# The Tactical Berth Allocation Problem with Quay Crane Assignment and Transshipment-related Quadratic Yard Costs

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### **Outline**

- Introduction
- Container Terminal Operations
- Berth Allocation Problem: Tactical vs Operational
- Tactical Berth Allocation with Quay Crane Assignment: MIQP / MILP models
- Computational preliminary results
- Final remarks



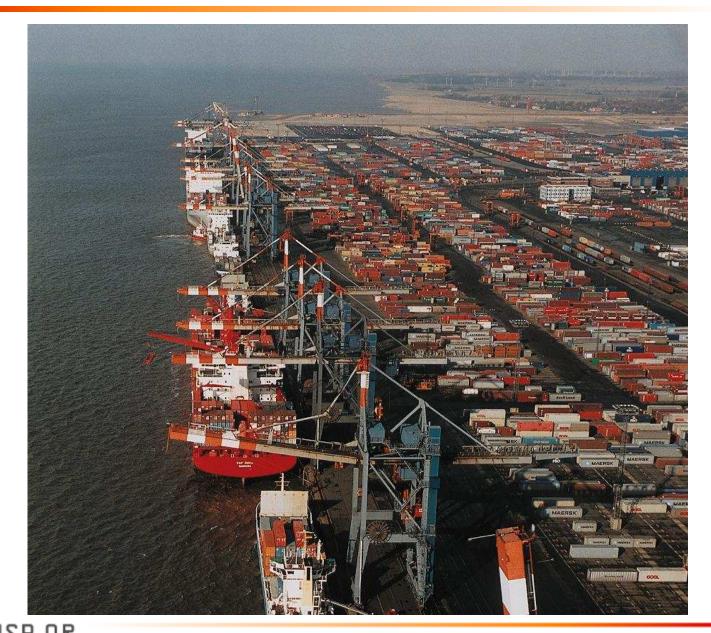
### Introduction

- Crucial role of *maritime transport* in the exchange of goods
- Growth of container traffic worldwide

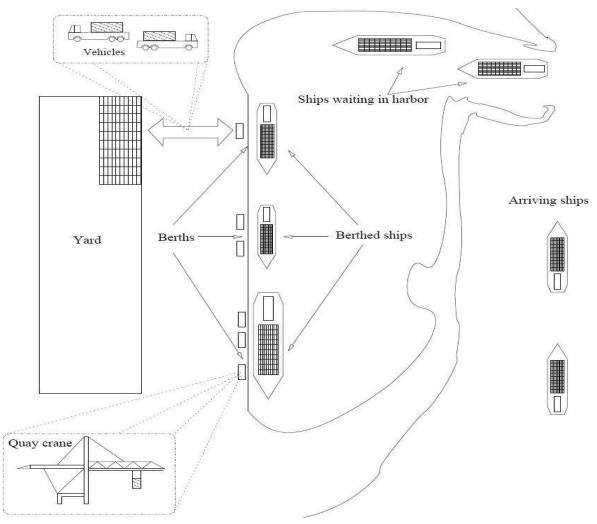
Worldwide	2005	2006	2007
1 Singapore	23,190,000	24,800,000 (+6.94%)	27,932,000 (+12.63%)
2 Shanghai	18,084,000	21,700,000 (+20.00%)	26,150,000 (+20.51%)
3 Hong Kong	22,602,000	23,230,000 (+2.78%)	23,881,000 (+2.80%)
Europe	2005	2006	2007
1 Rotterdam	9,287,000	9,690,000 (+4.34%)	10,790,000 (+11.35%)
2 Hamburg	8,087,550	8,861,804 (+9.57%)	9,900,000 (+11.72%)
3 Antwerp	6,482,030	7,018,799 (+8.28%)	8,176,614 (+16.50%)



### **Container terminal overview**



# **Container terminal operations**







### **Motivation**

### **Focus on Transshipment**

 Collaboration with Medcenter Container Terminal (MTC), port of Gioia Tauro, Italy.

#### **Context**

- Hub-and-spoke
- Mother vessels and feeders
- Terminal operations
  - Berth Allocation Problem (BAP)
  - Quay Crane Assignment Problem (QCAP)

### **Approach**

Tactical viewpoint: support the terminal in the negotiation with shipping lines.



### The Berth Allocation Problem (BAP)

#### Aim

Assign and schedule incoming ships to berthing positions

#### **Constraints**

- Depth of the water (allowable draft)
- Distance from the most favorable location
- Time windows on completion time
- Handling times depend on berthing point and on the number of QCs assigned

#### Standard scenario

QCAP solved before BAP

We remark that this approach is not efficient, because terminal resources are not taken into account in an integrated way.



### **Operational vs Tactical BAP**

### **Operational BAP**

 The objective is to comply with a predetermined plan (in terms of expected handling times and favourite berths) as much as possible.

#### **Tactical BAP**

- The template used at the operational level is determined at the tactical decision level.
- In addition to favourite berthing positions, the concept of quay cranes
   assignment profile, i.e. the number of QCs per shift assigned to a vessel, is
   used to determine expected handling times.
- Service levels are negotiated with shipping lines at this stage.



### **BAP & QCAP: literature review**

### **Operational BAP + QCAP**

- Park & Kim (2003)
- Meisel & Bierwirth (2006, 2008)
- Imai et al. (2008)

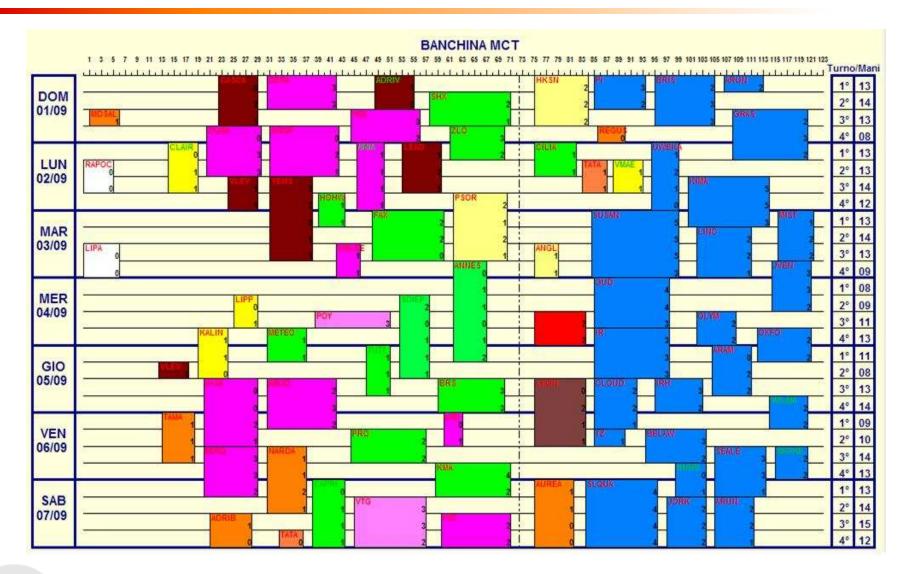
### Tactical BAP (with no QCAP)

- Moorthy & Teo (2006)
- Cordeau et al. (2007)





### **Berth Allocation Plan**







### **Berth Allocation Plan**

	h=1	h=2	h=3	h=4	h=5	h=6	h=7	h=8
		Ship 1			5	Shi	p 2	
berth 1	3	2	2		4	4	5	5
		Shi	Ship 3 S				Ship 4	L
berth 2		4	5			3	3	3
			Ship 5					
berth 3	8		3	3	3	2	2	
QCs TOT	3	6	10	3	7	9	10	8



# TBAP with QCs assignment

### **Combination of 2 decision problems**

- Berth Allocation Problem (BAP)
- Quay-Cranes Assignment Problem (QCAP)

#### **Tactical decision level**

 the amount of quay crane hours is negotiated months in advance with shipping lines

#### **Issues**

- the chosen profile determines the ship's handling time and thus impacts on the scheduling;
- feasible profiles can vary in length (number of shifts dedicated to the ship) and in size (number of QCs dedicated to the ship in each active shift).



# TBAP with QCs assignment

#### **Find**

- A berth allocation
- A schedule
- A quay crane assignment

#### Given

- Time windows on availability of berths
- Time windows on arrival of ships
- Handling times dependent on QC profiles
- Values of QC profiles

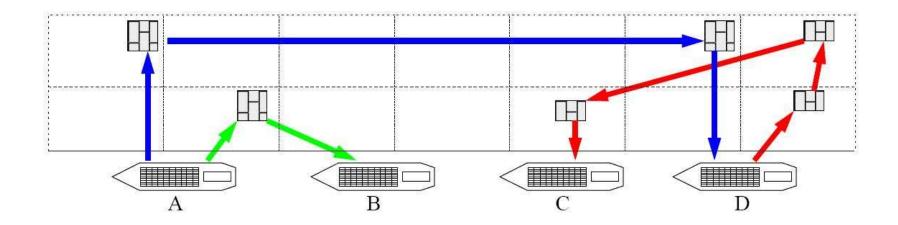
### Aiming to

- Maximize total value of QC assignment
- Minimize housekeeping costs of transshipment flows between ships



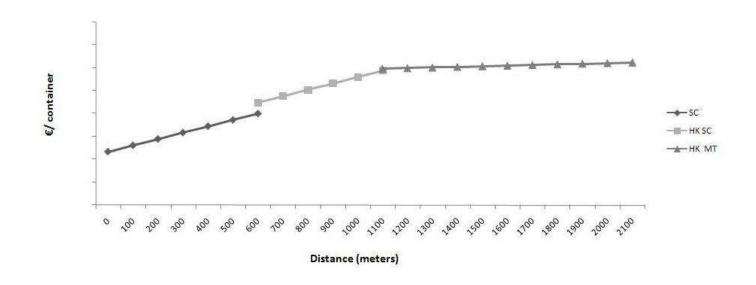
### Housekeeping yard costs

- the analysis refers to the Medcenter Container Terminal
- transshipment context
- the cost function depends on the distance between the incoming and the outgoing berths





### Housekeeping yard costs



Piecewise linear function depending on the distance and on the type of carrier used:

- < 600m : no housekeeping, straddle carriers</li>
- 600 1100 m: housekeeping, straddle carriers
- > 1100 m : housekeeping, multi-trailer



- N = set of vessels;
- M = set of berths;
- H = set of time steps (each time step  $h \in H$  is submultiple of the work shift length);
- $S = \text{set of the time step indexes } \{1, ..., \bar{s}\}$  relative to a work shift; ( $\bar{s}$  represents the number of time steps in a work shift);
- $H^s$  = subset of H which contains all the time steps corresponding to the same time step  $s \in S$  within a work shift;
- $P_i^s$  = set of feasible QC assignment profiles for the vessel  $i \in N$  when vessel arrives at a time step with index  $s \in S$  within a work shift;
- $P_i$  = set of quay crane assignment profiles for the vessel  $i \in N$ , where  $P_i = \bigcup_{s \in S} P_i^s$ ;



- $t_i^p$  = handling time of ship  $i \in N$  under the QC profile  $p \in P_i$  expressed as multiple of the time step length;
- $v_i^p$  = the value of serving the ship  $i \in N$  by the quay crane profile  $p \in P_i$ ;
- $q_i^{pu}$  = number of quay cranes assigned to the vessel  $i \in N$  under the profile  $p \in P_i$  at the time step  $u \in (1, ..., t_i^p)$ , where u = 1 corresponds to the ship arrival time;
- $Q^h$  = maximum number of quay cranes available at the time step  $h \in H$ ;
- $f_{ij}$  = flow of containers exchanged between vessels  $i, j \in N$ ;
- $d_{kw}$  = unit housekeeping cost between yard slots corresponding to berths  $k, w \in M$ ;
- $[a_i, b_i]$  = [earliest, latest] feasible arrival time of ship  $i \in N$ ;
- $[a^k, b^k]$  = [start, end] of availability time of berth  $k \in M$ ;
- $[a^h, b^h]$  = [start, end] of the time step  $h \in H$ .



Consider a graph  $G^k = (V^k, A^k) \ \forall k \in M$ , where  $V^k = N \cup \{o(k), d(k)\}$ , with o(k) and d(k) additional vertices representing berth k, and  $A^k \subseteq V^k \times V^k$ .

- $x_{ij}^k \in \{0,1\} \ \forall k \in M, \, \forall (i,j) \in A^k$ , set to 1 if ship j is scheduled after ship i at berth k;
- $y_i^k \in \{0,1\} \ \forall k \in M, \forall i \in N$ , set to 1 if ship i is assigned to berth k;
- $\gamma_i^h \in \{0,1\} \ \forall h \in H, \forall i \in N$ , set to 1 if ship i arrives at time step h;
- $\lambda_i^p \in \{0,1\}$   $\forall p \in P_i, \forall i \in N$ , set to 1 if ship i is served by the profile p;
- $\rho_i^{ph} \in \{0,1\} \ \forall p \in P_i, \forall h \in H, \forall i \in N$ , set to 1 if ship i is served by profile p and arrives at time step h;
- $T_i^k \ge 0 \ \forall k \in M, \, \forall i \in N$ , representing the berthing time of ship i at the berth k i.e. the time when the ship moors;
- $T_{o(k)}^k \ge 0 \ \forall k \in M$ , representing the starting operation time of berth k i.e. the time when the first ship moors at the berth;
- $T_{d(k)}^k \ge 0 \ \forall k \in M$ , representing the ending operation time of berth k i.e. the time when the last ship departs from the berth.



### **Objective function**

Maximize total value of QC profile assignments + Minimize the (quadratic) housekeeping yard cost of transshipment flows between ships:

$$\max \sum_{i \in N} \sum_{p \in P_i} v_i^p \lambda_i^p - \frac{1}{2} \sum_{i \in N} \sum_{k \in M} y_i^k \sum_{j \in N} \sum_{w \in M} f_{ij} d_{kw} y_j^w \tag{1}$$



### **Berth covering constraints**

$$\sum_{k \in M} y_i^k = 1 \qquad \forall i \in N, \tag{2}$$

#### Flow and linking constraints

$$\sum_{j \in N \cup \{d(k)\}} x_{o(k),j}^k = 1 \qquad \forall k \in M, \tag{3}$$

$$\sum_{i \in N \cup \{o(k)\}} x_{i,d(k)}^k = 1 \qquad \forall k \in M, \tag{4}$$

$$\sum_{j\in N\cup\{d(k)\}}x_{ij}^k-\sum_{j\in N\cup\{o(k)\}}x_{ji}^k=0 \qquad \forall k\in M,\,\forall i\in N, \tag{5}$$

$$\sum_{j \in N \cup \{d(k)\}} x_{ij}^k = y_i^k \qquad \forall k \in M, \, \forall i \in N,$$
 (6)





#### **Precedence constraints**

$$T_i^k + \sum_{p \in P_i} t_i^p \lambda_i^p - T_j^k \le (1 - x_{ij}^k) M \qquad \forall k \in M, \ \forall i \in N, \forall j \in N \cup d(k) \tag{8}$$

$$T_{o(k)}^k - T_j^k \le (1 - x_{o(k),j}^k) M \qquad \forall k \in M, \ \forall j \in N, \tag{8}$$

### **Ship and Berth time windows**

$$a_i y_i^k \le T_i^k \qquad \forall k \in M, \, \forall i \in N,$$
 (9)

$$T_i^k \le b_i y_i^k \qquad \forall k \in M, \, \forall i \in N, \tag{10}$$

$$a^k \le T_{o(k)}^k \qquad \forall k \in M, \tag{11}$$

$$T_{d(k)}^k \le b^k \qquad \forall k \in M, \tag{12}$$





### **Profile covering & linking constraints**

$$\sum_{p \in P_i} \lambda_i^p = 1 \qquad \forall i \in N, \tag{13}$$

$$\sum_{h \in H^s} \gamma_i^h = \sum_{p \in P_i^s} \lambda_i^p \qquad \forall i \in N, \forall s \in S, \tag{14}$$

$$\sum_{k \in M} T_i^k - b^h \le (1 - \gamma_i^h) M \qquad \forall h \in H, \, \forall i \in N, \tag{15}$$

$$a^{h} - \sum_{k \in M} T_{i}^{k} \le (1 - \gamma_{i}^{h})M \qquad \forall h \in H, \, \forall i \in N, \tag{16}$$

$$\rho_i^{ph} \ge \lambda_i^p + \gamma_i^h - 1 \qquad \forall h \in H, \, \forall i \in N, \, \forall p \in P_i, \tag{17}$$

### Quay crane and profile feasibility

$$\sum_{i \in N} \sum_{p \in P_i} \sum_{u = max\{h - t_i^p + 1; 1\}}^{h} \rho_i^{pu} q_i^{p(h - u + 1)} \le Q^h \qquad \forall h \in H^{\bar{s}}$$
 (18)





#### Additional decision variable

 $z_{ij}^{kw} \in \{0,1\} \ \forall i,j \in N, \ \forall k,w \in M$ , set to 1 if  $y_i^k = y_j^w = 1$  and 0 otherwise.

### **Linearized objective function**

$$\max \sum_{i \in N} \sum_{p \in P_i} v_i^p \lambda_i^p - \frac{1}{2} \sum_{i \in N} \sum_{j \in N} \sum_{k \in M} \sum_{w \in M} f_{ij} d_{kw} z_{ij}^{kw}$$
 (19)

#### **Additional constraints**

$$\sum_{k \in K} \sum_{w \in K} z_{ij}^{kw} = g_{ij} \qquad \forall i, j \in N, \tag{20}$$

$$z_{ij}^{kw} \le y_i^k \qquad \forall i, j \in N, \forall k, w \in M \tag{21}$$

$$z_{ij}^{kw} \le y_j^w \qquad \forall i, j \in N, \forall k, w \in M$$
 (22)



### Generation of test instances

- Based on real data provided by MCT
  - container flows
  - housekeeping yard costs
  - vessel's arrival times
- Crane productivity of 24 containers per hours
- Set of feasible profiles synthetically generated:

Class	min QC	max QC	min HT	max HT	volume (min,max)
Mother	3	5	3	6	(1296, 4320)
Feeder	1	3	2	4	(288, 1728)



### Generation of test instances

- 24 instances organized in 3 classes: E (easy), M (medium) and D (difficult)
  - Class E: 9 instances, 10 ships, 3 berths, 8 QCs
  - Class M: 9 instances, 20 ships, 5 berths, 13 QCs
  - Class D: 6 instances, 30 ships, 5 berths, 13 QCs
- Different traffic volumes in scenarios A, B, C
- Each scenario is tested with a set of  $\bar{p}=10,20,30$  feasible profiles for each ship

MIQP and MILP formulations tested with CPLEX 10.2 on an Intel 3GHz workstation



### **Numerical results**

- Class E: always solved at optimality (MILP 8/9, MIQP 4/9) or near-optimality
- Class M and D: even a feasible solution is hardly found (MILP finds one feasible solution for class M)
- As expected:
  - the quadratic term in the objective function adds complexity (comparison with MaxTotalValue formulation)
  - the higher the number of feasible profiles, the higher complexity
- Interesting findings:
  - MILP provides better bounds than MIQP
  - MIQP seems to be independent from time granularity
  - Symmetry in the problem



### **Conclusions and future work**

#### Contribution

- Tactical viewpoint: Integration between BAP and QCAP
- QC profiles
- Analysis of yard costs
- MIQP/MILP models
- Preliminary numerical results

### **Forthcoming**

- Heuristics
- Decomposition methods
- Analysis of value functions for QC profiles

