Physical limits to broadening compensation in a linear slow light system

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Abstract: The dispersion experienced by a signal in a slow light system leads to a significant pulse broadening and sets a limit to the maximum delay actually achievable by the system. To overcome this limitation, a substantial research effort is currently being carried out, and successful strategies to reduce distortion in linear slow light systems have already been demonstrated. Recent theoretical and experimental works have even claimed the achievement of zero-broadening of pulses in these systems. In this work we obtain some physical limits to broadening compensation in linear slow light systems based on simple Fourier analysis. We show that gain and dispersion broadening can never compensate in such a system. Additionally, it is simply proven that all the linear slow light systems that introduce a low-pass filtering of the signal (a reduction in the signal root-mean-square spectral width), will always cause pulse broadening. These demonstrations are done using a rigorous shape-independent definition of pulse width (the root-mean-square temporal width) and arguments borrowed from time-frequency analysis.

OCIS codes: (999.9999) Slow light; (060.4370) Nonlinear optics, fibers; (290.5900) Scattering, stimulated Brillouin; (350.5500) Propagation

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1. Introduction

The use of Stimulated Brillouin Scattering (SBS) for producing slow light systems presents two main advantages: first, given its large efficiency, an extremely wide control of the group velocity can be carried out at still reasonable power levels [1]; second, the spectrum of the interaction can be engineered to fit different requests in terms of bandwidth and distortion of the signal [2,3]. For example, the bandwidth of SBS-induced gain can be increased arbitrarily by actively broadening the pump spectrum using a random modulation of the pump laser current [2]. The unique spectral tailoring capability in SBS has also been used to manage the distortion of the pulses travelling in the slow light system [3]. The idea is to optimize the pump spectrum so as to minimize the distortion in the signal pulse. This possibility was thoroughly explored by independent teams [4,5,6], even finding an "optimum" gain and loss profile to yield the best performance in terms of delay and distortion.

Following the same research direction, a recent study [7] has reported the achievement of gain profiles that would yield "zero-broadening" of the pulses travelling through it. The gain profiles suggested in this report are fairly similar to the ones already proposed in preceding works [4,5]. This approach was further developed by an independent team and experimentally implemented, even showing pulse compression in the slow light medium [8]. The principles for broadening compensation stated in [7] are based on the assumption that gain and dispersion broadenings add arithmetically to eventually cancel when they show opposite signs. This statement was also used to back up the later experimental work in [8].

In this paper we would like to discuss some issues regarding these and other demonstrations of "zero-broadening" slow light. Specifically, we will demonstrate the impossibility to compensate amplitude and phase broadenings in any stationary linear system (SLS) such as those proposed in [7,8]. At first glance this can be viewed very simply by representing the SLS transfer function by $\exp\{f(\nu)\}$ and by considering that the signal amplitude is changed by the real part of $f(\nu)$ and the phase by the imaginary part of $f(\nu)$. Since a purely real and a purely imaginary quantity cannot mutually cancel, their global effect is scaled by the modulus of $f(\nu)$ and is thus given by the geometrical sum instead of the simple arithmetical one stated in [7]. This is illustrated by the repeatedly observed effect that linear chromatic dispersion of any sign can never lead to a compressed pulse narrower than the original un-chirped pulse.

Additionally, we will demonstrate that all linear slow light systems (like the ones proposed in references [7] and [8]), that introduce a low-pass filtering of the input spectrum (a reduction in the signal root-mean-square spectral width), will necessarily introduce a temporal broadening of a Gaussian un-chirped pulse. This means that such media will never be good candidates for making "zero-broadening" slow light. This is the case of most of the amplifier-based slow light systems proposed in the literature up to now.

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2. Discussion and demonstration

In this section we would like to demonstrate (1) the impossibility to obtain mutual compensation of phase and amplitude broadenings in a SLS and (2) that all linear slow light systems that introduce a low-pass filtering of the input spectrum (a reduction in the signal RMS spectral width), will necessarily introduce a temporal broadening in the pulses propagating through it. For this purpose, we will use the root-mean-square (RMS) width of the pulses instead of the full-width at half maximum (FWHM) used in the demonstrations of zero-broadening slow light [7,8]. While both measures of pulse width are valid and widely used, we will show that the FWHM is less suitable from the point of view of information transmission and capture.

This section is structured as follows: first we will give a detailed explanation of the "zero-broadening" experiments developed in [8], with a comparison of the FWHM and RMS measures of pulse broadening. We will show that the zero-broadening condition is only met using the FWHM measure of broadening, but not with the RMS measure, and we will explain why. Next, we will use the definition of RMS temporal width of a pulse to demonstrate the impossibility of a mutual cancellation of amplitude and phase distortions. Lastly, we will use the uncertainty principle to show that low-pass systems (such as most of the amplifier-based slow light systems proposed in the literature) will always cause pulse broadening. This kind of systems will therefore never be candidates for making "zero-broadening" slow light.

2.1 Review of the" zero-broadening" experiments

For completeness in the data, we refer now to the work by Schneider et al [8], which additionally show experimental measures of pulse propagation in the proposed profile (in Fig. 3, for instance). The proposed gain medium consists of a broad Gaussian-shaped gain with two narrow losses superposed in its wings. The propagated pulses are narrower in spectral width than the broad gain, and the frequency difference between the two narrowband losses is scanned for further optimization. As a distortion measure, the width of the output pulses at the full-width at half maximum (FWHM) is measured and compared to that at the input. By calculating the quotient between the output and input widths, the FWHM broadening is evaluated. When the frequency difference between the loss resonances is reduced below a certain quantity, the FWHM broadening of the pulses remains in the order of or below one. This means that the pulses are not broadened or even compressed. However, it must be pointed out that the condition of "zero-broadening" propagation comes at the expense of a strong distortion in the pulse waveform. The main pulse ejects some satellite sub-pulses, which are neglected in the calculation of the pulse broadening, since they are below the half-maximum. This neglected part, however, can lead to a severe intersymbol interference in a real communication system, as we will show below. This is one of the reasons why the FWHM broadening is not a convenient measure of distortion when the pulses change the shape strongly, as it is the case in this system. A more detailed discussion on the convenience of this measure can be found in [9].

The experimental pulse distributions containing a pre-shoot as illustrated in Fig. 3 of reference [8] can be fully explained by the combined effect of the strong 3^{rd} order dispersion induced by the linear slow light system, and the notch filtering effect caused by the narrowband losses. To further understand these effects, we simulated the response of the system described in [8] using a conventional transfer-function approach [10]. The parameters used have been as close as possible to the experimental ones used in [8] while trying to obtain the same qualitative response. In Fig. 1 we show the calculated temporal pulse shapes obtained as we scan the frequency separation between the two loss resonances. As it can be observed, the obtained response is qualitatively identical to the one shown in [8]. We can distinguish two regimes: when the frequency separation between the resonances is more than 180 MHz, a "broadening" regime, in which the pulse is plainly broadened at the output; when

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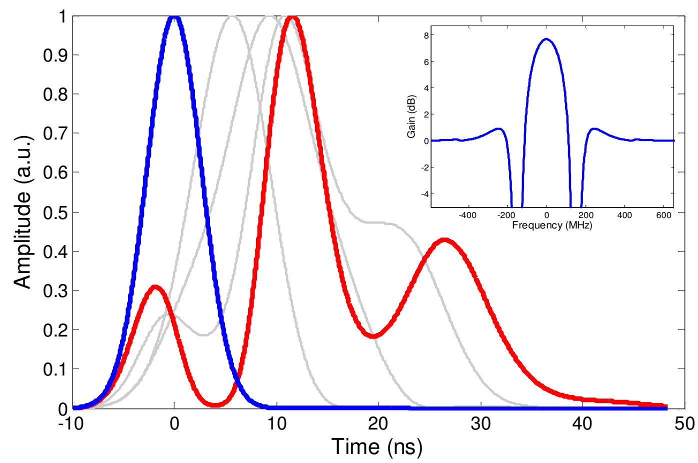

Fig. 1. Input (blue) and output (grey and red) waveforms for the gain medium combining a broadband gain and 2 lateral narrowband losses (sample spectrum shown in the inset). The frequency separation between the loss resonances is varied from 100 to 400 MHz. Red curve corresponds to a separation of 130 MHz between the loss resonances, grey curves to larger separations.

the frequency separation is below 180 MHz, a "pulse splitting" regime, in which the pulse basically breaks up into a train of short pulses. This ringing effect is caused by the combined effect of third-order dispersion and the notch filtering effect caused by the loss resonances. The main source of the ringing is the frequency slicing induced by the loss resonances in the pulse spectrum. As a matter of fact, it is true that, in some instances, the third-order dispersion and the low-pass filtering effect may be combined to result in a pulse with a somewhat cleaner appearance (this is indeed what happens in [11]). However, as we will show below, the phase dispersion introduced in the pulse can never lead to a good compensation of the low-pass filtering effect. In Fig. 2 we show the corresponding broadening of the pulses in such medium for the conditions given above, as a function of the separation between the loss resonances. The broadening is calculated with the FWHM and RMS measurements of pulse width. We

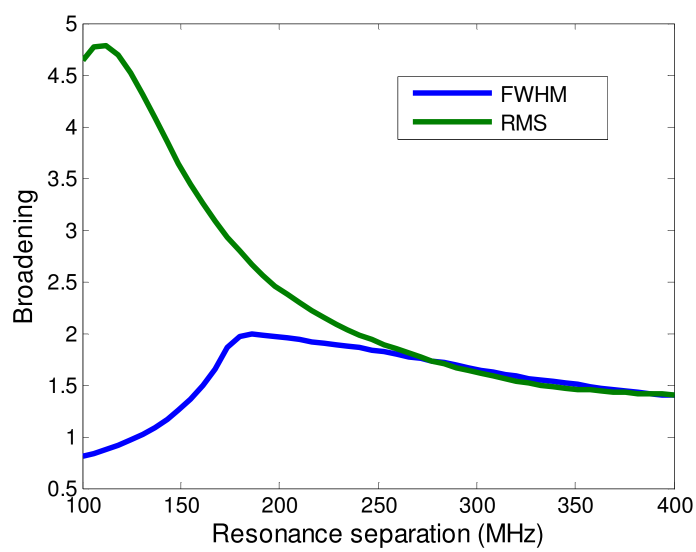

Fig. 2. Pulse broadening as a function of the frequency separation between the two loss resonances. The measurement of pulse broadening based on the RMS or the FWHM values results in very different estimations when the system operates in the "pulse splitting" regime (frequency separations below180 MHz).

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can see that for the "pulse splitting" regime, the FWHM and RMS measurements of broadening deviate strongly, the FWHM measurement even dropping below 1 for some cases. The reason is to be found in the simple fact that the FWHM neglects all the satellite pulses showing amplitude below the half-maximum. However, in a real transmission system these satellite pulses turn out to have a very strong impact on the performance. We sketch this situation in Fig. 3, in which the transmission of a "101" sequence in the medium is simulated. Again, the resonance separation is swept from 400 MHz to 100 MHz. Two cases are plotted in the figure: the case of 400 MHz separation and the case of 130 MHz separation. For the 400 MHz case, the sequence, although distorted, is well transmitted and could be potentially well detected. For the case of 130 MHz, however, the original sequence is completely jammed, and the information cannot be retrieved properly.

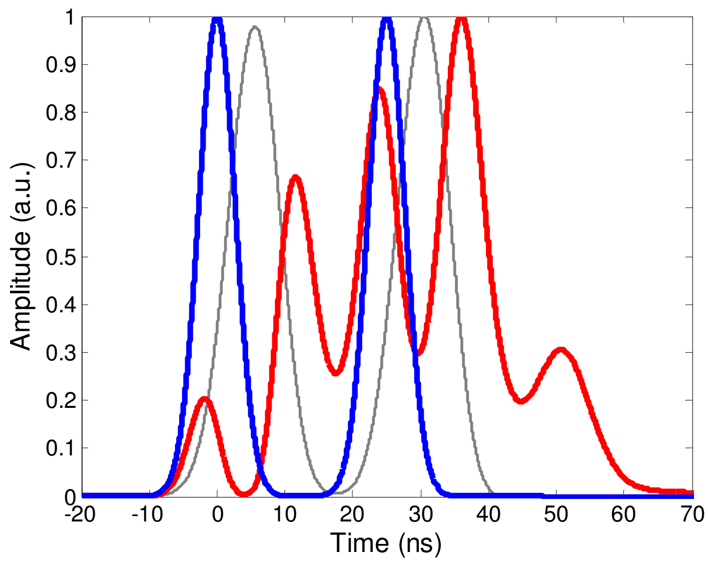

Fig. 3. Transmission of a "101" sequence in the slow light medium described above for the cases of loss resonances separation of 400 MHz (grey curve) and 130 MHz (red curve). The input sequence is shown in blue. In the "pulse splitting" regime in red ("FWHM zero-broadening"), the distortion experienced is so strong that the extraction of the information turns out to be impossible and leads to a severe intersymbol interference.

2.2 Gain and dispersion broadenings cannot mutually compensate

In previous demonstrations of "zero-broadening" slow light [7,8], it was claimed that the "zero broadening" condition was met because the amplitude distortion introduced by the low-pass filtering of the slow-light element could be compensated by the group velocity dispersion introduced by it. As sketched in our introduction we show below that this statement cannot be justified in a linear system representation. Moreover, it is also suggested in [7] that the phase broadening in the linear slow light system is due to second-order group velocity dispersion like observed during the passive propagation in single mode fibers. For symmetry reasons and as already clearly pointed out in a former work [5] the phase broadening can only originate from the third-order dispersion, as clarified below, and therefore cannot be compensated by ordinary second-order dispersion.

To develop our proof, we start by defining a generalized metric of pulse width and pulse broadening. The definition that we use of pulse width is the RMS temporal width of the pulse:

$$\sigma_t^2 = \frac{1}{E}\int_{-\infty}^{+\infty} t^2 \left|A(t)\right|^2 dt \tag{1}$$

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where $A(t)$ is the temporal amplitude of the pulse and

$$E = \int_{-\infty}^{+\infty} \left|A(t)\right|^2 dt \tag{2}$$

is the total energy inside the pulse.

For simplicity, we have considered that the pulse is always centered at t=0. This simplifies the relations, and the results do not lose any generality. The RMS pulse width is the definition of pulse width that is conventionally used in fiber-optic communication systems. This definition of pulse width measures how much the pulse energy is temporally "spread", regardless of the shape.

Let us now consider that the Fourier transform of the pulse envelope reads $A(\omega)\exp[i\varphi(\omega)]$, where $\varphi(\omega)$ has a complicated shape owing to the dispersion introduced by the slow light element, nevertheless showing an odd symmetry. Using the well-known properties of the Fourier transform, we can rewrite the RMS pulse width in a way that shows the relative contributions to it by the spectral amplitude $A(\omega)$ and the spectral phase $\varphi(\omega)$ [9,12]:

$$\sigma_t^2 = \frac{1}{E}\left[\int_{-\infty}^{+\infty}\left|\frac{dA(\omega)}{d\omega}\right|^2 d\omega + \int_{-\infty}^{+\infty}\left|A(\omega)\right|^2\left(\frac{d\phi(\omega)}{d\omega}\right)^2 d\omega\right] \tag{3}$$

where, for simplicity, it has been assumed that the mean group delay has been subtracted from $\frac{d\varphi(\omega)}{d\omega}$ (as stated before, the pulse is centered in t=0). Equation (3) shows that there are two main contributions to the temporal broadening of the pulse: one is due to the variations in the spectral amplitude of the signal, that can be modified through the slow light system by the spectral filtering effect, the so-called gain broadening. The second is due to the variations of the spectral phase in the pulse, and can thus be related to the phase distortion introduced by the medium. It is frequently designated as dispersion broadening in the literature. Both contributions are positive, and therefore, they cannot cancel each other. In other words, for a given spectral amplitude of the output pulse, the best spectral phase distribution to get the shorter pulse width is the flat one, corresponding to an un-chirped pulse. We can equivalently say that the group velocity dispersion will never be responsible of a temporal narrowing in the output pulse unless it is pre-chirped at the input.

We should point out here that the correct methodology to study the combined impact of gain (or loss) and dispersion broadenings in slow light systems was already developed in [13], and applied to the analysis of coupled resonator structures.

2.3 Low-pass systems always cause RMS broadening of the pulses

The result given above proves that the phase response of the linear slow light system can never compensate, in terms of RMS broadening, the distortion introduced by the amplitude response. It would nevertheless be interesting to know if there is any possibility of achieving such a zero-broadening slow light system with any kind of gain profile similar to the ones used in [7,8]. Although we cannot give a general response to this question, we can easily show that, if a system introduces a low-pass filtering effect in the signal (a reduction in the RMS spectral width), any Gaussian pulse at the input will be broadened at the output. To show this, we can define the RMS spectral width:

$$\sigma_\omega^2 = \frac{1}{2\pi E}\int_{-\infty}^{+\infty}\omega^2\left|A(\omega)\right|^2 d\omega \tag{4}$$

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The well-known uncertainty principle [14] states that as long as $A(t)$ is absolutely continuous and the functions $tE(t)$ and $E'(t)$ are square integrable:

$$\sigma_t \sigma_\omega \geq \frac{1}{2} \tag{5}$$

where the equality *only* holds for a Gaussian pulse, therefore $A(t)=C\exp(-t^2/\sigma_{0t}^2)$. Let us consider this case as the input pulse to our linear slow light medium. The input spectral width is therefore $\sigma_{0\omega} = 1/(2\sigma_{0t})$. One common thing to all the slow light systems proposed in [7] and [8] is that their transfer function $H(\omega)$ is inherently low-pass. Therefore, the output RMS spectral width will necessarily be smaller than the input spectral width, thus $\sigma_\omega < \sigma_{0\omega}$. Now, if we consider the uncertainty principle stated in Eq. (5), the output spectral width will be $\sigma_t \geq 1/(2\sigma_\omega)$. Since $\sigma_\omega < \sigma_{0\omega}$, it follows obviously that $\sigma_t > 1/(2\sigma_{0\omega}) = \sigma_{0t}$. Thus, the output pulse width is always larger than the input pulse. This means that, in general, low-pass linear elements (which have been, so far, all the amplifier-based slow light elements proposed in the literature) will never be candidates to make zero-broadening slow light. There remains an open question on whether any other kind of profile would be able to achieve zero-broadening of pulses. We must remind here, nevertheless, that an extensive, computer-based search for the "optimum" gain profile (that introduces the maximum fractional delay with the minimum distortion in the pulse) was already carried out by Pant et al. [4], finding that the square-shaped low-pass response was the best possible in SBS-based slow light systems.

We would like also to stress that ordinary group velocity dispersion as observed in single mode fibers can in no way lead to a good compensation of phase broadening in a slow light linear system. For a pulse placed at the center of the gain spectrum and for symmetry reasons the amplitude part of the transfer function $A(\omega)$ will show a symmetric distribution around the central frequency ω_0 while the phase part $\varphi(\omega)$ will be anti-symmetric and thus only contains odd terms in its polynomial expansion. The first term in this odd expansion is linear and gives the group delaying effect, while the following term, responsible for distortion and phase broadening, is the third-order dispersion that depends cubically on the frequency. It can thus never be cancelled by a quadratic term as induced by ordinary group velocity dispersion and suggested in [7]. We should mention that the same authors had eventually drawn the same conclusion about the symmetry of the distribution and the parity of the gain and phase terms in a nearly simultaneously submitted paper [15]. However they unfortunately maintained the same arithmetic summation for the two types of broadening to present their final results and consequently the impact of phase broadening turns out to be strongly overestimated. In this paper the claim that gain broadening can be compensated by group velocity dispersion was unfortunately not clearly revisited.

3. Conclusion

In conclusion, we have shown that phase distortion can never lead to the compensation of the low pass filtering effect in slow light systems. Additionally, we have shown that all the linear slow light systems showing a low-pass response (such as all the amplifier-based ones) will never be candidates for making zero-broadening slow light. We should stress that this impossibility is limited to linear systems and it might be possible, however, to compensate the broadening after the slow light system by inserting suitable nonlinear elements (such as, for instance, saturable absorbers). The role of these nonlinear elements would be, eventually, to "re-generate" the frequencies that have been filtered out by the slow light element. A preliminary exploration of this approach using gain saturation in SBS has already been presented in the literature [16]. This approach could reduce substantially the broadening, but had the unwanted outcome of reducing also the delaying effect.

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