The Tactical Berth Allocation Problem with Quay-Crane Assignment and Transshipment Quadratic Costs

Models and Heuristics

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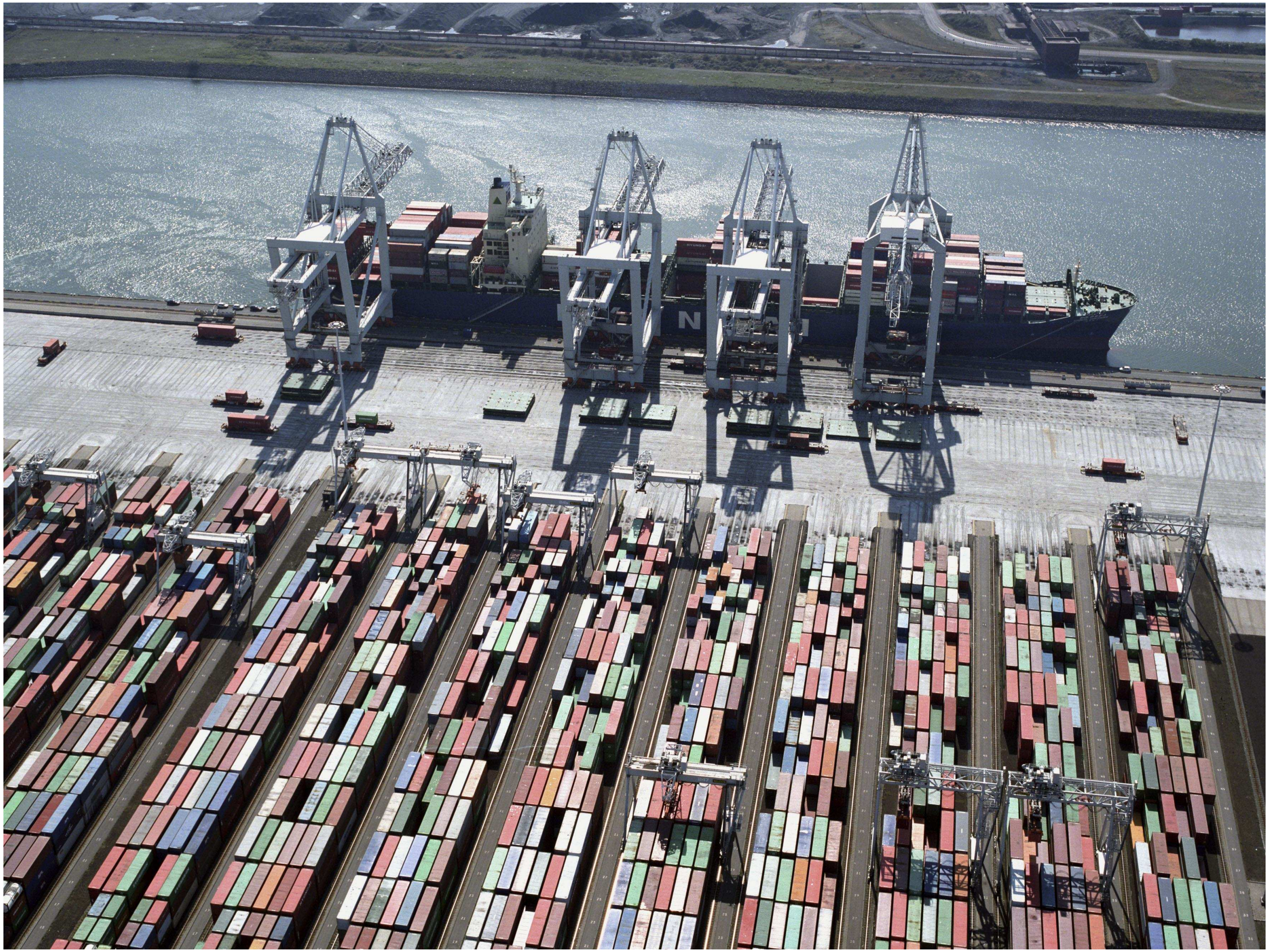
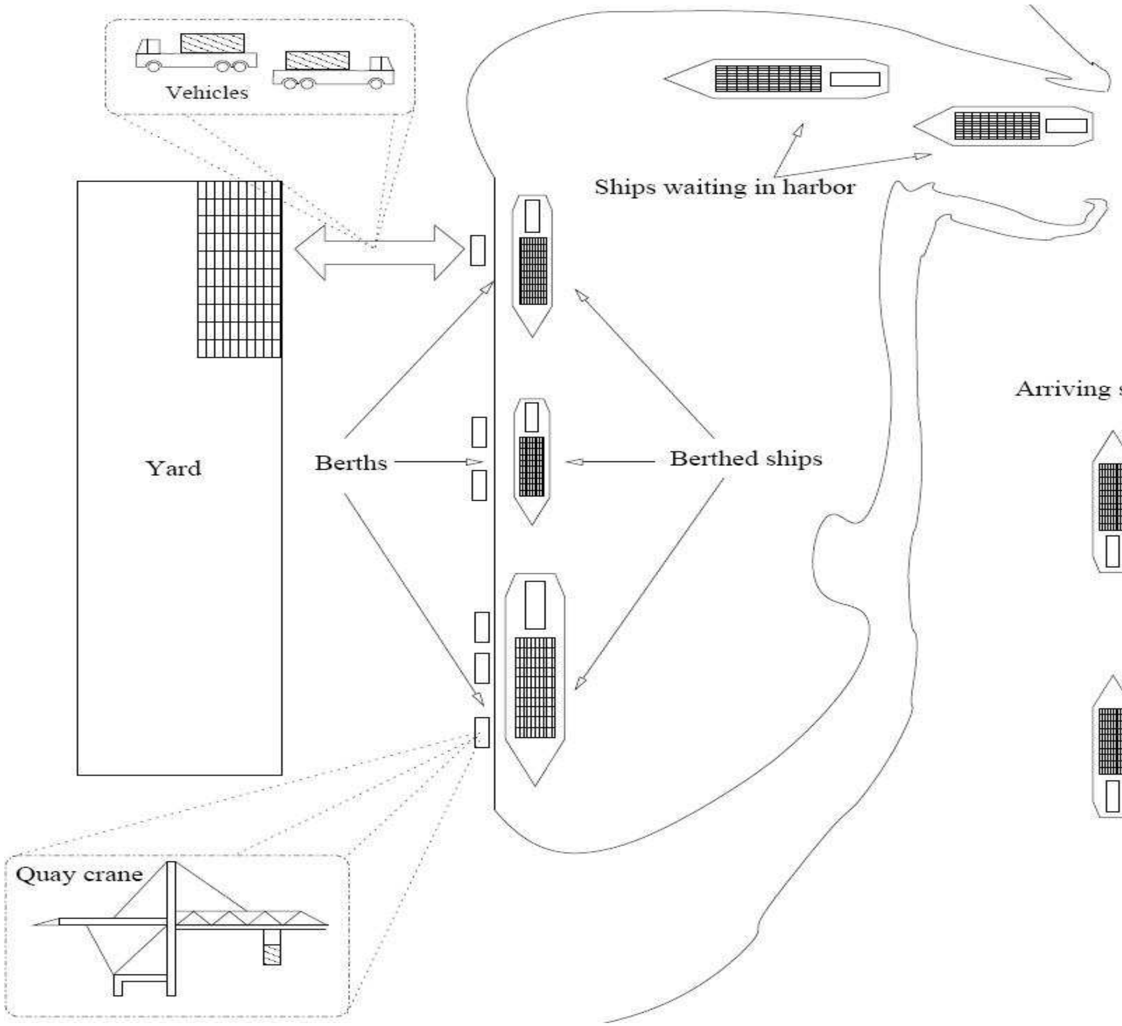
Outline

- Container terminals
- Tactical Berth Allocation Problem (TBAP) with Quay Crane Assignment
- MILP and MIQP models
- Heuristics for TBAP: Tabu Search & Math Programming
- Computational results
- Conclusions & future work

Context: container terminals



Container terminal operations



Tactical Berth Allocation with QCs Assignment

Giallombardo, Moccia, Salani and Vacca (2008)

Problem description

- *Tactical Berth Allocation Problem* (TBAP): assignment and scheduling of ships to berths, according to time windows for both berths and ships; tactical decision level, w.r.t. negotiation between terminal and shipping lines;
- *Quay-Cranes Assignment Problem* (QCAP): a quay crane (QC) profile (number of cranes per shift, ex. 332) is assigned to each ship;
- *Quadratic Yard Costs*: take into account the exchange of containers between ships, in the context of transshipment container terminals.

Issues

- the chosen profile determines the ship's handling time and thus impacts on the scheduling;
- feasible profiles can vary in length (number of shifts dedicated to the ship) and in size (number of QCs dedicated to the ship in each active shift).

Tactical Berth Allocation with QCs Assignment

Find

- a berth allocation
- a schedule
- a quay crane assignment

Given

- time windows on availability of berths
- time windows on arrival of ships
- *handling times dependent on QC profiles*
- values of QC profiles

Aiming to

- maximize total value of QC assignment
- minimize housekeeping costs of transshipment flows between ships

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TBAP with QCs assignment: the model

- N = set of vessels;
- M = set of berths;
- H = set of time steps (each time step $h \in H$ is submultiple of the work shift length);
- S = set of the time step indexes $\{1, ..., \bar{s}\}$ relative to a work shift; ($\bar{s}$ represents the number of time steps in a work shift);
- H^s = subset of H which contains all the time steps corresponding to the same time step $s \in S$ within a work shift;
- P_i^s = set of feasible QC assignment profiles for the vessel $i \in N$ when vessel arrives at a time step with index $s \in S$ within a work shift;
- P_i = set of quay crane assignment profiles for the vessel $i \in N$, where $P_i = \cup_{s \in S} P_i^s$;

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TBAP with QCs assignment: the model

- t_i^p = handling time of ship $i \in N$ under the QC profile $p \in P_i$ expressed as multiple of the time step length;
- v_i^p = the value of serving the ship $i \in N$ by the quay crane profile $p \in P_i$;
- q_i^{pu} = number of quay cranes assigned to the vessel $i \in N$ under the profile $p \in P_i$ at the time step $u \in (1, ..., t_i^p)$, where $u = 1$ corresponds to the ship arrival time;
- Q^h = maximum number of quay cranes available at the time step $h \in H$;
- f_{ij} = flow of containers exchanged between vessels $i, j \in N$;
- d_{kw} = unit housekeeping cost between yard slots corresponding to berths $k, w \in M$;
- $[a_i, b_i]$ = [earliest, latest] feasible arrival time of ship $i \in N$;
- $[a^k, b^k]$ = [start, end] of availability time of berth $k \in M$;
- $[a^h, b^h]$ = [start, end] of the time step $h \in H$.

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TBAP with QCs assignment: the model

Consider a graph $G^k = (V^k,\ A^k)\ \ \forall k \in M$, where $V^k = N \cup \{o(k),\ d(k)\}$, with $o(k)$ and $d(k)$ additional vertices representing berth k, and $A^k \subseteq V^k \times V^k$.

- $x_{ij}^k \in \{0,1\}\ \ \forall k \in M,\ \forall (i,j) \in A^k$, set to 1 if ship j is scheduled after ship i at berth k;
- $y_i^k \in \{0,1\}\ \ \forall k \in M,\ \forall i \in N$, set to 1 if ship i is assigned to berth k;
- $\gamma_i^h \in \{0,1\}\ \ \forall h \in H, \forall i \in N$, set to 1 if ship i arrives at time step h;
- $\lambda_i^p \in \{0,1\}\ \ \forall p \in P_i, \forall i \in N$, set to 1 if ship i is served by the profile p;
- $\rho_i^{ph} \in \{0,1\}\ \ \forall p \in P_i, \forall h \in H, \forall i \in N$, set to 1 if ship i is served by profile p and arrives at time step h;
- $T_i^k \geq 0\ \ \forall k \in M,\ \forall i \in N$, representing the berthing time of ship i at the berth k i.e. the time when the ship moors;
- $T_{o(k)}^k \geq 0\ \ \forall k \in M$, representing the starting operation time of berth k i.e. the time when the first ship moors at the berth;
- $T_{d(k)}^k \geq 0\ \ \forall k \in M$, representing the ending operation time of berth k i.e. the time when the last ship departs from the berth.

TBAP with QCs assignment: the MIQP model

Objective function

Maximize total value of QC profile assignments + Minimize the (quadratic) housekeeping yard cost of transshipment flows between ships:

$$\max \sum_{i \in N} \sum_{p \in P_i} v_i^p \lambda_i^p - \frac{1}{2} \sum_{i \in N} \sum_{k \in M} y_i^k \sum_{j \in N} \sum_{w \in M} f_{ij} d_{kw} y_j^w \qquad (1)$$

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TBAP with QCs assignment: the MIQP model

Berth covering constraints

$$\sum_{k \in M} y_i^k = 1 \qquad \forall i \in N, \tag{2}$$

Flow and linking constraints

$$\sum_{j \in N \cup \{d(k)\}} x_{o(k),j}^k = 1 \qquad \forall k \in M, \tag{3}$$

$$\sum_{i \in N \cup \{o(k)\}} x_{i,d(k)}^k = 1 \qquad \forall k \in M, \tag{4}$$

$$\sum_{j \in N \cup \{d(k)\}} x_{ij}^k - \sum_{j \in N \cup \{o(k)\}} x_{ji}^k = 0 \qquad \forall k \in M, \forall i \in N, \tag{5}$$

$$\sum_{j \in N \cup \{d(k)\}} x_{ij}^k = y_i^k \qquad \forall k \in M, \forall i \in N, \tag{6}$$

TBAP with QCs assignment: the MIQP model

Precedence constraints

$$T_i^k + \sum_{p \in P_i} t_i^p \lambda_i^p - T_j^k \leq (1 - x_{ij}^k)M \qquad \forall k \in M, \forall i \in N, \forall j \in N \cup d(k) \quad (7)$$

$$T_{o(k)}^k - T_j^k \leq (1 - x_{o(k),j}^k)M \qquad \forall k \in M, \forall j \in N, \quad (8)$$

Ship and Berth time windows

$$a_i y_i^k \leq T_i^k \qquad \forall k \in M, \forall i \in N, \quad (9)$$

$$T_i^k \leq b_i y_i^k \qquad \forall k \in M, \forall i \in N, \quad (10)$$

$$a^k \leq T_{o(k)}^k \qquad \forall k \in M, \quad (11)$$

$$T_{d(k)}^k \leq b^k \qquad \forall k \in M, \quad (12)$$

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TBAP with QCs assignment: the MIQP model

Profile covering & linking constraints

$$\sum_{p \in P_i} \lambda_i^p = 1 \qquad \forall i \in N, \tag{13}$$

$$\sum_{h \in H^s} \gamma_i^h = \sum_{p \in P_i^s} \lambda_i^p \qquad \forall i \in N, \forall s \in S, \tag{14}$$

$$\sum_{k \in M} T_i^k - b^h \leq (1 - \gamma_i^h) M \qquad \forall h \in H,\ \forall i \in N, \tag{15}$$

$$a^h - \sum_{k \in M} T_i^k \leq (1 - \gamma_i^h) M \qquad \forall h \in H,\ \forall i \in N, \tag{16}$$

$$\rho_i^{ph} \geq \lambda_i^p + \gamma_i^h - 1 \qquad \forall h \in H,\ \forall i \in N,\ \forall p \in P_i, \tag{17}$$

Quay crane and profile feasibility

$$\sum_{i \in N} \sum_{p \in P_i} \sum_{u = max\{h - t_i^p + 1; 1\}}^{h} \rho_i^{pu} q_i^{p(h-u+1)} \leq Q^h \qquad \forall h \in H^{\bar{s}} \tag{18}$$

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TBAP with QCs assignment: the MILP model

Additional decision variable

$z_{ij}^{kw} \in \{0,1\}\ \forall i,j \in N,\ \forall k,w \in M$, set to 1 if $y_i^k = y_j^w = 1$ and 0 otherwise.

Linearized objective function

$$\max \quad \sum_{i \in N} \sum_{p \in P_i} v_i^p \lambda_i^p - \frac{1}{2} \sum_{i \in N} \sum_{j \in N} \sum_{k \in M} \sum_{w \in M} f_{ij} d_{kw} z_{ij}^{kw} \qquad (19)$$

Additional constraints

$$\sum_{k \in K} \sum_{w \in K} z_{ij}^{kw} = g_{ij} \qquad \forall i,j \in N, \qquad (20)$$

$$z_{ij}^{kw} \leq y_i^k \qquad \forall i,j \in N, \forall k,w \in M \qquad (21)$$

$$z_{ij}^{kw} \leq y_j^w \qquad \forall i,j \in N, \forall k,w \in M \qquad (22)$$

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Generation of test instances

- Based on real data provided by MCT, Port of Gioia Tauro, Italy:
  - container flows
  - housekeeping yard costs
  - vessel's arrival times

- Crane productivity of 24 containers per hours

- Set of feasible profiles synthetically generated, according to ranges given by practitioners:

Class	min QC	max QC	min HT	max HT	volume (min,max)
Mother	3	5	3	6	(1296, 4320)
Feeder	1	3	2	4	(288, 1728)

Generation of test instances

- 18 instances organized in 2 classes:
  - "Easy": 9 instances, 10 ships, 3 berths, 8 QCs
  - "Difficult": 9 instances, 20 ships, 5 berths, 13 QCs

- Different traffic volumes in scenarios A, B, C

- Each scenario is tested with a set of $\bar{p} = 10, 20, 30$ feasible profiles for each ship

MIQP and MILP formulations tested with CPLEX 10.2 on an Intel 3GHz workstation.

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CPLEX results

10 x 3		MILP FORMULATION			MIQP FORMULATION		
Set	$\bar{p}$	OBJ	GAP (%)	CPU (sec)	OBJ	GAP (%)	CPU (sec)
A	10	645995	0	99.07	643871	0.33	3600
A	20	646029	0	2.78	642263	0.59	3600
A	30	641402	0.72	3600	646029	0	1018.26
B	10	387855	0	6.71	387855	0	1008.69
B	20	387855	0	25.92	386252	0.42	3600
B	30	387855	0	1457.3	386252	0.42	3600
C	10	611219	0	16.34	608650	0.42	3600
C	20	611287	0	36.97	611287	0	1018.43
C	30	611287	0	2.08	611287	0	3384.06

CPLEX results

20 x 5		MILP FORMULATION				MIQP FORMULATION			
Set	$\bar{p}$	OBJ	GAP (%)	UB	CPU (sec)	OBJ	GAP (%)	UB	CPU (sec)
A	10	-	∞	1122068	7200	-	∞	1409782	7200
A	20	-	∞	1122807	7200	-	∞	1444628	7200
A	30	-	∞	1122807	7200	-	∞	1498501	7200
B	10	-	∞	843126	7200	-	∞	1088668	7200
B	20	-	∞	843160	7200	-	∞	1117253	7200
B	30	-	∞	843160	7200	-	∞	1158170	7200
C	10	1269372	7.55	1365148	7200	-	∞	1664112	7200
C	20	-	∞	1365697	7200	-	∞	1699890	7200
C	30	-	∞	1365697	7200	-	∞	1744295	7200

Gap ∞: no integer solution has been found by the solver; only UB has been provided.

A New Heuristics for TBAP

Algorithm 1: TBAP Bi-level Heuristics

Initialization : Assign a QC profile to each ship

repeat

1. solve BAP
2. update profiles

until *stop criterion* ;

TBAP Bi-level Heuristics:

1. BAP solution via Tabu Search
2. Profiles' updating via Math Programming

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1. Tabu Search for BAP

Adapted from Cordeau, Laporte, Legato and Moccia (2005).

- New objective function: minimization of yard-related transshipment quadratic costs
- New constraints: QCs availability
- Each solution $s \in S$ is represented by a set of m berth sequences such that every ship belongs to exactly one sequence.
- Penalized cost function:

$$f(s) = c(s) + \alpha_1 w_1(s) + \alpha_2 w_2(s) + \alpha_3 w_3(s)$$

  where $w_1(s)$ is the total violation of ships' TWs, $w_2(s)$ is the total violation of berths' TWs and $w_3(s)$ is the total violation of QCs availability.
- "Move": ship i is removed from sequence k and inserted in sequence $k' \neq k$. The new position in k' is such that $f(s)$ is minimized.
- Initial solution: randomly built assigning ships to berths and relaxing the QCs availability constraint.

2. Profiles' Updating via Math Programming

Basic idea: use information of reduced costs to update the vector of assigned QC profiles in a "smart" way.

- Let $\bar{X} = [\bar{x}, \bar{y}, \bar{T}]$ be the BAP solution found by the Tabu Search for a given QC profile assignment $\bar{\lambda}$.
- We solve the linear relaxation of the TBAP MILP formulation, with the additional constraints:

$$\bar{X} - \epsilon \leq X \leq \bar{X} + \epsilon \qquad (23)$$

$$\bar{\lambda} - \epsilon \leq \lambda \leq \bar{\lambda} + \epsilon \qquad (24)$$

- As suggested by Desrosiers and Lübbecke (2005), the shadow prices of these constraints are the reduced costs of original variables X and λ.
- We identify the λ_i^p variable with the maximum reduced cost and we assign this new profile p to ship i.
- If all reduced costs are ≤ 0, then we stop.

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Computational results

10 x 3		MILP FORMULATION			HEURISTICS		
Set	$\bar{p}$	OBJ	GAP (%)	CPU (sec)	OBJ	GAP (%)	CPU (sec)
A	10	645995	0	99.07	638428	1.17	22
A	20	646029	0	2.78	635693	1.60	53
A	30	641402	0.72	3600	631514	1.54	86
B	10	387855	0	6.71	383730	1.06	22
B	20	387855	0	25.92	382449	1.39	49
B	30	387855	0	1457.3	380200	1.97	80
C	10	611219	0	16.34	605628	0.91	23
C	20	611287	0	36.97	602171	1.49	51
C	30	611287	0	2.08	597833	2.20	85

Stop criterion for the Heuristics: maximum number of iterations ($n \times \bar{p}$).

Computational results

20 x 5		MILP FORMULATION				HEURISTICS		
Set	$\bar{p}$	OBJ	GAP (%)	UB	CPU (sec)	OBJ	GAP (%)	CPU (sec)
A	10	-	∞	1122068	7200	1095720	2.35	166
A	20	-	∞	1122807	7200	1089910	2.93	358
A	30	-	∞	1122807	7200	1077340	4.05	527
B	10	-	∞	843126	7200	821428	2.57	164
B	20	-	∞	843160	7200	818634	2.91	348
B	30	-	∞	843160	7200	812697	3.61	562
C	10	1269372	7.55	1365148	7200	1332990	2.36	160
C	20	-	∞	1365697	7200	1328240	2.74	340
C	30	-	∞	1365697	7200	1324930	2.99	539

Gap ∞: no integer solution has been found by the solver; only UB has been is provided.

Stop criterion for the Heuristics: maximum number of iterations ($n \times \bar{p}$).

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Computational results

10 x 3

- CPLEX solves at optimality and fast;
- Heuristics finds good solutions (gap 1-2%) pretty fast.

20 x 5

- CPLEX cannot provide any feasible integer solution;
- Heuristics finds good solutions (gap 2-4%) pretty fast.

Summing up:

- Heuristics provides satisfactory results in terms of:
  - quality of the solution;
  - speed.

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Conclusions and future work

Contribution

- Integration of two decision problems (BAP and QCAP)
- MIQP/MILP models
- Heuristics

Next steps

- Tests on bigger instances
- Improve quality of the solutions

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Thanks for your attention!

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