The Tactical Berth Allocation Problem with QC Assignment and Transshipment Costs

Models and Heuristics

Ilaria Vacca
Transport and Mobility Laboratory, EPFL

joint work with Giovanni Giallombardo, Luigi Moccia & Matteo Salani

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Outline

- Container terminals
- Tactical Berth Allocation Problem (TBAP) with Quay Crane Assignment
- MILP and MIQP models
- Heuristics for TBAP: Tabu Search & Math Programming
- Computational results
- Conclusions
Context: container terminals
Container terminal operations

- Yard
- Berths
- Berthed ships
- Ships waiting in harbor
- Arriving ships
- Quay crane
- Vehicles
Problem description

- **Tactical Berth Allocation Problem** (TBAP): assignment and scheduling of ships to berths, according to time windows for both berths and ships; tactical decision level, w.r.t. negotiation between terminal and shipping lines;

- **Quay-Cranes Assignment Problem** (QCAP): a quay crane (QC) profile (number of cranes per shift, ex. 332) is assigned to each ship;

- **Housekeeping Quadratic Yard Costs**: take into account the exchange of containers between ships, in the context of transshipment container terminals.
### The concept of QC assignment profile

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<tr>
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<tr>
<td>QCs</td>
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<td>3</td>
<td>7</td>
<td>9</td>
<td>10</td>
<td>8</td>
</tr>
</tbody>
</table>
Transshipment-related housekeeping yard costs

- Vessels A-B: no housekeeping, straddle carriers
- Vessels C-D: housekeeping, straddle carriers
- Vessels A-D: housekeeping, multi-trailers
Tactical Berth Allocation with QCs Assignment

Issues

- the chosen profile determines the ship’s handling time and thus impacts on the scheduling;

- feasible profiles can vary in length (number of shifts dedicated to the ship) and in size (number of QCs dedicated to the ship in each active shift).
Tactical Berth Allocation with QCs Assignment

Find

- a berth allocation
- a schedule
- a quay crane assignment

Given

- time windows on availability of berths
- time windows on arrival of ships
- handling times dependent on QC profiles
- values of QC profiles

Aiming to

- maximize total value of QC assignment
- minimize housekeeping costs of transshipment flows between ships
TBAP with QCs assignment: the model

- $N$ = set of vessels;
- $M$ = set of berths;
- $H$ = set of time steps (each time step $h \in H$ is submultiple of the work shift length);
- $S$ = set of the time step indexes $\{1, \ldots, \bar{s}\}$ relative to a work shift; ($\bar{s}$ represents the number of time steps in a work shift);
- $H^s$ = subset of $H$ which contains all the time steps corresponding to the same time step $s \in S$ within a work shift;
- $P^s_i$ = set of feasible QC assignment profiles for the vessel $i \in N$ when vessel arrives at a time step with index $s \in S$ within a work shift;
- $P_i$ = set of quay crane assignment profiles for the vessel $i \in N$, where $P_i = \bigcup_{s \in S} P^s_i$. 
TBAP with QCs assignment: the model

- $t^p_i =$ handling time of ship $i \in N$ under the QC profile $p \in P_i$ expressed as multiple of the time step length;
- $v^p_i =$ the value of serving the ship $i \in N$ by the quay crane profile $p \in P_i$;
- $q^{pu}_i =$ number of quay cranes assigned to the vessel $i \in N$ under the profile $p \in P_i$ at the time step $u \in (1, ..., t^p_i)$, where $u = 1$ corresponds to the ship arrival time;
- $Q^h =$ maximum number of quay cranes available at the time step $h \in H$;
- $f_{ij} =$ flow of containers exchanged between vessels $i, j \in N$;
- $d_{kw} =$ unit housekeeping cost between yard slots corresponding to berths $k, w \in M$;
- $[a_i, b_i] =$ [earliest, latest] feasible arrival time of ship $i \in N$;
- $[a^k, b^k] =$ [start, end] of availability time of berth $k \in M$;
- $[a^h, b^h] =$ [start, end] of the time step $h \in H$. 
TBAP with QCs assignment: the model

Consider a graph \( G^k = (V^k, A^k) \) \( \forall k \in M \), where \( V^k = N \cup \{o(k), d(k)\} \), with \( o(k) \) and \( d(k) \) additional vertices representing berth \( k \), and \( A^k \subseteq V^k \times V^k \).

- \( x_{ij}^k \in \{0, 1\} \ \forall k \in M, \forall (i, j) \in A^k \), set to 1 if ship \( j \) is scheduled after ship \( i \) at berth \( k \);
- \( y_i^k \in \{0, 1\} \ \forall k \in M, \forall i \in N \), set to 1 if ship \( i \) is assigned to berth \( k \);
- \( \gamma_i^h \in \{0, 1\} \ \forall h \in H, \forall i \in N \), set to 1 if ship \( i \) arrives at time step \( h \);
- \( \lambda_i^p \in \{0, 1\} \ \forall p \in P_i, \forall i \in N \), set to 1 if ship \( i \) is served by the profile \( p \);
- \( \rho_i^{ph} \in \{0, 1\} \ \forall p \in P_i, \forall h \in H, \forall i \in N \), set to 1 if ship \( i \) is served by profile \( p \) and arrives at time step \( h \);
- \( T_{ik}^k \geq 0 \ \forall k \in M, \forall i \in N \), representing the berthing time of ship \( i \) at the berth \( k \) i.e. the time when the ship moors;
- \( T_{o(k)}^k \geq 0 \ \forall k \in M \), representing the starting operation time of berth \( k \) i.e. the time when the first ship moors at the berth;
- \( T_{d(k)}^k \geq 0 \ \forall k \in M \), representing the ending operation time of berth \( k \) i.e. the time when the last ship departs from the berth.
TBAP with QCs assignment: the MIQP model

Objective function

Maximize total value of QC profile assignments + Minimize the (quadratic) housekeeping yard cost of transshipment flows between ships:

$$\max \sum_{i \in N} \sum_{p \in P_i} v_i^p \lambda_i^p - \frac{1}{2} \sum_{i \in N} \sum_{k \in M} y_i^k \sum_{j \in N} \sum_{w \in M} f_{ij}d_{kw}y_j^w$$

(1)
TBAP with QCs assignment: the MIQP model

Berth covering constraints

\[ \sum_{k \in M} y_i^k = 1 \quad \forall i \in N, \]  

(2)

Flow and linking constraints

\[ \sum_{j \in N \cup \{d(k)\}} x_{o(k),j}^k = 1 \quad \forall k \in M, \]  

(3)

\[ \sum_{i \in N \cup \{o(k)\}} x_{i,d(k)}^k = 1 \quad \forall k \in M, \]  

(4)

\[ \sum_{j \in N \cup \{d(k)\}} x_{ij}^k - \sum_{j \in N \cup \{o(k)\}} x_{ji}^k = 0 \quad \forall k \in M, \forall i \in N, \]  

(5)

\[ \sum_{j \in N \cup \{d(k)\}} x_{ij}^k = y_i^k \quad \forall k \in M, \forall i \in N, \]  

(6)
TBAP with QCs assignment: the MIQP model

Precedence constraints

\[ T^k_i + \sum_{p \in P_i} t^p_i \lambda^p_i - T^k_j \leq (1 - x^k_{ij})M \quad \forall k \in M, \forall i \in N, \forall j \in N \cup d(k) \quad (7) \]

\[ T^k_{o(k)} - T^k_j \leq (1 - x^k_{o(k),j})M \quad \forall k \in M, \forall j \in N, \quad (8) \]

Ship and Berth time windows

\[ a_i y^k_i \leq T^k_i \quad \forall k \in M, \forall i \in N, \quad (9) \]

\[ T^k_i \leq b_i y^k_i \quad \forall k \in M, \forall i \in N, \quad (10) \]

\[ a^k \leq T^k_{o(k)} \quad \forall k \in M, \quad (11) \]

\[ T^k_{d(k)} \leq b^k \quad \forall k \in M, \quad (12) \]
TBAP with QCs assignment: the MIQP model

Profile covering & linking constraints

\[ \sum_{p \in P_i} \lambda^p_i = 1 \quad \forall i \in N, \]  
(13)

\[ \sum_{h \in H^s} \gamma^h_i = \sum_{p \in P^s_i} \lambda^p_i \quad \forall i \in N, \forall s \in S, \]  
(14)

\[ \sum_{k \in M} T^k_i - b^h \leq (1 - \gamma^h_i)M \quad \forall h \in H, \forall i \in N, \]  
(15)

\[ a^h - \sum_{k \in M} T^k_i \leq (1 - \gamma^h_i)M \quad \forall h \in H, \forall i \in N, \]  
(16)

\[ \rho^p_{ih} \geq \lambda^p_i + \gamma^h_i - 1 \quad \forall h \in H, \forall i \in N, \forall p \in P_i, \]  
(17)

Quay crane and profile feasibility

\[ \sum_{i \in N} \sum_{p \in P_i} \sum_{u = \max \{h - t^p_i + 1; 1\}}^{h} \rho^p_{iu} q^p_i (h - u + 1) \leq Q^h \quad \forall h \in H^s \]  
(18)
TBAP with QCs assignment: the MILP model

Additional decision variable

\[ z_{ij}^{kw} \in \{0, 1\} \quad \forall i, j \in N, \forall k, w \in M, \text{ set to 1 if } y_i^k = y_j^w = 1 \text{ and 0 otherwise.} \]

Linearized objective function

\[
\max \sum_{i \in N} \sum_{p \in P_i} v_i^p \lambda_i^p - \frac{1}{2} \sum_{i \in N} \sum_{j \in N} \sum_{k \in M} \sum_{w \in M} f_{ij} d_{kw} z_{ij}^{kw}
\]  

(19)

Additional constraints

\[
\sum_{k \in K} \sum_{w \in K} z_{ij}^{kw} = g_{ij} \quad \forall i, j \in N,
\]  

(20)

\[
z_{ij}^{kw} \leq y_i^k \quad \forall i, j \in N, \forall k, w \in M
\]  

(21)

\[
z_{ij}^{kw} \leq y_j^w \quad \forall i, j \in N, \forall k, w \in M
\]  

(22)
Generation of test instances

- Based on real data provided by MCT, Port of Gioia Tauro, Italy:
  - container flows
  - housekeeping yard costs
  - vessel's arrival times

- Crane productivity of 24 containers per hours

- Set of feasible profiles synthetically generated, according to ranges given by practitioners:

<table>
<thead>
<tr>
<th>Class</th>
<th>min QC</th>
<th>max QC</th>
<th>min HT</th>
<th>max HT</th>
<th>volume (min,max)</th>
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<td>Mother</td>
<td>3</td>
<td>5</td>
<td>3</td>
<td>6</td>
<td>(1296, 4320)</td>
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<tr>
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<td>2</td>
<td>4</td>
<td>(288, 1728)</td>
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</tbody>
</table>
Generation of test instances

- 6 classes of instances:
  - 10 ships and 3 berths, 1 week, 8 quay cranes;
  - 20 ships and 5 berths, 1 week, 13 quay cranes;
  - 30 ships and 5 berths, 1 week, 13 quay cranes;
  - 40 ships and 5 berths, 2 weeks, 13 quay cranes;
  - 50 ships and 8 berths, 2 weeks, 13 quay cranes;
  - 60 ships and 13 berths, 2 weeks, 13 quay cranes.

- 12 scenarios for each class, with high (H) and low (L) traffic volumes;

- each scenario is tested with a set of $\bar{p} = 10, 20, 30$ feasible profiles for each ship;

- CPLEX 10.2 solver for MILP and MIQP formulations.
## CPLEX results

<table>
<thead>
<tr>
<th>Instance</th>
<th>MILP</th>
<th>MIQP</th>
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<td>H1_10</td>
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<td>96.97</td>
<td>96.91</td>
</tr>
<tr>
<td>H2_30</td>
<td>96.79</td>
<td>-</td>
</tr>
<tr>
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<tr>
<th>Instance</th>
<th>MILP</th>
<th>MIQP</th>
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<tbody>
<tr>
<td>H1_10</td>
<td>94.33</td>
<td>-</td>
</tr>
<tr>
<td>H1_20</td>
<td>93.74</td>
<td>-</td>
</tr>
<tr>
<td>H2_10</td>
<td>93.52</td>
<td>96.66</td>
</tr>
<tr>
<td>L2_10</td>
<td>93.87</td>
<td>96.74</td>
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<tr>
<th>Instance</th>
<th>MILP</th>
<th>MIQP</th>
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<tr>
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<td>94.47</td>
<td>-</td>
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<td>-</td>
</tr>
<tr>
<td>L2_30</td>
<td>94.61</td>
<td>-</td>
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</table>
CPLEX results

- Time limits:
  - 1 hour for class 10x3;
  - 2 hours for classes 20x5 and 30x5;
  - 3 hours for classes 40x5, 50x8 and 60x13.

- The objective function value is scaled to 100 with respect to the upper bound:

\[
scaled \ obj = \frac{obj \times 100}{UB}
\]

A value of 100 means that the solution is certified to be optimal.

- No feasible solution was found for classes 30x5, 50x8 and 60x13;

- However, an upper bound is always provided.
## CPLEX results

<table>
<thead>
<tr>
<th>Instance</th>
<th>MILP UB</th>
<th>MIQP UB</th>
<th>Instance</th>
<th>MILP UB</th>
<th>MIQP UB</th>
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<tbody>
<tr>
<td>H1_10</td>
<td>1 754 291</td>
<td>2 288 451</td>
<td>H1_10</td>
<td>3 227 542</td>
<td>5 939 357</td>
</tr>
<tr>
<td>H1_20</td>
<td>1 754 633</td>
<td>2 288 793</td>
<td>H1_20</td>
<td>3 228 422</td>
<td>6 038 925</td>
</tr>
<tr>
<td>H1_30</td>
<td>1 754 669</td>
<td>2 288 829</td>
<td>H1_30</td>
<td>3 228 709</td>
<td>5 941 943</td>
</tr>
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<td>H2_10</td>
<td>1 708 485</td>
<td>2 256 299</td>
<td>H2_10</td>
<td>3 130 833</td>
<td>5 965 539</td>
</tr>
<tr>
<td>H2_20</td>
<td>1 709 020</td>
<td>2 256 834</td>
<td>H2_20</td>
<td>3 131 431</td>
<td>5 966 137</td>
</tr>
<tr>
<td>H2_30</td>
<td>1 709 230</td>
<td>2 257 044</td>
<td>H2_30</td>
<td>3 131 677</td>
<td>5 966 383</td>
</tr>
<tr>
<td>L1_10</td>
<td>1 420 485</td>
<td>1 787 983</td>
<td>L1_10</td>
<td>3 014 276</td>
<td>5 668 646</td>
</tr>
<tr>
<td>L1_20</td>
<td>1 420 713</td>
<td>1 817 824</td>
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<td>3 014 877</td>
<td>5 669 247</td>
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<tr>
<td>L1_30</td>
<td>1 420 819</td>
<td>1 842 700</td>
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<td>3 015 054</td>
<td>5 669 424</td>
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<td>L2_10</td>
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<td>1 948 130</td>
<td>L2_10</td>
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<td>5 749 854</td>
</tr>
<tr>
<td>L2_20</td>
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<td>1 973 914</td>
<td>L2_20</td>
<td>3 085 121</td>
<td>5 750 560</td>
</tr>
<tr>
<td>L2_30</td>
<td>1 613 805</td>
<td>2 008 053</td>
<td>L2_30</td>
<td>3 085 364</td>
<td>5 750 803</td>
</tr>
</tbody>
</table>
A New Heuristics for TBAP

- Our heuristic algorithm is organized in two stages:
  1. identify a QC profile assignment for the ships;
  2. solve the resulting berth allocation problem for the given QC assignment.

- The procedure is iterated over several sets of QC profiles;

- New profiles are chosen via reduced costs arguments (MILP formulation).
A New Heuristics for TBAP

Algorithm 1: TBAP Bi-level Heuristics

Initialization: Assign a QC profile to each ship

repeat
  1. solve BAP
  2. update profiles
until stop criterion

TBAP Bi-level Heuristics:

1. BAP solution via Tabu Search
2. Profiles’ updating via Math Programming
1. Tabu Search for BAP

Adapted from Cordeau, Laporte, Legato and Moccia (2005).

- New objective function: minimization of yard-related transshipment quadratic costs
- New constraints: QCs availability
- Each solution $s \in S$ is represented by a set of $m$ berth sequences such that every ship belongs to exactly one sequence.
- Penalized cost function:

$$f(s) = c(s) + \alpha_1 w_1(s) + \alpha_2 w_2(s) + \alpha_3 w_3(s)$$

where $w_1(s)$ is the total violation of ships’ TWs, $w_2(s)$ is the total violation of berths’ TWs and $w_3(s)$ is the total violation of QCs availability.

- “Move”: ship $i$ is removed from sequence $k$ and inserted in sequence $k' \neq k$. The new position in $k'$ is such that $f(s)$ is minimized.
- Initial solution: randomly built assigning ships to berths and relaxing the QCs availability constraint.
2. Profiles’ Updating via Math Programming

Basic idea: use information of reduced costs to update the vector of assigned QC profiles in a “smart” way.

- Let $\bar{X} = [\bar{x}, \bar{y}, \bar{T}]$ be the BAP solution found by the Tabu Search for a given QC profile assignment $\bar{\lambda}$.
- We solve the linear relaxation of the TBAP MILP formulation, with the additional constraints:

$$\bar{X} - \epsilon \leq X \leq \bar{X} + \epsilon \quad (23)$$
$$\bar{\lambda} - \epsilon \leq \lambda \leq \bar{\lambda} + \epsilon \quad (24)$$

- As suggested by Desrosiers and Lübbecke (2005), the shadow prices of these constraints are the reduced costs of original variables $X$ and $\lambda$.
- We identify the $\lambda^p_i$ variable with the maximum reduced cost and we assign this new profile $p$ to ship $i$.
- If all reduced costs are $\leq 0$, we stop.
Computational results

- The heuristic has been implemented in C++ using GLPK 4.31.

- Stopping criteria:
  - $n \times \bar{p}$ iterations;
  - time limit of 1 hour for classes 10x3, 20x5 and 30x5;
  - time limit of 2 hours for classes 40x5, 50x8 and 60x13.

- Results are compared to the best solution found by CPLEX for either the MILP or MIQP formulation.
### Computational results

<table>
<thead>
<tr>
<th>Instance</th>
<th>10x3 CPLEX</th>
<th>10x3 HEUR</th>
<th>Time (sec)</th>
<th>20x5 CPLEX</th>
<th>20x5 HEUR</th>
<th>Time (sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>H1_10</td>
<td>99.17</td>
<td>98.52</td>
<td>7</td>
<td>H1_10</td>
<td>-</td>
<td>97.26</td>
</tr>
<tr>
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<td>97.26</td>
</tr>
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Conclusions

- The heuristics is able to find feasible solutions in 70 out of 72 instances, whereas CPLEX succeeds at that only on 20 instances, the smaller ones.

- Our algorithm is up to 2 order of magnitude faster than CPLEX, especially on small instances.

- The heuristics performs very well also on the instances of bigger size, where CPLEX generally fails.

- Next step: improve upper bounds using decomposition techniques.
Thanks for your attention!
References

