Toroidal mode number analysis of degenerate Alfvén Eigenmodes in the active MHD spectroscopy on JET

A. Klein[1], D. Testa[2], J. Snipes[1], A. Fasoli[2], H. Carfantan[3], JET-EFDA contributors*
[1]*MIT Plasma Science and Fusion Center, Cambridge, USA*
[2]*CRPP, Association EURATOM –EPFL, Lausanne, Switzerland*
[3]*CNRS/Université Paul Sabatier Toulouse 3, Toulouse, France*

The active MHD spectroscopy program at JET incorporates eight antennas, designed to excite and characterize stable Alfvén Eigenmodes (AE's) with medium toroidal mode numbers ($-10 < n < 10$) in a variety of plasma conditions [1,2]. The antennas consist of simple rectangular loops (18 turns per antenna) which cover less than 0.5 m^2 of the plasma surface. Due to the compact antenna geometry, the generated magnetic perturbations are composed of a broad spectrum of mode numbers (in vacuum, $-100 < n < 100$). In the presence of a strong, single AE plasma resonance, traditional methods such as straight line phase fitting of signals from an array of Mirnov coils easily reveals the n-number. Often however, several degenerate AE's with multiple mode numbers may be excited by the antennas, and subsequent n-number analysis of the plasma response becomes difficult. A new method based on the SPARSPEC code [3], used in Astronomy, is applied to resolve individual mode numbers in this data.

It is easily seen that temporal frequencies in astronomical data can correspond to spatial toroidal mode numbers in tokamaks, and that unevenly sampled data in time is the analog of data from unevenly distributed Mirnov sensors in the toroidal coordinate. As it turns out, SPARSPEC is ideally suited for toroidal mode number analysis because it models data as a large number (possibly larger than the data size) of pure sinusoidal modes, discretized on a fixed, arbitrarily thin, grid, which means that it can be configured to search for only integer mode numbers. Among the many representations fitting the data, SPARSPEC seeks the one with the fewest non-zero amplitudes, i.e. the sparsest spectrum, by assigning a penalty to solutions which invoke larger numbers of modes to fit the data. The minimization in SparSpec to detect frequencies and estimate the amplitude of the detected modes is performed on what is known as the ℓ^1-norm, which bypasses the requirement to test every possible combination of mode vectors as is the case with the ℓ^0-norm, and which results in very rapid computation.

Many theoretical works have been done to determine conditions of equivalence of both settings, one example is found in [4]. We adapted the SPARSPEC code to take complex data as input, and Monte-Carlo style simulations were run to benchmark the method with the JET style data and sensor geometry.

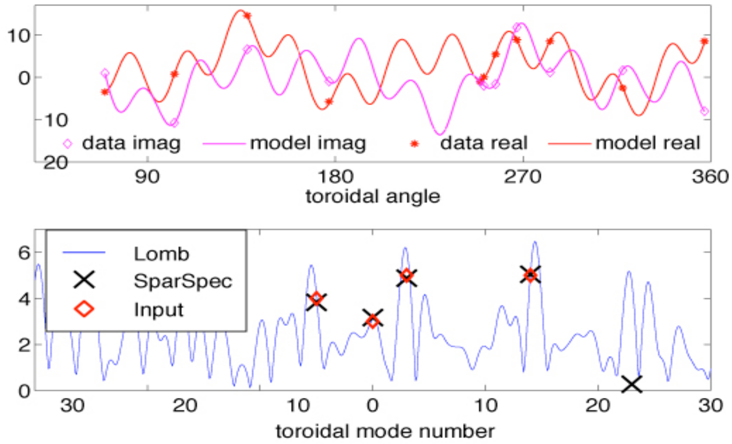


Figure 1: SparSpec calculation using simulated data: top: data (points) and model fit (lines). Bottom: input mode amplitudes vs. estimation from SparSpec, along with Lomb periodogram. 11 positions (corresponding to JET Mirnov coils), complex data with 4 random modes + 5% Gaussian noise. Mode number grid restricted to –40 < n < 40. Computation time: < 6 msec on a 2.2 GHz processor.

The code was then applied to real data consisting of unstable AEs at frequencies up to 500 kHz, very good agreement was found with straight line phase fitting methods, and SPARSPEC was found to be much more immune to noise and false n-number detection. We believe that this new method of processing fast-fluctuation data from toroidal plasmas will be found useful for all experiments in which spatial harmonic analysis is desireable.

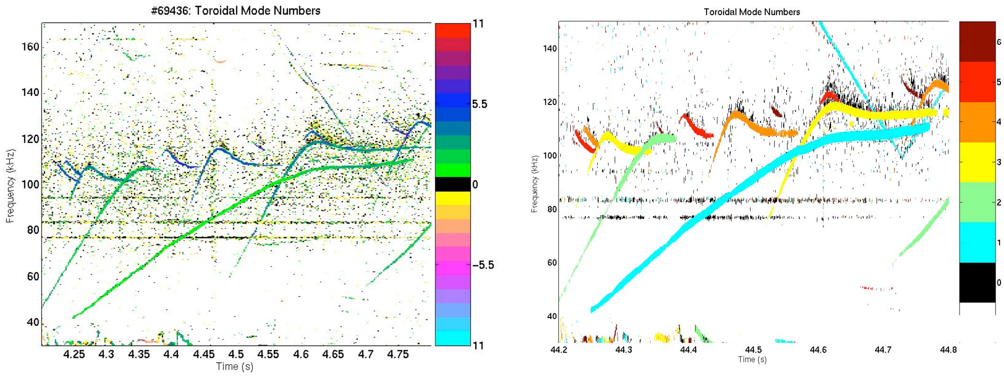
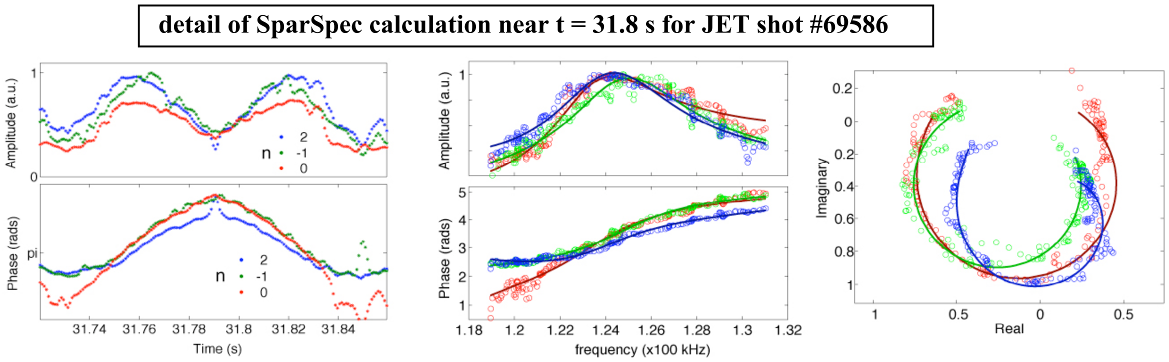


Figure 2: Toroidal mode analysis for unstable Alfvén Cascades with straight line phase fitting (left) and SPARSPEC analysis (right) show SPARSPEC is somewhat less sensitive to noise and errors.

Experimentally, the antenna excitation fields are strongly edge localized, so that AEs with long wavelengths ($n \le 2$) couple much more efficiently to the antennas, because their radial eigenfunctions tend to be broad and extend all the way to the plasma edge. As a consequence, data from initial experiments which utilized only one octant of antennas in 2007 consist predominantly of low-n damping measurements, which are not the goal of the project and for which a large data base already exists. However, despite the tendency for the resonance detection and tracking system to lock onto low-n AEs, many weak medium-n resonances could be extracted from the data using SPARSPEC.

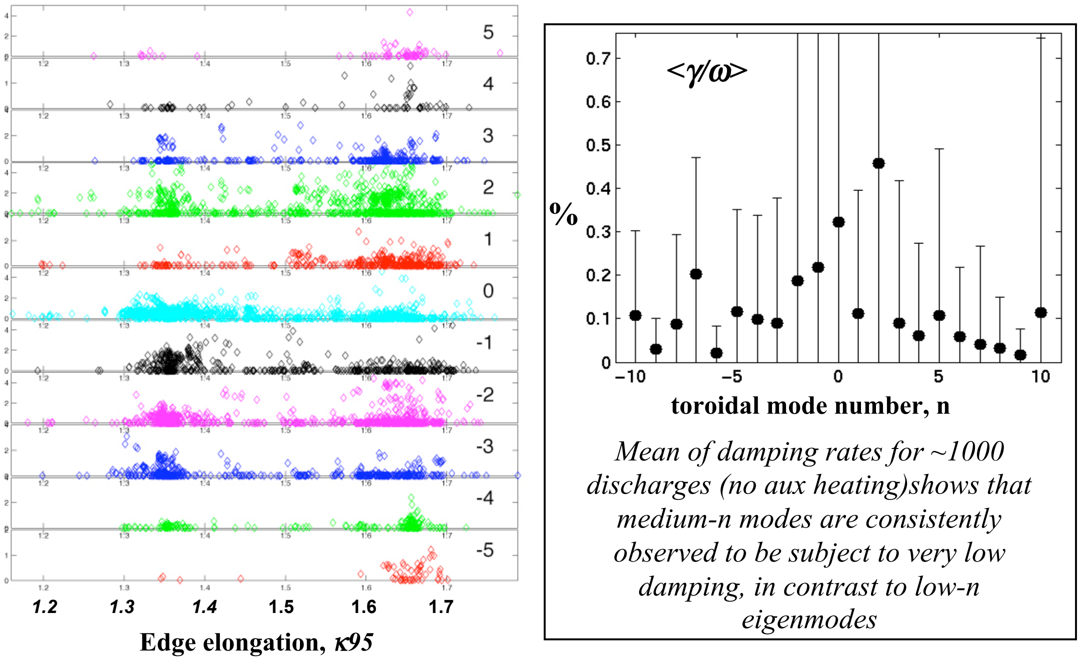


detail of SparSpec calculation near t = 31.8 s for JET shot #69586

Figure 3: Left: . Three modes can be seen to have slightly different resonant frequencies. Middle: normalized amplitude and phase each mode vs. antenna frequency. Right: damping calculation the three modes: for n =0: freqres = 123 kHz, $\gamma/\omega = 1.96\%$, for n =-1 : freqres = 125 kHz, $\gamma/\omega = 2.11\%$, for n =2 : freqres = 124 kHz, $\gamma/\omega = 2.12\%$.

In the 2007 campaigns, the TAE spectroscopy system was active for about 75% of all discharges in a large variety of plasma conditions, filling a data base with over 56,000 damping measurements. Clear scaling and physics trends are not easily extracted however. AEs are ubiquitous in toroidal plasmas, and very often they can be essentially undamped. The resonance detection algorithm is able to detect modes with damping rates between $0.1\% < \gamma/\omega = 10\%$.

Theoretical predictions calculations show that small differences in equilibrium parameters such as the q profile can cause wild variation in the damping rates [5], so that overall trends may be obscured by the drastic effect of very subtle differences in the MHD environment. Some preliminary results are shown in fig.4.



Mean of damping rates for ~1000 discharges (no aux heating)shows that medium-n modes are consistently observed to be subject to very low damping, in contrast to low-n eigenmodes

Figure 4:Left: damping rate γ/ω (in %) vs. edge elongation, κ95, for –5 < n < 5. Medium n modes (n > 2) semm to be more easily excited at higher elongation. Right: Mean of damping rates for ~1000 discharges (no aux heating)shows that medium-n modes are consistently observed to be subject to very low damping, in contrast to low-n eigenmodes.

In addition to post-pulse analysis, the efficiency of the SparSpec method makes it suitable for real-time application, allowing for selective targeting of individual AEs in the tracking algorithm which controls the antennas. For the resonance detection and tracking computer on JET, looprates of 1 msec are required, which is easily accomplished when SPARSPEC is applied to synchronously detected signals from eight Mirnov coils. With the ability to ignore the strong low-n response in selected Mirnov coils, the range of the active antenna system can be functionally extended to allow for medium-n specific experiments. New experiments with improved mode selection and detection, are planned in 2008.

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