2 = \mathbf{o}_2 \circ \mathbf{R} \\ \mathbf{d}_3 = \mathbf{o}_3 \circ \mathbf{R} \\ \mathbf{d}_4 = \mathbf{o}_4 \circ \mathbf{R}, \end{cases} \quad (2)$$

where  $\mathbf{o}_i = [O_i(s_1), O_i(s_2), \dots, O_i(s_{12})]$  and  $\mathbf{d}_i = [D_i(f_1), D_i(f_2), \dots, D_i(f_{12})]$ ,  $i \in \{1, 2, 3, 4\}$ .

The selection of one of these two approaches depends on the application and cannot be predetermined, but the fuzzy relation equations based approach has such advantages as theoretical soundness and clearly defined properties [15], providing reliable models of processes and making the whole design process more consistent and systematic. Its main disadvantage is the potential nonexistence of the solution of (2). However in last decades a variety of necessary and sufficient conditions of the solution existence were derived [16], [20] as well as methods allowing to increase the solvability of equations by modifying a rule base [17] and techniques to find approximate solutions [18], [19].

A rule base and a fuzzy inference system using this rule base are said to be *consistent* when there is a common solution for all the equations of (2).

Let an antecedents matrix  $\mathbf{O}$  is defined as

$$\mathbf{O} = \begin{bmatrix} \mathbf{o}_1 \\ \mathbf{o}_2 \\ \mathbf{o}_3 \\ \mathbf{o}_4 \end{bmatrix} = \begin{bmatrix} O_1(s_1) & O_1(s_2) & \dots & O_1(s_{12}) \\ O_2(s_1) & O_2(s_2) & \dots & O_2(s_{12}) \\ O_3(s_1) & O_3(s_2) & \dots & O_3(s_{12}) \\ O_4(s_1) & O_4(s_2) & \dots & O_4(s_{12}) \end{bmatrix},$$

and a consequences matrix  $\mathbf{D}$  is defined as

$$\mathbf{D} = \begin{bmatrix} \mathbf{d}_1 \\ \mathbf{d}_2 \\ \mathbf{d}_3 \\ \mathbf{d}_4 \end{bmatrix} = \begin{bmatrix} D_1(f_1) & D_1(f_2) & \dots & D_1(f_{12}) \\ D_2(f_1) & D_2(f_2) & \dots & D_2(f_{12}) \\ D_3(f_1) & D_3(f_2) & \dots & D_3(f_{12}) \\ D_4(f_1) & D_4(f_2) & \dots & D_4(f_{12}) \end{bmatrix}.$$

Then the system of equations (2) can be rewritten in the form of one fuzzy relation equation

$$\mathbf{O} \circ \mathbf{R} = \mathbf{D}. \quad (3)$$

Thus the rule base is consistent iff equation (3) is solvable.

For the rule base that we designed for our obstacle avoidance system the antecedents and consequences matrices have the following forms:

$$\mathbf{O} = \begin{bmatrix} 1 & 1/2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1/2 & 1 \\ 0 & 1/2 & 1 & 1/2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1/2 & 1 & 1 & 1 & 1 & 1/2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1/2 & 1 & 1/2 & 0 \end{bmatrix},$$

$$\mathbf{D} = \begin{bmatrix} 1 & 1 & 1/2 & 0 & 0 & 0 & 0 & 0 & 0 & 1/2 & 1 & 1 \\ 1/2 & 1 & 1 & 1 & 1/2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1/2 & 1 & 1 & 1 & 1 & 1 & 1/2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1/2 & 1 & 1 & 1 & 1/2 \end{bmatrix}.$$

For these given values of  $\mathbf{O}$  and  $\mathbf{D}$  equation (3) is solvable as the following solvability criteria holds true: if the input fuzzy sets  $A_i$  are semi-overlapped, satisfying the conditions

$$\text{height}(A_i \wedge A_{i+1}) = 1/2 \text{ and } \forall x \in X \sum_i A_i(x) = 1,$$

where  $A_i$  and  $A_{i+1}$  are adjacent normal fuzzy sets, then equation  $\mathbf{A} \circ \mathbf{R} = \mathbf{B}$  can be exactly solved [28].

If a fuzzy relation equation is solvable, a number of numerical methods can be used to find its solution set [21], [29], [30]. We found  $\mathbf{R}$  as

$$\mathbf{R} = \min_{\forall_i} (\mathbf{o}_i \varphi \mathbf{d}_i), \quad (4)$$

where  $\varphi$  is a residual implication to a  $T$ -norm used in (3). For the min  $T$ -norm

$$\varphi(x, y) = \begin{cases} 1 & x \leq y \\ y & x > y \end{cases}.$$

Equation (4) gives a maximum solution of the equation (3) [31]. In our case

$$\mathbf{R} = \begin{bmatrix} 1 & 1 & 1/2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1/2 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1/2 & 1 & 1 & 1 & 1/2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1/2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1/2 & 0 & 0 \\ 0 & 0 & 1/2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1/2 & 0 & 0 \\ 0 & 0 & 1/2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1/2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1/2 & 1 & 1 & 1 & 1/2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1/2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1/2 & 1 & 1 \end{bmatrix}.$$

After  $\mathbf{R}$  is one time computed, we can use it in the control system to calculate the output. The proposed obstacle avoidance system is presented on Figure 7. The fuzzification procedure computes the input vector  $\mathbf{o}$  using the following formula

$$o_i = \frac{v_i - T}{V_{max} - T},$$

where  $v_i$  – proximity value from the  $i$ -th sensor pair (we take the largest of two sensors' values),  $V_{max}$  is the known largest proximity value and  $T$  is the obstacle avoidance activation threshold. The disallowed directions are calculated as

$$\mathbf{d} = \mathbf{o} \circ \mathbf{R}.$$

To find the steering angle we calculate an allowed direction fuzzy subset as  $\bar{\mathbf{d}} = 1 - \mathbf{d}$  and then use the centroid of the largest area defuzzification method [10]. The resulted steering angle is sent to microcontrollers responsible for the motor control, where it is translated to the left and the right motor speed values.

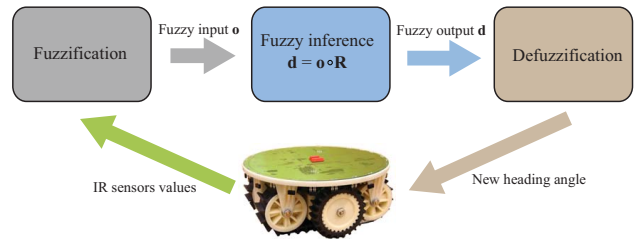


Fig. 7: The overview of the obstacle avoidance system.

### B. Computational complexity estimation

We will show that the use of the fuzzy relation  $R$  in the inference process allows to reduce the computational load comparing with the conventional approach to fuzzy control. Let  $N$  be the cardinality of the input set,  $M$  be the cardinality of the output set,  $\mathbf{o}' = [O'(s_1), \dots, O'(s_N)]$  be an input vector, and vectors  $\mathbf{o}_k$  and  $\mathbf{d}_k$  represent the left and the right parts of the  $k$ -th rule, then the output vector



field; its another problem is that arbitration of behaviors by vector addition can result in the command, which does not satisfy to any of the contributing behaviors. For example, a robot cannot pass through closely spaced obstacles. The fuzzy behaviors based method does not have this limitation, but similar to the potential field approach it allows to combine different behaviors to carry out complex tasks [33]. At the same time the vector field histogram method is able to produce shorter while less smooth paths than the fuzzy behaviors based one does [34]. **Therefore**, the approach based on the fuzzy behaviors is an optimal choice for the reactive mobile navigation system in the case when the flexibility to add new behaviors is demanded.

In our work several issues still remain open for improvement. The rule base that we employ is rather simple, using it we intended to test and illustrate the proposed approach. In future we are planning to expand the rule base, providing more detailed relationship between the input and output sets, and also to replace the current wall avoidance system by the fuzzy one, using the same design approach. Finally, the evaluation of our collision avoidance system on real animals in animal-robot experiments will be a part of the future work.

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