MODELING OF MULTI-WINDING PHASE SHIFTING TRANSFORMERS
APPLICATIONS TO DC AND MULTI-LEVEL VSI SUPPLIES

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ABSTRACT

This paper deals with the modeling of multi-winding transformers. Based on the real example of a railway DC supply, a modeling methodology is presented as the original part of the paper. Afterwards two application examples of multi-winding phase shifting transformers (18 and 24 pulse) are described, these applications have been simulated using the SIMSEN simulation software package [1,2]. Harmonics analysis of simulation results (elimination of low order harmonics 5, 7, 11, 13) has validated the proposed methodology.

Keywords: Multi-winding transformer, phase shifting, harmonics reduction, DC supply, simulation, harmonics analysis.

1 INTRODUCTION

The DC power supplies are made of rectifiers [3]. Those are providing harmonics leading to a great inconvenience for the power networks. Even if additional L-C filters are able to absorb the most important part of the harmonics content, they may introduce resonance problems. A more suitable solution is to increase the pulse number of the rectifiers. This requires phase shifting transformers having multi-w windings [4]. The most complex part to modelize such special transformers is the determination of all the needed parameters: inductance and resistance values for each winding, coupling coefficients between all the considered windings. These parameters must be determined using the measurements obtained from the typical tests under no-load and short circuit conditions.

Vector group T1 = Dd11.75d0.75

Figure 1 - Multi-winding phase shifting transformer example
2 METHODOLOGY

2.1 Systematic names definition

The developed methodology is explained using the multi-winding phase shifting transformer example given in the left part of Fig. 1. The initial stage is to define the different names and indexes for inductances and to use for the development, as shown in the right part of Fig. 1:

L

X

I

Y/D

Inductance

phase x (a, b, c)

winding I (1, 2, 3…)

star/delta connection

Example: LB2D inductance, phase b, winding 2, delta-connected.

The different coupling coefficients also have to be defined carefully:

k

i

j

y/d

y/d

Coefficient

winding i (1, 2, 3…)

star/delta connection

winding j (1, 2, 3…)

star/delta connection

Example: kl1d2y coupling coefficient between delta-connected winding 1 and star-connected winding 2.

2.2 Necessary input data

The necessary input data correspond to the data plate of the transformer as well as some additional measurements coming from the typical tests under no-load and short circuit conditions. These input data are:

<table>
<thead>
<tr>
<th>SN1</th>
<th>3146 kVA</th>
<th>apparent power.</th>
</tr>
</thead>
<tbody>
<tr>
<td>UN1</td>
<td>22 kV</td>
<td>primary line voltage.</td>
</tr>
<tr>
<td>FN</td>
<td>60 Hz</td>
<td>frequency.</td>
</tr>
<tr>
<td>UN20</td>
<td>569 V</td>
<td>secondary 1 line voltage (no-load)</td>
</tr>
<tr>
<td>UN30</td>
<td>569 V</td>
<td>secondary 2 line voltage (no-load)</td>
</tr>
<tr>
<td>i0</td>
<td>0.00569 P.U.</td>
<td>no-load primary side current.</td>
</tr>
<tr>
<td>xcc12</td>
<td>0.0878 P.U.</td>
<td>short circuit reactance 1-2, 3 open.</td>
</tr>
<tr>
<td>xcc13</td>
<td>0.0878 P.U.</td>
<td>short circuit reactance 1-3, 2 open.</td>
</tr>
<tr>
<td>xcc23</td>
<td>0.0112 P.U.</td>
<td>short circuit reactance 2-3, 1 open</td>
</tr>
<tr>
<td>R1</td>
<td>0.611 Ohm</td>
<td>primary terminal resistance.</td>
</tr>
<tr>
<td>R2</td>
<td>0.00088 Ohm</td>
<td>secondary 1 terminal resist.</td>
</tr>
<tr>
<td>R3</td>
<td>0.00104 Ohm</td>
<td>secondary 2 terminal resist.</td>
</tr>
</tbody>
</table>

2.3 Stray reactances in per unit

In a first stage, the stray reactances (xs1, xs2 and xs3) of each terminal are calculated in [p.u.]:

\[
\begin{align*}
xs1 &= \frac{xcc12 + xcc13 - xcc23}{2} = 0.0822 \text{ p.u.} \\
xs2 &= \frac{xcc12 + xcc23 - xcc13}{2} = 0.0056 \text{ p.u.} \\
xs3 &= \frac{xcc13 + xcc23 - xcc12}{2} = 0.0056 \text{ p.u.}
\end{align*}
\]

2.4 Primary winding inductances

For the primary winding, the rated impedance ZN1Y is first calculated according to equation (4):

\[
ZN1Y = \frac{UN1^2}{SN1} = 153.8 \text{ Ohms}
\]

Using equations (1) and (4), the equivalent star-connected stray reactance XS1 is:

\[
XS1 = xs1 \cdot ZN1Y = 12.64 \text{ Ohms}
\]

The star-connected equivalent no-load impedance ZIY is calculated taking into account the no-load current i0:

\[
ZIY = \frac{1}{i0} \cdot ZN1Y = 27.04 \text{ kOhms}
\]

Using the angular frequency, the equivalent star-connected inductance LIY is calculated:

\[
LIY = \frac{ZIY}{2 \cdot \pi \cdot FN} = 71.72 \text{ H}
\]

To obtain the delta-connected inductance L1D, the previous value has to be multiplied by 3:

\[
L1D = 3 \cdot LIY = 215.2 \text{ H}
\]

Doing the same operation for the delta-connected stray inductance LS1D, we obtain:

\[
LS1D = \frac{3 \cdot XS1}{2 \cdot \pi \cdot FN} = 0.1006 \text{ H}
\]

Finally, the main delta-connected inductance LH1D is:

\[
LH1D = L1D - LS1D = 215.1 \text{ H}
\]

2.5 Secondary 1 winding inductances

The mathematical development is based on the elements represented in Fig. 2.

For the secondary 1 winding (index 2), the desired phase shifting angle is equal to \(-7.5^\circ\). This means that this secondary winding has to be a combination of Dy11 (-30°) and Dd0 (0°). The first step is to define the ratios n2y/n1d
and \( \frac{n2y}{n1d} \) between the turns numbers of the windings 2Y, 2D and 1D according to the equations (11) and (12):

\[
\frac{n2y}{n1d} = a \tag{11}
\]
\[
\frac{n2d}{n1d} = b \tag{12}
\]

The phase angle between the line voltages \( U_{1U-1V} \) and \( U_{2U-2V} \) must be 7.5°. The line voltage \( U_{2U-2V} \) is the sum of the three partial voltages: \( U_{2U-2V}^2 \), \( U_{2U-2V}^3 \), and \( U_{2V-2V} \), as represented in Fig. 2.

\[
U_{2U-2V} = aE_{U-1V}^2 + bE_{U-1V}^1 - aE_{1V-1W}^2 \tag{13}
\]

**Figure 2 – Vector diagram for the secondary 1 winding**

Under the assumption that primary and secondary line voltages are equal (\( a \) and \( b \) become \( a' \) and \( b' \) in equations 11 and 12) and by taking into account the angles at points 2V’ and 2V equal to 60° and 60° − 7.5° = 52.5°, the projections of equation (13) on both horizontal and vertical axis lead to:

\[
\cos 60° \cdot a' + \cos 60° \cdot b' = \cos 52.5° \tag{14}
\]
\[
\sin 60° \cdot a' + \sin 60° \cdot b' = \sin 52.5° \tag{15}
\]

and finally:

\[
a = \frac{n2y}{n1d} = \frac{UN20}{UN1} = 0.00389819 \tag{15}
\]
\[
b = \frac{n2d}{n1d} = \frac{UN20}{UN1} = 0.019795 \tag{16}
\]

The delta-connected inductance L2D is:

\[
L2D = \left( \frac{n2d}{n1d} \right)^2 \cdot L1D = 84.31 \text{ mH} \tag{17}
\]

Doing the same for the star-connected inductance L2Y, we obtain:

\[
L2Y = \left( \frac{n2y}{n2d} \right)^2 \cdot L2D = 3.269 \text{ mH} \tag{18}
\]

The equivalent star-connected stray reactance is calculated using equations (2):

\[
X_{S2} = \frac{X_{S2}}{2 \cdot \pi \cdot FN} = 0.5763 \text{ mH} \tag{19}
\]

The corresponding inductance is:

\[
L_{S2} = \frac{L2Y}{L2Y + \frac{1}{3} \cdot L2D} = 0.159 \text{ mH} \tag{20}
\]

Now, the global stray inductance LS2 has to be distributed on both star/delta-connected part of the secondary 1 winding. Therefore, a total equivalent star-connected inductance L2YTOT has to be defined:

\[
L_{2YTOT} = L2Y + \frac{1}{3} \cdot L2D = 31.37 \text{ mH} \tag{21}
\]

Then, the star-connected stray inductance is:

\[
L_{S2} = \frac{L2Y}{L2YTOT} \cdot L_{S2} = 0.159 \text{ mH} \tag{22}
\]

The remaining delta-connected stray inductance LS2D is:

\[
L_{S2D} = \frac{1}{3} \cdot (L_{S2} - L_{S2Y}) = 4.108 \text{ mH} \tag{23}
\]

Finally, the two main inductances of the secondary 1 winding are deduced:

\[
L_{H2D} = L2D - L_{S2D} = 84.31 \text{ mH} \tag{24}
\]
\[
L_{H2Y} = L2Y - L_{S2Y} = 3.269 \text{ mH} \tag{25}
\]

For all the secondary 1 winding, the total, main and stray inductances have been defined for both star/delta-connected windings. The same operations can be applied to the secondary 2 winding.

**2.6 Coupling coefficients**

The last step is to define the coupling coefficients between all the windings. The coefficients are calculated with the assumption that the 3 magnetic columns of the transformer are symmetrical. This leads to a magnetic coupling coefficient \( k_{1d}2D \) equal to 0.5 between the columns. Two examples of coupling coefficients are calculated in equations (26,27):

\[
k_{1d}2D = \frac{L1H1D \cdot L1H2D}{L1H1D \cdot L1H2D} = 0.9997 \tag{26}
\]
\[
k_{1d}2y = \frac{L1H1D \cdot L1H2Y}{L1H1D \cdot L1H2Y} = 0.9997 \tag{27}
\]
2.7 Winding resistances

Finally, the resistance of each winding is calculated according to the measured terminal resistance and the related connections. For the primary winding, this is easy, because the terminal resistance is equal to the winding resistance (delta-connection).

\[ R_{1D} = R_1 = 0.611 \text{ Ohm} \]  \hspace{1cm} (28)

With the assumption that the resistance is proportional to the square root of the inductance, we can write:

\[ R_{2Y} = \frac{R_2}{2 + \sqrt{L_2/D}} = 0.1243 \text{ mOhm} \]  \hspace{1cm} (29)

\[ R_{2D} = \frac{L_2/D}{2L_2Y} \cdot R_{2Y} = 0.6313 \text{ mOhm} \]  \hspace{1cm} (30)

3 MODELING AND IMPLEMENTATION IN SIMSEN

The SIMSEN simulation software has been developed to simulate electrical power and adjustable speed drive systems having an arbitrary topology [1,2]. A given system topology (i.e. fig. 3 and 6) is built by choosing and linking the different necessary elements or modules from a module’s list. The actual module’s list of SIMSEN offers about one hundred modules, one of them is the so-called “linked inductor”, this module is suitable for modeling multi-winding transformers.

Table 1. shows the input data for one linked inductor. In the section - LINKED INDUCTORS - all the coupled linked inductors as well as their related coupling coefficients are mentioned.

![Diagram](image)

Figure 3 - 24-pulse DC supply for railway substation.

- GENERAL DATA :
  - Name = TILA1D
  - Comment = 
  - Writing = SI

- LINKED INDUCTORS :
  - TILA1D
    - TILB1D - kld1d*km
    - TILC1D - kld1d*km
    - TILA2Y kld2y
    - TILB2Y - kld2y*km
    - TILC2Y - kld2y*km
  - TILA2D kld2d
    - TILB2D - kld2d*km
    - TILC2D - kld2d*km
  - TILA3Y - kld3y*km
    - TILB3Y kld3y
    - TILC3Y - kld3y*km
  - TILA3D - kld3d*km
    - TILB3D kld3d
    - TILC3D - kld3d*km

- RATED VALUES:
  - Sn [VA] = 0.000000000000000000E+0000
  - Un [V] = 0.000000000000000000E+0000
  - Fn [Hz] = 0.000000000000000000E+0000

- PARAMETERS :
  - R [Ohm] = 6.11000000000000000E-0001
  - L [H] = 2.15161475046E+0002

- INITIAL CONDITIONS:
  - I [A] = 1.37001402709E+0001

- CALCULATED VALUES:
  - Lefh [H] = 2.15161475046E+0002

Table 1. Input data for linked inductor TILA1D

The SIMSEN simulation software automatically takes into account all the differential equations of a multi-winding transformer defined with linked inductors. The fact that a linked inductor can be coupled with an infinite number of other linked inductors is one of the SIMSEN’s features, it makes possible combining any kind of magnetic coupled circuits.

For further information: [http://simsen.epfl.ch](http://simsen.epfl.ch)
4 APPLICATION EXAMPLES

4.1 24-pulse transformer for a railway DC supply

The first application example deals with the 24-pulse DC supply of a railway substation. The corresponding circuit is represented in Fig. 3.

The DC supply is made of two 12-pulse transformers parallel connected. Each of them is modeled by using 15 linked inductors. The first transformer has the connection Dd11.75d0.75 and the second one has the connection Dd0.25d11.25. The parallel connection allows supplying the 4 diode rectifiers with shifted angles -22.5°, -7.5°, +7.5° and +22.5°. This leads to a 24-pulse rectifier system. Note that the rectifiers are parallel connected. To equalize the load of both 12-pulse transformers, additional chokes are connected in the DC-link. The next figures present simulation results in steady state at 100% load with symmetrical transformers, as calculated in point 2.

As expected, the first dominant AC supply phase current harmonics are the 23rd and the 25th. The harmonics analysis results confirm the proposed 24-pulse modeling.

Figure 4 - AC supply phase current.

Figure 5 - Spectrum of AC supply phase current.

4.2 Multi-level Voltage Source Inverter 18-pulse DC supply

The studied circuit (see Fig. 6) corresponds to a multi-level Voltage Source Inverter (VSI) feeding an AC motor. This topology has been presented in [5].

The VSI is made of several DC-cells, each supplied through a 6-pulse diode rectifier. Regarding the present paper, the most interesting part of the above topology is the 18-pulse multi-winding transformer. The VSI requires many floating voltage potentials. A special transformer having many output or secondary windings fulfills this specification.

Figure 6 - 18-pulse DC supply for multi-level VSI.

Figure 7 - AC supply phase current.
5 CONCLUSIONS

Multi-winding phase shifting transformers are a suitable solution to supply DC systems with low harmonics content in the AC supply phase current. Based on a real example, a modeling methodology has been developed. This method requires the main data of the transformer, the phase shifting angles as well as the windings connections. Multi-winding phase shifting transformers have been simulated using the SIMSEN simulation software, which proved to be a powerful tool to simulate such complex transformers. Harmonics analysis of simulation results (elimination of low order harmonics 5, 7, 11, 13) has validated the proposed methodology.

REFERENCES