**What is Aperture Synthesis?**

- Using **N** telescopes, \( \binom{N}{2} \) possible pairing (visibilities).
- And baselines undergo Earth rotation!
- Example:
  - Each telescope pair = one elliptical path.

**CLEAN Mathematical Model (Högjom, 1974 [1]):**

- Problem: In matrix notation, find \( I \in \mathbb{R}^{N^2} \) from
  \[ d = B I, \quad \text{with} \quad B = F' MF \]
  - Dirty map \( d \)
  - Circulant matrix, i.e., convolution \( B \)
  - Diagonal matrix, i.e., the visibility mask \( F' \)
  - Fourier basis, i.e., \( F' = F.u \)
- Assume \( I \) sparse in space, i.e., in the canonical (Dirac) basis.
- CLEAN is a (\( \gamma \) damped) Matching Pursuit in the Dictionary B.
- Other methods: Multi-scale CLEAN, MR CLEAN, MEM, ...

**BP and BP\(^+\) Reconstruction**

- **Compressed Sensing Model: Fourier Acquisition**
  \[ y = \Phi I = SFI, \quad \text{with} \quad I = \Phi \Psi u \quad \text{sparse in} \quad \Psi \]
  - Sensing matrix, \( y \in \mathbb{C}^m \)
- Visibility Selection
  \[ M \in \mathbb{R}^{8 \times 8} \]
  \[ S \in \mathbb{R}^{m \times N^2} \]
- Context similar to Magnetic Resonance Imaging MRI
- We may use Basis Pursuit [2]:
  \[ \alpha_{est} = \arg \min_u ||u||_1 \quad \text{s.t.} \quad y = \Phi \Psi u \quad \text{(BP)} \]
  - Or, if positive image (additional prior)
  \[ \alpha_{est} = \arg \min_u ||u||_1 \quad \text{s.t.} \quad y = \Phi \Psi u, \quad \Psi u \geq 0 \quad \text{(BP\(^+\))} \]
- Or, noisy version:
  \[ y = \Phi \Psi u + n, \quad n \sim N(0, \sigma^2), \quad y = \Phi \Psi u \quad \Leftrightarrow \quad \|y - \Phi \Psi u\|_2 \leq \epsilon \]
- Solver: Proximal Methods and Douglas-Rachford Splitting [3]
- **Simulations**: random interferometer, \( \Psi = \text{Dirac}, 1.8^\circ \times 1.8^\circ\)

**Cosmic String Enhancement in AS**

- **Cosmic Microwave Background (CMB) signal** = \( C_{\text{Gaussian Noise}} \) + String signal (no gradient here)
- Laboratory to test cosmological models
  - Very low SNR for string signal (i.e. low string tension) : -30 dB !
  - String signals not yet observed but simulated [4]
- Prior Information: string signal follows GGD in wavelet space
  - GGD scale and shape parameters deduced in steerable wavelets [5]
  \[ \pi_j(u_w) \sim \exp \left| w_j \rho_b \right|^{\alpha_j}, \quad \text{with} \quad u = (j, \theta_k, \phi_l) \]
- Reconstruction: Statistical BP DeNoise (with some \( s_j < 1 \))
  \[ \arg \min_u ||u||_2 \quad \text{s.t.} \quad ||Wy - W\Phi\Psi u||_2 \leq \epsilon \quad \text{(SBP\(_j\))} \]
  \[ \text{with} \quad ||u||_2 = \sum_w |w_j|^2 |\rho_j|^2, \quad \epsilon = 99\% \text{ percentile } \chi^2(2m) \]
- **Solver:** re-weighted \( \ell_1 \) with SPGL1 toolbox

**Conclusion**

- CS is a flexible framework for image reconstruction from radio-interferometric data through convex optimization.
- The inclusion of prior knowledge on the signal under scrutiny improve the quality of signal reconstruction.
- In progress: control of the actual visibility coverage, inclusion of TV sparsity term, mosaicking.

**References**