Cooperative Localization for Autonomous Underwater Vehicles

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1 Motivation

The absence of GPS underwater makes navigation for Autonomous Underwater Vehicles (AUVs) a difficult challenge. Without an external reference in the form of acoustic beacons at known positions, the vehicle has to rely on proprioceptive information obtained through a compass, a Doppler Velocity Logger (DVL) or an Inertial Navigation System (INS) [1]. Independent of the quality of the sensors used, the error in the position estimate based on dead-reckoning information grows without bound. Typical navigation errors are 0.5% to 2% of distance traveled for vehicles traveling within a few hundred meters of the sea floor such that their DVL has a lock on the bottom. Errors as low as 0.1% can be obtained with large and expensive INS systems, but for vehicles relying only on a compass and a speed estimate can be as high as 10%. By surfacing the AUV can obtain a position update through its GPS, but this is impossible (under ice) or undesirable for many applications. The use of static beacons in the form of a Long Baseline (LBL) array limits the operation area to a few km^2 and requires a substantial deployment effort before operations, especially in deep water.

As underwater vehicles become more reliable and affordable the simultaneous use of several AUVs recently became a viable option and multi-vehicle deployments will become standard in the upcoming years. This will not only make entirely new types of missions which rely on cooperation possible, but will also allow each individual member of the group to benefit from navigation information obtained from other members. For optimal cooperative localization a few dedicated Communication and Navigation Aid-AUVs (CNAs), which maintain an accurate estimate of their position through sophisticated DVL and INS sensors, can enable a much larger group of vehicles with less sophisticated navigation suites to maintain an accurate position, as described in [2].

2 Problem Statement and Related Work

The subject of cooperative navigation has been addressed for land robots or moving nodes in sensor networks. The assumption of a fast and reliable communication channel between all participants of the cooperative navigation effort, as made in [3] and [4], does not hold underwater. Due to the strong attenuation of electro-magnetic waves underwater, radio or optical communication is not practically feasible except for distances of a few meters. As a result acoustic modems, typically operating between 15 and 30 kHz, provide the only possible means of communicating at long ranges underwater. Data rates are typically several orders of magnitude below those achieved with radio-based communication channels [5]. With sound propagation being dependent on temperature and salinity, which can both vary strongly within the water column, the acoustic communication channel is unreliable and its performance hard to predict. This is especially true in shallow water, where severe multipath is often encountered. The concept of portable landmarks as outlined in [6] is not feasible as it is often difficult for an AUV to hold its position, especially in strong currents.

The objective for our work is to develop and test an algorithm for cooperative positioning of multiple mobile undersea vehicles that can use acoustic modems concurrently for both ranging and for communication [7]. The solution must be robust to the errors and time delays that are inherent to acoustic range measurements and must take into account the severe bandwidth constraints of state-of-the-art undersea acoustic modems. This restriction prevents the transfer of full state information between vehicles.

3 Technical Approach

In order to cooperate during their mission the AUVs will be outfitted with acoustic modems. Data rates on the order of 100 bytes/s over distances of up to 5 km have been achieved, but given varying channel quality, multi-path propagation and possible interference with other acoustic sources, these can drop to as low as 32 byte data packets sent every ten seconds. Furthermore, the small bandwidth of the frequency spectrum which is usable for acoustic communication restricts the use of Frequency-Division-Multiple-Access (FDMA) schemes for multiple channels. The modem which is used throughout these experiments has been developed by the Acoustics Group of the Woods Hole Oceanographic Institution [7]. A special feature of this modem is its ability to embed a time stamp into the data packet and transmit messages which are synced to a pulse-per-second (PPS) signal if such a signal is provided. This signal can be obtained from a GPS receiver and thereby allows all modems to be synced to the same global reference clock. When the AUV is submerged and no GPS is available, the PPS signal is obtained from a precise timer which is synchronized to the GPS clock at the surface. If the transmitting and receiving modem have a PPS signal the receiving modem knows when the message has been sent. This feature is particularly useful for cooperative navigation as each listener overhearing a transmitted data package can now estimate its distance to the transmitting vehicle based on the time of flight (TOF).

While any asset in the water outfitted with an acoustic modem (AUV, ship, Autonomous Surface Craft, fixed mooring) can participate actively (by transmitting navigation information) or passively (by receiving) in cooperative localization we assume for the remaining discussion that an AUV navigates by receiving multiple messages from a CNA. It is important to note that it does not matter if the transmissions are all sent by the same CNA or each time by a different one. The localization algorithm is decentralized and each node incorporates every overheard data packet which contains an estimate of the transmitting vehicle's position (latitude, longitude and depth) as well as uncertainty information. Assuming that most data packets transmitted contain this information, it is not necessary to transmit data packets dedicated to cooperative navigation, which is crucial given the small available bandwidth.

With each successful transmission at time k the AUV receives an estimate of the CNA's position $\boldsymbol{x}^C(k) = [x^C(k), y^C(k)]^T$, the covariance matrix, $\boldsymbol{P}^C(k)$, which accounts for the confidence the CNA has in each component of $\boldsymbol{x}^C(k)$, a depth $z^C(k)$ and a range $r(k)$.

$$\boldsymbol{P}^C(k) = \begin{bmatrix} \sigma^C_{xx}(k) & \sigma^C_{xy}(k) \\ \sigma^C_{yx}(k) & \sigma^C_{yy}(k) \end{bmatrix}$$

$\boldsymbol{x}^C(k)$ and $\boldsymbol{P}^C(k)$ can be a snapshot from the navigation filter running on the CNA or from the GPS in case the CNA is at the surface. The range $r(k)$ is directly obtained by the AUV through the PPS-synced transmission feature with a fixed variance of σ_r^2. As depth can be accurately measured with a pressure sensor, the AUV can use its depth $z^A(k)$ and the depth received from the CNA $z^C(k)$ to project the CNA's position into a 2D plane at $z^A(k)$ and thereby reducing the cooperative localization from a 3D to a 2D problem.

Furthermore, the AUV builds a matrix $\boldsymbol{D}$ where each entry $\boldsymbol{D}(n, m)$ contains the distance traveled $\boldsymbol{d}_{\mathbf{n,m}} = [dx_{n,m}, dy_{n,m}]^T$ between receiving a transmission at $t(n)$ and at $t(m)$ as obtained from proprioceptive measurements as well as the covariance matrix $\boldsymbol{Q}_{\mathbf{n,m}}$ associated with that measurement. Figure 1 shows how the AUV uses information received at $t(n)$ and $t(m)$ to compute two possible solutions for its position at $t(m)$: The circle with radius $r(n)$ defines all possible positions at $t(n)$. Shifting the center of this circle by $[dx_{n,m}, dy_{n,m}]^T$ and solving the resulting quadratic equation, we obtain a set $\boldsymbol{X}^A(m)$ of 0, 1 or 2 intersections with the circle around $\boldsymbol{x}^C(m)$ with radius $r(m)$.

$$\boldsymbol{X}^A(m) = \mathcal{F}(\boldsymbol{x}(n)^C, \boldsymbol{x}(m)^C, r(n), r(m), \boldsymbol{d}_{\mathbf{n,m}}) \tag{1}$$

with

$$\boldsymbol{X}^A(m) = \emptyset \qquad \text{or} \qquad \boldsymbol{X}^A(m) = \boldsymbol{x}_1^A(m) \qquad \text{or} \qquad \boldsymbol{X}^A(m) = \begin{pmatrix} \boldsymbol{x}_1^A(m) \\ \boldsymbol{x}_2^A(m) \end{pmatrix}$$

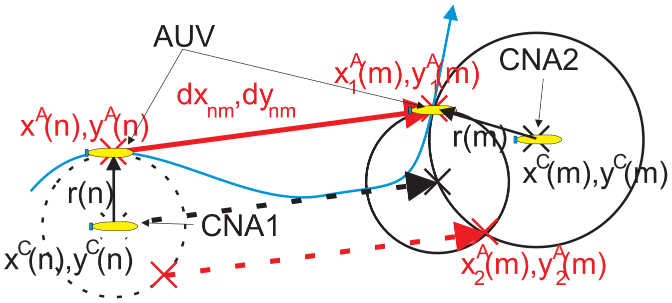

Fig. 1. Computing two possible positions of the AUV using information received at $t(n)$ and $t(m)$

Using other values for n ($n = [1, \ldots, m-1]$), we can compute up to $2(m-1)$ solutions for $\boldsymbol{x}^A(m)$. For the upcoming computations we assume that we use q solutions. The Jacobian of the intersection function $\mathcal{F}$ with respect to the measured and transmitted parameters $\boldsymbol{x}(n)^C$, $\boldsymbol{x}^C(m)$, $r(n)$, $r(m)$, $\boldsymbol{d_{n,m}}$ is $\boldsymbol{J}_{n,m}$ and can be used to compute $\boldsymbol{P}^A(m)$ the covariance of $\boldsymbol{x}^A(m)$. $\boldsymbol{P}^A(m)$ is given by

$$\boldsymbol{P}^A(m) = \begin{bmatrix} \sigma_{xx}^A(m) & \sigma_{xy}^A(m) \\ \sigma_{yx}^A(m) & \sigma_{yy}^A(m) \end{bmatrix} = \boldsymbol{J}_{n,m} \boldsymbol{G}_{n,m} \boldsymbol{J}_{n,m}^T \tag{2}$$

with

$$\boldsymbol{G}_{n,m} = \begin{bmatrix} \sigma_{xx}^C(n) & \sigma_{xy}^C(n) & 0 & 0 & 0 & 0 & 0 & 0 \\ \sigma_{yx}^C(n) & \sigma_{yy}^C(n) & & & & & & \\ 0 & 0 & \sigma_{xx}^C(m) & \sigma_{xy}^C(m) & 0 & 0 & 0 & 0 \\ 0 & 0 & \sigma_{yx}^C(m) & \sigma_{yy}^C(m) & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \sigma_r(n) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \sigma_r(m) & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \sigma_{dx}(n,m) & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sigma_{dy}(n,m) \end{bmatrix}$$

and

$$\boldsymbol{J}_{n,m} = \begin{bmatrix} \frac{\partial x^A(m)}{\partial x^C(n)} & \frac{\partial x^A(m)}{\partial y^C(n)} & \frac{\partial x^A(m)}{\partial x^C(m)} & \frac{\partial x^A(m)}{\partial y^C(m)} & \frac{\partial x^A(m)}{\partial r(n)} & \frac{\partial x^A(m)}{\partial r(m)} & \frac{\partial x^A(m)}{\partial dx_{n,m}} & \frac{\partial x^A(m)}{\partial dy_{n,m}} \\ \frac{\partial y^A(m)}{\partial x^C(n)} & \frac{\partial y^A(m)}{\partial y^C(n)} & \frac{\partial y^A(m)}{\partial x^C(m)} & \frac{\partial y^A(m)}{\partial y^C(m)} & \frac{\partial y^A(m)}{\partial r(n)} & \frac{\partial y^A(m)}{\partial r(m)} & \frac{\partial y^A(m)}{\partial dx_{n,m}} & \frac{\partial y^A(m)}{\partial dy_{n,m}} \end{bmatrix}$$

All possible solutions for $\boldsymbol{x}_v^A(m)$ and their respective covariances $\boldsymbol{P}_v^A(m)$ are combined into a matrix $\boldsymbol{S}_v(m)$.

$$\boldsymbol{S}_v(m) = \begin{bmatrix} x_1^A(m) & y_1^A(m) & \sigma_{xx}^A(m) & \sigma_{xy}^A(m) & \sigma_{yx}^A(m) & \sigma_{yy}^A(m) \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ x_q^A(m) & y_q^A(m) & \sigma_{xx}^A(m) & \sigma_{xy}^A(m) & \sigma_{yx}^A(m) & \sigma_{yy}^A(m) \end{bmatrix}, \ v = [1 \dots q]$$

We also define a position matrix $\boldsymbol{T}_u(m-1)$ which stores all possible past positions $\boldsymbol{x}_u^A(m-1)$ and their respective covariances $\boldsymbol{P}_u^A(m-1)$ of the AUV and an associated accumulated transition cost $c_u(m-1)$ at $t(m-1)$.

$$\boldsymbol{T}_u(m-1) = \begin{bmatrix} x_1^A(m-1) & \dots & \sigma_{yy}^A(m-1) & c_1(m-1) \\ \vdots & \vdots & \vdots & \vdots \\ x_q^A(m-1) & \dots & \sigma_{yy}^A(m-1) & c_q(m-1) \end{bmatrix}, \ u = [1 \dots q]$$

The cost function $\mathcal{C}_{m-1,m}$ which computes the cost (inverse of likelihood) of the AUV having traveled from $\boldsymbol{x}_u^A(m-1)$ to $\boldsymbol{x}_v^A(m)$ given $\boldsymbol{x}_u^A(m-1)$, $\boldsymbol{P}_u^A(m-1)$, $\boldsymbol{x}_v^A(m)$, $\boldsymbol{P}_v^A(m)$, $\boldsymbol{d}_{\mathbf{m-1,m}}$, $\boldsymbol{Q}_{\mathbf{m-1,m}}$ is given by (time indices m,$m-1$ omitted)

$$\begin{aligned} \mathcal{C}_{m-1,m}(u,v) = & \left((\boldsymbol{P}_u^A + \boldsymbol{Q}_{\mathbf{m-1,m}})^{-1} + (\boldsymbol{P}_v^A)^{-1}\right)^{-1} \cdot \\ & \left((\boldsymbol{P}_u^A + \boldsymbol{Q}_{\mathbf{m-1,m}})^{-1}(\boldsymbol{x}_u^A + \boldsymbol{d}_{\mathbf{m-1,m}}) + (\boldsymbol{P}_v^A)^{-1}\boldsymbol{x}_v^A\right) \end{aligned} \tag{3}$$

Using 3 we now compute the cost $c_{u,v}(m-1,m)$ for all q^2 possible transitions from $\boldsymbol{T}_u(m-1)$ to $\boldsymbol{S}_v(m)$.

$$c_{u,v}(m-1,m) = \mathcal{C}_{m-1,m}(u,v) + c_u(m-1) \qquad \forall\, u = [1 \dots q], v = [1 \dots q] \tag{4}$$

We then form a new position matrix $\boldsymbol{T}_v(m)$

$$\boldsymbol{T}_v(m) = \begin{bmatrix} x_1^A(m) & y_1^A(m) & \sigma_{xx}^A(m) & \sigma_{xy}^A(m) & \sigma_{yx}^A(m) & \sigma_{yy}^A(m) & c_1(m) \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ x_q^A(m) & y_q^A(m) & \sigma_{xx}^A(m) & \sigma_{xy}^A(m) & \sigma_{yx}^A(m) & \sigma_{yy}^A(m) & c_q(m) \end{bmatrix}, \ v = [1 \dots q]$$

where $c_v(m)$ is the smallest accumulated cost with the transition to solution $x_v^A(m)$ of all possible transitions from $x_u^A(m-1)$ to $x_v^A(m)$.

$$c_v(m) = \min_{\forall u}(c_{m-1,m}(u,v)) \qquad v = [1 \dots q] \tag{5}$$

The likeliest position $\boldsymbol{x}_w^A(m)$, i.e. our computed solution for $t(m)$ is

```
Initialize position matrix $\boldsymbol{T}(0) = [\boldsymbol{x}^A(0) \;\; c(0) = 0]$
loop {compute position}
    $m++$
    Wait for new range/position pair $\boldsymbol{x}^C(m)$,$z^C(m)$,$\boldsymbol{P}^C(m)$,$r(m)$ from CNA
    Use $z^C(m)$ to project $\boldsymbol{x}^C(m)$ to a plane at the AUV's depth $z^A(m)$
    for $j = 1$ to $q$ do
        Calculate solution and its covariance:
        $n = m - j$
        $\boldsymbol{x}_j^A(m) \leftarrow (1)|_{\boldsymbol{x}(n)^C,\boldsymbol{x}(m)^C,r(n),r(m),\boldsymbol{d}_{\mathbf{n},\mathbf{m}}}$
        $\boldsymbol{P}_j^A(m) = \boldsymbol{J}_{n,m}\boldsymbol{G}_{n,m}\boldsymbol{J}_{n,m}^T$
        Add solution $\boldsymbol{x}_j^A(m)$ and its covariance $\boldsymbol{P}_j^A(m)$ to solution matrix:
        $\boldsymbol{S}(m) \leftarrow \boldsymbol{x}_j^A(m), \boldsymbol{P}_j^A(m)$
    end for
    Compute transition cost from all possible positions at $\boldsymbol{T}(m-1)$ to all solutions at $\boldsymbol{S}(m)$ and for each element add the cost which accumulated up to $m - 1$:
    $c_{u,v}(m-1, m) \leftarrow c_u(m-1)+(3)|_{\boldsymbol{x}_u^A(m-1),\boldsymbol{P}_u^A(m-1),\boldsymbol{x}_v^A(m),\boldsymbol{P}_v^A(m),\boldsymbol{d}_{\mathbf{m-1,m}},\boldsymbol{Q}_{\mathbf{m-1,m}}}$
    Move all solutions from $\boldsymbol{S}(m)$ and the accumulated transition cost into $\boldsymbol{T}(m)$
    $\boldsymbol{T}(m) \overset{c_v(m)=\min_{\forall u}(c_{u,v}(m-1,m))}{\longleftarrow} [\boldsymbol{x}_v^A(m) \;\; \boldsymbol{P}_v^A(m) \;\; c_v(m)]$
    Retrieve
end loop
```

Algorithm 1: Summary of cooperative navigation algorithm.

$$\boldsymbol{x}_w^A(m) \text{ with } w \text{ such that } c_w(m) = \min_{\forall v}(c_v(m)) \qquad v = [1 \ldots q] \tag{6}$$

Figure 2 shows a snapshot at $t(m)$ during a cooperative navigation experiment with a depth of two (i.e. using two past measurements). The AUV (blue) has just received a position/range-pair from the red CNA (full red circle). This circle intersects with the position/range-pair received at $t(m-1)$ (dashed green circle) and forward propagated by the dead-reckoned distance $\boldsymbol{d}_{\mathbf{m-1,m}}$ and the position/range-pair received at $t(m-2)$ (dashed red circle) forward propagated by the dead-reckoned distance $\boldsymbol{d}_{\mathbf{m-2,m}}$. All intersections and therefore possible solutions at $t(m)$ are marked by a small black "x". The likeliest solution, taking past computed positions (not shown) into account, is given by equation 1 and is marked by the large black "X".

The complexity to compute a single position is $O(q^2)$ where q is the number of past measurements taken into account. The maximum frequency at which this computation step is invoked is $f = 0.1\ Hz$, as each packet is 10 seconds long. For $q \approx 10$ the time to compute a new position is $t = 0.01\ s$ on a 1 GHz PC. This makes this algorithm well suited to run on the Main Vehicle Computer (MVC) of today's AUVs.

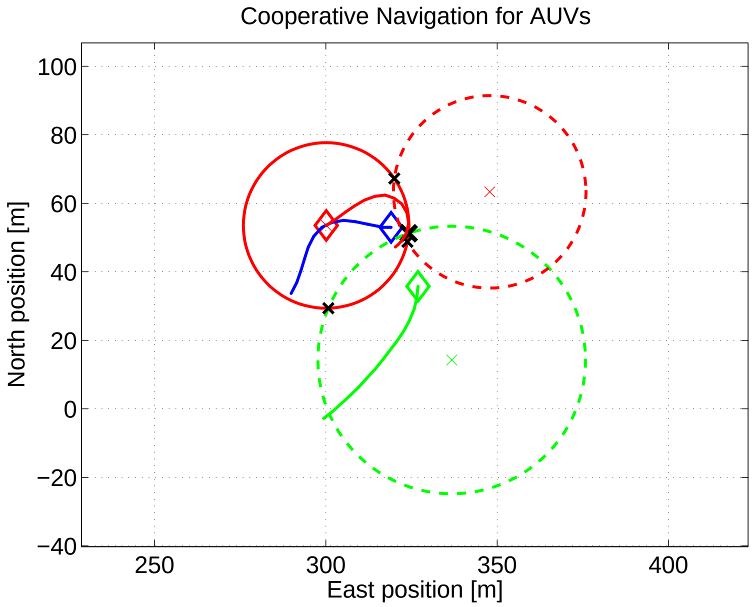

Fig. 2. All possible solutions ("x") and the likeliest ("X") at $t(m)$

4 Experiments

To obtain data using real acoustic range data in a realistic environment, we performed an experiment using several low-cost Autonomous Surface Crafts (ASCs) as a replacement for AUVs. The ASC is shown in Figure 3 and described in [8]. It is a kayak hull outfitted with a thruster, a mini-ATX PC, GPS and the same acoustic modem which is also used on our AUVs. The vehicle dynamics of the ASC are comparable to those of an AUV. By using only the acoustic modem to exchange information and estimate ranges between the two vehicles, we have applied the same restrictions which are encountered in an AUV-only scenario while at the same time being able to compare the algorithm's navigation performance against the "true" GPS position.

For this experiment three ASCs were set up to run in formation along a trackline while broadcasting their position information over the acoustic modem. Each ASC in the formation was able to participate actively, by sending information, and passively by computing its position estimate based on the information obtained from the other two, but the results are only shown for one ASC of the formation. In this case two kayaks act as the "CNAs" while the other kayak acts as the "AUV" . In the setup shown in Figure 3 the center kayak ran a preprogrammed mission using its GPS for navigation. The other two kayaks followed in a predetermined formation in order to stay within range of the acoustic modems. The range and position obtained from the two CNAs over the acoustic modem were logged by the AUV-kayak.

Fig. 3. 3 kayaks navigating cooperatively

5 Results

Post-processing the data logged on the AUV we computed the position estimate whenever a broadcast from any of the two CNAs was successfully received. Figure 4 shows the GPS track of the AUV (red) and the computed positions (black). The tracks of the CNAs are not shown.

6 Future Work

As a next step we will replace the center kayak by a real AUV. The dock-side testing for this experiment is underway. Picture 5 shows the preparation for the experiment. The algorithm is anticipated to be used in a series of experiments for the PlusNet program, which will incorporate a variety of AUVs, ASCs, gliders and buoys outfitted with acoustic modems. These experiments will provide an opportunity to test our algorithm in an ocean-deployed network of various underwater vehicles.

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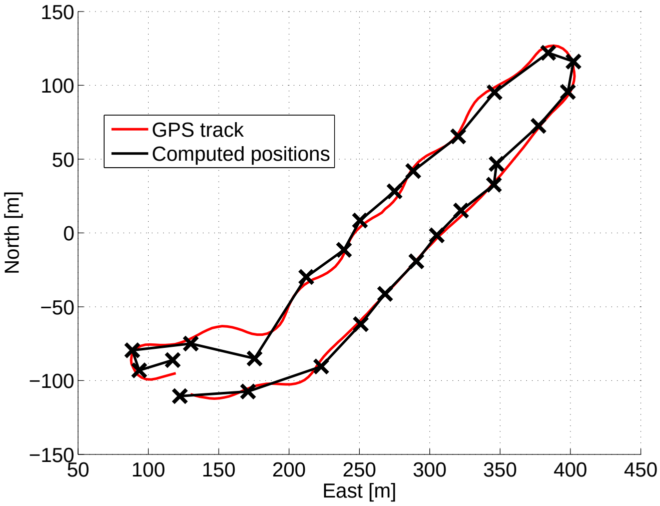

Fig. 4. GPS tracks of AUV (red) and computed track of AUV (black)

Fig. 5. 2 kayaks and a Bluefin 12” getting ready for a cooperative localization experiment

the data processed in this paper was obtained. This work was supported in part by ONR grants N00014-02-C-0210, N00014-97-1-0202 and N00014-05-G-0106, and by the MIT Sea Grant College Program under grant NA86RG0074 (project RD-24).

References

1. L. Whitcomb, D. Yoerger, H. Singh, and J. Howland, "Advances in Underwater Robot Vehicles for Deep Ocean Exploration: Navigation, Control and Survery Operations," in *The Ninth International Symposium on Robotics Research*, Springer-Verlag, London, 2000.
2. J. Vaganay, J. Leonard, J. Curcio, and S. Willcox, "AOFNC - experimental validation of the moving long base line navigation concept," in *AUV 2004*, Nagoya, Japan, June 2004, pp. 555–556.
3. S. Roumeliotis and G. Bekey, "Synergetic localization for groups of mobile robots," in *Proc. 39th IEEE Conference on Decision and Control*, Sydney, Australia, Dec. 2000, pp. 3477–3482.
4. D. Moore, J. Leonard, D. Rus, and S. Teller, "Robust distributed network localization with noisy range measurements," in *SenSys '04: Proceedings of the 2nd international conference on Embedded networked sensor systems.* New York, NY, USA: ACM Press, 2004, pp. 50–61.
5. D. B. Kilfoyle and A. B. Baggeroer, "The current state-of-the-art in underwater acoustic telemetry," *IEEE J. Ocean Engineering*, vol. 25, no. 1, pp. 4–27, 2000.
6. R. Kurazume, S. Nagata, and S. Hirose, "Cooperative positioning with multiple robots," in *Proc. IEEE International Conference in Robotics and Automation*, Los Alamitos, CA, USA, May 1994, pp. 1250–1257.
7. L. Freitag, M. Johnson, M. Grund, S. Singha, and J. Preisig, "Integrated acoustic communication and navigation for multiple UUVs," in *Proc. MTS/IEEE Oceans 2001*, Honolulu, HI, USA, Sept. 2001, pp. 290–294.
8. J. Curcio, J. Leonard, J. Vaganay, A. Patrikalakis, A. Bahr, D. Battle, H. Schmidt, and M. Grund, "Experiments in moving baseline navigation using autonomous surface craft," in *Proc. MTS/IEEE Oceans 2005*, Washington DC, USA, Sept. 2005, pp. 290–294.