

# Elastic Transactions

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## Abstract

This paper presents *elastic transactions*, a variant of the transactional model. Upon conflict detection, an elastic transaction might drop what it did so far within a separate transaction that immediately commits, and initiate a new transaction which might itself be elastic. Elastic transactions are a complementary alternative to traditional transactions, particularly appealing when implementing search structures. Both forms of transactions can safely be combined within the same application.

We implemented software support for elastic transactions and evaluated them on four common data structure applications, namely *linked list*, *skip list*, *red-black tree* and *hash table*. Our implementation is faster than a state-of-the-art software transactional memory in various workloads and with an improvement of 36% on average. It also presents an improvement over lock-based solutions of 89% on average.

## 1. Introduction

**Background.** Transactional memory (TM) is an appealing synchronization paradigm for leveraging modern multicore architectures. The power of the paradigm lies in its abstract nature: no need to know the internals of shared object implementations, it suffices to delimit any critical sequence of shared object accesses using transactional boundaries. Not surprisingly, however, this abstraction sometimes severely hampers parallelism. This is particularly true for search data structures where transactions do not know a priori where to add an element unless it explores a large part of the data structure. Consider for instance an integer set that supports search, insert, and remove operations. Assume furthermore that the set is implemented with a bucket hash table. A bucket, implemented with a sorted linked list, indicates where an integer should be stored. Consider a situation where one transaction searches for an

integer whereas another one seeks to insert an integer after a node that has been read by the first transaction: in a strict sense, there is a read-write conflict, yet this is a false (search-insert) conflict.

We propose *elastic transactions*, a new type of transactions that enables to efficiently implement search data structures and use them with regular transactional applications. As for a regular transaction, the programmer must simply delimit the blocks of code that represent elastic transactions. Nevertheless, during its execution, an elastic transaction can be cut into multiple normal transactions, depending on the conflicts detected. We show that this model is very effective whenever operations parse a large part of the structure while their effective update is localized.

**Elastic transactions: a primer.** To give an intuition of the idea behind elastic transactions, consider again the integer set abstraction. Each of the insert, remove, and search operations consists of lower-level operations: some reads and possibly some writes. Consider an execution in which two transactions,  $i$  and  $j$ , try to insert keys 3 and 1 concurrently in the same linked list. Each insert transaction parses the nodes in ascending order up to the node before which they should insert their key. Let  $\{2\}$  be the initial state of the integer set and let  $h, n, t$  denote respectively the memory locations where the head pointer, the single node (its key and next pointer) and the tail key are stored. Let  $\mathcal{H}$  be the following resulting history of operations where transaction  $j$  inserts 1 while transaction  $i$  is parsing the data structure to insert 3 at its end. (In the following history examples we indicate only operations of non-aborting transactions, thus, commit events have been omitted for simplicity.)

$$\mathcal{H} = r(h)^i, r(n)^i, r(h)^j, r(n)^j, w(h)^j, r(t)^i, w(n)^i.$$

This history is clearly not serializable since there is no sequential history that allows  $r(h)^i$  to occur before  $w(h)^j$  and also  $r(n)^j$  to occur before  $w(n)^i$ . A traditional transactional model would detect a conflict between transactions  $i$  and  $j$ , and the transactions could not both commit. Nonetheless, history  $\mathcal{H}$  does not violate the high-level linearizability of the integer set: 1 appears to be inserted before 3 in the linked list and both are present at the end of the execution.

To make a transaction elastic, the programmer has simply to label this transaction as being so and use its associated operations to access the shared memory. Assume indeed that transaction  $i$  has been labelled as elastic. History  $\mathcal{H}$  can now be viewed as a slightly different history,  $f(\mathcal{H})$ :

$$\boxed{r(h)^i, r(n)^i}^{s_1}, r(h)^j, r(n)^j, w(h)^j, \boxed{r(t)^i, w(n)^i}^{s_2}.$$

The elastic transaction has been cut in two transactions  $s_1$  and  $s_2$ , each being atomic. The cut is only possible because the value returned by the read of  $t$  has been the successor of  $n$  at some point in time. More precisely, the specific operations inside the elastic transaction ensure that no modifications on  $n$  and  $t$  have occurred between  $r(n)^{s_1}$  and  $r(t)^{s_2}$ . Otherwise the transaction would have to abort.

Even though a read value has been freshly modified by another transaction, it might not be necessary to abort and restart from the beginning. Assume that a transaction  $i$  searches for a key that is not in the linked list while a transaction  $j$  is inserting a node after the  $k^{\text{th}}$  node. Let  $h, n_1, \dots, n_\ell, t$  denote respectively the memory locations of the linked list:  $n_k$  denotes the memory location of the  $k^{\text{th}}$  node key and its next pointer. In the following history  $\mathcal{H}'$ , transaction  $i$  reads node  $n_k$  and can detect that it has freshly been modified by another transaction  $j$ .

$$\dots, r(n_k)^j, r(n_{k-1})^i, w(n_k)^j, r(n_k)^i, r(n_{k+1})^i, \dots$$

In this example, transaction  $i$  does not have to abort and restart from the beginning because it is the first time it accesses  $n_k$  and because the preceding node accessed by  $i$  has not been overwritten since then. Hence, after making sure that the previously read node  $n_{k-1}$  has not been modified, transaction  $i$  can resume and commit, as if its read of  $n_k$  was part of a new transaction  $s_k$ , serialized after  $j$ . Hence, we get the following history.

$$\dots, r(n_k)^j, r(n_{k-1})^i, w(n_k)^j, \boxed{r(n_k)^i, r(n_{k+1})^i}^{s_k}, \dots$$

**$\mathcal{E}$ -STM.** We propose  $\mathcal{E}$ -STM, an implementation of our transactional model that uses timestamps, two-phase-locking, and universal atomic primitives. It provides both normal transactions and elastic transactions, allowing the latter ones to be cut to achieve high concurrency, but it retains the abstraction simplicity of transactional memory.

To evaluate the performance and simplicity of our solution, we implement it on four data structure applications: (i) linked list, (ii) skip list, (iii) red-black tree, and (iv) hash table. We compared  $\mathcal{E}$ -STM with three other synchronization techniques: (i) regular STM transactions, (ii) lock-based, and (iii) lock-free. The regular STM technique relies on TinySTM [2], the fastest STM for micro-benchmarks we know of [2, 5]. The lock-based and lock-free implementations are based on the algorithms of Herlihy, Luchangco, Shavit et al. [8, 10], and of Fraser, Harris, and Michael [3, 6, 12], respectively. We also implemented complex operations, move and sum, to illustrate how transactions can be combined.

The results we obtained indicate that  $\mathcal{E}$ -STM speeds up regular transactions on all workloads and with an average improvement of 36%.  $\mathcal{E}$ -STM presents competitive performance compared to lock-based techniques and its average improvement is 89%. Although it is less efficient than lock-free techniques, (regression of 47% on average), it is significantly simpler and, as we will argue in Section 6, does not hamper extensibility.

**Roadmap.** In the remainder of this paper, we present our system model (Section 2) and our transactional model (Section 3), and we give an implementation of it, called  $\mathcal{E}$ -STM (Section 4) that we prove correct (Section 5). Then, we elaborate on the advantage of using  $\mathcal{E}$ -STM, we present four data structure applications that take advantage of the simplicity of our implementation, and we compare their performance with other synchronization techniques (Section 6). Finally, we present the related work (Section 7) before concluding (Section 8). The optional appendix includes the correctness proof of our  $\mathcal{E}$ -STM-based linked list implementation (Appendix A).

## 2. System Model

Our system comprises transactions and objects similarly to [19]. The states of all objects define the state of the system. A transaction is a sequence of read and write operations that can examine and modify, respectively, the state of the objects. More precisely, it consists of a sequence of events that are an *operation invocation*, an *operation response*, a *commit invocation*, a *commit response*, and an *abort event*.

An operation whose response event occurred is considered as *terminated* while a transaction whose commit response or abort event occurred is considered as *completed*.

The set of transactions is denoted by  $T$  and we consider two types of transactions: normal and elastic. We assume that the type of all transactions is initially known. The sets of normal transactions and elastic transactions are denoted by  $\mathcal{N}$  and  $\mathcal{E}$ , respectively. The set of possible objects is denoted by  $X$  and the set of possible values is  $V$ . An *operation* accessing an object  $x$  and belonging to a transaction  $t$ , can be of two *types* (read or write), and either takes as an argument or returns a value  $v$ . Hence, an operation is denoted by a tuple in  $X \times T \times V \times \text{type}$ .

### 2.1 Histories

We consider only well-formed sequences of events that consist of a set of transactions, each satisfying the following constraints: (i) a transaction must wait until its operation terminates before invoking a new one, (ii) no transaction both commit and abort, and (iii) a transaction cannot invoke an operation after having completed. We refer to these well-formed sequences as *histories*.

A history  $\mathcal{H}$  is complete if all its transactions are completed. We define a completing function *complete* that maps any history  $\mathcal{H}$  to a set of complete histories by appending an event  $q$  to each non-completed transaction  $t$  of  $\mathcal{H}$  such that:

- $q$  is an abort event if there is no commit invocation for  $t$  in  $\mathcal{H}$ ;
- $q$  is a commit or an abort event if there is a commit invocation for  $t$  in  $\mathcal{H}$ .

Given a set of transactions  $T$  and a history  $\mathcal{H}$ , we define  $\mathcal{H}|T$ , the restriction of  $\mathcal{H}$  to  $T$ , to be the subsequence of  $\mathcal{H}$  consisting of all events of any transaction  $t \in T$ . We refer to the set of transactions that have committed (resp. aborted) in  $\mathcal{H}$  as *committed*( $\mathcal{H}$ ) (resp.

*aborted*( $\mathcal{H}$ )). The history of all committed transactions of a given history  $\mathcal{H}$  is denoted by  $\text{permanent}(\mathcal{H}) = \mathcal{H}|_{\text{committed}(\mathcal{H})}$ . Similarly, for a set of objects  $X$  we denote by  $\mathcal{H}|X$  the subsequence of  $\mathcal{H}$  restricted to  $X$ . For the sake of simplicity, to denote  $\mathcal{H}|\{x\}$ , for  $x \in X$  (resp.  $\mathcal{H}|\{t\}$ , for  $t \in T$ ) we simply write  $\mathcal{H}|x$  (resp.  $\mathcal{H}|t$ ).

Let  $\rightarrow_{\mathcal{H}}$  be the total order on the events in  $\mathcal{H}$ . We say that  $t$  *precedes*  $t'$  in  $\mathcal{H}$  (denoted by  $t \rightarrow_{\mathcal{H}} t'$ ) if there are no events  $q \in \mathcal{H}|t$  and  $q' \in \mathcal{H}|t'$  such that  $q' \rightarrow_{\mathcal{H}} q$ . Two transactions  $t$  and  $t'$  are called *concurrent* if none precedes the other, i.e.,  $t \not\rightarrow_{\mathcal{H}} t'$  and  $t' \not\rightarrow_{\mathcal{H}} t$ . A history  $\mathcal{H}$  is *sequential* if no two transactions of  $\mathcal{H}$  are concurrent.

### 2.2 Operation sequences

For simplicity, we consider a sequence of operations instead of a sequence of events to describe histories and transactions. An operation  $\pi$  is a pair of invocation event and response event such that the invocation and response correspond to the same operation, accessing the same object and being part of the same transaction. A given history  $\mathcal{H}$  is thus an operation sequence  $\mathcal{S}_{\mathcal{H}} = \pi_1, \dots, \pi_n$  resulting from  $\mathcal{H}$  where commit invocations, commit responses, and invocations that do not have a matching response have been omitted. Concurrent operations ordering is determined by the object serial specification described below. We say that two histories  $\mathcal{H}$  and  $\mathcal{H}'$  are *equivalent* if for any transaction  $t$ ,  $\mathcal{H}|t = \mathcal{H}'|t$ .

The serial specification of an object is the set of acceptable sequences of its operations. Each object  $x$  is initialized with a default value  $v_x$  and accessed either by a write operation,  $\pi(x, v)$ , that writes a value  $v$  or by a read operation,  $\pi(x) : v$ , that returns a value  $v$ . That is, we only focus on read/write objects the serial specification of which requires that a read operation on  $x$  returns the last value written on  $x$ , or its default value  $v_x$  (if the value has not been written before). Without loss of generality, we assume that each written value is unique, hence: let  $\pi(x, v)$  and  $\pi'(x', v')$  be two write operations, if  $v = v'$  then  $x = x'$  and  $\pi = \pi'$ .

Next, we define three binary relations on the read and write operations of transactions. First,  $\pi_1$  precedes directly  $\pi_2$ , denoted by  $\pi_1 \prec_d \pi_2$  if  $\pi_1$  and  $\pi_2$  are two consecutive operations of the same transaction  $t$ . Second,  $\pi_1$  precedes  $\pi_2$  on object  $x$ , denoted by  $\pi_1 \prec_x \pi_2$  if  $\pi_1(x, v)$  is a write operation and  $\pi_2(x) : v$  is a read

operation that returns the value written by  $\pi_1$  ( $\pi_2$  reads from  $\pi_1$ ) or  $\pi_1$  is a read or a write operation and  $\pi_2$  is a write operation that overwrites the value accessed by  $\pi_1$ . The transitive closure of the union of these two precedence relations is denoted  $\prec$ . More precisely, let  $\prec_*$  be the union of the two relations  $\prec_d$  and  $\prec_x$ , hence  $\pi_1 \prec_* \pi_2$  if and only if either  $\pi_1 \prec_d \pi_2$  or  $\pi_1 \prec_x \pi_2$ . We obtain the following recursive definition for the precedence relation  $\prec$ . We say that  $\pi_1 \prec \pi_2$  if and only if one of the two following properties hold:

- either  $\pi_1 \prec_* \pi_2$ ,
- or there exists  $\pi_3$  such that  $\pi_1 \prec_* \pi_3$  and  $\pi_3 \prec \pi_2$ .

A sequential history  $\mathcal{H}$  is *legal* if each read operation  $\pi$  on some object  $x$  returns either the value written by the last write operation on  $x$ , preceding  $\pi$ , or the default value  $v_x$  if no such write operation exists. More precisely,  $\mathcal{H}$  is legal if the value  $v$  returned by any  $\pi(x) : v \in \mathcal{H}$  is either such that  $\pi'(x, v) = \max_{\pi' \rightarrow_{\mathcal{H}} \pi} \{ \pi'(x, *) \in \mathcal{H} \text{ s.t. } \pi' \rightarrow_{\mathcal{H}} \pi \}$  or  $v_x$  if there is no  $\pi'(x, *) \in \mathcal{H}$  such that  $\pi' \rightarrow_{\mathcal{H}} \pi$ .

We refer to a transaction that never writes an object value in the shared memory as an *invisible* transaction. Observe that an invisible transaction may, however, write some metadata (e.g., lock ownership) in the shared memory. An example is a transaction that acquires some locks before aborting.

### 3. Elastic Transactions: Definition

An *elastic* transaction is a transaction the size of which may vary depending on conflicts. More precisely, such transaction may cut itself upon conflict detection as if the start of the transaction has moved forward, hence the name elastic. Next, we explain how a cut is achieved.

First, note that a sequence of operations is a totally ordered set, hence, we refer to a history  $\mathcal{H}$  as a tuple  $\langle S_{\mathcal{H}}, \rightarrow_{\mathcal{H}} \rangle$  where  $S_{\mathcal{H}}$  is the corresponding set of operations and  $\rightarrow_{\mathcal{H}}$  a total order defined over  $S_{\mathcal{H}}$ .

A *sub-history*  $\mathcal{H}'$  of history  $\mathcal{H} = \langle S_{\mathcal{H}}, \rightarrow_{\mathcal{H}} \rangle$  is a history  $\mathcal{H}' = \langle S_{\mathcal{H}'}, \rightarrow_{\mathcal{H}'} \rangle$  such that  $S_{\mathcal{H}'} \subseteq S_{\mathcal{H}}$  and  $\rightarrow_{\mathcal{H}'} \subseteq \rightarrow_{\mathcal{H}}$ . Next, we define the notion of cut and its well-formedness.

**DEFINITION 3.1 (Cut).** A cut of a history  $\mathcal{H}$  is a sequence  $\mathcal{C} = \langle S_{\mathcal{C}}, \rightarrow_{\mathcal{C}} \rangle$  of sub-histories of  $\mathcal{H}$  such that:

1. each of the cut sub-histories contains only consecutive operations of  $\mathcal{H}$ : for any sub-history  $\mathcal{H}' =$

$\pi_1, \dots, \pi_n$  in  $S_{\mathcal{C}}$ , if there exists  $\pi_i \in \mathcal{H}$  such that  $\pi_1 \rightarrow_{\mathcal{H}} \pi_i \rightarrow_{\mathcal{H}} \pi_n$ , then  $\pi_i \in \mathcal{H}'$ ;

2. if one sub-history precedes another in  $\mathcal{C}$  then the operations of the first precede the operations of the second in  $\mathcal{H}$ : for any sub-histories  $\mathcal{H}_1$  and  $\mathcal{H}_2$  in  $S_{\mathcal{C}}$  and two operations  $\pi_1 \in \mathcal{H}_1$  and  $\pi_2 \in \mathcal{H}_2$ , if  $\mathcal{H}_1 \rightarrow_{\mathcal{C}} \mathcal{H}_2$  then  $\pi_1 \rightarrow_{\mathcal{H}} \pi_2$ ;
3. any operation of  $\mathcal{H}$  is in exactly one sub-history of the cut:  $\bigcup_{\mathcal{H}' \in S_{\mathcal{C}}} S_{\mathcal{H}'} = S_{\mathcal{H}}$  and for any  $\mathcal{H}_1, \mathcal{H}_2 \in S_{\mathcal{C}}$ , we have  $S_{\mathcal{H}_1} \cap S_{\mathcal{H}_2} = \emptyset$ .

For example, there are four cuts of history  $a, b, c$ , denoted by  $\mathcal{C}1 = \{a, b ; c\}$ ,  $\mathcal{C}2 = \{a ; b, c\}$ ,  $\mathcal{C}3 = \{a ; b ; c\}$ , and  $\mathcal{C}4 = \{a, b, c\}$ , where semi-colons are used to separate consecutive sub-histories of the cut and braces are used for clarity to enclose a cut. In contrast, neither  $\{a, c ; b\}$  nor  $\{a ; a, b, c\}$  are cuts of  $\mathcal{H}$ . The reason is that the former violates property (1) while the latter violates property (3) of Definition 3.1.

**DEFINITION 3.2 (Well-formed cut).** A cut  $\mathcal{C}_t$  of history  $\mathcal{H}|t$ , where  $t$  is a transaction, is well-formed if for any of its sub-histories  $s_i$  the following properties are satisfied:

1. if  $s_i$  contains only one operation, then there is no other  $s_j \in S_{\mathcal{C}_t}$ ;
2. if  $\pi_i \in s_i$  and  $\pi_j \in s_j$  are two write operations of  $t$ , then  $s_i = s_j$ ;
3. if  $\pi_i$  is the first operation of  $s_i$ , then either  $\pi_i$  is a read operation or  $\pi_i$  is the first operation of  $t$ .

For example, consider the following history  $\mathcal{H}_1|t$  where  $t$  is an elastic transaction, and where  $r(x)$  and  $w(x)$  refer to a read and a write operation on  $x$ . (For the following examples, we omit the values returned by the read operations and consider that the object serial specification is satisfied.)

$$\mathcal{H}_1|t = r(u), r(v), w(x), r(y), r(z).$$

There are two well-formed cuts of history  $\mathcal{H}_1|t$  that are  $\mathcal{C}1' = \{r(u), r(v), w(x), r(y), r(z)\}$  and  $\mathcal{C}2' = \{r(u), r(v), w(x) ; r(y), r(z)\}$ , however, neither  $\mathcal{C}3' = \{r(u) ; r(v), w(x) ; r(y), r(z)\}$  nor  $\mathcal{C}4' = \{r(u), r(v) ; w(x), r(y), r(z)\}$  are well-formed. More precisely, the first sub-history of  $\mathcal{C}3'$  contains only one operation violating property (1) of Definition 3.2 and the second sub-history of  $\mathcal{C}4'$  starts with a write operation, that is, property (3) of Definition 3.2 is violated.

In the remainder of this paper, we only consider well-formed cuts.

Next, we define a consistent cut with respect to a history of potentially concurrent transactions. This definition is crucial as it indicates the singularity of elastic transactions. The programmer can label a transaction as elastic if he/she does not need this transaction to appear as atomic, but still he/she requires that a set of consecutive operations in this transaction appear as atomic, as formalized below. In a history  $\mathcal{H}$ , a cut is consistent if there are no writes separating two of its sub-histories each accessing one of the object written by these writes.

**DEFINITION 3.3 (Consistent cut).** *A cut  $\mathcal{C}_t$  of  $\mathcal{H}|t$  is consistent with respect to history  $\mathcal{H}$  if, for any operation  $\pi_i$  and  $\pi_j$  of any two of its sub-histories  $s_i$  and  $s_j$  respectively ( $s_i \neq s_j$ ), the two following properties hold:*

- *there is no write operation  $\pi'(x)$  from a transaction  $t' \neq t$  such that  $\pi_i(x) \rightarrow_{\mathcal{H}} \pi'(x) \rightarrow_{\mathcal{H}} \pi_j(x)$ ;*
- *there are no two write operations  $\pi'(x)$  and  $\pi''(y)$  from transactions  $t' \neq t$  and  $t'' \neq t$  such that  $\pi_i(x) \rightarrow_{\mathcal{H}} \pi'(x) \rightarrow_{\mathcal{H}} \pi_j(y)$  and  $\pi_i(x) \rightarrow_{\mathcal{H}} \pi''(y) \rightarrow_{\mathcal{H}} \pi_j(y)$ .*

For example, consider the following history  $\mathcal{H}_2$  where  $e$  is an elastic transaction and  $n$  is a normal transaction, and where  $r(x)^t$  and  $w(x)^t$  refer to a read and a write operation on  $x$  in transaction  $t$ .

$$\mathcal{H}_2 = r(x)^e, r(y)^e, w(y)^n, r(z)^e, w(u)^e.$$

Two consistent cuts of  $\mathcal{H}_2|e$  with respect to  $\mathcal{H}_2$  are possible. One contains two sub-histories  $\mathcal{C}_1 = \{r(x)^e, r(y)^e; r(z)^e, w(u)^e\}$  while the other contains one sub-history  $\mathcal{C}_2 = \{r(x)^e, r(y)^e, r(z)^e, w(u)^e\}$ . Observe that  $\mathcal{C}_1$  is consistent because there are no two writes from other transactions that occur at objects between the accesses of  $e$  to these objects, hence  $r(y)^e$  and  $r(z)^e$  seem to execute atomically at the time  $r(y)^e$  occurs. In contrast, consider history  $\mathcal{H}_3$  where  $e$  is elastic and  $n$  is normal.

$$\mathcal{H}_3 = r(x)^e, r(y)^e, w(y)^n, w(z)^n, r(z)^e, w(u)^e.$$

There is no consistent cut of  $\mathcal{H}_3|e$  with respect to  $\mathcal{H}_3$  because  $n$  writes  $y$  and  $z$  between the times  $e$  reads each of them.

Given a cut  $\mathcal{C}_t = s_1^t, \dots, s_n^t$  of  $\mathcal{H}|t$  for each elastic transaction  $t \in \mathcal{H}|\mathcal{E}$ , we define a *cutting function*

$f_{\mathcal{C}_t}$  that replaces an elastic transaction  $t$  by the transactions  $s_i^t$  resulting from its cut. More precisely,  $f_{\mathcal{C}_t}$  maps a history  $\mathcal{H} = \pi_1, \dots, \pi_n$  to a history  $f_{\mathcal{C}_t}(\mathcal{H}) = \pi'_1, \dots, \pi'_n$  where if  $\pi_i = \langle x, t, v, type \rangle \in s_i^t$  then  $\pi'_i = \langle x, s_i^t, v, type \rangle$ , otherwise  $\pi_i = \pi'_i$ , and if  $t \in committed(\mathcal{H})$  then  $s_i^t \in committed(f_{\mathcal{C}_t}(\mathcal{H}))$ , otherwise  $s_i^t \in aborted(f_{\mathcal{C}_t}(\mathcal{H}))$ . We denote the composition of  $f$  for a set of cuts  $\mathcal{C} = \{\mathcal{C}_1, \dots, \mathcal{C}_m\}$  by  $f_{\mathcal{C}} = f_{\mathcal{C}_1} \circ \dots \circ f_{\mathcal{C}_m}$ .

Next, we define an elastic-opaque transactional system, which combines normal and elastic transactions; this definition relies on Definition 3.3 of consistent cut, and the definition of opacity [4]. More precisely, it states that a system is elastic-opaque if, for any history of this system, there exists a consistent cut for each elastic transaction such that the history resulting from the composition of these cuts is opaque.

**DEFINITION 3.4 (Elastic-opacity).** *A transactional system is elastic-opaque if, for any history  $\mathcal{H}$  of this system, there exists a consistent cut  $\mathcal{C}_t$  for each elastic transaction  $t$  of  $\mathcal{H}|\mathcal{E}$  with  $\mathcal{C} = \{\mathcal{C}_t\}$ , such that  $f_{\mathcal{C}}(\mathcal{H})$  is opaque.*

As an example, consider the following history  $\mathcal{H}_4$  and assume  $e$  is elastic while  $n$  is normal and both transactions commit:

$$r(x)^e, r(y)^e, r(x)^n, r(y)^n, r(z)^n, w(x)^n, r(t)^e, w(z)^e.$$

This history would clearly not be serializable in a traditional model (with  $e$  and  $n$  two normal transactions) since there is no sequential histories that allow not only  $r(x)^e$  to occur before  $w(x)^n$  but also  $r(z)^n$  to occur before  $w(z)^e$ . However, there exists one consistent cut  $\mathcal{C}_e$  of  $\mathcal{H}_4|e$  with respect to  $\mathcal{H}_4$ ,  $\mathcal{C}_e = s_1, s_2$  where  $s_1 = r(x)^e, r(y)^e$  and  $s_2 = r(t)^e, w(z)^e$  such that, for  $\mathcal{C} = \{\mathcal{C}_e\}$ , we have  $f_{\mathcal{C}}(\mathcal{H}_4)$ :

$$r(x)^{s_1}, r(y)^{s_1}, r(x)^n, r(y)^n, r(z)^n, w(x)^n, r(t)^{s_2}, w(z)^{s_2}.$$

And  $\mathcal{H}_4$  is elastic-opaque as  $f_{\mathcal{C}}(\mathcal{H}_4)$  is equivalent to a sequential history:  $s_1, n, s_2$  (and  $f_{\mathcal{C}}(\mathcal{H}_4)$  is opaque).

## 4. Elastic Transactions: Implementation

This section introduces  $\mathcal{E}$ -STM, a software transactional memory system that implements elastic transactions. The corresponding pseudocode appears in Algorithm 1.  $\mathcal{E}$ -STM combines two-phase locking, timestamp mechanism, and atomic operations that

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**Algorithm 1**  $\mathcal{E}$ -STM

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1: clock  $\in \mathbb{N}$ , initially 0
2: State of variable  $x$ :
3: val  $\in V$ 
4: tlk a record with fields: // time-based lock
5: owner  $\in T$ , the lock owner, initially  $\perp$ 
6: time  $\in \mathbb{N}$ , a version counter, initially 0
7: w-entry  $\in X \times V \times \mathbb{N}$ , an entry address
8:   initially  $\perp$ 
9: // time/w-entry share the same location

10: State of transaction  $t$ :
11: type  $\in \{\text{elastic, normal}\}$ , initially the
12:   type of the ancestor transaction or  $\perp$ 
13: r-set and w-set, sets of entries with fields:
14:   addr  $\in X$ , an address
15:   val  $\in V$ , its value
16:   ts  $\in \mathbb{N}$ , its version timestamp
17: last-r-entry  $\in X \times \mathbb{N}$ , an entry,
18:   initially  $\perp$ 
19: lb  $\in \mathbb{N}$ , initially 0 // time lower bound
20: ub  $\in \mathbb{N}$ , initially 0 // time upper bound

21: begin( $tx$ -type) $_t$ :
22: ub  $\leftarrow clock$ 
23: lb  $\leftarrow clock$ 
24: // if it is nested inside a normal, be normal
25: if type  $\neq$  normal then type  $\leftarrow tx$ -type

26: try-extend() $_t$ :
27: // make sure read values have not changed
28: now  $\leftarrow clock$ 
29: for all  $\langle y, *, ts \rangle \in r$ -set do
30:   ow  $\leftarrow y.tlk.owner$ 
31:   last  $\leftarrow y.tlk.time$ 
32:   if ow  $\notin \{t, \perp\} \vee$ 
33:     (ow  $= \perp \wedge last \neq ts$ ) then
34:     abort()
35:   ub  $\leftarrow now$ 

36: ver-val-ver( $x$ , evenlocked) $_t$ :
37: // load a versioned value from memory
38: repeat:
39:    $\ell_1 \leftarrow x.tlk$ 
40:   v  $\leftarrow x.val$ 
41:    $\ell_2 \leftarrow x.tlk$ 
42: until ( $\ell_1 = \ell_2 \wedge$ 
43:   ( $\ell_1.owner = \perp \vee evenlocked$ ))
44: return  $\langle \ell_1, v \rangle$ 

45: read( $x$ ) $_t$ :
46: // log normal reads for later extensions
47: if type  $=$  normal  $\vee w$ -set  $\neq \emptyset$  then
48:    $\langle \ell_x, v_x \rangle \leftarrow ver\text{-}val\text{-}ver(x, true)$ 
49:   if  $\ell_x.owner \notin \{t, \perp\}$  then ctn_mgt()
50:   else if  $\ell_x.owner = t$  then
51:     vx  $\leftarrow \ell_x.w\text{-}entry.val$ 
52:   else //  $\ell_x.owner = \perp$ 
53:     if  $\ell_x.time > ub$  then try-extend()
54:     r-set  $\leftarrow r\text{-}set \cup \{ \langle x, v_x, \ell_x.time \rangle \}$ 
55: // ...or log only the most recent elastic read
56: if type  $=$  elastic  $\wedge w$ -set  $= \emptyset$  then
57:    $\langle \ell_x, v_x \rangle \leftarrow ver\text{-}val\text{-}ver(x, false)$ 
58:   if  $\ell_x.time > ub$  then
59:     if last-r-entry  $\neq \perp$  then
60:        $\langle y, * \rangle \leftarrow last\text{-}r\text{-}entry$ 
61:        $\langle \ell_y, * \rangle \leftarrow ver\text{-}val\text{-}ver(y, false)$ 
62:       if  $\ell_y.time > ub$  then abort()
63:       ub  $\leftarrow \ell_x.time$ 
64:       last-r-entry  $\leftarrow \langle x, \ell_x.time \rangle$ 
65: return vx

66: write( $x, v$ ) $_t$ :
67: // lock and postpone the write until commit
68: repeat:
69:    $\ell \leftarrow x.tlk$ 
70:   if  $\ell.owner \notin \{\perp, t\}$  then ctn_mgt()
71:   else if  $\ell.time > ub$  then
72:     if type  $=$  normal then try-extend()
73:     else abort()
74:   w-entry  $\leftarrow \langle x, v, \ell.time \rangle$ 
75:   x.tlk  $\leftarrow \langle t, *, w\text{-}entry \rangle$  // cas
76:   until (x.tlk.owner  $= t$ )
77:   lb  $\leftarrow max(lb, \ell.time)$ 
78:   w-set  $\leftarrow (w\text{-}set \setminus \{ \langle x, *, * \rangle \})$ 
79:      $\cup \{ w\text{-}entry \}$ 
80: // make sure last value read is unchanged
81: if type  $=$  elastic  $\wedge last\text{-}r\text{-}entry \neq \perp$  then
82:    $\langle e, t_e \rangle \leftarrow last\text{-}r\text{-}entry$ 
83:    $\langle \ell_e, * \rangle \leftarrow ver\text{-}val\text{-}ver(e, true)$ 
84:   ow  $\leftarrow \ell_e.owner$ 
85:   last  $\leftarrow \ell_e.time$ 
86:   if ow  $\neq \perp \vee last \neq t_e$  then abort()
87:   last-r-entry  $\leftarrow \perp$ 

88: abort() $_t$ :
89: for all  $\langle x, *, * \rangle \in write\text{-}set$  do
90:   x.tlk.owner  $\leftarrow \perp$ 
91: begin(type) // restart from the beginning

92: commit() $_t$ :
93: // apply writes to memory and release locks
94: if w-set  $\neq \emptyset$  then
95:   ts  $\leftarrow clock++$  // fetch&increment
96:   if lb  $\neq ts - 1$  then try-extend()
97:   for all  $\langle x, v, ts \rangle \in write\text{-}set$  do
98:     x.val  $\leftarrow v$ 
99:     x.tlk.time  $\leftarrow ts$ 
100:    x.tlk.owner  $\leftarrow \perp$ 

```

---

are supported at the hardware level by most common architectures: compare-and-swap (Lines 75), fetch-and-increment (Line 95), and atomic loads and stores.

#### 4.1 Transaction and Variable State

A transaction  $t$  starts with a `begin(type)` indicating whether its type is elastic or normal. Then, it accesses the memory locations using read or write operations. Finally, it completes either by a `commit` call or by an `abort` that restarts the same transaction. The `try-extend` and `ver-val-ver` are helper functions. A transaction  $t$  may keep track of the variable it has accessed since it has lastly started using a *r-set* to log the reads and a *w-set* to log the writes. More precisely, the entries of these sets contain the variable address, *addr*, its value *val*, and its version *ts* (Lines 13–16). If  $t$  is elastic, it

may only need to keep track of the last read operation, so it uses *last-r-entry* (Lines 17 and 18) to log a single address and its version instead of the entire set *r-set*. The two last fields of  $t$  indicate a lower-bound *lb* and an upper-bound *ub* on the logical times at which  $t$  can be serialized (Lines 19 and 20).

For the sake of clarity in the pseudocode presentation, we consider that each memory location is protected by a distinct lock. We call it the associated memory location of the lock. More precisely, each shared variable  $x$  can be represented by a value (Line 3) *val* and a timestamped lock *tlk*, also called versioned write-lock [1]. A timestamped lock has three fields: (i) the *owner* indicating which transaction has acquired the lock, if any, (ii) the *time* the associated memory location of the lock has the most recently been written, and (iii) *w-entry*, a reference to the

corresponding entry in the owner’s write set (Lines 4–8). Timestamps are given by a global counter, *clock* (Line 1), that does not hamper scalability [1, 2, 15].

## 4.2 Normal Transactions

The algorithm restricted to normal transactions builds upon TinySTM [2] logging all operations. All transactions use two-phase locking when writing to a memory location. While the location is locked by the transaction at the time it executes the write, all updates are buffered into a write-set, *w-set*, until the commit time at which these updates are applied to the memory. When a transaction performs a write( $x, *$ ), it acquires the lock of  $x$  using a compare-and-swap (Line 75) and holds it until it commits or aborts. When accessing a locked variable, the transaction detects a conflict and calls the contention manager, which typically aborts the current transaction (Lines 49 and 70). Various contention management policies could be used instead to handle conflicts between normal transactions.

When a read request on variable  $x$  as part of transaction  $t$  is received by  $\mathcal{E}$ -STM, the value of  $x$  is read in a three-step process called ver-val-ver, which consists in loading its timestamped lock  $x.tlk$ , loading its value  $x.val$ , and re-loading its lock  $x.tlk$ . This read-version-value-version is repeated until the two versions read are identical (Line 42) indicating that the value corresponds to that version. Only in some cases needs the value be returned unlocked, hence the use of the boolean *evenlocked*. The transactions of  $\mathcal{E}$ -STM use the extension mechanism of LSA [2, 15]. Each transaction  $t$  maintains an interval of time  $[lb, ub]$  indicating the time during which  $t$  can be serialized. More precisely, for a given transaction  $t$ ,  $lb$  and  $ub$  represent respectively lower and upper bounds on the versions of values accessed by  $t$  during its execution. When  $t$  reads  $x$ , it records the last time  $x$  has been modified in its read-set, *r-set*, for future potential check. Later on, if  $t$  accesses a variable  $y$  that has been recently updated ( $y.tlk.time > ub$ ),  $t$  first tries to extend its interval of time by calling *try-extend()*. Transaction  $t$  detects a conflict only if this extension is impossible (Lines 34), meaning that some variables, among the ones  $t$  has read, have been updated by another transaction since then.

## 4.3 Elastic Transactions

An important difference between normal and elastic transactions is that elastic transactions never use the *r-set* until they read after a write, as there is at most one read operation the transaction has to keep track of: the most recent one. Hence, elastic transactions use the *last-r-entry* field to log the last read operation. In our implementation all reads following a write in an elastic transaction will use the *r-set* like normal transactions (Lines 46–54), however, the implementation could be improved using static analysis to require this only for reads that are both preceded and succeeded by write operations in the same transaction.

Upon reading  $x$  (without having written before) an elastic transaction must make sure that the value  $v_x$  it reads was present at the time the immediately preceding read occurs. This typically ensures that a thread does not return an inconsistent value  $v_x$  after having been pre-empted, for example. If the version  $v_x$  of the value is too recent,  $\ell_x.time > ub$ , then the read operation must recheck the value logged in *last-r-entry* to be sure that the value read has not been overwritten since then (Lines 58–63). This can be viewed as a partial roll-back similar to the one provided by nested models, except that no on-abort definition is necessary and only a single operation would have to be re-executed here. Upon writing  $x$ , a similar verification regarding the last value read is made. If the lock corresponding to this address has been acquired,  $ow \neq \perp$ , or if the version has changed since then,  $last \neq time_e$ , then the transaction aborts (Line 86). If, however, no other transaction tried to update this address since it has been read, then the write executes as normal (Lines 67–79).

## 5. Correctness of $\mathcal{E}$ -STM

Here, we show that  $\mathcal{E}$ -STM is elastic-opaque in three steps. First, we give preliminary definitions. Second, we show that for each committed elastic transaction of  $\mathcal{E}$ -STM, there exists a consistent cut so that  $f_C$  is well-defined. Third, we show that  $\mathcal{E}$ -STM is elastic-opaque by differentiating histories restricted to aborting transactions from histories restricted to committed transactions.

**THEOREM 5.1.**  *$\mathcal{E}$ -STM is elastic-opaque.*

**DEFINITION 5.2 (Opacity).** *A history  $\mathcal{H}$  is opaque if it satisfies the following properties:*

1. All transactions that abort in  $\text{complete}(\mathcal{H})$  are invisible.
2. The history  $\text{permanent}(\text{complete}(\mathcal{H}))$  is equivalent to a sequential history (where all non-concurrent transactions are ordered as in  $\mathcal{H}$ ) that is legal.

We define *sub-complete* as a mapping between history  $\mathcal{H}$  and a history  $\mathcal{H}' = \text{sub-complete}(\mathcal{H})$  in which (i) all non-completed transactions of  $\mathcal{H}$  that have a commit invocation and that do not write any object value into the memory are aborted in  $\mathcal{H}'$ , (ii) all the transactions of  $\mathcal{H}$  that have written an object value into the memory are committed in  $\mathcal{H}'$ , and (iii) all non-completed transactions of  $\mathcal{H}$  with no commit-request are aborted in  $\mathcal{H}'$ . Observe that for any  $\mathcal{H}$ ,  $\text{sub-complete}(\mathcal{H}) \in \text{complete}(\mathcal{H})$ .

First-of-all, we show that there exists a consistent cut for any elastic transaction  $t$  in  $\mathcal{H}|\mathcal{E}$ . This ensures that the  $f_{C_t}(\mathcal{H}|t)$  is well defined and more generally, that  $f_C(\mathcal{H})$  is well-defined.

LEMMA 5.3. *Let  $\mathcal{H}$  be any history of  $\mathcal{E}$ -STM. There exists a consistent cut  $C_t$  of  $\mathcal{H}|t$  with respect to  $\mathcal{H}$ .*

**Proof.** The proof relies essentially on the definition of a consistent cut (Definition 3.3). First, if  $t$  contains a single operation, then  $C_t$  contains only  $t$  and the definition is straightforwardly satisfied.

Now consider the case where  $t$  has more than one operation. We show that there can neither be a write operation  $\pi(x)^{t'}$  from transaction  $t' \neq t$  such that  $\pi(x)^t \rightarrow_{\mathcal{H}} \pi(x)^{t'} \rightarrow_{\mathcal{H}} \pi(x)^t$  nor be two writes operations  $\pi(x)^{t'} \pi(y)^{t''}$  from transaction  $t' \neq t$  and  $t'' \neq t$  such that  $\pi(y)^t \rightarrow_{\mathcal{H}} \pi(x)^{t'} \rightarrow_{\mathcal{H}} \pi(x)^t$  and  $\pi(y)^t \rightarrow_{\mathcal{H}} \pi(y)^{t''} \rightarrow_{\mathcal{H}} \pi(x)^t$ . First, we start by showing that the latter case cannot happen, the impossibility of the former case will follow.

Assume that such writes  $\pi(x)^{t'}$  and  $\pi(y)^{t''}$  exist, we show that  $t$  would abort leading to a consistent cut. When  $\pi(x)^{t'}$  executes, it locks  $x$  by setting  $x.\text{tlk.owner}$  to  $t'$  until it commits and sets the associated timestamp  $x.\text{tlk.time}$  to a new strictly higher clock value than  $t.\text{ub}$ . For the same reason,  $y.\text{tlk.time} > t.\text{ub}$  just after the execution of  $\pi(y)^{t''}$ . Hence,  $t$  reads  $x$  after  $t'$  and  $t''$  have committed and  $t$  observes that  $x.\text{tlk.time}$  is larger than its  $t.\text{ub}$ . Since  $t$  is elastic, this observation leads it to verify that the version of  $y$  has not changed (Line 58). Since, we have

$y.\text{tlk.time} > t.\text{ub}$ , the transaction aborts as indicated Line 62 and the resulting sequence of operations of  $t$  is a consistent cut. The same proof holds also for the case where  $x = y$ .

As a result, there always exists a consistent cut  $C_t$  of an elastic transaction  $t$  of  $\mathcal{H}$ .  $\square$

In the remainder of the proof, we refer to the cut history  $f_C(\mathcal{H})$  as the history  $\mathcal{H}$  where each committed elastic transaction has been replaced by its sub-transactions in one of its consistent cut. Property (1) of Definition 5.2 comes from the fact that no value is written in memory unless the transaction is ensured to commit. Hence, the first Lemma shows that an aborting transaction is invisible to other transactions.

LEMMA 5.4. *Let  $\mathcal{H}$  be any history of  $\mathcal{E}$ -STM and let  $\mathcal{H}' = \text{sub-complete}(f_C(\mathcal{H}))$ . If  $t' \in \text{aborted}(\mathcal{H}')$  then  $t'$  is an invisible transaction.*

**Proof.** The proof is divided in two parts whether we consider the completed transactions of  $\mathcal{H}$  or the transactions that were not completed in  $\mathcal{H}$  but that are completed in  $\mathcal{H}'$ .

1. First, we show that if  $t \in \text{aborted}(\mathcal{H})$ , then  $t$  is invisible. By transaction well-formedness, no  $\text{abort}()_t$  can occur after a  $\text{commit}()_t$  completes. By examination of the code, we know that memory can only be updated during the for-loop of the  $\text{commit}()_t$  function (Line 97). With no loss of generality let  $\tau_1$  be the starting time of the for-loop. A transaction that issued a commit invocation can only abort at time  $\tau_2$ , before the  $\text{try-extend}()$  call returns at Line 96. Since Line 96 is before the beginning of the for-loop,  $\tau_2 < \tau_1$  and the result follows.
2. Second, we show that if  $t \in \text{aborted}(\mathcal{H}') \setminus \text{aborted}(\mathcal{H})$  then  $t$  is invisible. Since a transaction can only write after a commit invocation, all abort events that are not in  $\mathcal{H}$  but that are in  $\mathcal{H}'$  are appended only to invisible transactions.

The conjunction of the two parts of the proof states that there exists  $\mathcal{H}' \in \text{complete}(\mathcal{H})$  such that if  $t \in \text{aborted}(\mathcal{H}')$  then  $t$  is invisible.  $\square$

To show Property (2) of Definition 5.2, we determine a serialization point for each operation and show that each transaction appears “as if” it was executed atomically at this point in time.



1. read operation  $\pi$ : its serialization point  $ser(\pi)$  is the time the last  $\ell_1 \leftarrow x.tlk$  of the loop occurs (Line 39).
2. write operation  $\pi$ : its serialization point  $ser(\pi)$  is the time its compare-and-swap occurs (Line 75).

Observe that serialization points are defined at the time an atomic operation occurs. Hence, two distinct operations on the same object cannot have the same serialization point.

**LEMMA 5.5.** *Let  $\mathcal{H}$  be any history of  $\mathcal{E}$ -STM and let  $\mathcal{H}' = sub\text{-complete}(f_C(\mathcal{H}))$ . Let  $t_1$  and  $t_2$  be two transactions in  $permanent(\mathcal{H}')$ . For any two distinct operations  $\pi_1$  and  $\pi_2$  executed respectively in  $t_1$  and  $t_2$  and accessing location  $x$ : if  $ser(\pi_1) < ser(\pi_2)$ , then  $\pi_2 \not\prec_d \pi_1$ .*

**Proof.** We show by contradiction that we cannot have  $ser(\pi_1) < ser(\pi_2)$  and  $\pi_2 \prec_d \pi_1$ . Assume by absurd that  $\pi_2 \prec_d \pi_1$ , there are three cases to consider.

- If  $\pi_1$  and  $\pi_2$  are executed in order by the same transaction, then the result follows directly by the well-formedness assumption.
- If  $\pi_1$  reads or overwrites the value written by  $\pi_2$ , then  $\pi_2 \prec_d \pi_1$  implies that  $ser(t_1)$  is after  $t_2$  releases its lock on  $x$  (Line 43 or 76) otherwise  $t_1$  would have aborted (Line 49 if  $\pi_1$  is a read or Line 70 if  $\pi_1$  is a write) contradicting  $t_1 \in permanent(\mathcal{H}')$ .
- If  $\pi_1$  overwrites the value read by  $\pi_2$ , then either  $\pi_2$  would detect that its transaction owns the lock and so it would return the value written by  $\pi_1$  contradicting that  $\pi_1 \prec_d \pi_2$ , or  $\pi_2$  would detect that another transaction owns the lock, so  $t_2$  would abort (Line 49) contradicting that  $t_2 \in permanent(\mathcal{H}')$ .

Hence, all cases assuming that  $\pi_2 \prec_d \pi_1$  lead to a contradiction, implying that  $ser(\pi_2) \leq ser(\pi_1)$ . Hence, the equivalent contrapositive  $ser(\pi_1) < ser(\pi_2) \Rightarrow \pi_2 \not\prec_d \pi_1$  gives the result.  $\square$

**INVARIANT 5.6.**  $clock \geq lb$ .

**Proof.** Initially,  $clock = x.tlk.time = 0$ , for any variable  $x$ . Since  $clock$  is monotonically increasing,  $x.tlk.time$  can only be set to  $clock$ , and  $lb$  can only be set to  $clock$  or  $x.tlk.time$ , the result follows.  $\square$

Next, we generalize the previous lemma to  $\prec$ , the transitive closure of  $\prec_d$ .

**COROLLARY 5.7.** *Let  $\mathcal{H}$  be any history of  $\mathcal{E}$ -STM and let  $\mathcal{H}' = sub\text{-complete}(f_C(\mathcal{H}))$ . Let  $t_1$  and  $t_2$  be two transactions in  $permanent(\mathcal{H}')$ . For any two operations  $\pi_1$  and  $\pi_2$  executed respectively in  $t_1$  and  $t_2$ : if  $ser(\pi_1) < ser(\pi_2)$ , then  $\pi_2 \not\prec \pi_1$ .*

Next lemma indicates that any history is equivalent to some sequential history. More precisely, it shows that all operations of a single transaction are ordered in the same manner with respect to distant transactional operations. For the proof, let  $a_\pi$  denote the state of  $a$  when it is set in operation  $\pi$  for the first time, or when  $\pi$  starts if it is never set by  $\pi$ .

**LEMMA 5.8.** *Let  $\mathcal{H}$  be any history of  $\mathcal{E}$ -STM and let  $\mathcal{H}' = sub\text{-complete}(f_C(\mathcal{H}))$ . Let  $t_1$  and  $t_2$  be two transactions in  $permanent(\mathcal{H}')$ . Let  $\pi_1$  and  $\pi_2$  be some operation of transaction  $t_1$  and  $t_2$ , respectively. If  $\pi_1 \prec \pi_2$ , then for any  $\pi'_1 \in t_1$ :  $\pi_2 \not\prec \pi'_1$ .*

**Proof.** By contradiction we assume that  $\pi_1 \prec \pi_2 \prec \pi'_1$  and we show that  $t'_1$  aborts. With no loss of generality, let  $\pi_1$  and  $\pi'_1$  access  $a$  and  $a'$  respectively, we first show that  $\pi_1$  can only be a read and then consider the two cases whether  $\pi'_1$  is a write or a read.

$\pi_1$  cannot be a write operation, otherwise there should be a read operation  $r(a)$  such that  $\pi_1(a) \prec_d r(a) \prec \pi'_1$ , but  $r(a)$  as a read would have aborted (because  $x.tlk.owner_r = t_1$ , Line 49), or would loop while  $x.tlk.owner_r = t_1$  (Line 43), leading in both cases to the contradiction  $r(a) \not\prec \pi'_1$ . Since  $\pi_1$  is a read  $\pi_1 \prec \pi_2$  implies that there is a write operation  $w$  such that  $\pi_1(a) \prec_d w(a) \prec \pi'_1$ .

There are two cases to consider whether  $\pi'_1$  is a write. First, assume that  $\pi'_1$  is a write. By Invariant 5.6 we know that  $lb_{\pi_1} \leq clock_{\pi_1}$  and by the write  $w$ :  $clock_{\pi_1} < clock_{\pi'_1}$  so that  $lb_{\pi_1} < clock_{\pi'_1}$  and try-extend occurs in  $t_1$  or  $t_1$  would have aborted (contradicting the assumption). Second, if  $\pi'_1$  is a read, there is a write  $w'(a')$  such that  $\pi_2 \prec w'(a') \prec_d \pi'_1(a')$ . Hence  $x.tlk.time_{\pi'_1} > clock_w \geq clock_{\pi_1}$ , and by Invariant 5.6 we have  $clock_{\pi_1} \geq ub_{\pi_1} \geq ub_{\pi'_1}$ , whose conjunction leads to  $x.tlk.time_{\pi'_1} > ub_{\pi'_1}$ . Now observe that either  $a'.tlk.owner \notin \{\perp, t_1\}$  and  $t_1$  aborts,  $a'.tlk.owner = t_1$ , and  $w'$  would have aborted, or try-extend occurs in  $t_1$ . Hence, whether  $\pi'_1$  is a read or a write, try-extend occurs at  $t_1$ .

Finally, because  $\pi_1$  is a read and  $w$  is a write that occurs between  $\pi_1$  and  $\pi'_1$ ,  $t_1$  aborts during the try-extend

occurrence of  $\pi'_1$ . This contradicts the assumption and the result follows.  $\square$

**COROLLARY 5.9.** *Let  $\mathcal{H}$  be any history of  $\mathcal{E}$ -STM and let  $\mathcal{H}' = \text{sub-complete}(f_c(\mathcal{H}))$ . History permanent( $\mathcal{H}'$ ) is equivalent to a sequential history that is legal.*

**Proof.** Let  $\prec_t$  be an ordering on the set of transactions of  $\mathcal{H}'$  such that  $\forall t_1 \neq t_2, t_1 \prec_t t_2$  if there exist operations  $\pi_1$  in  $t_1$  and  $\pi_2$  in  $t_2$  such that  $\pi_1 \prec \pi_2$ . This ordering  $\prec_t$  is an irreflexive partial order because (i) it is antisymmetric by Lemma 5.8, and (ii) it is irreflexive and transitive by definition of  $\prec$ . This ordering  $\prec_t$  defines a set  $S$  of histories that are equivalent to  $\mathcal{H}'$ . This set is non-empty because  $\prec_t$  is a partial order. It is easy to see that for any  $s \in S$ ,  $s$  is sequential by the antisymmetry property of  $\prec_t$ . Finally and because  $\prec_t \subseteq \prec$ ,  $s$  is legal as well.  $\square$

## 6. Elastic Transactions: Evaluations

We evaluate here the performance of elastic transactions on four data structure applications.

### 6.1 Qualitative Assessment

$\mathcal{E}$ -STM is simple to program with for two reasons: (i) it indeed provides a high-level abstraction that do not expose synchronization mechanisms to the programmer, and (ii) it enables code composition.

#### 6.1.1 Abstraction

As with a classical transactional model, the programmer can use  $\mathcal{E}$ -STM to write a concurrent program almost as if he (she) was writing a sequential program. Like all TMs,  $\mathcal{E}$ -STM provides the programmer with labels *begin* and *commit* that can delimit the transactions. Hence, all calls to reading and writing the shared memory are redirected to the wrappers *read* and *write* of  $\mathcal{E}$ -STM, but this redirection does not incur efforts from the programmer and can be made automatic: some compilers already detect transaction labels and redirect memory accesses of these transactions automatically even though it is known that over instrumentation of accesses may unnecessarily impact performance.

To illustrate this, consider the sorted linked list implementation of an integer set, where integers (node *keys*) can be searched, removed, and inserted. Algorithm 2 depicts the entire program that uses  $\mathcal{E}$ -STM plus a lock-free *harris-ll-find* function for comparison

purpose. This function is at the core of the lock-free linked list of Harris [6]. It is pretty clear that the *harris-ll-find* function is more complex than its *ll-find* counterpart based on  $\mathcal{E}$ -STM. In fact, *harris-ll-find* relies on the use of a mark bit to indicate that a node is logically deleted, and must physically delete the nodes that have been logically deleted to ensure that the size of the list does not grow with each operation. Unlike the Harris lock-free function,  $\mathcal{E}$ -STM-based functions are very simple, as all synchronizations are handled transparently underneath by  $\mathcal{E}$ -STM. The pseudocode is the same as the non-thread-safe version, except that *begin(elastic)*, and *commit* have been added at the right places in the code.

#### 6.1.2 Extensibility

$\mathcal{E}$ -STM combines elastic transactions with normal transactions. As a result the code that uses  $\mathcal{E}$ -STM is easily extensible. To illustrate this, we implemented the hash table example presented in Section 1 extended with operations *move* and *sum*. The pseudocode is presented in Algorithm 3.

More specifically, we implemented the *insert*, *search*, and *remove* operations using elastic transactions. Since each bucket of the hash table is implemented with a linked list, we re-used (Lines 12, 18, and 24) the program of the linked list written above. More complex operations like *move* and *sum* have been implemented using normal transactions. The elastic transactions nested inside the normal transactions of *move* (Lines 22 and 16) execute in the normal mode. Although it is also possible to implement an elastic version of *move*, *sum* cannot be elastic as it requires an atomic snapshot of all elements of the data structure. This example illustrates the way elastic and normal transactions can be combined.

Observe that, although moving a value from one node to one of its predecessors in the same linked list may lead an elastic search not to see the moved value, the two operations remain correct. Indeed, the search looks for a key associated with a value while the *move* changes the key of a value  $v$ . Hence, if the search looks for the initial key  $k$  of  $v$  and fails in finding it, then search will be serialized after *move*, if search looks for the targeted key  $k'$  of  $v$  and does not find it, then search will be serialized before *move*. In contrast, a less usual search-value operation looking for the associated value rather than the key of an element would have to be im-

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**Algorithm 2** Linked list implementation built on  $\mathcal{E}$ -STM (the lock-free harris-ll-find is given for comparison)

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<pre> 1: <b>State of process <math>p</math>:</b> 2:   <math>node</math> a record with fields: 3:     <math>key</math>, an integer 4:     <math>next</math>, a node 5:   <math>set</math> a linked-list of <math>nodes</math> with: 6:     <math>head</math> at the beginning, 7:     <math>tail</math> at the end. 8:   Initially, the <math>set</math> contains head and 9:     tail nodes, and <math>head.key = \min</math>, 10:    and <math>tail.key = \max</math>.  11: <b>free(<math>x</math>)<sub><math>t</math></sub>: 12:   // memory disposal is postponed 13:   write(<math>x</math>, 0)  14: <b>ll-find</b>(<math>i</math>)<sub><math>p</math></sub>: 15:   <math>curr \leftarrow set.head</math> 16:   <b>while true do</b> 17:     <math>next \leftarrow read(curr.next)</math> 18:     <b>if</b> <math>next.key \geq i</math> <b>then break</b> 19:     <math>curr \leftarrow next</math> 20:   <b>return</b> (<math>curr, next</math>) </b></pre>	<pre> 21: <b>ll-insert</b>(<math>i</math>)<sub><math>p</math></sub>: 22:   <b>begin</b>(elastic) 23:   (<math>curr, next</math>) <math>\leftarrow</math> ll-find(<math>i</math>) 24:   <math>in \leftarrow (next.key = i)</math> 25:   <b>if</b> !<math>in</math> <b>then</b> 26:     <math>new-node \leftarrow \langle i, next \rangle</math> 27:     write(<math>curr.next, new-node</math>) 28:   <b>commit</b>() 29:   <b>return</b> (!<math>in</math>)  30: <b>ll-search</b>(<math>i</math>)<sub><math>p</math></sub>: 31:   <b>begin</b>(elastic) 32:   (<math>curr, next</math>) <math>\leftarrow</math> ll-find(<math>i</math>) 33:   <math>in \leftarrow (next.key = i)</math> 34:   <b>commit</b>() 35:   <b>return</b> (<math>in</math>)  36: <b>ll-remove</b>(<math>i</math>)<sub><math>p</math></sub>: 37:   <b>begin</b>(elastic) 38:   (<math>curr, next</math>) <math>\leftarrow</math> ll-find(<math>i</math>) 39:   <math>in \leftarrow (next.key = i)</math> 40:   <b>if</b> <math>in</math> <b>then</b> 41:     <math>n \leftarrow read(next.next)</math> 42:     write(<math>curr.next, n</math>) 43:     <b>free</b>(<math>next</math>) 44:   <b>commit</b>() 45:   <b>return</b> (<math>in</math>) </pre>	<pre> 46: <b>harris-ll-find</b>(<math>i</math>)<sub><math>p</math></sub>: 47:   <b>loop</b> 48:     <math>t \leftarrow set.head</math> 49:     <math>t\_next \leftarrow read(curr.next)</math> 50:     // 1. find left and right nodes 51:     <b>repeat:</b> 52:       <b>if</b> !is_marked(<math>t\_next</math>) <b>then</b> 53:         <math>curr \leftarrow t</math> 54:         <math>c\_next \leftarrow t\_next</math> 55:         <math>curr \leftarrow unmarked(next)</math> 56:       <b>if</b> !<math>t\_next</math> <b>then break</b> 57:       <math>t\_next \leftarrow t.next</math> 58:     <b>until</b> is_marked(<math>t\_next</math>) <math>\vee</math> (<math>t.key &lt; i</math>) 59:     <math>next = t</math> 60:     // 2. check nodes are adjacent 61:     <b>if</b> <math>c\_next = next</math> <b>then</b> 62:       <b>if</b> (<math>next.next \wedge</math> 63:         is_marked(<math>next.next</math>)) <b>then</b> 64:         goto line 48 65:       <b>else return</b> (<math>curr, next</math>) 66:     // 3. remove one or more marked node 67:     <b>if</b> cas(<math>curr.next, c\_next, next</math>) <b>then</b> 68:       <b>if</b> (<math>next.next \wedge</math> 69:         is_marked(<math>next.next</math>)) <b>then</b> 70:         goto line 48 71:       <b>else return</b> (<math>curr, next</math>) 72:   <b>end loop</b> </pre>
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plemented using normal transactions, otherwise, a concurrent move may lead to an inconsistent state. Another issue, pointed out in [17], may arise when one transaction inserts  $x$  if  $y$  is absent and another inserts  $y$  if  $x$  is absent. If executed concurrently, these two transactions may lead to an inconsistent state where both  $x$  and  $y$  are present. Again, our model copes with this issue as the programmer can use a normal transaction to encapsulate each conditional insertion. All these normal and elastic transactions are safely combined.

Unlike elastic transactions, existing synchronization techniques (e.g., based on locks or compare-and-swap) cannot be easily combined with normal transactions. They furthermore introduce a significant complexity. Using a coarse-grained lock to make the hash table move operation atomic would prevent concurrent accesses to the data structure. In contrast, using fine-grained locks may lead to a deadlock if one process moves from bucket  $\ell_1$  to bucket  $\ell_2$  while another moves from  $\ell_2$  to  $\ell_1$ . With a lock-free approach (e.g., based on an underlying compare-and-swap), one could either modify a copy of the data structure before switching a pointer from one copy to another, or use a multi-word compare-and-swap instruction. Unfortunately, the former solution is costly in memory usage whereas the

latter solution requires a rarely supported instruction that is also considered as inefficient. Extending a lock-free hash table further to provide a resize operation reveals even more as this requires to replace its internal bucket linked lists by a single linked list imposing to re-implement the whole data structure [16].

## 6.2 Quantitative Assessment

In this section, we present the performance results obtained when running  $\mathcal{E}$ -STM, locks, regular STM transactions, and lock-free algorithms on our data structure applications. We also compare the obtained results to the non-thread-safe code executed sequentially.

### 6.2.1 Testbed data structures

A linked list is an appealing data structure: it is simple yet flexible, and it provides insert and remove operations that only affect a localized part of the data structure, as opposed to arrays. The pseudocode of our  $\mathcal{E}$ -STM-based linked list have been given in Algorithm 2 and its proof of consistency has been deferred to Appendix A.

Skip lists are known to provide logarithmic search time complexity while being simpler to program than balanced trees. For instance, insert and remove in an

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**Algorithm 3** Hash table implementation built on  $\mathcal{E}$ -STM and linked-list

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1: <b>State of process <math>p</math>:</b>	15: <b>ht-insert(<math>i</math>)<math>_p</math>:</b>	27: <b>ht-sum()<math>_p</math>:</b>
2: $node$ a record with fields:	16: <b>begin(elastic)</b>	28: <b>begin(normal)</b>
3: $key$ , an integer	17: $a \leftarrow \text{hash}(i)$	29: <b>for each bucket in set do</b>
4: $next$ , a node	18: $result \leftarrow \text{set}[a].\text{ll-insert}(i)$	30: $next \leftarrow \text{read}(bucket.\text{head}.next)$
5: $set$ a mapping from an integer to a	19: <b>commit()</b>	31: <b>while <math>next.next \neq \perp</math> do</b>
6: $linkedlist$ representing a bucket.	20: <b>return result</b>	32: $sum \leftarrow sum + \text{read}(next.val)$
7: Initially, all buckets of the $set$ are empty		33: $next \leftarrow \text{read}(next.next)$
8: lists.	21: <b>ht-remove(<math>i</math>)<math>_p</math>:</b>	34: <b>commit()</b>
	22: <b>begin(elastic)</b>	35: <b>return sum</b>
9: <b>ht-search(<math>i</math>)<math>_p</math>:</b>	23: $a \leftarrow \text{hash}(i)$	
10: <b>begin(elastic)</b>	24: $result \leftarrow \text{set}[a].\text{ll-remove}(i)$	36: <b>ht-move(<math>i, j</math>)<math>_p</math>:</b>
11: $a \leftarrow \text{hash}(i)$	25: <b>commit()</b>	37: <b>begin(normal)</b>
12: $result \leftarrow \text{set}[a].\text{ll-search}(i)$	26: <b>return result</b>	38: <b>ht-remove(<math>i</math>)</b>
13: <b>commit()</b>		39: <b>ht-insert(<math>j</math>)</b>
14: <b>return result</b>		40: <b>commit()</b>
		41: <b>return result</b>

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AVL tree induce complex re-balancing operations. Red-black trees are binary trees where each node is colored (red or black) and where leaves have no keys. The coloring is such that the root and the leaves are black, all paths from the root to leaves contain the same number of black nodes, and all children of a red node are black. As a result, a red-black tree remains balanced enough to provide logarithmic operation complexity and relaxes the strict balancing of AVLs. Red-black trees have been largely used as micro-benchmarks in concurrent programming and especially for the evaluation of numerous STMs [1, 2, 11]. Hash tables provide constant access complexity. Our corresponding pseudocode is given in Algorithm 3. Due to lack of space, the pseudocode of the skip list and the red-black tree has been omitted here. Our implementations consist simply of a non-thread-safe code simply enriched with elastic transaction calls (`begin(elastic)`, `begin(normal)`, `read`, `write`, `commit`) as for the linked list.

### 6.2.2 Lock-based, lock-free, and STM alternatives

Our lock-based linked list implements the lazy algorithm which is more efficient than fine-grained alternatives [8] (e.g., lock-coupling or hand-over-hand locking); our lock-based version of the hash table builds upon it. Using a similar technique, our lock-based skip list is a C version of the optimistic algorithm of [10], the most recent lock-based skip list algorithm we know of.

We consider the Harris-Michael implementation of the lock-free linked list as presented in [6] and the hash table version that is based on it [12]. Unfortunately,

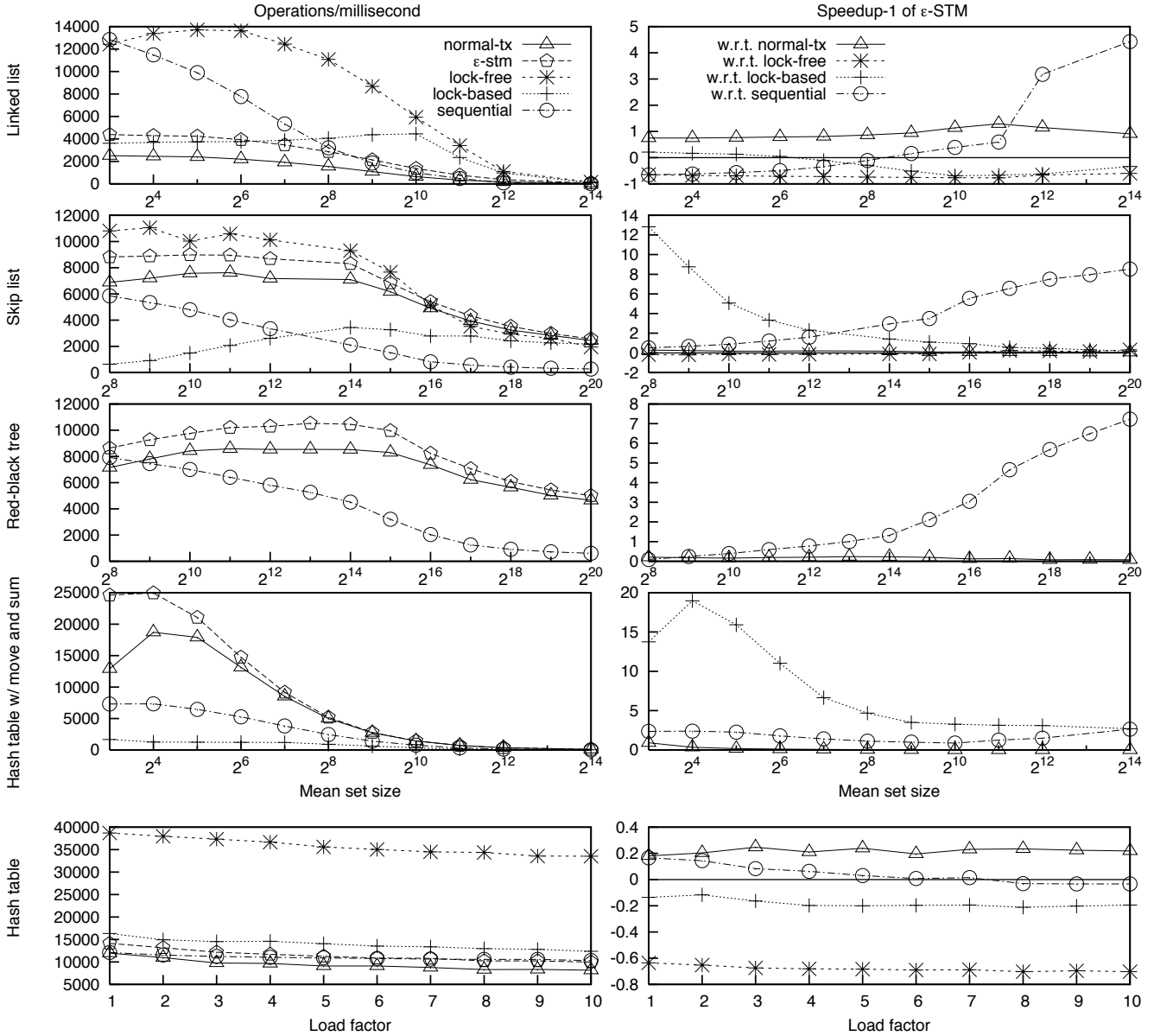
this hash table algorithm cannot support complex operations like sum and move, so we ran two distinct sets of experiment on hash tables. For our needs (x86-64), we re-implemented the Fraser lock-free skip list that uses the low-order bit marking technique of Harris-Michael’s linked list [3]. Only miss the lock-free and lock-based versions of the red-black tree.

We chose TinySTM [2] v0.9.8 (in its default mode) to compare regular transactions with  $\mathcal{E}$ -STM because TinySTM is, as far as we know, the most efficient STM on micro-benchmarks such as linked lists and red-black trees [2, 5].

### 6.2.3 Experimental results

We tested the performance of these data structure implementations on a 4 quad-core AMD Opteron machine, i.e., including 16 cores in total. We ran the aforementioned algorithms using 16 threads and their non-thread-safe counterpart using a single thread, all written in C. Each point on the graphs represents the average value obtained from 5 runs of 10 seconds each.

Figure 1 depicts the results obtained when running 90% of search operations, 5% of insert, and 5% of remove, except in the 4th row. We varied the initial size of the sets, and we insert and remove the same integers to keep the set size roughly constant in each experiment. As we do not know of any lock-free implementation of a move, we ran two sets of hash table experiments: (i) experiments of row 4 include 10% of move, 10% of sum with 80% of search while the load factor is fixed to 5 (the load factor corresponds to the mean ratio of the number of nodes over the number



**Figure 1.** Performance results when running integer set operations. *Left side:* operation throughput of all synchronization techniques (with 16 threads) compared to non-thread-safe throughput (with a single thread) on linked list (1st row), skip list (2nd row), red-black tree (3rd row), and hash table (5th row) with 5% insert, 5% remove, and 90% search operations; and hash table with 10% move, 10% sum, and 80% search (4th row). *Right side:* resulting improvement  $i$  (or speedup-1) of throughput  $u$  of  $\mathcal{E}$ -STM over the throughput  $v$  of each other techniques (where  $i = \frac{u}{v} - 1$ ), the solid lines representing  $f(x) = 0$  is the limit above which  $\mathcal{E}$ -STM presents better performance than other techniques.

of buckets); (ii) experiments of row 5 include only the three basic integer set operations in the same proportion as above (5-5-90) while the load factor varies and the hash table contains 256 buckets. We chose a reasonably low amount of updating operations (10%) in these experiments because transactions are already known to perform well in highly-contended environment.

The results show that  $\mathcal{E}$ -STM always improves the performance of the regular STM and can speed up the regular STM by more than 2.3x in some circumstances, with an averaged speedup of 1.36x.<sup>1</sup> We averaged the speedups over linked list, skip list, and hash table using only insert, remove, and search operations for which we had values regarding each implementation. We observe that  $\mathcal{E}$ -STM performance competes with lock-based performance: the improvement factor of  $\mathcal{E}$ -STM over lock-based implementations is 1.89x on average. Our implementation is on average slower than lock-free implementations (averaged slowdown of 1.47x) yet significantly simpler. It is noteworthy that, unlike all other implementations, the Harris lock-free algorithms have been implemented “as is” without any memory re-allocation, which may impact the performance. In addition, the  $\mathcal{E}$ -STM average speedup over the sequential performance is of 2.8x. Finally,  $\mathcal{E}$ -STM is on average 1.2x faster than regular STM transactions and up to 8.2x faster than sequential, on red-black trees.

More particularly, the fourth row indicates a speedup of  $\mathcal{E}$ -STM over regular STM transactions that goes up to 1.9x, over the sequential version that goes up to 3.6x, and over lock-based that goes up to 20x, outlining the inherent efficiency obtained when combining normal and elastic transactions like  $\mathcal{E}$ -STM does.

## 7. Discussion and Related Work

One programmer may think of cutting normal transactions himself (herself) instead of using elastic transactions. Nevertheless, hand-crafted cuts must be defined prior to execution which may lead to inconsistencies. As an example, consider that a transaction  $t$  searches a linked list. A hand-crafted cut of  $t$  between two read operations on  $x$  and  $y$  may lead to an inconsistent state if another transaction deletes  $y$  (by modifying the next

pointers of  $x$  and  $y$ ) between those reads:  $t$  does not detect that it stops parsing the data structure as soon as it tries to access  $y$ . In contrast, elastic transactions avoid this issue by checking dynamically if a transaction can be safely cut and aborting otherwise.

Besides elastic transactions, there have been several attempts to extend the classical transactional model. Open nesting [13] provides sub-transactions that can commit while the outermost transaction is not completed yet. More precisely, open nesting makes sub-transactions visible before the outermost transaction commits. This requires the programmer to define complex roll-backs [14].

Transactional boosting [9] is a methodology for transforming linearizable objects into transactional objects, which builds upon techniques from the database literature. Although transactional boosting enhances concurrency by relaxing constraints imposed by read/write semantics at low-level, it requires the programmer to identify the commutative operations and to define inverse operations for non-commutative ones.

Abstract nesting [7] allows to abort partially in case of low-level conflict. As the authors illustrate, abstract nested transactions can encapsulate independent sub-parts of regular transactions like insert and remove sub-parts of a move transaction. In contrast, abstract nested transactions cannot encapsulate sub-parts of the parsing (as in search/insert/remove) of a data structure. Moreover, abstract nested transactions aim at reducing the roll-back cost due to low-level conflicts, but not at reducing the amount of low-level conflicts.

Early release [11] is the action of forgetting past reads before a transaction ends. This mechanism, presented for DSTM, enhances concurrency by decreasing the number of low-level conflicts for some pointer structures. It requires the programmer to carefully determine when and which objects in every transaction can be safely released [17]: if an object is released too early then the same inconsistency problem as with hand-crafted cuts arises. Finally, early release provides less concurrency than elastic transactions. Consider a transaction  $t$  that accesses  $x$  and  $y$  before releasing  $x$ . If  $y$  is modified between  $t$  accessing  $x$  and  $y$  then a conflict is always detected. In contrast, if  $t$  is an elastic transaction then a conflict is detected only if  $x$  and  $y$  are consecutively accessed by  $t$  and both  $x$  and  $y$  are modified between those accesses, which is very unlikely in practice.

<sup>1</sup>The improvement (or speedup-1) represented in the graphs is  $i = \frac{u}{v} - 1$  and corresponds to the improvement of  $u$  over  $v$  so that positive values indicate gains and negative values indicate losses, while in the text we refer to the speedup  $s = \frac{u}{v}$  as the multiplying factor between  $u$  and  $v$ .

## 8. Conclusion

We have proposed a new transactional model that enhances concurrency in a simple fashion. The core idea relies on the combination of traditional transactions with a new type of transactions that are elastic in the sense that their size evolves dynamically depending on conflict detection.

We implemented this model in an STM, called  $\mathcal{E}$ -STM, that only requires to differentiate elastic from traditional transactions, making it simple to program with. Comparisons on four well-known data structures have confirmed that elastic transactions perform significantly better than traditional transactions and lock-based alternatives, and is much simpler to program with than existing lock-free solutions.

We have evaluated the performance of our approach with data structures where operations parsing the structure are loosely dependent on the subsequent operations modifying a region of it. It could be interesting to investigate how much performance other applications could gain from using this model. For example, the counter increment on which the rest of the transaction does not depend.

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### A. Proof of Correctness of the Linked list

Here, we prove that our linked list algorithm implements an integer set. The proof relies on elastic opacity (Definition 3.4) and the pseudocode of Algorithm 2. First-of-all, we recall the semantics of the integer set. Given a set  $S$ :

- $\text{search}(i)$  operation returns true if the node  $i$  is present in  $S$ , false otherwise;
- $\text{insert}(i)$  operation augments the set  $S$  with the node  $i$  if  $i$  is not in  $S$ ,  $S$  is unchanged otherwise;
- $\text{remove}(i)$  operation removes the node  $i$  from  $S$  if  $i$  is in  $S$ ,  $S$  is unchanged otherwise.

In the following, we refer to such an integer set operation as  $\Pi$ . Next, we state preliminary definitions.

**DEFINITION A.1.** *A node  $n$  is reachable if one of the two following properties holds:*

- $\text{set.head.next} = n$  or
- *there exists a reachable node  $m$  such that  $m.\text{next} = n$ .*

**DEFINITION A.2.** *Integer  $i$  is in the set if and only if there is a reachable node  $n$  such that  $n.\text{key} = i$ .*

First lemma gives an important result for proving the correctness of operation  $\text{search}(\ast)$  and relies on the definition of consistent cut (Definition 3.3).

**LEMMA A.3.** *Operation  $\text{find}(i)$  returns a pair of nodes  $\langle \text{curr}, \text{next} \rangle$  such that*

1.  $\text{curr.key} < i \leq \text{next.key}$  and

2.  $\text{curr}$  and  $\text{next}$  are consecutive nodes of the list at some point in time during the corresponding execution of  $\text{find}$ .

**Proof.** We show the two points separately.

1. First, we show that  $\text{curr.key} < i \leq \text{next.key}$ . At the beginning  $\text{curr}$  is initialized at the head of the linked list, while  $\text{next}$  is initialized to its successor in the linked list. As the loop iterates, the  $\text{curr}$  and  $\text{next}$  parses the linked list in ascending order, unless the transaction aborts. Observe that the function exits the main loop (and returns) if  $\text{next.key} \geq i$  at Line 18. If so,  $\text{curr}$  and  $\text{next}$  are returned as is. By absurd, if  $\text{curr.key} \geq i$  then the loop would have ended at least one iteration before, so  $\text{curr.key} < i \leq \text{next.key}$ . Finally, observe that  $\text{next} \leftarrow \text{read}(\text{curr.next})$ . Hence, during the last iteration of the main loop of  $\text{read}$ ,  $\text{curr.next} = \text{next}$ .
2. Second, we show that  $\text{curr}$  and  $\text{next}$  are consecutive nodes of the list at some point in time during the corresponding execution of  $\text{find}$ . Observe that a  $\text{find}$  operation is always called as part of an elastic transaction. As a consequence of Definition 3.3, there is no two write operations on  $\text{curr}$  and  $\text{next}$  between  $\text{read}(\text{curr})$  and  $\text{read}(\text{next})$  executed by  $\text{find}$ . Hence, there are two cases to consider whether one of these two writes is missing. If there is no write on  $\text{curr}$ , then at the time the second read occurs on  $\text{next}$ , a read of  $\text{curr}$  would return the same value as the one that has been returned before by  $\text{read}(\text{curr})$ . If there is no write on  $\text{next}$ , then at the time the first read on  $\text{curr}$  occurs, the second would have returned the same value as the one it will return with  $\text{read}(\text{next})$ . Note that both cases are true for the case  $\text{next} = \text{curr}$ , because none of these writes can occur between the reads by Definition 3.3. It follows that each of the two nodes are consecutive at some time during the execution of this  $\text{find}$ .

The result follows.  $\square$

We know by elastic-opacity (Definition 3.4) that no aborting transactions modify the state of the system hence if a transaction aborts, we are ensured that safety is preserved (only committing transactions can violate safety). The major difficulties when implementing integer set stem from the concurrent updates problem [18]. For instance, when attempting to insert two distinct values concurrently between nodes  $a$  and  $b$ , particular ef-



forts are required to prevent one of these insert from hiding the result of the other. Another critical example is when one inserts a value between  $a$  and  $b$  while another is concurrently deleting  $a$ , special care is needed to ensure that the insert will be effective. Finally deleting two successive nodes  $a$  and  $b$  concurrently may easily result in the sole deletion of node  $a$ . Next, we show that no update can annihilate the effect of another update on our linked list implementation.

LEMMA A.4. *Let  $\pi$  be the  $\text{write}(\text{curr.next}, *)$  of an insert or a  $\text{read}(\text{curr.next})$  or a  $\text{write}(\text{curr.next}, *)$  of a remove operation. While  $\pi$  executes:*

- either  $\text{curr.next} = \text{next}$ ,
- or  $\pi$  aborts its transaction.

**Proof.** Assume that  $\text{curr.next} \neq \text{next}$ . The proof relies on the fact that  $\pi$  detects that  $\text{curr}$  and  $\text{next}$  are not consecutive and aborts its transaction.

By functions  $\text{find}()$  and  $\text{read}()$ , at some time  $t$  during the last iteration of its main loop we have  $\text{curr.next} = \text{next}$ . At this time  $t$ , either  $\text{curr.tlk.time} \leq ub$  and  $\text{next.tlk.time} \leq ub$  or  $ub$  is updated to  $\text{next.tlk.time}$  such that  $\text{next.tlk.time} > \text{curr.tlk.time}$  before  $\text{find}()$  returns. Hence  $\text{curr.tlk.time} \leq ub$  and  $\text{next.tlk.time} \leq ub$  when the normal transaction of  $\pi$  starts.

Since any modification uses a two-phase locking mechanism to modify the value and the timestamp of a location, either  $\pi$  detects that  $\text{curr}$  is locked, i.e.,  $\text{curr.tlk.owner} \neq \perp$  or it detects that its version is too recent, i.e.,  $\text{curr.tlk.time} > ub$ . In the former case, since no previous write operation occurs in the same transaction,  $\text{curr.tlk.owner} \neq t$  and this transaction aborts as indicated Lines 49 and 70 of Algorithm 1. In the latter case, it cannot extend and aborts because the transaction type is elastic (Line 73).

Consequently, either  $\text{curr.next} = \text{next}$  or  $\pi$  aborts its enclosing transaction (Line 70 or Line 73 of Algorithm 1).  $\square$

Next, we show that the time at which insert or remove operation acquires its lock during its write operation serves as a serialization point for the operation.

LEMMA A.5. *Let  $\Pi_1$  and  $\Pi_2$  be two update operations on the linked list. Then among them, the first to acquire the lock in its write appears to have executed before the other.*

**Proof.** There are only two types of operations to consider here:  $\text{insert}(*)$  and  $\text{remove}(*)$ . It is easy to see that if these operations do not conflict then they can be arbitrarily ordered and if these operations are not concurrent then the first to execute can not see the result of the second one.

Consider the cases where they conflict, hence either (i)  $\text{insert}(*)$  inserts a node between nodes  $a$  and  $b$  and  $\text{remove}(a)$  deletes  $a$  or deletes  $b$ , or (ii) two  $\text{insert}(a)$  insert at node  $a$ , or (iii) two  $\text{remove}(a)$  delete node  $a$ , or finally, (iv)  $\text{remove}(a)$  deletes node  $a$  and  $\text{remove}(b)$  deletes node  $b$ .

If insert acquires the lock first before remove acquires it then insert acquires the lock on  $a$  either before the read of remove occurs, or between the read and write of remove. In the former case, the read detects it and aborts as indicated in Lemma A.4. In the latter case, by the elastic-opacity of transactions (Theorem 5.1), one must abort (i.e, when the transaction of the remove commits, it sees that  $\text{clock}$  has been increased leading to an abort). Hence insert appears as occurring before the remove if it acquires the lock first.

In the other cases, the operations conflict on  $\text{write}(a.\text{next}, *)$  so that one acquires the lock before the other, modifying  $a.\text{next}$  and the other aborts as indicated by Lemma A.4. Again the first to acquire the lock executes while the other aborts and restarts later.  $\square$

LEMMA A.6. *A  $\text{search}(i)$  operation appears as if it is executed atomically at the time the  $\text{read}(\text{next})$  of its find occurs such that  $\text{curr.next.key} \geq i$ .*

**Proof.** If a search occurs immediately after an update acquires a lock on  $\text{curr}$ , then search will spin over the lock until  $\text{curr.tlk.owner} = \perp$ . Hence the read will observe the result of the update and will appear to be ordered after, so as the enclosing search. If however, it consults  $\text{curr}$  before any operation acquires a lock on it, then it will be ordered before. Finally, by Lemma A.3,  $\text{curr.next.key} \geq i$ .  $\square$

INVARIANT A.7. *For any node  $\text{curr}$  in the linked list, if  $\text{curr.next} = \text{next}$  then  $\text{curr.key} < \text{next.key}$ .*

**Proof.** By Lemma A.3, the  $\text{curr}$  and  $\text{next}$  returned by  $\text{find}(i)$  are such that  $\text{curr.key} < i \leq \text{next.key}$ . By examination of the code of  $\text{insert}(i)$  node  $i$  can only be

inserted between  $a$  and  $b$  such that  $a.key < i < b.key$ .  
The result follows.  $\square$

**THEOREM A.8.** *The linked list implementation presented in Algorithm 2 is correct.*

**Proof.** By Invariant A.7 the linked list is a sorted linked list with no duplicates. By Lemma A.5, an update op-

eration admits a serialization point at which it appears to be atomic. Hence the semantics of remove and insert are satisfied. Finally, by Lemma A.3 the search operation respects its semantics and by Lemma A.6, search is linearizable.  $\square$