

Summary
 This research project presents a corporation of architects, mathematicians and computer scientists. The team researches new methods for the efficient realization of complex architectural shapes. The aim is to develop computer-aided solutions which optimize the design and production of free-form surfaces. Therefore, the team worked out a new geometric design method. The method studied provides new form-finding possibilities while satisfying a certain number of material and construction constraints.

Definition of a controlled IFS

Automaton (Q, Σ, δ) :

- Finite set of states $Q = \{A, B, N\}$
- Finite set of symbols (alphabet) $\Sigma = \{a, b, c, d, e, f, g, h, i\}$
- Transition function $\delta(A, a) = B, \delta(A, f) = N, \dots$

Each state $x \in Q$ has :

- A geometrical figure $K^{x, \sigma}$ a space E^3 (a vector space, or a projective space)
- Each transition $\delta(x, a) = y$ has :
- A contractive function $T_a^x : E^3 \rightarrow E^3$ (A matrix)

Topologically constrained automaton

Curve e

- Subdivided into 2 curves e by T0 an T1
- bounded by 2 vertices v by B0 and B1

Automaton :

Path tree :

Path tree with constraints :

- B0 T = T0 B0
- B1 T = T1 B1
- T0 B1 = T1 B0

Relation between the transformation matrices and the control points

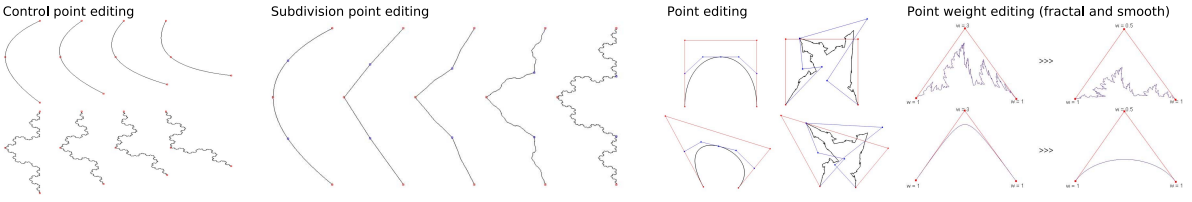
$$P = (p_1, p_2, p_3) = \begin{pmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ z_1 & z_2 & z_3 \end{pmatrix}$$

$$T_0^e = (a, b, c) = \begin{pmatrix} 1 & b_1 & c_1 \\ 0 & b_2 & c_2 \\ 0 & b_3 & c_3 \end{pmatrix}$$

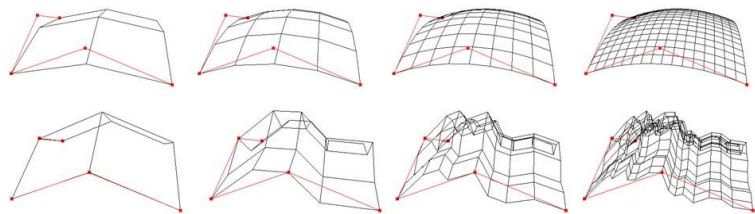
$$T_1^e = (c, d, e) = \begin{pmatrix} c_1 & d_1 & 0 \\ c_2 & d_2 & 0 \\ c_3 & d_3 & 1 \end{pmatrix}$$

The mathematical Formalism

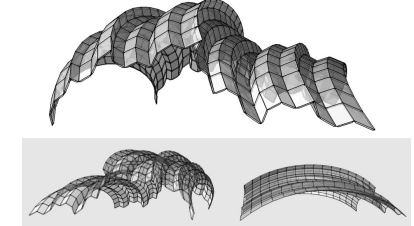
The developed mathematical model is a **Boundary Controlled Iterated Function System (BCIFS)**. It is a fairly general formalism, which is able to describe iterative processes while integrating several models (L-systems, Subdivision modelling, IFS and generalised models). Iterative models use a set of subdivision rules, which are applied on an initial object (primitiv) to generate more complex figures. The BCIFS model consists in a description of an automaton. Within the definition of a BCIFS, topological constraints can be set by equalizing paths in the construction tree. This allows us to model objects of different topologies; smooth (classical) or rough (fractal). The topological constraints in the BCIFS induce conditioned transformation matrices (transitions).



Iterative surface modelling (smooth and fractal)



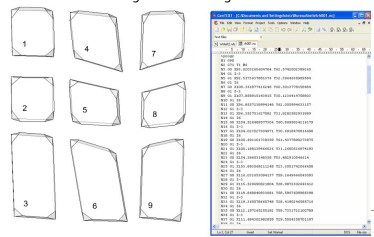
Constructional elements based on the discrete geometric figure



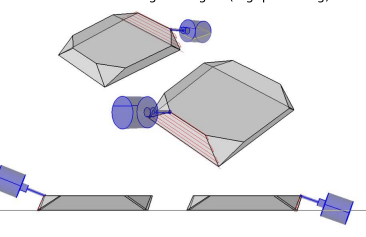
Software implementation of BCIFS

The software implementation of the BCIFS provides large design possibilities. Once the definition of the automaton is loaded (presets), the user can act on the geometrical values of the control points (global shape control) and on the subdivision points, which define the values of the geometric transformation matrices (local shape control). The BCIFS model is general enough to be combined with usual geometric design methods. For example, it is easy change the degree of an object or use projective geometry, which allows assigning different weights to the control- and subdivision-points.

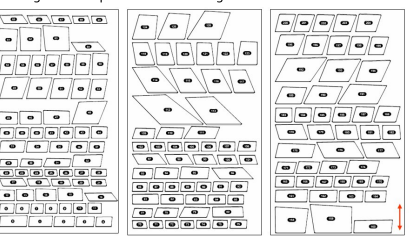
Automated addressing and G-Code generation



Simulation of machining strategies (e.g. pocketing)



Nesting for multipart manufacturing



From discrete geometry to constructional elements

In order to realise physical buildings out of discrete virtual geometries, the discrete geometric elements, which constitute the 3D-models, are replaced by constructional elements. In the presented examples, the faces that compose the surface are replaced by planar timber panels. The initial 3D-surface does not present any thickness. A volume model has to be derived from the surface model. First, we generate a parallel offset surface, which holds a constant distance to the initial surface. The distance corresponds to the thickness of the timber panel. Second, the bisector planes are calculated, we will use them later for the chamfer cut of the panels.

Production of the constructional elements

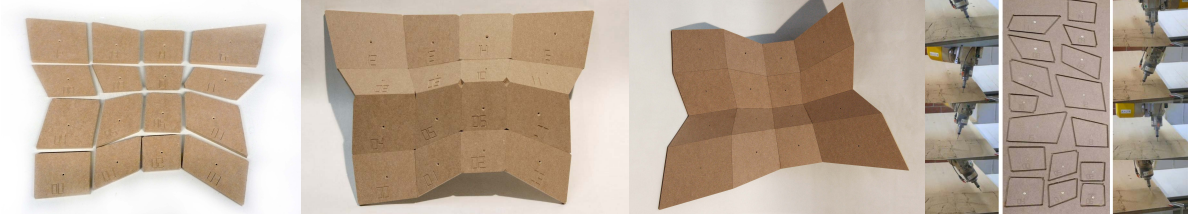


Integrated Manufacturing

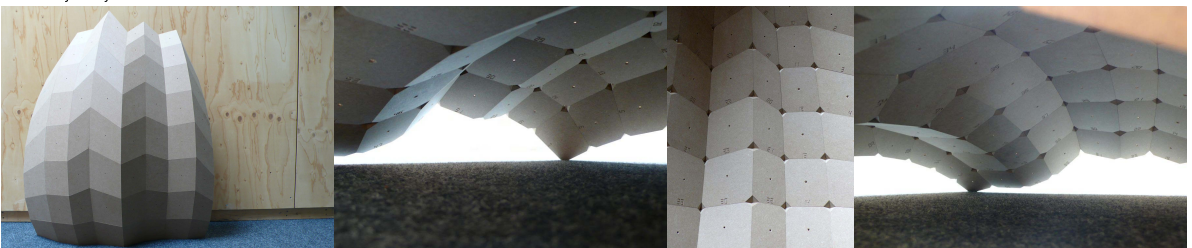
Following steps have been considered in order to get from the geometric data to the produced elements, which are all different in size and shape:

- A unique address for each constructional element was necessary for the logistical reason that the different elements can be assembled in the right place. This address is given by the index function of the BCIFS.
- Each element had to be oriented according to the coordinate system of the CNC-machine. Nesting for raw material savings was applied.
- The generation of the machine code for each element has been automated taking in account material properties (wood fibre direction), the type of 5-axis-machine, the nature of the cutting tools and a given set of machining strategies.

Feasibility study I: 4x4 Model



Feasibility study II: 8x8 Model



Feasibility study III: 16x16 Prototypel



Outlook

The assembled manufactured elements give an accurate rendering of the surfaces designed on the computer screen. This shows that practical realization of iteratively constructed surfaces becomes possible. It requires a relatively small planning effort. The efficiency of the method presented is proved insofar as the processing of the data, from design to production, last only a few moments. In the future, we will develop bigger and more complex objects. We hope to be able to apply our findings on applications such as suspended ceilings, free-form facades, climbing walls, halls, etc.

Aknoledgements

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