A dynamic Game Model of Strategic RD&D Cooperation and GHG Emission Mitigation

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Abstract

This deliverable describes the game structures implemented in the TOCSIN project to find self-enforcing and stable international environmental agreements. It presents the first results obtained with the use of these models. The document starts with a review of the different approaches that have been proposed in the literature to represent in a game theoretic framework the concept of self-enforcing or stable international environmental agreement.

1 Introduction

The aim of the TOCSIN research program is to assess the benefits and costs of possible self-enforcing technology-based international agreements involving the EU, China and India with the aim of stabilizing the long term atmospheric concentrations of GHGs. One specific objective of the project is

> *To define the possible self-enforcing international agreements on GHG emission abatement, taking into account their economic impacts, including terms of trade changes, as well as the possible gains of multilateral and bilateral collaborations, Clean Development Mechanisms (CDM) and international emission trading (IET) in order to stimulate RD&D cooperation and technology transfers toward China and India.*

The purpose of this deliverable is twofold: (i) to explore and compare different game theoretic models already proposed to study *Self-Enforcing and Stable Climate-Change Treaties* and, (ii) to propose different ways to adapt these approaches to the needs of the TOCSIN project.

The deliverable is organized as follows. Part-1 discusses the different game theoretic approaches already proposed to study self enforcing International Environmental Agreements (IEAs); Part-2 is a model of financial constraint in intertemporal cooperative pollution management games, proposed by the TOCSIN partners of HKBU; Part-3 discusses in more details the game models that are implemented in the TOCSIN project. Part-4 shows the first experiments with a budget sharing game implemented with the TOCSIN relevant data.

Part I
Review of IEA games

Self-enforcing[1] or stable treaties on IEA have the property that no signatory has a temptation to leave the agreement, which means in general that the terms of the agreement must be either (i) a Nash (or Stackelberg) equilibrium in a properly defined noncooperative game of strategy or (ii) a stable solution in a suitably defined cooperative game. It appears that there is considerable flexibility in the way one can design these games and, depending on the design adopted, very different classes of equilibrium and stable solutions can be obtained, with outcomes varying from very inefficient (that is very far from Pareto optimality) to "quasi Paretian" or Pareto optimal.

In the forthcoming sections we propose a rapid survey of the game-theoretic frameworks that have been proposed to study self-enforcing IEAs.

2 Players, payoffs, rules of the game

In this section we consider the general characteristics of the game models used to represent an IEA. First we shall describe the players, their payoffs and the proposed rules of the game. The players are countries or groups of countries. Their actions are their GHG emission levels over a long term planning horizon or the emission caps that they agree to observe (their actual emissions being the result of emissions trading markets). The payoffs of the players are evaluated in terms of welfare gains (or losses) associated with their common actions. These welfare criteria can include the economic cost of climate change (cost-benefit (CB) framework) or not, in which case a limit is imposed on the gobal cumulated emissions (cost-effectiveness (CE) framework). The rules of the game can be diverse. They are described in the following sections.

[1]This part has been contributed by Alain Haurie from ORDECSYS.

2.1 Single level game (CB)

This is the basic model used by many authors to represent status quo (Chander and Tulkens Chander and Tulkens (1992, 1995, 1997), Labriet and Loulou (2007)) as a reference point. There are I countries, indexed $i = 1, \ldots, I$, that generate emissions $e_i \in \Re_+$. Benefits of emissions are denoted $\pi_i(e_i)$, with $\pi_i'(e_i) > 0$, $\pi_i''(e_i) < 0$. Furthermore, associated with the total emission level, $e = \sum_{i=1}^{I} e_i$, one defines the environmental damage cost of each country $\nu_i(e) \geq 0$, with $\nu_i'(e) > 0$, $\nu_i''(e) \geq 0$. [2]. For the game with payoffs,

$$J_i(e) = \pi_i(e_i) - \nu_i(e), \quad i \in I. \tag{1}$$

A Nash equilibrium solution is an emission schedule $e^* = (e_i : i \in I)$ such that

$$J_i(e^*) = \max_{e_i} J_i([e^{*-i}, ei]) \quad i \in I, \tag{2}$$

where $[e^{*-j}, ej]$ denotes the emission schedule where all players $j \neq i$ observe e_j^* and player i chooses e_i.

2.2 Two-level (two-stage) game with coalitions (CB)

This is the game theoretic framework introduced by Barrett Barrett (1994), used by Botteon and Carraro (1998); Bosello et al. (2001); Buchner et al. (2005); Carraro et al. (2003) and Kempfert (2005) and recently revisited by Kolstad and Ulph (2008) from which we quote:

> "...In stage (or level) 1, (membership game) each country decides whether or not to join the agreement (an IEA). The result of this is a set of signatories to the IEA and a set of fringe members, outside the IEA. We seek a Nash equilibrium in "announcements" (ie, "in" or "out") in which no country wishes to unilaterally leave or join the coalition. In stage (or level) 2, (emission game) each non-signatory, or fringe, country, denoted by superscript f, takes as given the emissions of all other countries and chooses its emissions to maximize its individual net benefit; the signatory countries, denoted by superscript s, collectively choose their emissions to maximize the aggregate net benefit of the signatory countries taking as given the emission strategy of the non-signatories. The outcome of stage 2 is a Stackelberg equilibrium involving the IEA (acting as one) as leader and the fringe

[2]Notice that this formulation can be used in a dynamic economic model (see Nordhaus and Yang Nordhaus and Yang (1996) or Labriet and Loulou (2007)). The solution is then an open-loop Nash equilibrium

countries (acting as individuals) as followers..." ... "There are two ways of defining the equilibrium of the membership game. The first, as presented in Barrett Barrett (1994) borrows the concept of a stable coalition from the literature on oligopoly and defines a stable IEA as follows:

i.e. no signatory country has any incentive to unilaterally leave the IEA, and no non-signatory has any incentive to unilaterally join the IEA, taking as given the membership decisions of all other countries. This definition is equivalent to saying that a stable IEA is a Nash equilibrium of the membership game.

This might seem to be a simplistic notion of stability. In fact, other authors have come up with more complex conditions for stability of an IEA (eg, Chander and Tulkens, Chander and Tulkens (1994)), including ways of committing countries to participate (eg, Carraro and Siniscalco, Carraro and Siniscalco (1993)). What this definition of stability gives us is a very basic, and perhaps weakest, concept of what it takes to hold an agreement together. It is a good starting point for explorations of the size of voluntary international environmental agreements."...

Note that in the papers by Carraro and Siniscalco (1993) or Kempfert (2005) the solution to the second (lower) level game is not a Stackelberg equilibrium but a Nash equilibrium where the signatory countries play as one player and each non signatory is also one player. In Kolstad and Ulph (2008) the simple strategic structure of the game considered makes also that the Stackelberg solution coincides with a Nash equilibrium.

2.3 Three-level (three-stage) game with IET (CB)

This is the framework proposed by Helm (2003) whom we quote below:

"...Allowing for the possibility of international emissions trading, countries interaction may be thought of as a three-stage game.

In the first stage (level), countries decide on the establishment of a trading system.

... In the second stage (level), ..., if countries have approved a trading system, they (non-cooperatively) choose tradable emission allowances, denoted $\omega_i \in \mathfrak{R}$. In this case, there is a third stage (level): the trading of allowances on an international permit market, leading to after-trade emission e_i^* such that $\sum_{i=1}^{I} e_i^* = \sum_{i=1}^{I} \omega_i = \omega$.

In the subgame perfect Nash equilibrium of this game, no country wants to unilaterally change its allowance choice nor its decision about the trading system so there is no need for binding agreements. The resulting game is solved by backwards induction..."

The introduction of the "caps" as the strategic variables and the definition of actual emissions as a result of an IET is very interesting as it permits the introduction of terms of trade effects in the payoffs of the players (see also section 2.6).

2.4 Cooperative game solutions (CB)

Cooperative game solution concepts have been used in the context of IEAs by Chander and Tulkens (1992, 1994) and implemented recently in the realm of a 15 region World MARKAL model by Labriet and Loulou (2007). Quoting from these authors:

> ...Chander and Tulkens (1992, 1997) analyze the formation of the grand coalition (cooperation among all countries). This "grand coalition approach" relies on the ability to make transfers to ensure that every region and possible coalition receives at least as much as it can earn on its own. However, transfers are often criticized as being unrealistic and some argue that transfers may be inadequate to offset the incentives to free-ride (Bosello et al., 2001). However, transfers do not need to be implemented through direct financial resource flows. They could also be translated in technology transfers or projects implemented jointly (investments from one region to another one), in an international carbon tax or in an international tradable permit scheme, where transfers are generated by the trade of carbon permits from the agreed initial allocation of carbon. Dutta and Radner (2004), Diamantoudi et al. (2002), Finus and Rundshagen (2002), Finus (2004), Ioannidis et al. (2000), Missfeldt (1999) and Tulkens (1998) provide very good reviews of the two approaches and help understand how their premises, such as coalition unanimity, farsightedness, coordination of defectors, contribute to their different conclusions.
>
> [...]
>
> The concepts of cooperative agreements have some normative appeal and possess some axiomatic properties, while the non-cooperative branch is concerned with a more positive analysis of coalition formation (Finus and Rundshagen, 2002; Missfeldt, 1999). The choice of a normative angle for the analysis of international climate agreement is

consistent with MARKAL's philosophy, which relies on optimal energy decision and is appropriate for prospective analysis. Moreover, cooperative cost-sharing solutions are important since they may act as focal points in negotiations.

The definition of transfers between countries ... requires the analysis of every possible coalition structure of the game, and therefore the behaviour of regions that are not members of the cooperative coalition (equivalent to the definition of the threat in case of defection). We adopt the γ-characteristic function proposed by Chander and Tulkens (1997): when a subcoalition S forms, outsiders do not take particular coalitional actions against or favouring S but adopt their individual best reply strategies (individual Nash) and enjoy the cleaner environment induced by S's actions. This means also that when a country deviates, the whole agreement collapses and each country sticks to its non-cooperative Nash strategy (Chander and Tulkens, 1992, 1997). The γ-characteristic function is defined by:

$$v_\gamma(S) = C_{PANE}(S) - \sum_{i \in S} C_{NASH}(i), \tag{3}$$

with $C_{PANE}(S)$ the total discounted costs of S under Partial Agreement Nash Equilibrium where regions of S cooperate and regions out of S play their individual Nash strategy $C_{NASH}(i)$ the cost borne by region i of S under its individual Nash strategy

In other words, the game assumes that emission leakage (more emissions by outsiders) is unappropriate since it is self-punishing in the context of global pollution. Moreover, the game implies for S a certain degree of pessimism, since S would be better off if the regions outside would form one or more non-singleton coalitions and then reduce more their emissions (Chander and Tulkens, 1997). The open-loop information structure that we use corresponds to negotiations that take place once: a binding agreement is signed in the first period and remains valid until the end of the horizon. This assumption is consistent with the perfect information and foresight characteristics of MARKAL, as well as with the long-term nature of some energy decisions. Thus, the problem is dynamic as regards MARKAL energy decisions, but it is static from the point of view of gains and transfers: gains and transfers are fixed for the entire time horizon and they can be not renegotiated later.

2.5 Correlated equilibrium and games with signaling (CB)

To our knowledge there are few models of IEAs that are using the paradigm of correlated equilibrium or of game with signaling, although these concepts have been demonstrated to lead to interesting conclusions in theoretical economics (Aumann (1974), Fudenberg and Tirole (1991)). An application to the allocation of permits in the EU ETS has been proposed by Viguier et al. (2006). In a correlated equilibrium one assumes that an umpire (the EU Commission or the UN IPCC) gives to each player a recommendation on the strategy to adopt. These recommendations constitute a correlated equilibrium if adopting them is a Nash equilibrium for the players. In a game with signaling an umpire sends a signal to the players with a recommendation on the strategy to adopt depending on the signal received. Again, playing according to these strategies would be a Nash equilibrium. These concepts are attractive for the modeling of IEAs since, obviously, although no international body can impose a solution to all countries, one can imagine the UN or a dominant coalition of nations to play the role of the umpire distributing strategic recommendations or signals.

2.6 Equilibrium with a coupled constraint (CE)

A different game format for IEA modeling has been proposed in Haurie et al. (2006) and Carbone et al. (2003) based on the paradigm of a game with coupled constraint introduced by Rosen (1965). The concept has the considerable advantage to reconcile game theory with the cost-effectiveness framework. Instead of evaluating the damages due to climate change one imposes a global limit on the GHG concentration that will have to be taken into consideration in the definition of the equilibrium solution. Let us quote from Drouet et al. (2007) where a dynamic (two period) game with uncertainty on climate sensitivity (CS) is formulated:

> The game is played over 2 periods $t = 0,1$. M is a set of m groups of countries hereafter called *players* which must decide on the caps they impose on their respective global emissions of GHGs in each period. Let Ω be the set of possible realizations of the climate sensitivity (CS) parameter values. We represent the uncertainty on this value as an event tree as shown in Fig. 1.
>
> Let $\pi(\omega)$ be the probability of realization $\omega \in \Omega$. We denote $\bar{e}_j(t,\omega)$ the cap decided by player j for period t and CS $\omega \in \Omega$. In period 0 the CS is unknown, therefore the following equalities must be satisfied
>
> $$\bar{e}_j(0,\omega) = \bar{e}_j(0,\omega') \quad \forall j \in M, \forall \omega, \omega' \in \Omega. \tag{4}$$

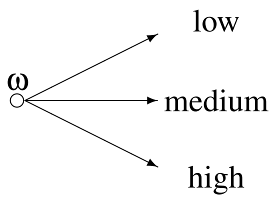

Figure 1: Climate sensitivity uncertainty

In period 1 the CS is known and each player adapts his decision $\bar{e}_j(1,\omega)$ to the observed realization ω. Depending on the realization $\omega \in \Omega$, a global limit $\bar{E}(\omega)$ will be imposed on the cumulative emissions of both periods $t = 0, 1$. Therefore the following coupled constraints are binding all players together

$$\sum_{j \in M} \sum_{t=0}^{1} \bar{e}_j(t,\omega) \leq \bar{E}(\omega) \quad \forall \omega \in \Omega. \tag{5}$$

Let $\bar{\mathbf{e}}(t,\omega) = \left\{\bar{e}_j(t,\omega)\right\}_{j \in M}$ denote the vector of emissions caps for all players in period t, and CS value ω. Given these cap values a general economic equilibrium is computed for the m-country which determines a welfare gain for each player, hereafter called its payoff at t,ω and denoted $W_j(\bar{\mathbf{e}}(t,\omega))$. Given a choice of emission programs $\bar{\mathbf{e}} = \{\bar{\mathbf{e}}(t,\omega)| : t = 0,1; \omega \in \Omega\}$ the expected payoff to player j is given by

$$J_j(\bar{\mathbf{e}}) = \sum_{t=0}^{1} \sum_{\omega \in \Omega} \pi(\omega)\, W_j(\bar{\mathbf{e}}(t,\omega)) \quad j \in M. \tag{6}$$

We assume that the players behave in a noncooperative way but are bound to satisfy the global cumulative emissions constraints (5) that are contingent to the realization ω of the CS.

Let us call $\mathcal{E}$ the set of emissions $\bar{\mathbf{e}}$ that satisfy the constraints (4) and (5). Denote also $[\bar{\mathbf{e}}^{*-j}, \bar{e}_j]$ the emission program obtained from $\bar{\mathbf{e}}^*$ by replacing only the emission programm $\bar{e}_j^*$ of player j by $\bar{e}_j$.

Definition 1 *The emission program $\bar{\mathbf{e}}^*$ is an equilibrium under the coupled constraints (5) if the following holds for each player $j \in M$*

$$\begin{aligned} \bar{\mathbf{e}}^* &\in \mathcal{E} && (7)\\ J_j(\bar{\mathbf{e}}^*) &\geq J_j([\bar{\mathbf{e}}^{*-j}, \bar{e}_j]) \quad \forall \bar{e}_j \; s.t. \; [\bar{\mathbf{e}}^{*-j}, \bar{e}_j] \in \mathcal{E}. && (8) \end{aligned}$$

Therefore, in this equilibrium, each player replies optimally to the emission program chosen by the other players, under the constraint that the global cumulative emission limits must be respected.

It is possible to characterize a class of such equilibria through a fixed point condition for a best reply mapping defined as follows. Let $\mathbf{r} = (r_j)_{j\in M}$ with $r_j > 0$ and $\sum_{j\in M} r_j = 1$ be a given weighting of the different players. Then introduce the combined response function

$$\theta(\bar{\mathbf{e}}^*, \bar{\mathbf{e}}; \mathbf{r}) = \sum_{j\in M} r_j J_j([\bar{\mathbf{e}}^{*-j}, \bar{e}_j]). \tag{9}$$

It is easy to verify that, if $\bar{\mathbf{e}}^*$ satisfies the fixed point condition

$$\theta(\bar{\mathbf{e}}^*, \bar{\mathbf{e}}^*; \mathbf{r}) = \max_{\bar{\mathbf{e}}\in \mathcal{E}} \theta(\bar{\mathbf{e}}^*, \bar{\mathbf{e}}; \mathbf{r}), \tag{10}$$

then it is an equilibrium under the coupled constraint.

Definition 2 *The emission program* $\bar{\mathbf{e}}^*$ *is a normalized equilibrium if it satisfies (10) for a weighting* $\mathbf{r}$ *and a combined response function defined as in (9).*

The RHS of (10) defines an optimization problem under constraint. Assuming the required regularity we can introduce a Kuhn-Tucker multiplier $\lambda^o(\omega)$ for each constraint $\sum_{t=0}^{1} \bar{e}_j(t,\omega) \leq \bar{E}(\omega)$ and form the Lagrangian

$$L = \theta(\bar{\mathbf{e}}^*, \bar{\mathbf{e}}; \mathbf{r}) + \sum_{\omega\in\Omega} \lambda^o(\omega)(\bar{E}(\omega) - \sum_{j\in M}\sum_{t=0}^{1} \bar{e}_j(t,\omega)). \tag{11}$$

Therefore, by applying the standard K-T optimality conditions we can see that the normalized equilibrium is also the Nash equilibrium solution for an auxiliary game with a payoff function defined for each player j by

$$J_j(\bar{\mathbf{e}}) + \sum_{\omega\in\Omega} \lambda^j(\omega)(\bar{E}(\omega) - \sum_{j\in M}\sum_{t=0}^{1} \bar{e}_j(t,\omega)), \tag{12}$$

where

$$\lambda^j(\omega) = \frac{1}{r_j}\lambda^o(\omega), \quad \omega \in \Omega. \tag{13}$$

This characterization has an interesting interpretation in terms of negotiation for a climate change policy. A common "tax" $\lambda^o(\omega)$ is defined and applied to each player with an intensity $\frac{1}{r_j}$ that depends on the weight given to this player in the global response function.

In Haurie et al. (2006) the concept is integrated in an economic growth model à la Ramsey. The interesting aspect of these models is that they reduce considerably the "Prisoner's dilemma" syndrome of the Nash equilibrium solution which is present in most environmental game models (see section 9). Also they provide a way to deal with the burden sharing issue, by distributing the weights given to the different players in the search for a normalized equilibrium.

2.7 Equilibrium in a dynamic game with trigger strategies (CB)

This is the game used by Dutta and Radner (2004) to model a self-enforcing IEA. The game is played over an infinite time horizon. It is dynamic as it takes into account the GHG accumulation process, the damages for each period being determined by the observed concentration. The threat strategy is a Markov (subgame perfect) Nash equilibrium (MNE). A Pareto optimal grand coalition strategy (GPO) is obtained by maximizing a convex combination of the welfare of all countries. For some choice of the weighting the vector of values for the welfare of all the countries will always dominate the vector of values for the MNE, whatever the initial state of the game in the admissible domain. This permits an umpire to propose the following trigger strategy which constitutes a subgame perfect equilibrium in the dynamic game:

signal "0": Start with the signal "0". As long as everybody cooperates (signal 0) play the GPO strategy;

signal "1": As soon as a defection has been noticed, switch the signal to "1" and recommend to play the MNE strategy forever (or for a sufficiently long time).

For a discount rate close enough to 0, the dominance of the GPO value vector over the MNE value vector will make such a strategy a subgame perfect equilibrium. Note that this game structures can be interpreted as a variant of the paradigm of a game with signaling.

The interesting feature of this model is that it takes into account the dynamic structure of the game and it proposes a subgame perfect solution, i.e. a solution which is "renegotiation proof". Furthermore, the stable equilibrium solution is also Pareto optimal! Unfortunately the computational aspects are not very attractive. The solution obtained in Dutta and Radner (2004) is based on a dynamic programming approach which is implementable only for models with very few "state variables".

2.8 Dynamic game with stable cooperative solutions (CB)

In a similar vein the models proposed by Petrosjan and Zaccour (2003) and Petrosjan and Yeung (2007) deal with the regularization of a cooperative solution in a dynamic game setting. Quoting from these authors:

> A particularly stringent condition (subgame consistency) is required for a dynamically stable cooperative solution in stochastic differential games. A cooperative solution is subgame consistent if the solution optimality principle is maintained in any subgame which starts at a later time with any feasible state brought about by prior optimal behaviors. Since all players are guided by the same optimality principle at each instant of time, they do not possess incentives to deviate from the previously adopted optimal behavior throughout the game. In this paper an optimality principle which shares the expected gain from cooperation proportional to the nations' relative sizes of expected noncooperative payoffs is adopted and a payment mechanism which ensures a subgame-consistent solution is explicitly derived.

The interesting aspect of this approach is that it proposes conditions for dynamic stability of the negotiated treaty. As for the Dutta-Radner model, this solution concept is based on a dynamic programming argument which is very difficult (impossible) to implement in a large scale model.

Part II

On Financial Constraint in Intertemporal Cooperative Pollution Management Games

Under dynamic cooperation, it has been shown that given subgame-consistent imputations satisfying group optimality and individual rationality throughout the cooperative trajectory, no rational players will deviate from the cooperative path. However, in reality low income nations may have financial constraint. In particular initial investment in pollution abatement and technology transfer under the new cooperation scheme may bring GDP down to a level below subsistence. Inability to borrow funds in the initial stage may force these nations to back off from cooperation. Refusal of participation by low income nations presents a stumbling block to successful cooperation in pollution management. This part of the deliverable examines conditions leading to the need for borrowing in intertemporal cooperative pollution management games.[3]

3 Motivation

Cooperative differential games represent a complex form of optimization analysis. This complexity leads to great difficulties in the derivation of satisfactory solutions. The recent work of Yeung and Petrosyan (2004 and 2006) developed a generalized method for the derivation of analytically tractable time-consistent solutions. One of the most commonly used assumptions to handle deviation of players from the cooperative path is that cooperation will break down and players would revert to non-cooperative behaviors if deviations occur. Given that subgame-consistent imputations satisfying group optimality and individual rationality throughout the cooperative trajectory, no rational players will deviate from the cooperative path.

In reality low income nations may have financial constraints. In particular initial investment in pollution abatement and technology transfer under the new cooperation scheme may bring GDP down to a level below subsistence. Inability to borrow funds in the initial stage may force these nations to back off from cooperation. Refusal of participation by low income nations presents a stumbling block to successful cooperation in pollution management.

In this article conditions leading to the need for borrowing in intertemporal cooperative pollution management games are examined. In Section 2 we present the issue of financial constraint in a discrete-time framework. Simple illustrations of financial constraint in cooperative pollution management are given in Section 3. Section 4 presents a continuous-time analog. In Section 5, the analysis is applied to an existing differential game of pollution management. Section 6 concludes the paper.

4 Financial Constraint in Cooperative Games Over Time

To present the issue of financial constraint in a comprehensive and rigorous framework we consider the general $n-$nation nonzero-sum dynamic game with initial state x_0 and played over T stages. The state space of the game is $\mathrm{X} \in \mathrm{R}^m$, and the

[3]This part has been contributed by David W. K. Yeung and Cynthia Y. X. Zhang of Center of Game Theory and Department of Decision Science, Hong Kong Baptist University.

state dynamics of the game is characterized by:

$$x_{k+1} = f_k[x_k, u_k^1, u_k^2, \cdots, u_k^n], \tag{14}$$

for $k \in \{1, 2, \cdots, T\} \equiv K$ and initial state x_0 given. Here $u_k^i \in R^{m_i}$ is the control vector of nation i at stage k.

The objective of nation i is

$$\sum_{k=1}^{T} g_k^i[x_k, u_k^1, u_k^2, \cdots, u_k^n, x_{k+1}] \left(\frac{1}{1+r}\right)^{k-1}, \quad i \in \{1, 2, \cdots, n\} \equiv N. \tag{15}$$

Let $\{\phi_k^i(x_k$, for $k \in K$ and $i \in N\}$ denote a set of strategies leading to a feedback Nash equilibrium, the game equilibrium state trajectory can be obtained as:

$$x_{k+1} = f_k[x_k, \phi_k^1(x_k), \phi_k^2(x_k), \cdots, \phi_k^n(x_k)], \tag{16}$$

for $k \in \{1, 2, \cdots, T\} \equiv K$ and x_0 is given,

We denote the process satisfying (16) by $\{\hat{x}_k\}_{k=1}^T$. The noncooperative payoff of nation i over the stages from h to T can be expressed as:

$$V^i(h, \hat{x}_h) = \sum_{k=h}^{T} g_k^i[\hat{x}_k, \phi_k^1(\hat{x}_k), \phi_k^2(\hat{x}_k), \cdots, \phi_k^n(\hat{x}_k)] \left(\frac{1}{1+r}\right)^{k-1}, \tag{17}$$

for $i \in N$ and $h \in \{1, 2, \cdots, T\}$.

Under cooperation group rationality required the nations to maximize their joint payoff

$$\sum_{k=1}^{T} \sum_{j=1}^{n} g_k^j[x_k, u_k^1, u_k^2, \cdots, u_k^n, x_{k+1}] \left(\frac{1}{1+r}\right)^{k-1} \tag{18}$$

subject to (14).

Let $\{\psi_k^i(x_k)$, for $i \in N\}$ denote a set of strategies leading to a solution of the optimal control problem (14) and (18), and let $\{x_k^*\}_{k=1}^T$ denote the optimal cooperative path. The total cooperative payoff over the stages from h to T can be expressed as:

$$W(h, x_h^*) = \sum_{k=h}^{T} \sum_{j=1}^{n} g_k^j[x_k^*, \psi_k^1(x_k^*), \psi_k^2(x_k^*), \cdots, \psi_k^n(x_k^*)] \left(\frac{1}{1+r}\right)^{k-1}, \tag{19}$$

for $i \in N$ and $h \in \{1, 2, \cdots, T\}$.

Let B_k^i denote the payment that nation i will received for stage k under the cooperative agreement. The imputation to nation i over the stages from h to T can be expressed as:

$$\xi^i(h, x_h^*) = \sum_{k=h}^{T} B_k^i \left(\frac{1}{1+r}\right)^{k-1}, \tag{20}$$

for $i \in N$ and $h \in \{1,2,\cdots,T\}$.

Often a side-payment

$$\varpi_k^i = B_k^i - g_k^i[x_k^*, \psi_k^1(x_k^*), \psi_k^2(x_k^*), \cdots, \psi_k^n(x_k^*)],$$

for $k \in \{1,2,\cdots,T\}$ will be given to nation i to yield the cooperative imputation.

Since an imputation satisfies group and individual rationalities, we have:

(i) $W(h,x_h^*) = \sum_{j=1}^{n} \xi^j(h,x_h^*)$, and

(ii) $\xi^i(h,x_h^*) \geq \quad V^i(h,x_h^*)$, for $i \in N$ and $h \in \{1,2,\cdots,T\}$.

In a noncooperative equilibrium, the payoff received by nation i over the stages from 1 to τ can be expressed as:

$$\begin{aligned} &\sum_{k=1}^{\tau} g_k^i[\hat{x}_k, \phi_k^1(\hat{x}_k), \phi_k^2(\hat{x}_k), \cdots, \phi_k^n(\hat{x}_k)] \left(\frac{1}{1+r}\right)^{k-1} \\ = \quad & V^i(t_0,x_0) - V^i(\tau,\hat{x}_\tau), \quad \tau \in \{1,2,\cdots,T\} \end{aligned} \tag{21}$$

The cooperative payoff received by nation i over the stages from 1 to τ can be expressed as:

$$\sum_{k=1}^{\tau} B_k^i \left(\frac{1}{1+r}\right)^{k-1} = \xi^i(t_0,x_0) - \xi^i(\tau,x_\tau^*). \tag{22}$$

If nation i's cooperative payoff i over the stages from 1 to τ is smaller than his noncooperative payoff i over the stages from 1 to τ, that is

$$\xi^i(t_0,x_0) - \xi^i(\tau,x_\tau^*) - [V^i(t_0,x_0) - V^i(\tau,\hat{x}_\tau)] \leq 0. \tag{23}$$

The present value of surplus /deficit of cooperative income over non-cooperative income of the low-income developing country after τ years is then represented by the Right-hand-side of (23). If the noncooperative payoff represents a subsistence level of payoff, the nation involved would have to borrow.

If the maximum borrowing that nation i can made is $\bar{M}_i$, and the condition

$$\xi^i(t_0,x_0 - \xi^i(\tau,x_\tau^*) - [V^i(t_0,x_0) - V^i(\tau,\hat{x}_\tau)] \leq -\bar{M}_i, \tag{24}$$

appears at $\tau \in \{1,2,\cdots,T\}$ it cannot finance the deficit and would reject the optimality principle leading to the imputation $\xi^i(\tau,x_\tau^*)$.

The failure of some (developing) nations to finance deficits may create severe strain on the cooperative scheme. Often these nations would request to be exempted from carrying out the optimal strategies (as in the case of the Kyoto Protocol). This is certainly a suboptimal arrangement and could reduce the gain from cooperation substantially. As a result financial aid may be considered to be given to these participants (with repayment made later) so that they can carry out the optimal strategies.

5 Financial Constraint in Cooperative Pollution Management: Simple Illustrations

Consider the case where there are four groups of nations IC1, IC2, DC, NIC involved in a cooperative pollution management scheme.

IC-1: North American Industrialized Country;

IC-2: EU industrialized Country;

NIC: Newly industrializing country, like China, Mexico, Brazil, Russia or Turkey

DC: Low income developing countries, like India or Pakistan.

The net benefits/revenue (in terms of billion dollars) under non-cooperation and cooperation are given in Table 1 which yield the following Table 2 showing the Surplus/Deficit for each player at each period.

Table 1: Net benefits/revenue in current value, with discount rate 0.05.

	2010	**2011**	**2012**	**2013**	**2014**	**2015**
IC1	3451	3400	3245	3189	3106	3100
IC2	2451	2344	2302	2291	2289	2280
DC	807	825	857	867	881	885
NIC	1103	1178	1227	1250	1261	1270
IC1(coop)	3300	3321	3500	3597	3612	3620
IC2(coop)	2339	2400	2412	2435	2456	2460
DC(coop)	737	799	950	1024	1033	1040
NIC(coop)	997	1214	1305	1387	1460	1470

Table 2: Surplus/deficit in current value, with discount rate 0.05.

	2010	**2011**	**2012**	**2013**	**2014**	**2015**
IC1	-151	-79	255	408	506	510
IC2	-112	56	110	143	167	180
DC	-70	-26	93	157	152	155
NIC	-106	36	78	137	199	200

The figures in Tables 1 and 2 are constructed to provide numerical examples for illustration. Given actual noncooperative payoffs and agreed upon cooperative payoffs, the financial constraint problem facing an individual country could be delineated as in subsections 5.1 and 5.2 below.

5.1 The low-income developing country

The low-income developing country's non-cooperative payoff for period from 2010 to 2015 is:

$$\begin{aligned}
&V^{DC}(2010,\hat{x}_{2010}) = \sum_{k=2010}^{2015} g_k^{DC}\left(\frac{1}{1+0.05}\right)^{k-2010} = 807+825\times(\frac{1}{1+0.05}) \\
&+857\times(\frac{1}{1+0.05})^2+867\times(\frac{1}{1+0.05})^3+881\times(\frac{1}{1+0.05})^4+885\times(\frac{1}{1+0.05})^5 \\
&=4537.
\end{aligned} \tag{25}$$

The low-income developing country's non-cooperative payoff for periods $[2011-2015]$, $[2012-2015]$, $[2013-2015]$ and $[2014-2015]$ are respectively:

$$\begin{aligned}
V^{DC}(2011,\hat{x}_{2011}) &= 3730 \\
V^{DC}(2012,\hat{x}_{2012}) &= 2944 \\
V^{DC}(2013,\hat{x}_{2013}) &= 2167 \\
V^{DC}(2014,\hat{x}_{2014}) &= 1418 \\
V^{DC}(2015,\hat{x}_{2015}) &= 693.
\end{aligned}$$

The low-income developing country's cooperative payoff for period from 2010 to 2015 is:

$$\begin{aligned}
\xi^{DC}(2010,x^*_{2010}) &= \sum_{k=2010}^{2015} B_k^{DC}\left(\frac{1}{1+0.05}\right)^{k-2010} \\
&= 737+799\times(\frac{1}{1+0.05})+950\times(\frac{1}{1+0.05})^2+1024\times(\frac{1}{1+0.05})^3 \\
&\quad +1033\times(\frac{1}{1+0.05})^4 \quad +1040\times(\frac{1}{1+0.05})^5 \\
&= 4910.
\end{aligned} \tag{26}$$

The low-income developing country's cooperative payoff for periods $[2011-2015]$, $[2012-2015]$, $[2013-2015]$ and $[2014-2015]$ are respectively:

$$\begin{aligned}
\xi^{DC}(2011,x^*_{2011}) &= 4173 \\
\xi^{DC}(2012,x^*_{2012}) &= 3412 \\
\xi^{DC}(2013,x^*_{2013}) &= 2550 \\
\xi^{DC}(2014,x^*_{2014}) &= 1665, \\
\xi^{DC}(2015,x^*_{2015}) &= 815.
\end{aligned}$$

The present value of surplus /deficit of cooperative income over non-cooperative income of the low-income developing country after years 1,2,3,4 and 5 are:

$$\xi^{DC}(2010,x^*_{2010}) - \xi^{DC}(2011,x^*_{2011}) - [V^{DC}(2010,\hat{x}_{2010} - V^{DC}(2011,\hat{x}_{2011})$$
$$= 4910 - 4173 - [4537 - 3730] = -70$$
$$\xi^{DC}(2010,x^*_{2010}) - \xi^{DC}(2012,x^*_{2011}) - [V^{DC}(2010,\hat{x}_{2010} - V^{DC}(2012,\hat{x}_{2011})$$
$$= 4910 - 3412 - [4537 - 2944] = -95$$
$$\xi^{DC}(2010,x^*_{2010}) - \xi^{DC}(2013,x^*_{2011}) - [V^{DC}(2010,\hat{x}_{2010} - V^{DC}(2013,\hat{x}_{2011})]$$
$$= 4910 - 2550 - [4537 - 2167] = -10$$
$$\xi^{DC}(2010,x^*_{2010}) - \xi^{DC}(2014,x^*_{2011}) - [V^{DC}(2010,\hat{x}_{2010} - V^{DC}(2014,\hat{x}_{2011})]$$
$$= 4910 - 1665 - [4537 - 1418] = 126$$
$$\xi^{DC}(2010,x^*_{2010}) - \xi^{DC}(2015,x^*_{2011}) - [V^{DC}(2010,\hat{x}_{2010} - V^{DC}(2015,\hat{x}_{2011})]$$
$$= 4910 - 815 - [4537 - 693] = 251.$$

If the low-income developing country's non-cooperative payoff represents the subsistence level the low-income developing country would have to borrow. If the borrowing constraint of the DC is 95 (billion dollars), the cooperative scheme is unaffordable.

5.2 North American industrialized country

The North American industrialized country's non-cooperative payoff for period from 2010 to 2015 is:

$$V^{IC1}(2010,\hat{x}_{2010} = \sum_{k=2010}^{2015} g_k^{IC1}\left(\frac{1}{1+0.05}\right)^{k-2010}$$
$$= 3451 + 3400 \times (\frac{1}{1+0.05}) + 3245 \times (\frac{1}{1+0.05})^2 + 3189 \times (\frac{1}{1+0.05})^3$$
$$+3106 \times (\frac{1}{1+0.05})^4 + 3100 \times (\frac{1}{1+0.05})^5 = 17370.$$

The North American industrialized country's non-cooperative payoff for periods $[2011-2015]$, $[2012-2015]$, $[2013-2015]$ and $[2014-2015]$ are respectively:

$$\begin{aligned} V^{IC1}(2011,\hat{x}_{2011} &= 13919 \\ V^{IC1}(2012,\hat{x}_{2012} &= 10681 \\ V^{IC1}(2013,\hat{x}_{2013} &= 7738 \\ V^{IC1}(2014,\hat{x}_{2014} &= 4984 \\ V^{IC1}(2015,\hat{x}_{2015} &= 2429. \end{aligned}$$

The North American industrialized country's cooperative payoff for period from 2010 to 2015 is:

$$\begin{aligned}\xi^{IC1}(2010,x^*_{2010}) &= \sum_{k=2010}^{2015} B^i_k \left(\frac{1}{1+0.05}\right)^{k-2010} = 3300+3321\times(\frac{1}{1+0.05})\\ &+3500\times(\frac{1}{1+0.05})^2+3597\times(\frac{1}{1+0.05})^3\\ &+3612\times(\frac{1}{1+0.05})^4+3620\times(\frac{1}{1+0.05})^5 = 18553.\end{aligned}$$

The North American industrialized country's cooperative payoff for periods $[2011-2015]$, $[2012-2015]$, $[2013-2015]$ and $[2014-2015]$ are respectively:

$$\begin{aligned}\xi^{IC1}(2011,x^*_{2011}) &= 15253\\ \xi^{IC1}(2012,x^*_{2012}) &= 12090\\ \xi^{IC1}(2013,x^*_{2013}) &= 8915\\ \xi^{IC1}(2014,x^*_{2014}) &= 5808\\ \xi^{IC1}(2015,x^*_{2015}) &= 2836.\end{aligned}$$

The present value of surplus /deficit of cooperative income over non-cooperative income of North American industrialized country after years 1,2,3,4 and 5 are:

$$\begin{aligned}&\xi^{IC1}(2010,x^*_{2010})-\xi^{IC1}(2011,x^*_{2011})-[V^{IC1}(2010,\hat{x}_{2010}-V^{IC1}(2011,\hat{x}_{2011})]\\ &=18553-15253-[17370-13919]=-151\\ &\xi^{IC1}(2010,x^*_{2010})-\xi^{IC1}(2012,x^*_{2012})-[V^{IC1}(2010,\hat{x}_{2010}-V^{IC1}(2012,\hat{x}_{2012})]\\ &=18553-12090-[17370-10681]=-226\\ &\xi^{IC1}(2010,x^*_{2010})-\xi^{IC1}(2013,x^*_{2013})-[V^{IC1}(2010,\hat{x}_{2010}-V^{IC1}(2013,\hat{x}_{2013})]\\ &=18553-8915-[17370-7738]=6\\ &\xi^{IC1}(2010,x^*_{2010})-\xi^{IC1}(2014,x^*_{2014})-[V^{IC1}(2010,\hat{x}_{2010}-V^{IC1}(2014,\hat{x}_{2014})]\\ &=18553-5808-[17370-4984]=359\\ &\xi^{IC1}(2010,x^*_{2010})-\xi^{IC1}(2015,x^*_{2015})-[V^{IC1}(2010,\hat{x}_{2010}-V^{IC1}(2015,\hat{x}_{2015})]\\ &=18553-2836-[17370-2429]=776.\end{aligned}$$

If the North American industrialized country are above subsistence level and its borrowing constraint is above 226, the cooperative scheme is feasible.

Similarly, the above results could be computed for the EU industrialized country (IC-2) and the newly industrialized country (NIC).

6 A Continuous-time Analog

Consider the general $n-$person nonzero-sum differential game with initial state x_0 and duration $T-t_0$ involving n nations. The state space of the game is $\mathrm{X} \in \mathrm{R}^m$, with permissible state trajectories $\{x(s),\, t_0 \le s \le T\}$. The state dynamics of the game is characterized by the vector-valued differential equations:

$$\dot{x}(s) = f[s, x(s), u_1(s), u_2(s), \cdots, u_n(s)], \quad x(t_0) = x_0, \tag{27}$$

where $u_i(s) \in R^{m_i}$ is the control vector of nation i.

The objective of nation i is

$$\int_{t_0}^{T} g^i[s, x(s), u_1(s), u_2(s), \cdots, u_n(s)]\, e^{-r(s-t_0)} ds + e^{-r(T-t_0)} q^i(x(T)), \tag{28}$$

for $i \in \{1, 2, \cdots, n\} \equiv N$

Let $\{\phi_i(s,x)$, for $i \in N\}$ denote a set of strategies leading to a feedback Nash equilibrium, the game equilibrium state trajectory can be obtained as:

$$\dot{x}(s) = f\{s, x(s), \phi_1[s, x(s)], \phi_2[s, x(s)], \cdots, \phi_n[s, x(s)]\},$$
$$x(t_0) = x_0. \tag{29}$$

We denote the solution to (29) by $\{\hat{x}(s)\}_{s=t_0}^{T}$, and use the terms $\hat{x}(s)$ and $\hat{x}_s$ interchangeably. The noncooperative payoff of nation i over the interval $[t, T]$ where $t \in [t_0, T]$ is:

$$V^i(t, \hat{x}_t) = \int_{t}^{T} g^i[s, \hat{x}(s), \phi_1(s, \hat{x}(s)), \phi_2(s, \hat{x}(s)), \cdots, \phi_n(s, \hat{x}(s))]\, e^{-r(s-t_0)} ds$$
$$+ e^{-r(T-t_0)} q^i(\hat{x}(T)), \tag{30}$$

for $i \in N$, and $x(t) = \hat{x}_t$.

Under cooperation group rationality required the nations to maximize their joint payoff

$$\int_{t_0}^{T} \sum_{j=1}^{n} g^j[s, x(s), u_1(s), u_2(s), \cdots, u_n(s)] e^{-r(s-t_0)}\, ds + \sum_{j=1}^{n} e^{-r(T-t_0)} q^j(x(T)) \tag{31}$$

subject to (27).

Let $\{\psi_i(s,x)$, for $i \in N\}$ denote a set of strategies leading to an optimal control solution of the problem (27) and (31), and let $\{x^*(s)\}_{s=t_0}^{T}$ denote the optimal cooperative path. The total cooperative payoff over the interval $[t, T]$ where $t \in [t_0, T]$ is:

$$W(t, x_t^*) = \int_{t}^{T} \sum_{j=1}^{n} g^j[s, x^*(s), \psi_1(s, x^*(s)), \psi_2(s, x^*(s)), \cdots, \psi_n(s, x^*(s))]\, e^{-r(s-t_0)} ds$$
$$+ \sum_{j=1}^{n} e^{-r(T-t_0)} q^j(x^*(T)). \tag{32}$$

Let $\xi^i(\tau, x^*_\tau) = \int_\tau^T B_i(s)\, e^{-r(s-\tau)} ds + q^i(x^*_T)$ denote the imputation to nation i under cooperation over the time interval $[\tau, T]$ along the cooperative path $\{x^*_\tau\}_{\tau=t_0}^T$ for $\tau \in [t_0, T]$. Often an instantaneous side-payment

$$\varpi_i(s) = B_i(s) - g^i[s, x^*(s), \psi_1(s, x^*(s)), \psi_2(s, x^*(s)), \cdots, \psi_n(s, x^*(s))],$$

for $i \in N$ and $s \in [t_0, T)$, will be given to nation i to yield the cooperative imputation.

Since an imputation satisfies group and individual rationalities, we have:

(i) $W(\tau, x^*_\tau) = \sum_{j=1}^{n} \xi^j(\tau, x^*_\tau)$, and

(ii) $\xi^i(\tau, x^*_\tau) \geq \quad V^i(\tau, x^*_\tau)$, for $i \in N$.

In a noncooperative equilibrium, the payoff received by nation i in the interval $[t_0, \tau]$ can be expressed as:

$$\int_{t_0}^{\tau} g^i[s, \hat{x}(s), \phi_1(s, \hat{x}(s)), \phi_2(s, \hat{x}(s)), \cdots, \phi_n(s, \hat{x}(s))]\, e^{-r(s-t_0)} ds$$
$$= V^i(t_0, x_0) - V^i(\tau, \hat{x}_\tau). \tag{33}$$

The cooperative payoff received by nation i in the interval $[t_0, \tau]$ can be expressed as:

$$\int_{t_0}^{\tau} B_i(s)\, e^{-r(s-t_0)} ds = \xi^i(t_0, x_0) - \xi^i(\tau, x^*_\tau). \tag{34}$$

If nation i's cooperative payoff in the interval $[t_0, \tau]$ is smaller than his noncooperative payoff in the interval $[t_0, \tau]$, that is

$$\xi^i(t_0, x_0) - \xi^i(\tau, x^*_\tau) - [V^i(t_0, x_0) - V^i(\tau, \hat{x}_\tau)] \leq 0. \tag{35}$$

The present value of surplus /deficit of cooperative income over non-cooperative income of the low-income developing country at time τ is then represented by the Right-hand-side of (23). If the noncooperative payoff represents a subsistence level of payoff, the nation involved would have to borrow.

If the maximum borrowing that nation i can made is $\bar{M}_i$, and the condition

$$\xi^i(t_0, x_0) - \xi^i(\tau, x^*_\tau) - [V^i(t_0, x_0) - V^i(\tau, \hat{x}_\tau)] \leq -\bar{M}_i, \tag{36}$$

appears at $\tau \in [t_0, T]$ it cannot finance the deficit and would reject the optimality principle under cooperation. This analysis was first proposed in Yeung (2008).

7 Financial Constraint in Cooperative Differential Game of Pollution Management

In this section we consider financial constraint in the deterministic version of the Yeung and Petrosyan (2008) cooperative stochastic differential game of trans-boundary industrial pollution.

7.1 Game Formulation

7.1.1 The Industrial Sector

Consider a multinational economy which is comprised of n nations. To allow different degrees of substitutability among the nations' outputs a differentiated products oligopoly model has to be adopted. The differentiated oligopoly model used by Dixit (1979) and Singh and Vives (1984)) in industrial organizations is adopted to characterize the interactions in this international market. In particular, the nations' outputs may range from a homogeneous product to n unrelated products. Specifically, the inverse demand function of the output of nation $i \in N \equiv \{1, 2, \cdots, n\}$ at time instant s is

$$P_i(s) = \alpha^i - \sum_{j=1}^{n} \beta^i_j q_j(s), \tag{37}$$

where $P_i(s)$ is the price of the output of nation i, $q_j(s)$is the output of nation j, $\alpha^i$and β^i_j for $i \in N$ and $j \in N$ are positive constants. The output choice $q_j(s) \in [0, \bar{q}_j]$ is nonnegative and bounded by a maximum output constraint $\bar{q}_j$. Output price equals zero if the right-hand-side of (37) becomes negative. The demand system (37) shows that the economy is a form of differentiated products oligopoly with substitute goods. In the case when $\alpha^i = \alpha^j$ and $\beta^i_j = \beta^j_i$ for all $i \in N$ and $j \in N$, the industrial outputs resemble a homogeneous good. In the case when $\beta^i_j = 0$ for $i \neq j$, the n nations produce n unrelated products. Moreover, the industry equilibrium generated by this oligopoly model is computable and fully tractable.

Industrial profits of nation i at time s can be expressed as:

$$\pi_i(s) = [\alpha^i - \sum_{j=1}^{n} \beta^i_j q_j(s)]q_i(s) - c_i q_i(s) - v_i(s) q_i(s), \quad i \in N. \tag{38}$$

where $v_i(s) \geq 0$ is the tax rate imposed by government i on its industrial output at time s and c_i is the unit cost of production. At each time instant s, the industrial sector of nation $i \in N$ seeks to maximize (38). Note that each industrial sector would consider the information on the demand structure, each other's cost structures and tax policies. The first order condition for a Nash equilibrium for the n nations economy yields

$$\sum_{j=1}^{n} \beta^i_j q_j(s) + \beta^i_i q_i(s) = \alpha^i - c_i - v_i(s), \quad i \in N. \tag{39}$$

With output tax rates $v(s) = \{v_1(s), v_2(s), \cdots, v_n(s)\}$ being regarded as parameters (5.3) becomes a system of equations linear in $q(s) = \{q_1(s), q_2(s), \cdots, q_n(s)\}$. Solving (39) yields an industry equilibrium:

$$q_i(s) = \phi^i_i(v(s)) = \bar{\alpha}^i + \sum_{j \in N} \bar{\beta}^i_j \, v_j(s), \tag{40}$$

where $\bar{\alpha}^i$ and $\bar{\beta}^i_j$, for $i \in N$ and $j \in N$, are constants involving the model parameters

$$\{\beta^1_1, \beta^1_2, \cdots, \beta^1_n; \beta^2_1, \beta^2_2, \cdots, \beta^2_n; \cdots; \beta^n_1, \beta^n_2, \cdots, \beta^n_n\},$$

$\{\alpha^1, \alpha^2, \cdots, \alpha^n\}$ and $\{c_1, c_2, \cdots, c_n\}$.

One can readily observe from (39) that an increase in the tax rate has the same effect of an increase in cost. Ceteris paribus, an increase in nation i's tax rate would depress the output of industrial sector i and vice versa. Given that outputs are substitutable products and the linear demand functions (37) industrial sector i's output and nation j's tax rate, where $j \neq i$, are positively related.

7.1.2 Local and Global Environmental Impacts

Industrial production emits pollutants into the environment. The emitted pollutants cause short term local impacts on neighboring areas of the origin of production in forms like passing-by waste in waterways, wind-driven suspended particles in air, unpleasant odour, noise, dust and heat. For an output of $q_i(s)$ produced by nation i, there will be a short-term local environmental impact (cost) of $\varepsilon^i_i q_i(s)$ on nation i itself and a local impact of $\varepsilon^i_j q_i(s)$ on its neighbor nation j. Nation i will receive short-term local environmental impacts from its adjacent nations measured as $\varepsilon^j_i q_j(s)$ for $j \in \bar{K}^i$. Thus $\bar{K}^i$ is the subset of nations whose outputs produce local environmental impacts to nation i. Moreover, industrial production would also create long-term global environmental impacts by building up existing pollution stocks like Green-house-gas, CFC and atmospheric particulates. Each government adopts its own pollution abatement policy to reduce the pollution stock. Let $x(s) \subset R^+$ denote the level of pollution at time s, the dynamics of pollution stock is governed by the differential equation:

$$\dot{x}(s) = \left[\quad \sum_{j=1}^{n} a_j q_j(s) - \sum_{j=1}^{n} b_j u_j(s) [x(s)]^{1/2} - \delta x(s) \quad \right], \quad x(t_0) = x_{t_0}, \tag{41}$$

where $a_j q_j$ is the amount added to the pollution stock by a unit of nation j's output,

$u_j(s)$ is the pollution abatement effort of nation j,

$b_j u_j(s)[x(s)]^{1/2}$ is the amount of pollution removed by $u_j(s)$ unit of abatement effort of nation j, and δ is the natural rate of decay of the pollutants.

7.1.3 The Governments' Objectives

The governments have to promote business interests and at the same time handle the financing of the costs brought about by pollution. In particular, each government maximizes the net gains in the industrial sector minus the sum of expenditures on pollution abatement and damages from pollution. The instantaneous objective of government i at time s can be expressed as:

$$[\alpha^i - \sum_{j=1}^{n} \beta^i_j q_j(s)]q_i(s) - c_i q_i(s) - c^a_i [u_i(s)]^2 - \sum_{j \in \bar{K}^i} \varepsilon^j_i [q_j(s)] - h_i x(s), i \in N, \quad (42)$$

where $c^a_i [u_i(s)]^2$ is the cost of employing $u_i$amount of pollution abatement effort, and $h_i x(s)$ is the value of damage to country i from $x(s)$ amount of pollution.

The governments' planning horizon is $[t_0, T]$. It is possible that T may be very large. At time T, the terminal appraisal associated with the state of pollution is $g^i[\bar{x}^i - x(T)]$ where $g^i \geq 0$ and $\bar{x}^i \geq 0$. The discount rate is r. Each one of the n governments seeks to maximize the integral of its instantaneous objective (42) over the planning horizon subject to pollution dynamics (41) with controls on the level of abatement effort and output tax.

By substituting $q_i(s)$, for $i \in N$, from (40) into (41) and (42) one obtains a differential game in which government $i \in N$ seeks to:

$$\max_{v_i(s), u_i(s)} \Bigg\{ \int_{t_0}^{T} \Bigg[\Bigg(\alpha^i - \sum_{j=1}^{n} \beta^i_j [\bar{\alpha}^j + \sum_{h \in N} \bar{\beta}^j_h \, v_h(s)] \Bigg)$$
$$[\bar{\alpha}^i + \sum_{h \in N} \bar{\beta}^i_h \, v_h(s)] - c_i[\bar{\alpha}^i + \sum_{j \in N} \bar{\beta}^i_j \, v_j(s)] - c^a_i [u_i(s)]^2$$
$$- \sum_{j \in \bar{K}^i} \varepsilon^j_i [\bar{\alpha}^j + \sum_{\ell \in N} \bar{\beta}^j_\ell \, v_\ell(s)] - h_i x(s) \Bigg] e^{-r(s-t_0)} ds - g^i[x(T) - \bar{x}^i] e^{-r(T-t_0)} \Bigg\} \quad (43)$$

subject to

$$\dot{x}(s) = [\, \sum_{j=1}^{n} a_j [\bar{\alpha}^j + \sum_{h \in N} \bar{\beta}^j_h \, v_h(s)] - \sum_{j=1}^{n} b_j u_j(s)[x(s)]^{1/2} - \delta x(s)\,] \quad (44)$$

with $x(t_0) = x_{t_0}$.

In the game (43)-(44) one can readily observe that government i's tax policy $v_i(s)$ is not only explicitly reflected in its own output but also on the outputs of other nations. This modeling formulation allows some intriguing scenario to arise. For instance, an increase of $v_i(s)$ may just cause a minor drop in nation i's industrial profit but may cause significant increases in its neighbors' outputs which produce large local negative environmental impacts to nation i. This results in nations' reluctance to increase or impose taxes on industrial outputs.

7.2 Noncooperative and Cooperative Outcomes

The payoff function $V^i(t,\hat{x}_t)$ of player i in a feedback Nash equilibrium of the game (43)-(44) can be obtained as

Proposition 1 *for $i \in N$ and $t \in [t_0, T]$*

$$V^i(t,\hat{x}_t) = [A_i(t)\hat{x}_t + C_i(t)]\, e^{-r(t-t_0)}, \tag{45}$$

where $\{A_1(t), A_2(t), \cdots, A_n(t)\}$ satisfying the following set of constant coefficient quadratic ordinary differential equations:

$$\dot{A}_i(t) = (r+\delta)\, A_i(t) - \frac{b_i^2}{4c_i^a}[A_i(t)]^2 - A_i(t) \sum_{\substack{j=1 \\ j \neq i}}^{n} \frac{b_j^2}{2c_j^a} A_j(t) + h_i,$$

$$A_i(T) = -g^i;$$

for $i \in N$,

and $\{C_i(t);\ i \in N\}$ is given by

$$C_i(t) = e^{r(t-t_0)} \left[\int_{t_0}^{t} F_i(y) e^{-r(y-t_0)} dy + C_i^0 \right], \tag{46}$$

where

$$C_i^0 = g^i \bar{x}^i e^{-r(T-t_0)} - \int_{t_0}^{T} F_i(y) e^{-r(y-t_0)} dy,$$

$$F_i(t) = -\Bigg(\alpha^i - \sum_{j=1}^{n} \beta_j^i \{\bar{\alpha}^j + \sum_{h \in Ni}^{n} \bar{\beta}_h^j\, [\hat{\alpha}^h + \sum_{k \in N} \hat{\beta}_k^h\, A_k(t)]\}\,)$$

$$(\, \bar{\alpha}^i + \sum_{h \in N} \bar{\beta}_h^i\, [\hat{\alpha}^h + \sum_{k \in N} \hat{\beta}_k^h\, A_k(t)]\,) + c_i \{\bar{\alpha}^i - \sum_{j \in N} \bar{\beta}_j^i\, [\hat{\alpha}^j + \sum_{k \in N} \hat{\beta}_k^j\, A_k(t)]\}$$

$$+\sum_{j\in\bar{K}^i}\varepsilon_i^j\{\bar{\alpha}^j+\sum_{\ell\in N}\bar{\beta}_\ell^j\,[\hat{\alpha}^\ell+\sum_{k\in N}\hat{\beta}_k^\ell\,A_k(t)]\}$$

$$-A_i(t)\,[\,\sum_{j=1}^{n}a_j\{\bar{\alpha}^j+\sum_{h\in N}\bar{\beta}_h^j\,[\hat{\alpha}^h+\sum_{k\in N}\hat{\beta}_k^h\,A_k(t)]\}\,]\,.$$

Proof. Follow Yeung and Petrosyan (2008).

Solving the feedback Nash equilibrium pollution dynamics yields the state trajectory:

$$\begin{aligned}\hat{x}(t) &= e^{\left[\int_{t_0}^{t}[\sum_{j=1}^{n}\frac{b_j^2}{2c_j^a}A_j(s)-\delta]ds\right]}\\ &\left[x_0+\int_{t_0}^{t}\sum_{j=1}^{n}a_j\{\bar{\alpha}^j+\sum_{h\in N}\bar{\beta}_h^j\,[\hat{\hat{\alpha}}^h+\hat{\hat{\beta}}^h\,A_h(s)]\}e^{\left[\int_{t_0}^{s}[\delta-\sum_{j=1}^{n}\frac{b_j^2}{2c_j^a}A_j(\tau)]d\tau\right]}ds\right],\end{aligned} \tag{47}$$

for $t\in[t_0,T]$.

Now consider the case when all the nations want to cooperate and agree to act so that an international optimum could be achieved.

To secure group optimality the participating nations seek to maximize their joint payoff by solving the following optimal control problem:

$$\begin{aligned}\max_{v_1,v_2,\cdots,v_n;u_1,u_2,\cdots,u_n}&\left\{\int_{t_0}^{T}\sum_{\ell=1}^{n}\left[\left(\alpha^\ell-\sum_{j=1}^{n}\beta_j^\ell[\bar{\alpha}^j+\sum_{h\in N}\bar{\beta}_h^j\,v_h(s)]\right)[\bar{\alpha}^\ell+\sum_{h\in N}\bar{\beta}_h^\ell\,v_h(s)]\right.\right.\\ &\left.-c_\ell[\bar{\alpha}^\ell+\sum_{j\in N}\bar{\beta}_j^\ell\,v_j(s)]-c_\ell^a[u_\ell(s)]^2-\sum_{j\in\bar{K}^\ell}\varepsilon_\ell^j[\bar{\alpha}^j+\sum_{k\in N}\bar{\beta}_k^j\,v_k(s)]-h_\ell x(s)\right]e^{-r(s-t_0)}ds\\ &\left.-\sum_{\ell=1}^{n}g^\ell[x(T)-\bar{x}^\ell]e^{-r(T-t_0)}\right\}\end{aligned} \tag{48}$$

subject to (44). The total cooperative payoff $W(t,x_t^*)$ can be obtained as

Proposition 2

$$W(t,x_t^*)=[A^*(t)x_\tau^*+C^*(t)]\,e^{-r(t-t_0)}, \tag{49}$$

with

$A^*(t)=A_*^P+\Phi^*(t)\,[\,\bar{C}^*-\int_{t_0}^{t}\sum_{j=1}^{n}\frac{b_j^2}{2c_j^a}\Phi^*(y)dy\,]^{-1}$, *and*

$$C^*(t)=e^{r(\,t-t_0)}\,[\int_{t_0}^{t}F^*(y)e^{-r(y-t_0)}dy+C_*^0\,]\,,$$

where $\Phi^*(t) = \exp\{ \int\limits_{t_0}^{t} [\sum\limits_{j=1}^{n} \frac{b_j^2}{2c_j^a} A_*^P + (r+\delta)] \ dy \}$,

$$\bar{C}^* = \frac{-\Phi^*(T)}{(A_*^P + \sum\limits_{j=1}^{n} g^j)} + \int\limits_{t_0}^{T} \sum_{j=1}^{n} \frac{b_j^2}{2c_j^a} \Phi^*(y) dy,$$

$$A_*^P(t) = \{ \ (r+\delta) \ - [\ (r+\delta)^2 + 4 \sum_{j=1}^{n} \frac{b_j^2}{2c_j^a} \sum_{j=1}^{n} h_j \ ]^{1/2} \ \} \Big/ \sum_{j=1}^{n} \frac{b_j^2}{c_j^a} \quad ,$$

$$F^*(t) =$$

$$- \sum_{\ell=1}^{n} [\ (\ \alpha^\ell - \sum_{j=1}^{n} \beta_j^\ell \{ \bar{\alpha}^j + \sum_{h\in N} \bar{\beta}_h^j \ [\hat{\alpha}^h + \hat{\hat{\beta}}^h \ A^*(t)] \} \ ) \{ \bar{\alpha}^\ell + \sum_{h\in N} \bar{\beta}_h^\ell \ [\hat{\alpha}^h + \hat{\hat{\beta}}^h \ A^*(t)] \}$$

$$-c_\ell^{\{} \bar{\alpha}^\ell + \sum_{j\in N} \bar{\beta}_j^\ell \ [\hat{\alpha}^j + \hat{\hat{\beta}}^j \ A^*(t)] \} - \sum_{j\in \bar{K}^\ell} \varepsilon_\ell^j \{ \bar{\alpha}^j + \sum_{k\in N} \bar{\beta}_k^j \ [\hat{\alpha}^k + \hat{\hat{\beta}}^{kj} \ A^*(t)] \} \ ]$$

$-A_x^*(t) \, [\sum\limits_{j=1}^{n} a_j^{\{} \bar{\alpha}^j + \sum\limits_{h\in N} \bar{\beta}_h^j \ [\hat{\alpha}^h + \hat{\hat{\beta}}^h \ A^*(t)] \} \ ]$, *and*

$$C_*^0 = \sum_{j=1}^{n} g^j \bar{x}^j e^{-r(\,T-t_0)} - \int\limits_{t_0}^{T} F^*(y) e^{-r(y-t_0)} dy.$$

Proof. Follow Yeung and Petrosyan (2008).

Solving the cooperative pollution dynamics yields the cooperative state trajectory:

$$x^*(t) = e^{\left[\int_{t_0}^{t} [\sum\limits_{j=1}^{n} \frac{b_j^2}{2c_j^a} A^*(s) - \delta] ds \right]}$$

$$\left[x_{t_0} + \int_{t_0}^{t} \sum_{j=1}^{n} a_j \{ \bar{\alpha}^j + \sum_{h\in N} \bar{\beta}_h^j \ [\hat{\alpha}^h + \hat{\hat{\beta}}^h \ A^*(s)] \} e^{\left[\int_{t_0}^{s} [\delta - \sum\limits_{j=1}^{n} \frac{b_j^2}{2c_j^a} A^*(\tau)] d\tau \right]} ds \right], \tag{50}$$

for $t \in [t_0, T]$.

Since nations are asymmetric and the number of nations may be large, a reasonable solution optimality principle for gain distribution is to share the expected gain from cooperation proportional to the nations' relative sizes of expected non-cooperative payoffs. As mentioned before, a stringent condition – subgame consistency – is required for a credible cooperative solution. In order to satisfy the property of subgame consistency, this optimality principle has to remain in effect throughout the cooperation period. Hence the solution imputation scheme $\{\xi^i(\tau, x_\tau^*);$ for $i \in N\}$ has to satisfy:

Condition 1

$$\xi^i(\tau,x_\tau^*) = \quad V^i(\tau,x_\tau^*) + \frac{V^i(\tau,x_\tau^*)}{\sum_{j=1}^{n} V^j(\tau,x_\tau^*)}\left[W(\tau,x_\tau^*) - \sum_{j=1}^{n} V^j(\tau,x_\tau^*)\right]$$

$$= \frac{V^i(\tau,x_\tau^*)}{\sum_{j=1}^{n} V^j(\tau,x_\tau^*)} W(\tau,x_\tau^*), \tag{51}$$

for $i \in N$, $x_\tau^* \in X_\tau^*$ *and* $\tau \in [t_0,T]$.

One can easily verify that the imputation scheme in Condition 1 satisfies individual rationality and group rationality.

7.3 Financial Constraint in Cooperative Pollution Management

Substituting $\xi^i(\tau,x_\tau^*) = \frac{V^i(\tau,x_\tau^*)}{\sum_{j=1}^{n} V^j(\tau,x_\tau^*)} W(\tau,x_\tau^*)$ into (35) the deficit/surplus condition at time $\tau \in [t_0,T]$ can be expressed as:

$$\xi^i(t_0,x_0 - \xi^i(\tau,x_\tau^*) - [V^i(t_0,x_0 - V^i(\tau,\hat{x}_\tau)], \tag{52}$$

for $i \in N$ and $\tau \in [t_0,T]$.

Invoking Propositions 4.1 and 4.2, (52) can be written as:

$$\frac{[A_i(t_0)x_0 + C_i(t_0)]}{\sum_{j=1}^{n} [A_j(t_0)x + C_j t_0)]} [A^*(t_0)x_0 + C^*(t_0)]$$

$$-e^{-r(\tau-t_0)} \frac{[A_i\tau)x_\tau^* + C_i\tau)]}{\sum_{j=1}^{n} [A_j\tau)x_\tau^* + C_j\tau)]} [A^*(\tau)x_\tau^* + C^*(\tau)]$$

$$+e^{-r(\tau-t_0)}[A_i(\tau)\hat{x}_\tau + C_i(\tau)] - [A_i(t_0)x_0 + C_i(t_0)] \geq 0, \quad i \in N. \tag{53}$$

Note that $A^*(\tau)$, $C^*(\tau)$, $x^*(\tau)$, $A_j(\tau)$, $C_j(\tau)$ and $\hat{x}(\tau)$ for $j \in N$ and $\tau \in [t_0,T]$ are obtained in explicitly computable form. Using (53) one can readily check the deficit/surplus of players along the cooperative trajectory $\{x^*(\tau)\}_{\tau=t_0}^{T}$.

8 Conclusions for Part-2

Cooperation in environmental control is urgently needed for the current damaging climate change scenario. Under dynamic cooperation, it has been shown that given subgame-consistent imputations satisfying group optimality and individual rationality throughout the cooperative trajectory, no rational players will deviate from the cooperative path.

However, in reality low income nations may have financial constraint. In particular initial investment in pollution abatement and technology transfer under the new cooperation scheme may bring GDP down to a level below subsistence. Inability to borrow funds in the initial stage may force these nations to back off from cooperation. Refusal of participation by low income nations presents a stumbling block to successful cooperation in pollution management.

This part has examined conditions leading to the need for borrowing in intertemporal cooperative pollution management games. The issue is expounded in both the continuous-time and discrete-time frameworks.

Part III
An operational dynamic game framework

9 Issues in a game theoretic approach to IEAs

9.1 Choice of a proper game structure

The rapid overview[4] of the different game formulations used to model IEAs shows different important issues that we summarize below.

Cost-benefit vs. cost-effectiveness. In general the game theoretic approaches to model IEAs use a cost-benefit framework where the economic cost of emissions abatement is balanced with the benefits obtained from a reduction of the economic loss due to climate change. In practice it is much easier to assess the abatement cost than the damage cost due to climate change[5].

[4]This part has been contributed by Alain Haurie, Laurent Drouet, Marc Vielle and Jean-Philippe Vial from ORDECSYS and EPFL and Valentina Bosetti from FEEM.

[5]It is particularly true when one considers that the major impact of climate change will be on eco-systems; the economic value of maintaining the current biodiversity is not assessed currently.

Prisoner's dilemma as a status quo. Normally, in a cost-benefit approach the modelers take a noncooperative Nash equilibrium solution as the status quo situation, and in general, the outcome is very bad in terms of global welfare. This disastrous situation when there is no agreement means that the equilibrium status quo can be taken as a credible threat to support an efficient IEA. This is the argument in the model of Dutta & Radner Dutta and Radner (2004). In Figs. 2 and 3 below we reproduce some outputs obtained from WITCH, a multi-region economic growth model designed for international climate policy assessment and which will be used in TOCSIN. One may observe that the non-cooperative solution with climate feedback is almost the same, in terms of global emissions, as the one without climate feedback, which is indeed an extreme case of prisoner's dilemma. The cooperative solution with climate feedback generates a much lower emission path.

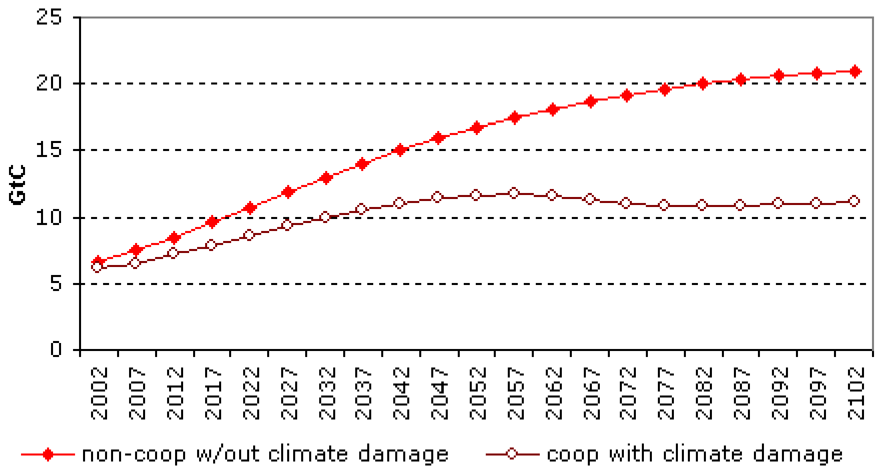

Figure 2: Non-coop. vs. cooperation in WITCH.

IETS. An important dimension of IEAs is the possibility to implement an international emissions trading scheme (IETS) which will redistribute the benefits of the common abatement policy. This is an important component of the results obtained when applying the concept of normalized equilibrium in a game with a coupled emission constraint (Haurie et al. (2006) or Drouet et al. (2007)). The key strategic variable is then the choice of a cap for a country. The actual emissions could then be at a level below or above the cap, the difference being sold or bought on the ET market.

Leader-follower relationship. Some actors will have the role of leaders, announcing their actions first and leaving the other actors react. OPEC is the

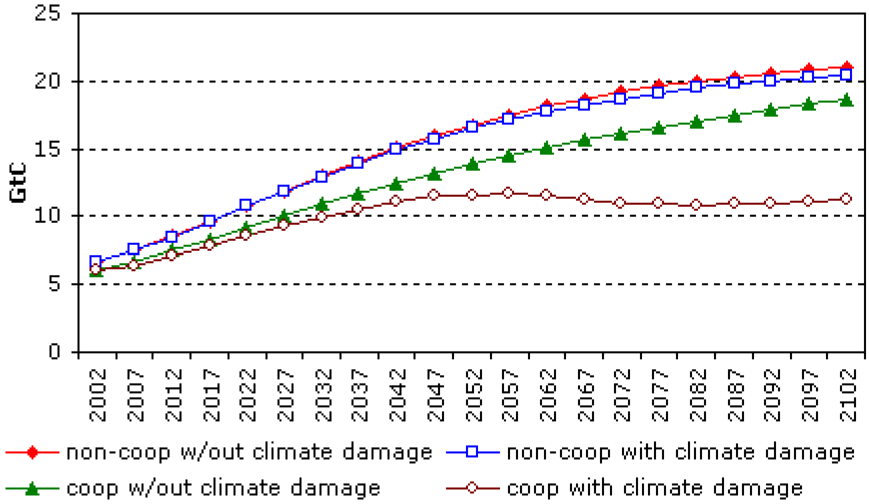

Figure 3: Effect of climate feedback.

archetypal leader in the World oil price game. As proposed in Kolstad and Ulph (2008), the signatories of an IEA could also be a Stackelberg leader in the emission game.

Uncertainty and timing. The climate sensitivity and hence the impact of anthropogenic climate change is still largely unknown, the access to a "backstop technology" permitting a carbon free economy at a low cost could occur in a not so distant future. These uncertain elements should be taken into account in an IEA. Stochastic or "robust" game models should therefore be introduced.

Umpire. IEAs negotiations take place in the framework of international organizations. EU Commission or UNO are possible "umpires" which could guide the equilibrium choice of competing countries in the reaching of a common global environmental target.

9.2 Choice of assessment tools

In TOCSIN we use a combination of a bottom-up, technology rich model (TIAM), a top-down macro-economic model (GEMINI-E3) and the WITCH aggregate multi-region optimal economic growth model with endogenous technical progress model.

Techno-economic model. TIAM is the last avatar in the MARKAL family of models. It includes a description of the carbon cycle and of the forcing of

GHG concentration, with a complete description of the energy systems of different world regions in order to satisfy demands in energy services that are elastic to the price of energy. The model will compute a partial economic equilibrium, via an ad-hoc linear program.

General equilibrium model. GEMINI-E3 is a computable general equilibrium model which represents also the economies in different world regions. A static general economic equilibrium is obtained at each time period. The links between periods is defined exogenously via the parameters determining savings and investments in the economies.

Optimal growth model. WITCH is a Regional Integrated Assessment Hard-Link Hybrid Model. Its top-down component consists of an intertemporal optimal growth model in which the energy input of the aggregate production function has been expanded to give a bottom-up like description of the energy sector. World countries are grouped in twelve regions that strategically interact following a game theoretic structure. A climate module and a damage function provide the feedback on the economy of carbon dioxide emissions into the atmosphere. The model is structured so as to provide normative information on the optimal responses of world economies to climate damages and to model the channels of transmission of climate policy to the economic system.

9.3 Computation of equilibrium solutions

In this section one recalls the different methods available for the computation of equilibrium solutions in different types of models.

Cobweb and relaxation Methods. It is the method used in WITCH to compute a Nash equilibrium in IEAs involving groups of countries. Although the method is not guaranteed to converge, it seems to work efficiently in WITCH (Kempfert , 2005).

The game theoretic setup of the WITCH model makes it possible to capture the non-cooperative nature of international relationships. Free riding behaviours and strategic inaction induced by the presence of a global externality are explicitly accounted for in the model. Climate change is the major global externality, as GHG emissions produced by each region indirectly impact on all other regions through the effect on global concentrations and thus global average temperature. The model features other economic externalities that provide additional channels of interaction. Energy prices depend on the extraction of fossil fuels, which in turn is affected by consumption patterns of all regions in the world. International knowledge and experience spillovers are two additional sources of externalities. By investing in energy R&D, each region accumulates a stock of knowledge that augments energy efficiency and reduces the cost of specific energy technologies. The effect of knowledge is not confined to the inventor region but it can spread to other regions. Finally, the diffusion of knowledge embodied in wind and solar experience is represented by learning curves linking investment costs with world, and not regional, cumulative capacity. Increasing capacity thus reduces investment costs for all regions. These externalities provide incentives to adopt strategic behaviours, both with respect to the environment (e.g. GHG emissions) and with respect to investments in knowledge and carbon free but costly technologies. In order to represent strategic behaviours, the model is solved as a non-cooperative game. The solution is found when all regions' strategies are a best response to other regions' best responses. The solution is found using an iterative algorithm that is solved recursively and yields an Open Loop Nash Equilibrium.

Using WITCH, we can evaluate the responsiveness of the economy to a global cap on concentration/radiative forcing or temperature. The problem is solved in two steps. In the first step the global path of emissions in line with the climate target has to be defined, by running the cooperative version of the model with the imposed climate cap. Once the optimal global path of emissions is defined, in the second step optimal emissions for each period are shared across regions on the basis of an allocation scheme (e.g. contraction and convergence, equal per capita, etc.). The model is run again now assuming that countries cooperate on the climate externality only (each country is constrained to the defined level of emissions in each period) while they do not cooperate on all the other externalities (each of the twelve region optimizes its own welfare given the constraint on emissions). Each country can optimally choose its own level of emissions giving the constraint by selling/buying emissions on a global carbon market. An additional possibility is to allow for banking of emissions, this enables us to model an optimal flexibility of emission reduction (borrowing being not allowed given the restrictions envisaged in the Kyoto protocol).

When a cap on emission (CAP) is included and banking is active an additional equation is allowed in the WITCH model, constraining emissions, given the possibility to save, sell and buy permits:

$$C_2(n,t) = CAP(n,t) + NIP(n,t) - SAV(n,t). \tag{54}$$

Saved permits can be banked and used in later periods. In addition, carbon permits revenues/expenses enter the budget constraint:

$$\begin{aligned} C(n,t) &= Y(n,t) - I_C(n,t) - I_{R\&D,EN} - \sum_j I_{R\&D,j}(n,t) \\ &\quad - \sum_j I_j(n,t) - \sum_j O\&M_j(n,t) - p(t)NIP(n,t). \end{aligned} \tag{55}$$

More can be found in Bosetti et al. (2008).

Linear Programming. MARKAL, TIMES and TIAM use linear programming to solve partial equilibrium problems. When different countries are represented within a linear programming framework, the Nash equilibrium solutions are easily obtained through reduced size linear programs. This is demonstrated in Labriet and Loulou (2007) where the damage functions for each country (group of countries) is approximated by a linear function of the cumulative global emissions. The Nash equilibrium in a CB framework is again an extreme example of prisoner's dilemma. When one uses a cost-effectiveness approach and one computes a normalized equilibrium, the equilibrium solution will coincide with a Pareto optimal solution. This is shown below for a game with m players ($I = 1, \ldots, m$):

Nash equilibrium in CB Let $c_j x_j$ and $A_j x_j = b_j$ be the payoff and the local constraints of country $j = 1, \ldots, m$. Let $\sum_{i=1}^m D_i^j x_i$ be the damage cost function for country j (in Labriet and Loulou (2007) this damage cost is proportional to the cumulative GHG emissions). In a Nash equilibrium solution, each player j solves an LP defined as

$$\max_{x_j : A_j x_j = b_j} c_j x_j - \sum_{i=1}^m D_i^j x_i \tag{56}$$

But, because of the decomposable structure of a linear cost function, this is equivalent to

$$\max_{x_j : A_j x_j = b_j} c_j x_j - D_j^j x_i \tag{57}$$

and the part of the damage cost due to the other emitters is a fixed cost that does not enter into the decision making. This explains the extreme "Prisoner's dilemma" behavior of Nash equilibrium in linear energy-environment models like MARKAL or TIAM.

Normalized Nash equilibrium in CE Let $c_j x_j$ and $A_j x_j = b_j$ be the payoff and the local constraints of country $j = 1, \ldots, m$. Let

$$\sum_{j=1}^m B_j x_j = e$$

be the coupled constraints. The optimality conditions for a normalized equilibrium, with weighting $r = (r_i : i \in I)$, are equivalent to solving the following problems

$$\begin{aligned} \max_{x_i : A_i x_i = b_i} \qquad & c_i x_i + \frac{1}{r_i}\lambda_0^T(\sum_{j=1}^m B_j x_j - e), \quad i \in I & (58) \\ 0 \quad = \quad & \lambda_0^T(\sum_{j=1}^m B_j x_j - e). & (59) \end{aligned}$$

Because of the linearity of the coupled constraint the maximization in (58) can be replaced by $\max_{x_i : A_i x_j = b_i} c_i x_i + \frac{1}{r_i}\lambda_0^T B_i x_i$ and the optimality conditions are equivalent to

$$\begin{aligned} \max_{x_i : A_i x_j = b_i} r_i c_i x_i + \lambda_0^T B_i x_i, \quad i \in I & \qquad (60) \\ 0 \quad = \quad \lambda_0^T(\sum_{j=1}^m B_j x_j - e). & \qquad (61) \end{aligned}$$

But this is exactly the optimality conditions for a Pareto optimal solution obtained with the weighting $r_i > 0 \quad i \in I$. Therefore, the normalized equilibrium solution has the property of being efficient when the game is formulated in a linear programming framework.

The LP framework pushed to the extreme the difference between Nash equilibrium in a CB context and normalized equilibrium in a CE context. The first one is very inefficient and the second one is Pareto optimal.

Nonlinear Complementarity Method. The optimizer PATH solves nonlinear complementarity problems. It is used in GEMINI-E3 to compute a general economic equilibrium.

PATH can be used also to compute normalized equilibrium solutions in games with coupled constraints. Indeed the equilibrium for the auxiliary game defined in Eqs. (12) –(13) is characterized by the following non linear complementarity problem

$$\begin{aligned} 0 \quad = \quad & \bar{e}_j(t,\omega))^T \frac{\partial}{\partial \bar{e}_j(t,\omega)} \left[J_j(\bar{\mathbf{e}}) + \sum_{\omega\in\Omega} \frac{1}{r_j}\lambda^o(\omega)(\bar{E}(\omega) - \sum_{i\in M}\sum_{t=0}^{1} \bar{e}_j(t,\omega)) \right], \\ & i \in M, \quad \omega \in \Omega \qquad (62) \end{aligned}$$

and

$$0 = \frac{1}{r_j}\lambda^o(\omega)(\bar{E}(\omega) - \sum_{j\in M}\sum_{t=0}^{1} \bar{e}_j(t,\omega)), \quad \omega \in \Omega. \qquad (63)$$

with

$$0 \geq \bar{E}(\omega) - \sum_{j \in M} \sum_{t=0}^{1} \bar{e}_j(t, \omega), \quad \omega \in \Omega. \tag{64}$$

This is a problem that PATH solves routinely.

Oracle method for VIs. In Drouet et al. (2007) a set of normalized equilibria has been computed for the problem described by Eqs. (4)–(13) when the payoff to each country is obtained from simulations of general economic equilibrium performed with GEMINI-E3. Notice that in this game formulation the players payoffs are nonlinear functions of the strategic cap decisions. Due to trade effects and in particular due to the IET implementation there are also cross effects and a normalized equilibrium will differ from a Pareto solution, but not very much.

An oracle based optimization (OBO) is used in Drouet et al. (2007) to solve the variational inequality which characterizes the normalized equilibrium associated with a given weighting of the players. This approach permits an extension of the method to a class of problems where the payoffs are computed via large-scale simulations obtained from bottom-up or top-down models.

In the last chapter of this report we present the implementation of the homogenous version of the oracle based method. This is a an improvement over the method that was originally used in Drouet et al. (2007). This homogenous version has better convergence properties and will provide an effective tool to solve non-cooperative games where the payoffs are obtained from simulations of TIMES and GEMINI-E3.

9.4 An interpretation in terms of distribution of a global allowance

Consider an m-player concave game à la Rosen with payoff functions

$$\psi_j(x_1, x_2, \ldots, x_m), \quad x_j \in X_j \quad j = 1, \ldots, m, \tag{65}$$

and a coupled constraint. When the coupled constraint is separable among players, i.e. when it takes the form

$$\sum_{j=1}^{m} \varphi_j(x_j) = e, \tag{66}$$

the coupled equilibrium can be interpreted in an interesting way.

Case1: e is scalar Consider e as being a global allowance and call $\varpi_j \geq 0$ the fraction of this allowance given to player j, with $\sum \varpi_j = 1$. Then define the game with payoffs and decoupled constraints

$$\psi_j(x_1, x_2, \dots, x_m), \quad x_j \in X_j \quad \varphi_j(x_j) \leq \varpi_j e \quad j = 1, \dots, m. \tag{67}$$

A Nash equilibrium for this game is characterized, under the usual regularity conditions, by the following conditions

$$\begin{aligned} & \max{}_j \quad \psi_j(x_1, \dots, x_j, \dots, x_m) - \lambda_j \varphi_j(x_j) && (68) \\ \lambda_j \quad & \geq \quad 0 && (69) \\ 0 \quad & = \quad \lambda_j(\varphi_j(x_j) - \varpi_j e). && (70) \end{aligned}$$

Now assume that at the equilibrium solution all the constraints are active and hence all the λ_j are > 0. Since the multipliers are scalars they can be written in the form

$$\lambda_j = \frac{\lambda_0}{r_j}, \tag{71}$$

by taking

$$\lambda_0 = \sum_{j=1}^{m} \lambda_j \tag{72}$$

and defining

$$r_j = \frac{\lambda_0}{\lambda_j}, \quad j = 1, \dots, m. \tag{73}$$

The assumption of active constraints at equilibrium leads to

$$\lambda_0 > 0 \quad \text{and} \quad \sum_{j=1}^{m} \varphi_j(x_j) - e = 0. \tag{74}$$

Therefore the conditions for a normalized coupled equilibrium are met.

Case 2: e is a vector When the constraint is non scalar we cannot check for a normalized equilibrium but we can show that the conditions for a coupled equilibrium still hold. Indeed, assume that $(x_1^*, x_2^*, \dots, x_m^*)$ is an equilibrium for the decentralized game associated with the repartition ϖ, and assume that this strategy vector is not an equilibrium for the game with coupled constraint. This means that, for at least one player j there exists a strategy $x_j \in X_j$ such that

$$\varphi_j(x_j^*) + \sum_{i \neq j}^{m} \varphi_i(x_i^*) \leq e \tag{75}$$

and for which

$$\psi_j(x_1^*,\ldots,x_j,\ldots,x_m^*) > \psi_j(x_1^*,\ldots,x_j^*,\ldots,x_m^*). \tag{76}$$

However, since all the constraints are active in the decentralized equilibrium, the condition (75) becomes

$$\varphi_j(x_j^*) \leq \varpi_j e.$$

Therefore one has now a contradiction with the decentralized equilibrium condition.

Part IV
Experiments with a budget sharing game

We implement a budget sharing game where a set of GEMINI-E3 simulations determine welfare gains for the regions that play the game. One uses the sharing of the total allowance or quotas as a way to introduce a coupled constraint in the game.

The regions are:

Regions	Countries
IC1	USA, Canada, Australia & New-Zealand
IC2	European Union, Switzerland and Japan
NIC	Russia, Rest of Europe, China, Brazil, Mexico Venezelua, Turkey, Middle-East
DCS	Africa, India, Latin America, Asia, South America

We assume that the international negociations decide a radiative forcing constraint in 2100 of a level of 3.5W/m^2. This constraint corresponds to about a 2.5 degrees Celsius temperature change from the preindustrial era. We computed the corresponding greenhouse gases cumulative emissions with the help of climate module of the TIAM model. We then computed the total emission budget over the period 2005–2050 which is around 519 GtC-eq. This total budget will be allocated to the 4 groups of nations, according to some equity rule.

Each region should decide how much quotas that it should have in each period while respecting a global limit on its cumulated emissions over the period 2005–2050 corresponding to its allocation of the global emission budget. As seen in

Table 3: Quotas Allocations per regions (GtC-eq)

Allocation rule	IC1	IC2	DCS	NIC	Total
Emission	129	95	133	162	519
Contract & Convergence in 2050	88	78	151	202	519
50%Pop + 50%Emi	79	72	200	168	519
Contract & Convergence in 2100	67	62	202	188	519
50%Pop + 25% Emi + 25%GDP	57	46	212	204	519

Table 3, we have defined several possible allocations following different rules, such as grandfathering, ability to pay, equity...

Figure 4 shows the global emissions when the regions are playing the game. A first interesting result is that the abatement dynamics do not depend on the initial allocations; we observe a similar curve for each allocation rule. CO_2 prices are not very high and are similar along the path as shown in Figure 5.

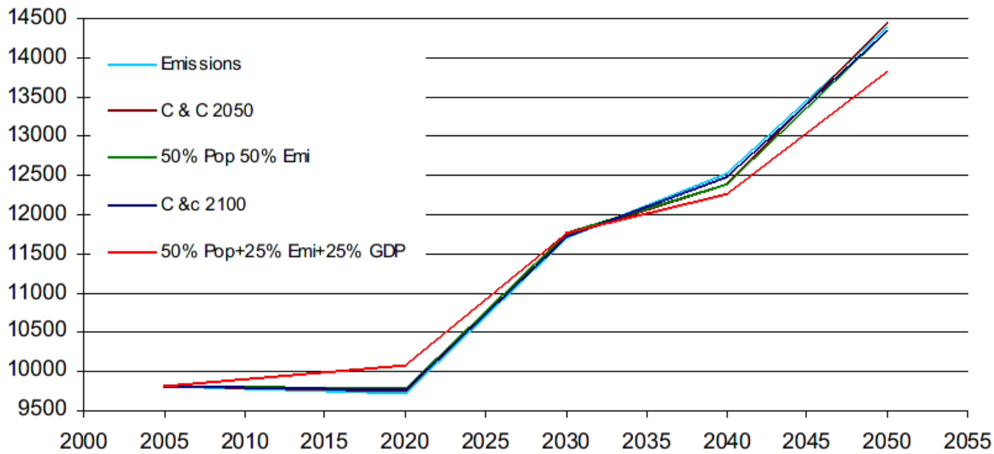

Figure 4: Emissions

We computed an indicator to highlight the temporal allocation of the different regions. If the indicator α_i^t is positive in period t, the region would like to allocate more in this period and conversely if negative.

$$\alpha_i^t = \frac{q_i^t}{\sum_j q_j^t} - 1, \forall i \in \{\mathrm{IC1}, \mathrm{IC2}, \mathrm{NIC}, \mathrm{DCS}\}$$

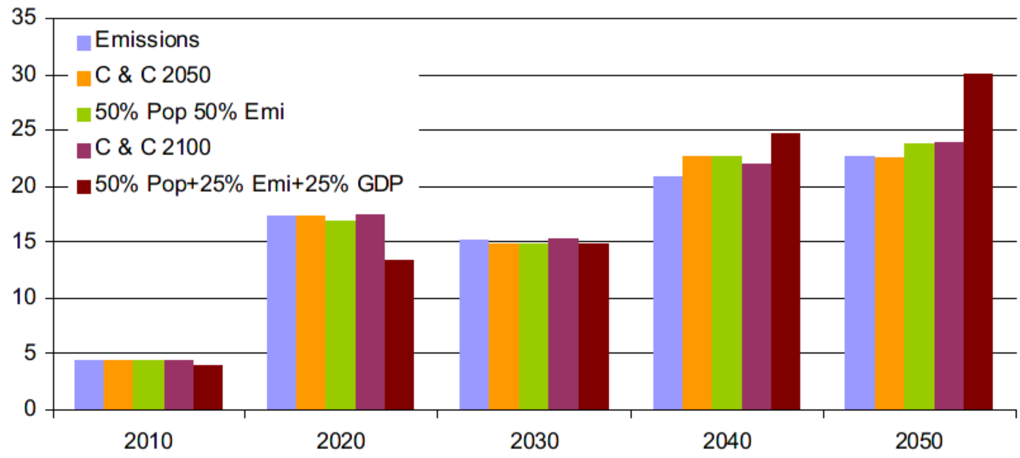

Figure 5: CO_2 prices in USD_{2001}

where q_i^t is the quotas of the region i at period t. Figure 6 shows this indicator for the chosen allocation rules. We can see that IC1 and IC2 want to allocate during the first period while the NIC and DCS regions prefer to allocate in the last period.

Figure 7 shows the total surplus of the regions. IC1 and C2 surpluses have a reduced variation domain between -0.5% and 0.5%. On the contrary NIC and DCS have very changing allocations. NIC have always a negative surplus which comes due the fact that Middle-East and Russia belong to this region and lose revenues anyway thanks to the diminishing overall energy demand.

We try to find an allocation that minimizes the losses of surplus for each region (called “min surplus”). We then obtain a new allocation. The resulting surpluses show a loss of less than 3% of total final consumption for all the regions, which seems acceptable for all.

The following table shows the “minmax loss-surplus” allocation. We can compare this allocation to the ones we have tested: the closest is “contract and convergence 2050”. The main difference is that we give more weight to the NIC region, and we then compensate the revenue losses from the energy exporters.

10 Conclusion

In this project we have identified two classes of dynamic games that could be used to assess the policies of R&D cooperation between EU and China or India in the context of the post 2012 climate negotiations:

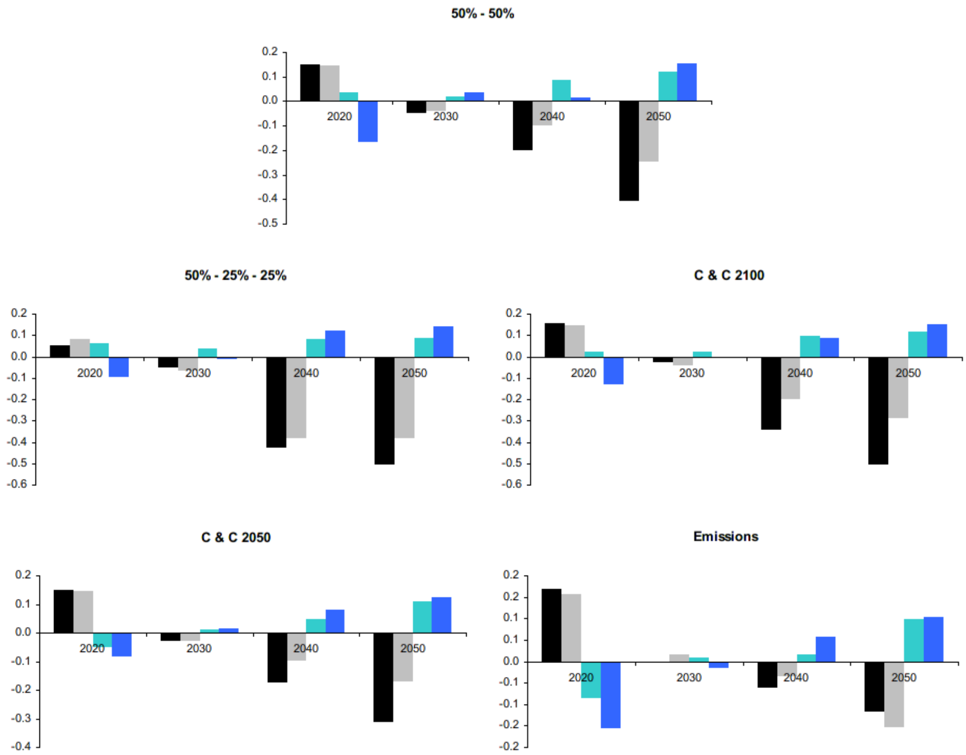

Figure 6: Allocation indicator

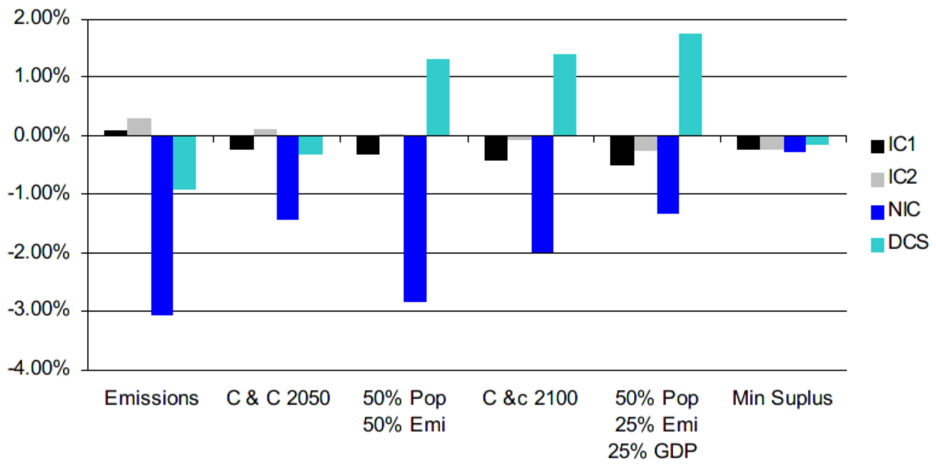

Figure 7: Surplus over consumption (%)

Table 4: New quotas allocations per regions (GtC-eq)

Allocation rule	IC1	IC2	DCS	NIC	Total
Loss surplus	86	49	155	229	519
Contract & Convergence in 2050	88	78	151	202	519

(i) A dynamic cooperative game with side payments permitting the establishment of a subgame consistent agreement that should not be rejected by the parties;

(ii) A two stage game where, in a first phase a global emissions budget is defined for the period 2005-2050 and this budget is split among different groups of countries sharing a common economic interest, while in a second phase, each group of countries decides in a non-cooperative manner, the timing of the use of these quotas on an international emissions trading scheme.

The first model could be implemented, using the cost-benefit version of TIAM. However for the purpose of TOCSIN we have implemented a Rosen normalized equilibrium solutions in a game where the payoffs are obtained from a large scale macro-economic simulation model.

The first simulations that we have realized show that the world GHG abatement schedule is independent of the quotas allocation, and thus we can have a multistage negotiation. The obtained welfare gains are very sensitive for newly industrialized countries (NIC) and developing countries (DCS) due to the loss of the terms of trade for energy exporting countries. Finally, with all these tested alloctions, we are able to find a solution which minimizes the maximum loss of surplus for all regions, thus creating a potentially attractive starting point for a stable post-Kyoto deal.

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