Compressed sensing imaging techniques for aperture synthesis by radio interferometry

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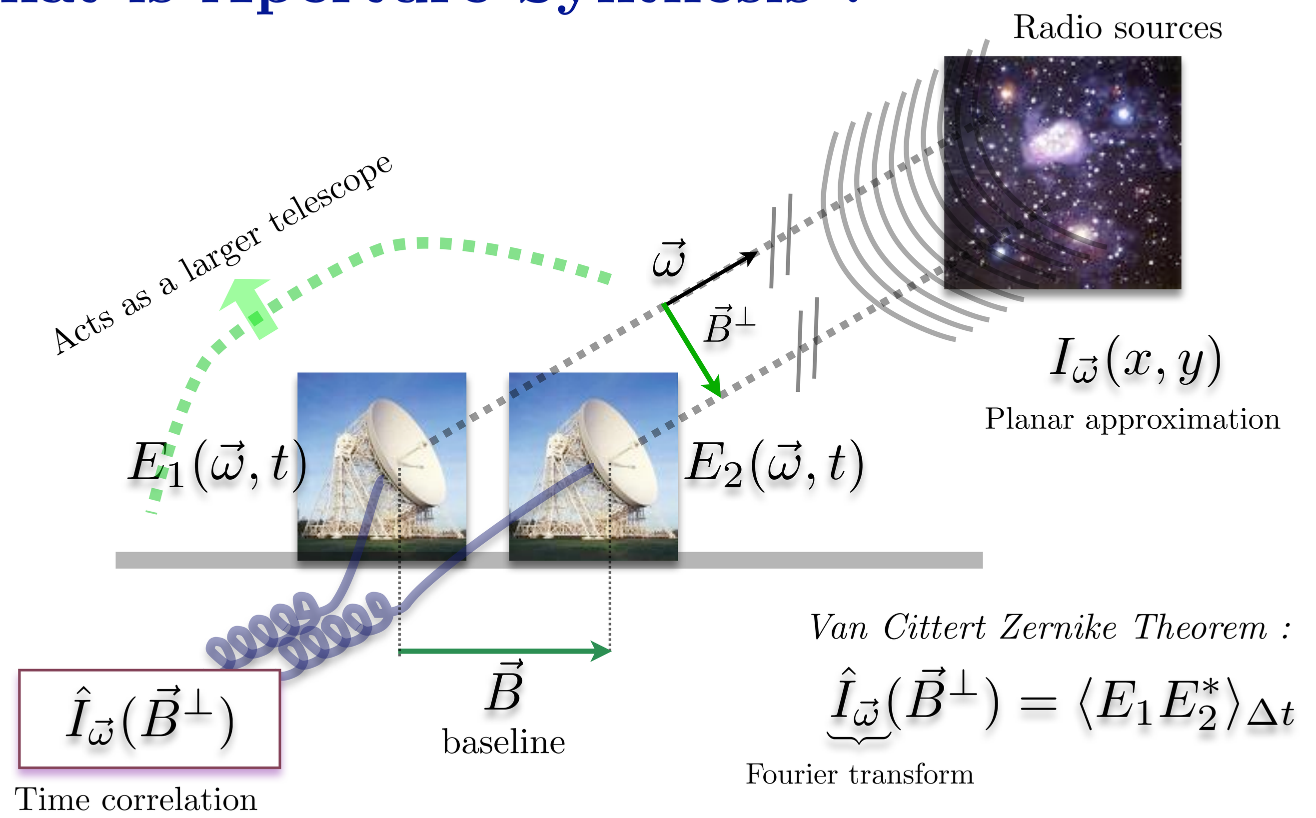
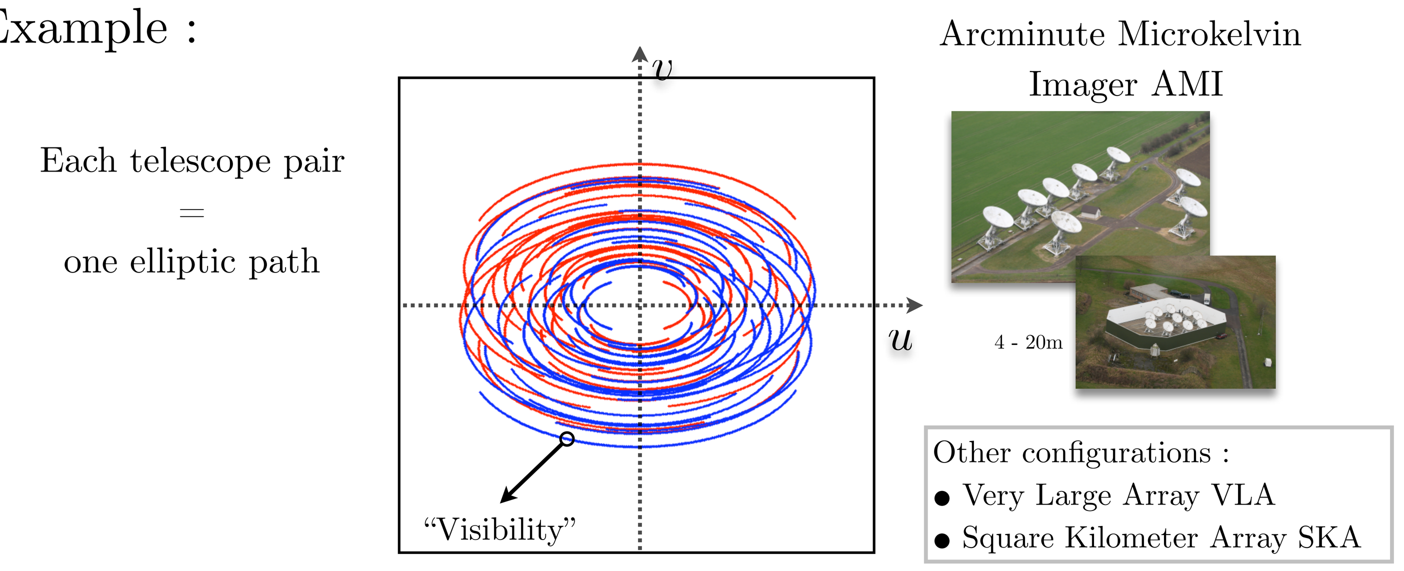
What is Aperture Synthesis ?



- using N telescopes, $\binom{N}{2}$ possible pairing (*visibilities*)
- and baselines undergo Earth rotation !
- Example :



CLEAN Mathematical Model (Högbom, 1974 [1]) :

- *Problem* : In matrix notations, find $I \in \mathbb{R}^{N^2}$ from

$$d = BI, \quad \text{with } B = F^* M F$$

Dirty map; $\mathbb{R}^{N^2 \times N^2}$ Circulant matrix, i.e. convolution; $\mathbb{R}^{N^2 \times N^2}$ Diagonal matrix, i.e. the visibility mask; Fourier basis, i.e. $\hat{u} = Fu$

- Assume I sparse in space, i.e. in the canonical (Dirac) basis.
- **CLEAN** is a (γ damped) **Matching Pursuit** in the **Dictionary** B
- Other methods : Multi-scale CLEAN, MR CLEAN, MEM, ...

BP and BP$^+$ Reconstruction

Compressed Sensing Model : *Fourier Acquisition*

$$y = \Phi I = SFI, \quad \text{with } I = \Psi\alpha \ \text{ sparse in } \Psi$$

m measurements $y \in \mathbb{C}^m$; Sensing matrix; Visibility Selection $M = S^T S$, $S \in \mathbb{R}^{m \times N^2}$

- Context similar to *Magnetic Resonance Imaging MRI*
- We may use *Basis Pursuit* [2] :

$$\alpha_{\text{est}} = \arg\min_u \|u\|_1 \ \text{ s.t. } \ y = \Phi\Psi u \qquad \text{(BP)}$$

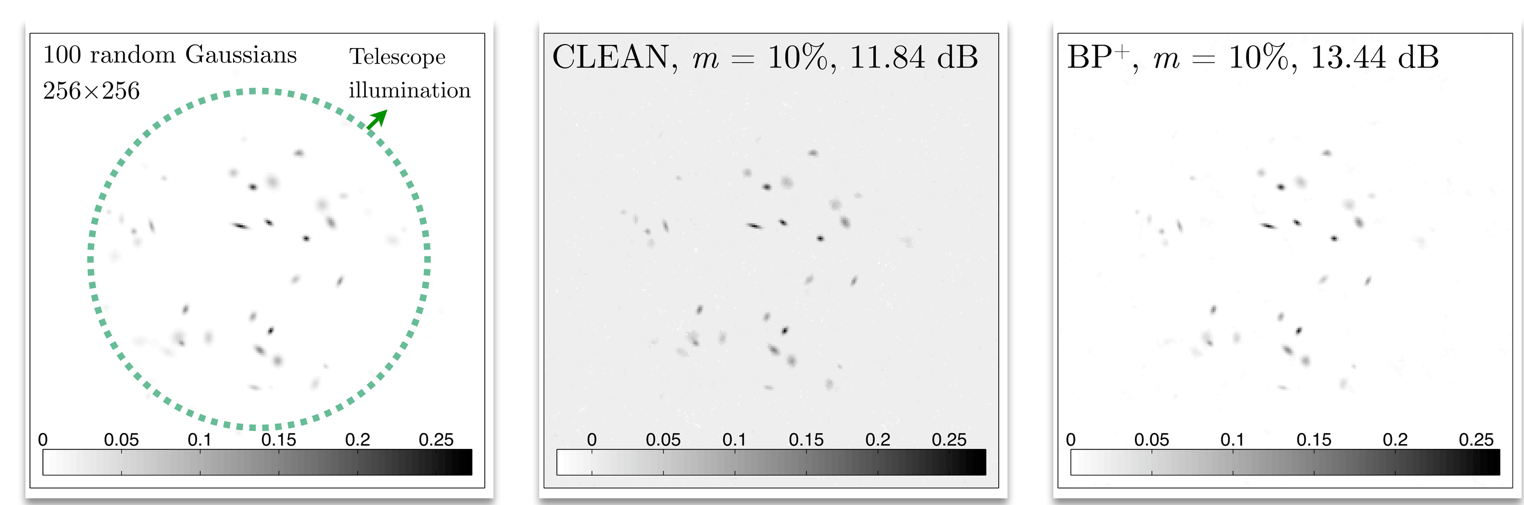
- or, if positive image (*additional prior*)

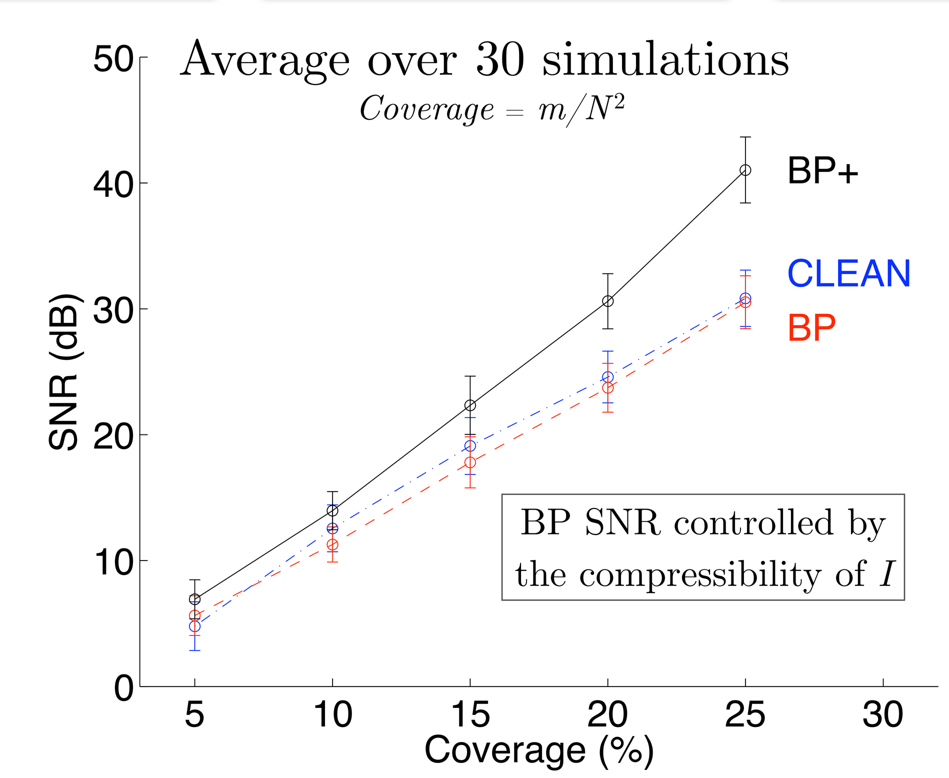
$$\alpha_{\text{est}} = \arg\min_u \|u\|_1 \ \text{ s.t. } \ y = \Phi\Psi u, \ \Psi u \geqslant 0 \qquad (\text{BP}^+)$$

- or, noisy version : $y = \Phi\Psi\alpha + n$, $n_i \sim N(0,\sigma^2)$, $y = \Phi\Psi u \ \rightarrow \ \|y - \Phi\Psi u\|_2 \leqslant \epsilon$
- Solver : Proximal Methods and Douglas-Rachford Splitting [3]

(acknowledgements to M. J. Fadili)

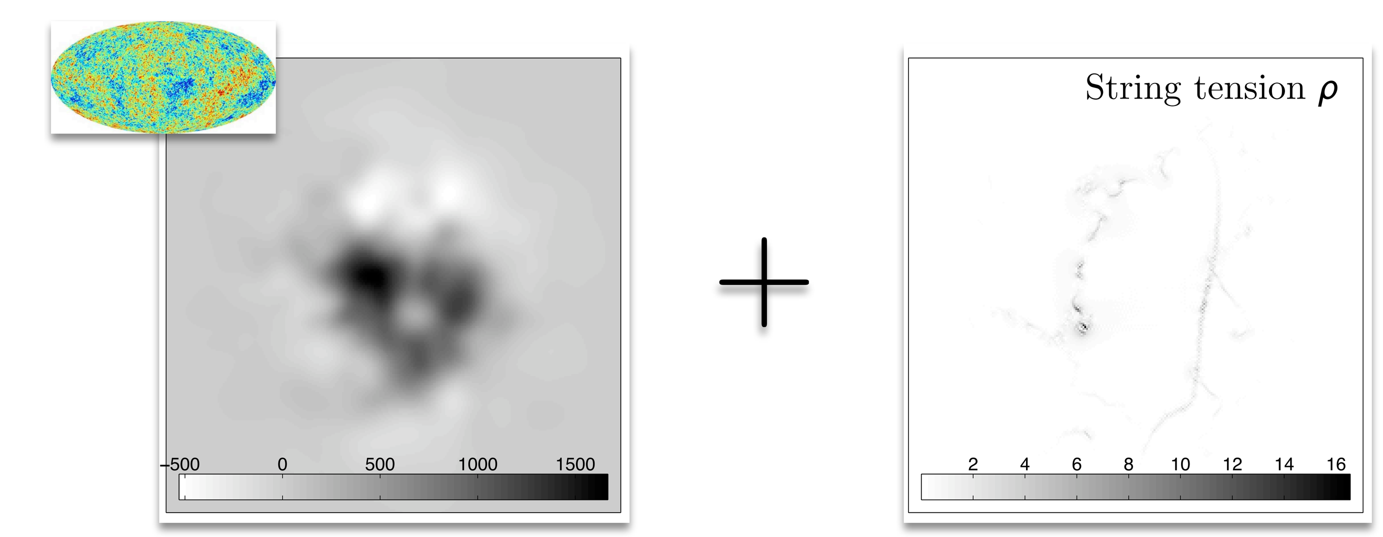
Simulations : random interferometer, Ψ = Dirac, 1.8°×1.8°





Cosmic String Enhancement in AS

- Cosmic Microwave Background (CMB) signal =



- Laboratory to test cosmological models
- Very **low SNR** for string signal (i.e. low string tension) : **-30 dB !**
- String signals not yet observed but simulated [4]
- Prior Information : string signal follows GGD in wavelet space
- GGD *scale* and *shape parameters* deduced in steerable wavelets [5]

$$\pi_j(u_w) \sim \exp |u_w/\rho b_j|^{s_j}, \ \text{ with } w = \{j, \theta_k, \vec{p}_i\}$$

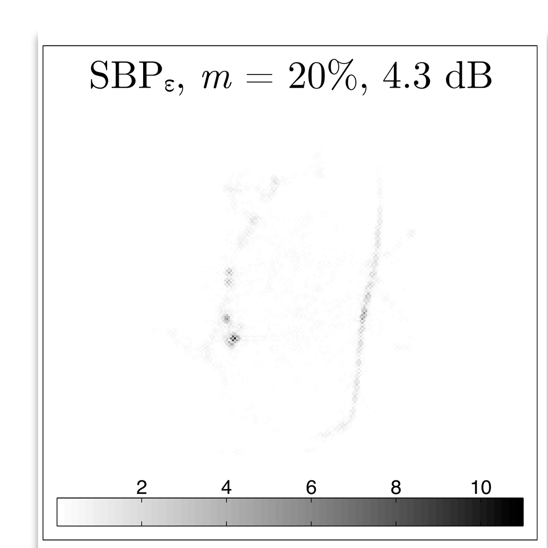
- Reconstruction : Statistical BP DeNoise (with some $s_j < 1$!)

$$\arg\min_u \|u\|_S \ \text{ s.t. } \ \|Wy - W\Phi\Psi u\|_2 \leqslant \epsilon, \qquad (\text{SBP}_\epsilon)$$

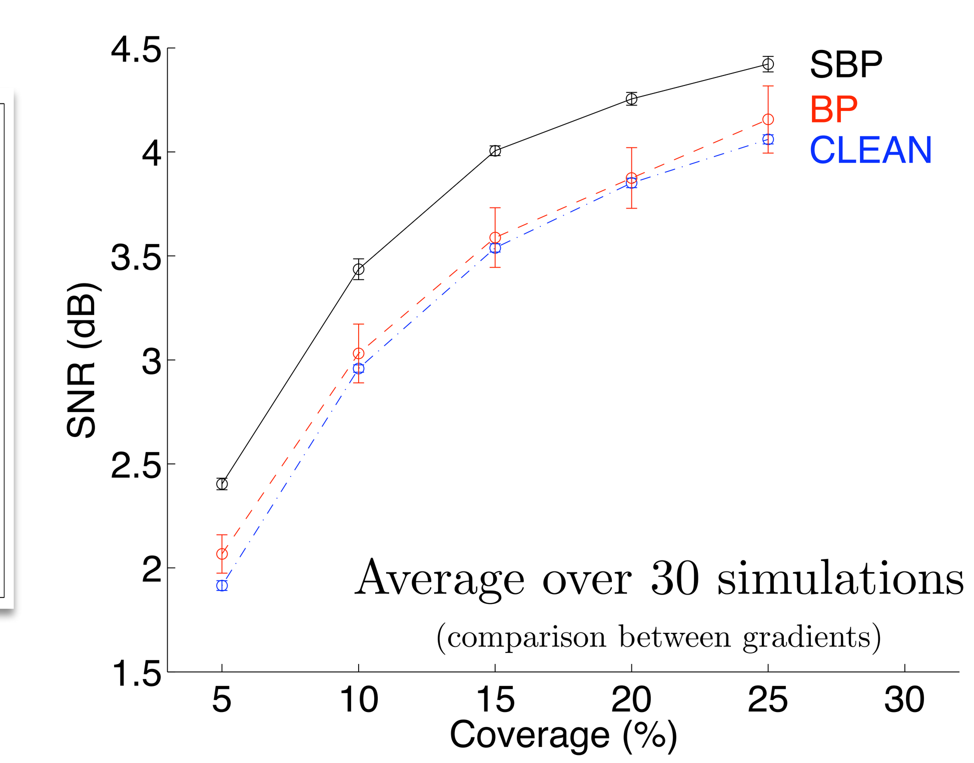
with $\|u\|_S = \sum_w |u_w/\rho b_j|^{s_j}$, $\varepsilon^2 = 99^{\text{th}}$ percentile $\chi^2(2m)$

W = whitening of the Gaussian Noise (known spectrum),

- Solver : re-weighted ℓ_1 with SPGL1 toolbox



(reconstruction gradient)



Conclusion :

- CS is a flexible framework for image reconstruction from radio-interferometric data through convex optimization.
- The inclusion of prior knowledge on the signal under scrutiny improve the quality of signal reconstruction.
- In progress : control of the actual visibility coverage, inclusion of TV sparsity term, mosaicking.

References :

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