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# Spectrum Sharing Games of Infrastructure-Based Cognitive Radio Networks

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***Abstract*—The IEEE 802.22 standard is the first proposed standard for the cognitive radio networks in which a set of *base stations* (BSs) make opportunistic spectrum access to provide wireless access to the *customer-premise equipments* (CPE) within their cells in *wireless regional area networks* (WRAN). The channel assignment and power control must be carried out in BSs and CPEs, such that no excessive interference is caused to the users of the primary network. We use a game-theoretic model to analyze the non-cooperative behavior of the secondary users in IEEE 802.22 networks. We first show the existence of Nash equilibrium in a 2-cell non-cooperative game model, where the players (BSs) want to increase their coverage range. Then we extend our game to an N-player non-cooperative game where the players aim at maximizing the number of subscribers (i.e., CPEs). We conclude that the non-cooperative behavior of the players might result in a small number of supported CPEs and this can be solved by cooperative techniques, such as the *Nash bargaining solution*. Numerical results show that our proposed Nash bargaining solution can significantly increase the efficiency of the opportunistic spectrum allocation.**

## I. Introduction

The traditional solution for the spectrum management of wireless networks is to let government agencies, such as the FCC in the USA, allocate communication frequencies to different wireless networks. The main problem with this approach is that the licenses are typically established for long periods of time. Recent performance studies have shown that this significantly affects efficiency [18], [19].

*Cognitive radio* [1], [9] (CR) is an emerging technology that enables devices to determine which part of the frequencies are unused, and to use them even if they are licensed to others. Cognitive radio devices can adapt to the actual frequency utilization and consequently increase the efficiency of wireless communications. One fundamental requirement of these devices is that they should not hamper the communication of the *primary users*, who obtained the license for the given frequency band.

Recently, there have been efforts to develop cognitive radios [2], [3], [20], [21] and the government agencies recognize these devices as a potential solution to increase the spectrum efficiency. For example, the FCC issued a Report and Order in 2004: it said that a radio spectrum allocated to TV but unused in a particular broadcast market can be used by cognitive radios as secondary users. In parallel, the IEEE 802.22 working group develops a standard for a cognitive radio-based PHY/MAC/Air_Interface for use by license-exempt devices on a non-interfering basis in spectrum that is allocated to the TV broadcast service [16]. One of the main problems of this new standard is the channel/power allocation among cognitive radios.[1] Many researchers are currently engaged in designing efficient protocols for channel/power allocation in these networks. They use several techniques, such as graph coloring and linear integer programming, which are appropriate for studying the behavior of this new networking environment. In this paper we use game theory, a useful tool to study the strategic behavior of network participants (i.e., secondary BSs and CPEs) in IEEE 802.22. We believe our paper to be one of the first steps towards a deeper understanding of the non-cooperative/selfish behaviors of IEEE 802.22 cognitive radios and we present some important criteria that should be considered for designing more efficient channel/power allocation schemes.

Our contributions in this paper are summarized as follows. First, we analyze the non-cooperative behavior of CRs in IEEE 802.22 environment with a simple 2-player non-cooperative game. We obtain the socially optimum Nash equilibrium of this game. We discuss the parameters (e.g., the distance of the primary user to CRs and BSs) that change the results of the power game between secondary users in this 2-cell network. Then we focus on the generalized model of channel and power allocation and we present the result of this allocation by using a game-theoretic framework. We show that the non-cooperative channel/power allocation may lead to poor performances (bad equilibria), where the secondary network cannot support many CPEs. Finally, we propose a cooperative scheme using the *Nash bargaining solution* (NBS) that significantly increases the performance of the IEEE 802.22 networks, avoiding the worst-case equilibria of channel/power allocation game.

The remainder of this paper is organized in the following way. In Section II, we present a brief review of the literature on spectrum sharing game. In Section III, we introduce IEEE 802.22 protocol. In Section IV, we describe the system model and the operational requirements. The game model and the results of a non-cooperative game in a 2-cell scenario are

[1]Note that *customer-premise equipments* (CPE) are the *cognitive radios* (CR) stations in IEEE 802.22 parlance. We use the two terms interchangeably in this paper.

presented in Section V, followed by a generalized game model of channel/power allocation in Section VI. Finally, we propose a cooperative scheme using the Nash bargaining solution in Section VII. We conclude the paper in Section VIII.

## II. Related Work

A brief description of several research contributions in the area of spectrum sharing games can be found in [13]. Halldorsson et al. study channel allocation strategies for Wi-Fi operators in [8]. They use the maximum graph coloring problem to identify Nash equilibria and they also provide a bound on the price of the anarchy of these equilibria. They also propose several local bargaining schemes to decrease the price of anarchy. But our solution is based on Nash bargaining solution and we also consider the power and channel allocation, simultaneously.

Hoang et al. [10], [11] propose a two-phase channel/power allocation scheme that improves the system throughput, defined as the total number of subscribers that can be simultaneously served. Their solution diverges from ours, because we consider a game theoretic approach and we analyze cooperative schemes using Nash bargaining.

Felegyhazi and Hubaux [5] consider the competition between different operators in terms of the pilot power control of their base stations. They show that in the pilot power control game a socially desirable Nash equilibrium exists and that it can be enforced by punishments. But in our game model, the spectrum belongs to the primary user and the players should not make interference to the primary user devices.

Game theory is also used in [17] and [7], to analyze the coexistence of licensed and unlicensed users. In [17], Sengupta et al. present a winner determining sealed-bid knapsack auction mechanism that dynamically allocates spectrum to the wireless service providers based on their bids. Finally in [7], the authors show that a basic auction and market interaction model based on the Anglo-Dutch split award auction, and a bargaining approach based on Rubinstein-Stahl bargaining, would be very suitable for revenue driven spectrum resource optimization.

## III. Wireless Regional Area Network: IEEE 802.22

In this section, we provide a short overview of the IEEE 802.22 standard. Basically, IEEE 802.22 targets the wireless broadband access in rural and remote areas, utilizing the spectrum holes in the allocated TV frequency spectrum without interfering with any TV channels [4], [16]. It specifies both the *medium access control* (MAC) and the *physical* (PHY) layers for WRANs. Although nothing has yet been specified regarding the particular functionalities of the PHY/MAC layers we know that IEEE 802.22 belongs to the *centralized cognitive radio networks* class, where the secondary network is infrastructure oriented. In these networks, the area is divided into cells and each cell is managed by one *base station* (BS). BSs can be equipped with a GPS and connected to a centralized server to obtain the information about the available free TV channels in the area at the given time. They can also count on their subscribers (i.e., CPEs) that can sense the channels regularly and report the available spectrums to the BSs. This will provide a suitable infrastructure for the future wireless Internet service providers.

As shown in Fig. 1 (a), IEEE 802.22 works in a point-to-multipoint basis where each cognitive radio is attached to a base station. *Orthogonal frequency division multiple access* (OFDMA) will be the modulation for uplink and downlink transmissions in this standard, because it provides an adaptive and flexible modulation to dynamically adjust the bandwidth. The frequency range of 54-862 MHz in the USA is a possible available bandwidth to this protocol. This might be extended to 41-910 MHz for international deployments. Note that by using one TV channel (i.e, 6, 7, or 8 MHz in different countries) the approximate maximum bit-rate is 19 Mbit/s at a 30 km distance. The maximum coverage range of the IEEE 802.22 BSs is expected to be around 100 km. In the next section, we propose a mathematical model that captures the most important features of the IEEE 802.22 protocol.

## IV. System Model for IEEE 802.22 Networks

We model the IEEE 802.22 network as a centralized cognitive radio network (similar to the model presented in [10]), as shown in Fig. 1 (b). The available free TV spectrum is divided into $K$ channels. These channels are licensed to $M$ *primary users* (PUs). In the same area, an IEEE 802.22 network is deployed. We divide the area into $L$ square cells. Within each cell, there is a base station serving a set of CPEs by using the spectrum opportunistically. We assume a free-space path-loss model with a path-loss exponent of $\alpha$. A complete list of notations used in this paper is introduced in Table I.

TABLE I
List of symbols and definition in our system model.

| Symbol | Definition |
|---|---|
| $L$ | Number of cells or base stations |
| $M$ | Number of primary users |
| $N$ | Number of cognitive radios |
| $K$ | Number of channels |
| $N_0$ | Noise power spectrum density |
| $\hat{G}^c_{pi}$ | The channel gain from BS serving $CR_i$ to primary user $p$ on channel $c$ |
| $G^c_{ij}$ | The channel power gain from the BS serving $CR_j$ to $CR_i$ on channel $c$ |
| $P^c_i$ | The transmit power from BS serving $CR_i$ toward $CR_i$ on channel $c$ |
| $P_{max}$ | Maximum transmission power on each channel |
| $\overline{\zeta}$ | The interference constraint for each primary user |
| $\overline{\gamma}$ | The SINR constraint for each supported CR |

### A. Operational Requirements

We consider downlink transmission from BSs to CRs. There are two conditions on power transmission from BSs to CRs. First, the total interference caused by all BSs to each PU must be lower than a threshold. Second, for each supported CR the received signal to interference and noise ratio (SINR) must be above a threshold.

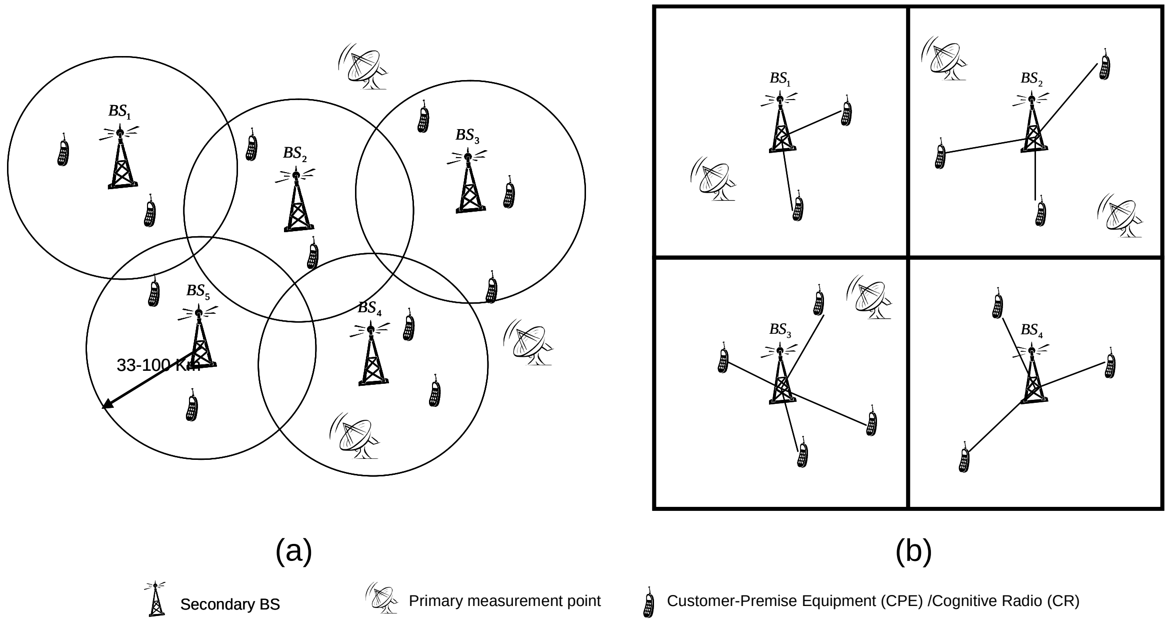



Fig. 1. The IEEE 802.22 deployment configuration. (a) An example of deployment of secondary base stations and cognitive radios. (b) Our proposed cell-based model for IEEE 802.22 networks.

As the spectrum is licensed to PUs, all BSs can use the spectrum opportunistically, i.e., whenever and wherever it is possible. We require that, for each PU, the total interference from all opportunistic transmissions by BSs does not exceed a predefined threshold $\overline{\zeta}$, i.e.,

$$\sum_{i=1}^{N} P_i^c \hat{G}_{pi}^c \leq \overline{\zeta}, \qquad \forall p, c \tag{1}$$

According to the physical model of signal propagation, SINR at $CR_i$ can be expressed as:

$$\gamma_i^c = \frac{G_{ii}^c P_i^c}{N_0 + \sum_{j=1, j\neq i}^{N} G_{ij}^c P_j^c}, \qquad \forall i, c \tag{2}$$

For reliable transmission toward $CR_i$, we require that

$$\gamma_i^c \geq \overline{\gamma}. \tag{3}$$

where $\overline{\gamma}$ is a predefined threshold. $\overline{\gamma}$ can be the minimum SINR required to achieve an acceptable bit error rate at CRs.

According to [10], the problem of finding the optimal channel/power allocation can be formulated as a linear mixed integer programming. As solving this problem for the optimal solution is an NP-hard problem, the authors present a heuristic scheme based on dynamic interference graph.

But in this paper, the game theory approach is proposed and examined in order to evaluate the channel/power assignment. In the next subsection we provide a brief overview of the main concepts in the game theory that we will use in our evaluations.

### *B. Game Model*

Game theory provides different methods for resource allocation in a distributed way. Our channel/power allocation problem can be modeled as a non-cooperative game $G$, in which each BS tries to maximize its payoff function. The *strategy* of players $s_i$, determines the channel allocation and the assigned power in each channel. Let $s$ be the strategy profile that is the set of strategies of all players. We assume that the players share the same strategy set $S$. $u_i$ is the payoff of player $i$

In order to gain an insight into the strategic behavior of the players, we apply the following game-theoretic concepts. First, let us introduce the concept of *best response*. We can write $br_i(s_j)$, the best response of player $i$ to the opponent's strategy $s_j$ as follows.

*Definition 1:* The *best response* of player $i$ to the profile of strategies $s_j$ is a strategy $s_i$ such that:

$$br_i(s_j) = \arg\max_{s_i \in S} u_i(s_i, s_j) \tag{4}$$

If two strategies are mutual best responses, then no player has any motivation to deviate from the given strategy profile. To identify such strategy profiles in general, Nash introduced the concept of Nash equilibrium [15]:

*Definition 2:* The pure-strategy profile $s^*$ constitutes a Nash equilibrium if, for each player $i$,

$$u_i(s_i^*, s_j^*) \geq u_i(s_i, s_j^*), \forall s_i \in S \tag{5}$$

where $s_i^*$ and $s_j^*$ are the Nash equilibrium strategies of player $i$ and $j$, respectively.

In other words, in a Nash equilibrium, none of the players can unilaterally change his strategy to increase his payoff. By carefully designing the payoff function and strategies, the game can be balanced at a unique socially optimal Nash equilibrium (NE), where the summation of all payoffs would be maximized. We introduce such a payoff function in Section

V. In some cases, the non-cooperative behavior may result in some undesirable Nash equilibria. We examine such a situation in Section VI. Finally, cooperation game theory introduces different methods of cooperation among players to improve the performances of the game. In Section VII, we investigate one method of cooperation, called Nash bargaining solution.

In this paper, we consider two different cases. First, we study non-cooperative behavior in a simplified IEEE 802.22 network, composed of two cells and one PU. In this case, the goal of each BS is to maximize its coverage area. Second, we study the problem of channel/power allocation in a general IEEE 802.22 network. The payoff of each BS is the number of supported CRs within the cell. We study both non-cooperative and cooperative behaviors in this case.

## V. 2-Cell Non-Cooperative Game

In this section, we develop a distributed game approach to adaptively assign power in a 2-cell IEEE 802.22 network. The goal is to maximize the coverage range under the constraint of the desirable SINR and the protection of primary user. We consider a 2-cell network (i.e., $L = 2$), as shown in Fig. 2. The number of available channels is one (i.e, $K = 1$). There is only one PU in the area. We model the problem as a two-player, non-cooperative game. The players of the game are the BSs. The strategy of each BS is its transmission power or equivalently its coverage area. The goal of each player is to maximize its own payoff. The payoff function of players depends on two parameters, the coverage range and the interference caused to the primary user. We consider a single-stage game, where both players simultaneously choose their radio range, once and for all. This corresponds to the case in which the base stations are not able to perform power control during the operation of the network.

### A. Operational Requirements and Feasibility Check

The SINR of a CR located in the first cell (for example, $CR_1$ in Fig. 2) can be written as:

$$\gamma_1 = \frac{P_1 \cdot d_{11}^{-\alpha}}{N_0 + P_2 \cdot d_{12}^{-\alpha}} \tag{6}$$

where $P_i$ is the transmission power of $BS_i$. Similarly, the SINR of a CR located in the second cell (for example, $CR_2$ in Fig. 2) can be written as:

$$\gamma_2 = \frac{P_2 \cdot d_{22}^{-\alpha}}{N_0 + P_1 \cdot d_{21}^{-\alpha}} \tag{7}$$

If a CR is inside the coverage area of its corresponding BS, it means that the BS supports it with high enough SINR, i.e., $\gamma_i \geq \overline{\gamma}$. Then, at the boundary of the coverage area, we have:

$$\gamma_i = \overline{\gamma}$$

We denote the maximum radio range of $BS_i$ by $r_i$. Assume $CR_1$ and $CR_2$ are located at the boundary of the coverage area of $BS_1$ and $BS_2$, respectively. From the SINR requirements (6) and (7), we obtain:

$$\overline{\gamma} = \frac{P_1 \cdot r_1^{-\alpha}}{N_0 + P_2 \cdot (D - r_1)^{-\alpha}} \tag{8}$$

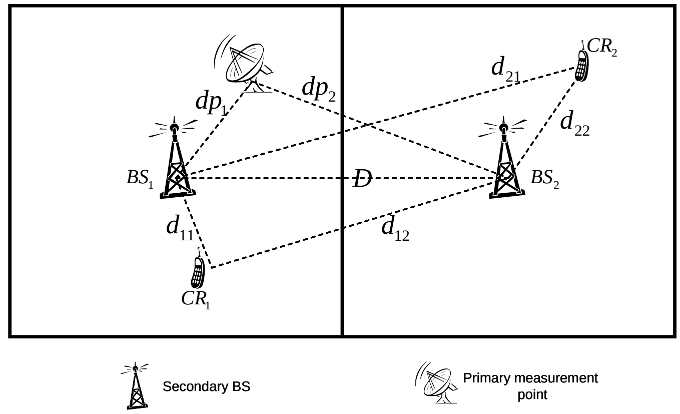



Fig. 2. 2-Cell game model parameters. Two secondary base stations provide wireless access to $CR_1$ and $CR_2$. The primary user measurement point is located at distance $dp_1$ and $dp_2$ from $BS_1$ and $BS_2$, respectively.

$$\overline{\gamma} = \frac{P_2 \cdot r_2^{-\alpha}}{N_0 + P_1 \cdot (D - r_2)^{-\alpha}} \tag{9}$$

When a CR is at distance $r_1$ from the first BS, its distance from the second BS can be any value between $D - r_1$ and $D + r_1$. A CR located in minimum distance, i.e., $D - r_1$, experiences the maximum interference from the other BS. So, if the SINR at this point is equal to $\overline{\gamma}$, we are guaranteed that the SINR at other points located at distance $r_1$ from the first BS, will be greater than $\overline{\gamma}$. This is the reason we replaced $d_{12}$ in Equation (6), by $D - r_1$ in Equation (8) and also $d_{21}$ in Equation (7), by $D - r_2$ in Equation (9).

In addition, we require that the total amount of interference caused by the two BSs to the PU must not exceed a predefined threshold ($\overline{\zeta}$), i.e.,

$$P_1 \cdot dp_1^{-\alpha} + P_2 \cdot dp_2^{-\alpha} \leq \overline{\zeta} \tag{10}$$

Let us address the question of whether there exists positive $P_1$ and $P_2$ such that Equations (8) and (9) are met. If

$$r_1^{-\alpha} r_2^{-\alpha} - \overline{\gamma}^2 (D - r_1)^{-\alpha} (D - r_2)^{-\alpha} \neq 0$$

then, the solution of (8) and (9) will be:

$$P_1 = \frac{\overline{\gamma} N_0 (r_2^{-\alpha} + \overline{\gamma}(D - r_1)^{-\alpha})}{r_1^{-\alpha} r_2^{-\alpha} - \overline{\gamma}^2 (D - r_1)^{-\alpha} (D - r_2)^{-\alpha}} \tag{11}$$

$$P_2 = \frac{\overline{\gamma} N_0 (r_1^{-\alpha} + \overline{\gamma}(D - r_2)^{-\alpha})}{r_1^{-\alpha} r_2^{-\alpha} - \overline{\gamma}^2 (D - r_1)^{-\alpha} (D - r_2)^{-\alpha}} \tag{12}$$

Note that the power of a BS (and consequently its coverage area) depends on the radio range of the other BS. As $r_1$ and $r_2$ are less than $D$, so the numerator of (11) and (12) are positive. Consequently, the denominator should be positive in order to have positive solutions $P_1$ and $P_2$, i.e.,

$$r_1^{-\alpha} r_2^{-\alpha} - \overline{\gamma}^2 (D - r_1)^{-\alpha} (D - r_2)^{-\alpha} > 0$$

Or, equivalently:

$$(\frac{D - r_1}{r_1})^{-\alpha} \cdot (\frac{D - r_2}{r_2})^{-\alpha} < \overline{\gamma}^{-2} \tag{13}$$

The above inequality introduces a feasible region for the coverage area of two BSs. If (13) holds, $P_1$ and $P_2$ can be calculated from Equation (11) and Equation (12). Furthermore, $P_1$ and $P_2$ should satisfy (10).

### B. 2-Cell Game Model and Results

We define the payoff function of each BS as follows:

$$u_i = \frac{r_i}{D/\sqrt{2}} - \frac{P_i \cdot dp_i^{-\alpha}}{\overline{\zeta}} \tag{14}$$

where $r_i$ is the coverage range of $BS_i$, $P_i$ is the transmission power of $BS_i$, $D/\sqrt{2}$ is the maximum radio range and $\overline{\zeta}$ is the threshold introduced in (1), i.e., the maximum tolerable interference caused by two BSs to the PU. The payoff function is composed of two terms. The positive term represents the normalized radio range of the BS. The negative term represents the normalized interference caused by this player to the PU. We choose the negative term in this way to allow the farther BS to operate at a higher power and consequently to have a larger coverage area. By introducing the payoff function in this way, the objective of each BS will be to maximize its coverage area, as well as to minimize its interference to the PU. Let's replace $P_1$ and $P_2$ calculated in Equation (11) and Equation (12) in Equation (14):

$$u_1 = \frac{r_1}{D/\sqrt{2}} - \frac{\gamma N_0 dp_1^{-\alpha}(r_2^{-\alpha} + \gamma(D-r_1)^{-\alpha})}{\zeta r_1^{-\alpha} r_2^{-\alpha} - \zeta\gamma^2 (D-r_1)^{-\alpha}(D-r_2)^{-\alpha}} \tag{15}$$

$$u_2 = \frac{r_2}{D/\sqrt{2}} - \frac{\gamma N_0 dp_2^{-\alpha}(r_1^{-\alpha} + \gamma(D-r_2)^{-\alpha})}{\zeta r_1^{-\alpha} r_2^{-\alpha} - \zeta\gamma^2 (D-r_1)^{-\alpha}(D-r_2)^{-\alpha}} \tag{16}$$

We make use of the concept of Nash equilibrium (Definition 2) to show stability points in the game. We first find the best response function for each player. Then we identify a set of strategies for which both players play their best response. We derive the best response of each player from the payoff functions presented in Equation (15) and Equation (16).

*Lemma 5.1:* In the feasible region, the payoff function of player $i$ is a concave function of $r_i$.

*Proof:* The proof is given in Appendix A. ■

Concavity means that the derivative of $u_i$ with respect to $r_i$ has only one real root. We denote this unique maximizer by $\hat{r}_i$. $\hat{r}_i$ is the best response of player $i$.

*Theorem 5.2:* There exists a unique Nash equilibrium for the 2-Cell game model with the payoff functions defined by Equation (14).

*Proof:* Considering Lemma 5.1 the proof is trivial [6]. ■

### C. Simulation Results and Discussions

To evaluate the non-cooperative behavior of BSs in a 2-cell scenario, we set up the simulations. We consider a service area of $100 \times 50\ km^2$, which is divided into two square cells. We assume a path-loss exponent of 4. The noise power spectrum density is $N_0 = -100dBm$. The required SINR for each CR is $15dB$. The maximum tolerable interference for the PU is $-110dBm$.

We obtain the NE of the game by looking at the best response curves of the two players. In order to obtain the best response of the first player, we change $r_2$ from zero to the maximum value. For each $r_2$, we find the best strategy of the first player (i.e., $\hat{r}_1$). The best response of the second player is obtained in a similar way. The NE of the game is the point of the intersection of the two curves.

In Fig. 3, we consider two different positions of PU. PU is located at the line connecting the center and the top right corner of the first cell. We change the location of PU along this line. So, in this case:

$$dp_2 = \sqrt{D^2 + dp_1^2 - \sqrt{2}Ddp_1}$$

We show the best response curves of the two BSs in Fig. 3 (a), where PU is near $BS_1$ at $dp1 = 15km$ and in Fig. 3 (b), where PU is located at $dp1 = 35km$. As we observe, there exists a unique NE that is the intersection point of the two curves. We also observe that the farther BS has a larger coverage range at the equilibrium point (see the NE point in Fig. 3 (a) where $r_2 \simeq 40km$ and $r_1 \simeq 10km$). This behavior is desirable, because the objective in our problem is to use the spectrum opportunistically, i.e., whenever and wherever it is possible. With the payoff function proposed at (14), we allow the farther BS to use the spectrum in a larger area.

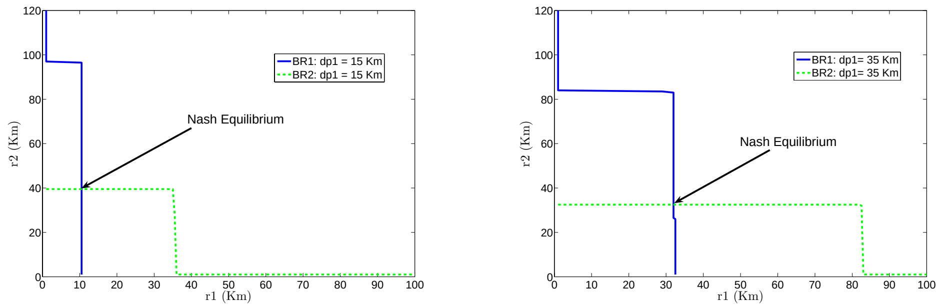
Similar simulation with $dp_2 = \sqrt{D^2 + dp_1^2 + \sqrt{2}Ddp_1}$ (i.e., where PU is located at the line connecting the center and the top left corner of the first cell) shows that when PU goes farther, both BSs obtain a larger coverage range at the equilibrium point.

We also look at the sum of the payoff of two players to find the social optimal strategy profile, where the overall payoff function is maximized. In Fig. 4, we plot $u_1 + u_2$ versus $r_1$ and $r_2$. The maximizer of $u_1 + u_2$ is the socially optimum strategy profile of the game. We observe that the 2-cell non-cooperative game has a unique social optimal strategy profile.

To summarize our simulation result shows that:

- There exists a unique NE in 2-cell non-cooperative game.
- The BS that is farther from PU gets a larger coverage area at NE.
- Changing the position of PU forces the BSs to adapt their coverage area accordingly.

The last two observations show that the 2-cell non-cooperative game provides BSs a distributed way to make use of the spectrum opportunistically, i.e., whenever and wherever it is possible. The above results highlight the properties of the non-cooperative resource allocation in IEEE 802.22 and will help us to design efficient mechanisms for channel/power allocation in these networks. Designing such mechanisms will be one of our future activities.




(a) (b)

Fig. 3. Effect of the location of primary user on the Nash equilibrium of 2-cells non-cooperative game for $dp_2 = \sqrt{D^2 + dp_1^2 - \sqrt{2}Ddp_1}$, where (a) $dp_1 = 15km$ and (b) $dp_1 = 35km$.

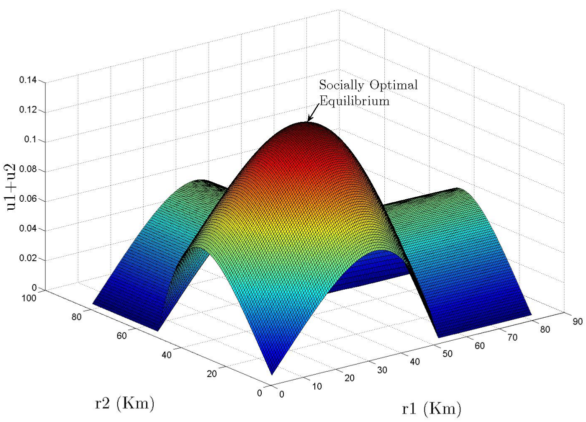



Fig. 4. Uniqueness of socially optimal strategy profile where $dp_1 = 35km$ and $dp_2 = \sqrt{D^2 + dp_1^2 - \sqrt{2}Ddp_1}$.

## VI. Non-Cooperative Channel/Power Allocation Game in IEEE 802.22 network

In this section, we consider the general problem of channel/power allocation in IEEE 802.22 networks. We develop a non-cooperative game to solve the problem in a distributed way. The objective of the problem is to find the optimal channel/power assignment, i.e., the assignment that enables BSs to support the maximum number of CRs. Similar to the previous game, there are two conditions on power transmission from BSs to CRs. First, the total interference caused by all BSs to each PU must be less than a threshold, i.e., Equation (1). Second, for each supported CR the received SINR must be above a threshold, i.e., Equation (3).

We model the problem as a L-player non-cooperative game (recall that $L$ is the number of cells in the network). The strategy of each player is its power on channels $1, 2, \cdots, K$. We define the payoff of each BS as the total number of supported CRs (subscribed CPEs) inside its cell. We consider a single-stage game again; without power control during the operation of the network.

We also use the *iterative water-filling* (IWF) algorithm to find the transmission power vector of BSs in a distributed way. As explained in [9], we formulate an iterative two-loop water filling algorithm. The inner loop of IWF finds the transmission powers in a distributed way. The outer loop adjusts the power vector to the minimum level needed to satisfy the target SINR of all CRs. As in this problem, we have the constraint of protecting PUs, we check this condition after the convergence of the IWF. The pseudo-code for the IWF that we use is given in Appendix B.

Different algorithms can be set up to be played among BSs to reach NE of the channel/power allocation game. One possible method is given in Appendix C. The performance of this algorithm is also analyzed through simulations. Here we are more interested in finding the potential Nash equilibria of this game and further developments of non-cooperative game algorithms are beyond the scope of this work.

We use Algorithm 1 to find all possible NE of the channel/power allocation game. We use two vectors: $channel$, $power$. $channel$ is equal to $(ch_1, ch_2, \cdots, ch_N)$, where $ch_i$, $i \in (1, 2, \cdots, K)$ denotes the assigned channel to $CR_i$. If $ch_i = 0$, this means that no channel is assigned to $CR_i$. Hence we are not considering $CR_i$ in power allocation. $power$ is equal to $(P_1, P_2, \cdots, P_N)$, where $P_i$ is the allocated power to $CR_i$. We use IWF, to calculate $power$. Note that here we write $P_i$ instead of $P_i^{ch_i}$ for notation abbreviation. We start with the all zero $channel$. $flag$ is the indicator of NE. If $flag = 1$, $channel$ is NE, otherwise it is not NE. For any possible $channel$, we first check if all CRs in one cell are assigned different channels (OFDMA). If this is not the case, we simply drop the $channel$. We then check if $ch_i$ is the best response of $CR_i$ or not for all $i$s. If all $ch_i$s are the best responses, then $channel$ is a NE. Otherwise it is not a NE.

### A. Simulation Results and Discussion

We consider one specific deployment of Fig. 1. The size of the service area is $100 \times 100$ $km^2$. The area is divided into 4 cells. The total number of CRs is $N = 6$. We vary $M$, the total number of PUs, from 1 to 5. CRs and PUs are randomly deployed over the area with uniform distribution. For each BS, the maximum transmission power on each channel is $P_{max} =$

**Algorithm 1** Calculation of Nash equilibria in Channel/Power Allocation Game

$channel \leftarrow (0, 0, \cdots, 0)$, $flag \leftarrow 1$
**for** $i = 1 : (K+1)^N$ **do**
  **if** two CRs in one cell has the same channel **then**
    $flag \leftarrow 0$, $break$
  **end if**
  **for** $k = 1 : K$ **do**
    find all CRs with allocated channel $k$
    call IWF
  **end for**
  put all outcomes of IWF together and obtain $power = (P_1, P_2, \cdots, P_N)$
  **for** $j = 1 : N$ **do**
    **if** $P_j = 0$ and by changing $ch_j$, $P_j$ can be $> 0$ **then**
      $flag \leftarrow 0$, $break$
    **end if**
  **end for**
  **if** flag=1 **then**
    $channel$ is a NE
  **else**
    not a NE
  **end if**
  $channel = channel + 1 \mod K$, $flag \leftarrow 1$
**end for**

$5W$. The number of channels is $K = 4$. Path-loss exponent, noise power spectrum, required SINR, and maximum tolerable interference for PU have the same values as in Section V.

Fig. 5 shows the number of NE of the game versus the number of PUs. As we see, there are many NE in all cases. We then consider the total number of supported CRs by all BSs in each NE. Fig. 6 shows the maximum and minimum number of supported CRs versus the number of PUs at all possible NE. We observe that when there is only one PU, the maximum and minimum are equal. This means that in all NE, the number of supported CRs are the same. So all NE of the game are optimum. But in other cases, the maximum and minimum of the supported CRs are different. This shows that out of many NE, some of them are optimal and some are not. Next we consider the total transmission power by all BSs in each NE. Fig. 7 shows the maximum total transmission power versus the number of PUs. When the number of PUs is increased, the total transmission power decreases. The reason is that BSs must fulfill the condition of protecting PUs (Equation (1)). When more PUs are present in the area, the total transmission power by all BSs is decreased in order to protect all PUs from excessive interference.

The above results show that the non-cooperative behavior in a general scenario, may result in non-convergence or many undesirable Nash equilibria with few supported CRs. The *Nash bargaining solution* (NBS), which requires the cooperation between BSs, is one method to enhance the performances. In the next section we study this method.

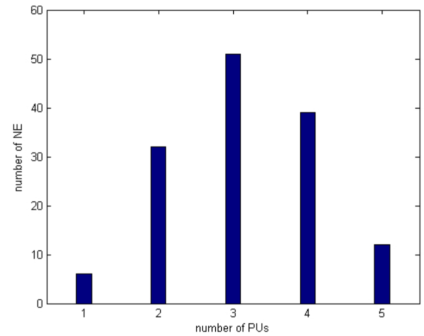



Fig. 5. Number of NE versus the number of primary users (PU).

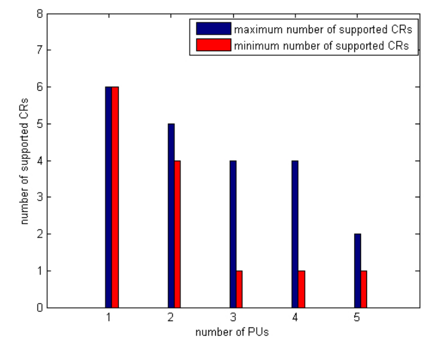



Fig. 6. Maximum and minimum number of supported CRs in NE versus the number of PUs.

## VII. Nash Bargaining Framework for Channel/Power Allocation in IEEE 802.22 Network

In this section, our approach is significantly different and is based on the general bargaining theory originally developed by Nash [14]. Non-cooperative games may lead to substantial loss to all players, compared to a cooperative strategy where players can cooperate. The main issue in this case is how to achieve the cooperation in a stable manner and which Nash equilibrium can be achieved through cooperation. One possible solution is Nash bargaining solution.

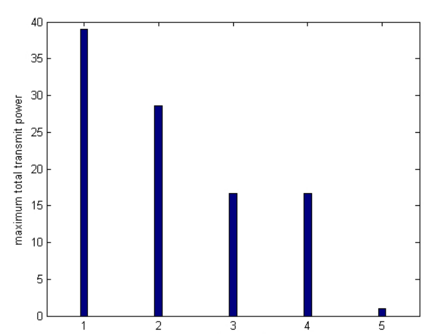



Fig. 7. Maximum total transmission power of base stations in Nash equilibrium points versus the number of deployed primary users (PUs).

The underlying structure for a Nash bargaining in an $N$ players game is a set of outcomes of the bargaining process $S$. Bargaining process $S$ is composed of:

1) A set of possible joint strategies or states
2) A designated disagreement outcome $d$, which represents the agreement to disagree and solve the problem competitively
3) A multiuser payoff function $U : S \cup \{d\} \rightarrow R^N$

The Nash bargaining is a function $F$ that assigns to each pair $(S \cup \{d\}, U)$ an element of $S \cup \{d\}$. Furthermore, the Nash solution is unique. In order to obtain the solution, Nash assumed four axioms: Linearity, Independence of irrelevant alternatives, Symmetry, and Pareto optimality. Nash proved that there exists a unique solution to the bargaining problem satisfying these 4 axioms. The solution is obtained by

$$s = \arg \max_{s \in S \cup \{d\}} \Pi_{n=1}^{N} (U_n(s) - U_n(d)). \tag{17}$$

We also define the Nash function $F(s) : S \cup \{d\} \rightarrow R$:

$$F(s) = \Pi_{n=1}^{N} (U_n(s) - U_n(d)). \tag{18}$$

The Nash bargaining solution is obtained by maximizing the Nash function over all possible states. Since the set of possible outcomes $U(S \cup \{d\})$ is convex, F(s) has a unique maximum on the boundary of $U(S \cup \{d\})$. It is common to define disagreement point for all players $i$ as follows:

$$d = \min_{s_{-i} \in S_{-i}} \max_{s_i \in S_i} u_i(s_i, s_{-i}). \tag{19}$$

We investigate NBS in our problem. First, BSs find all possible channel assignments. For each channel assignment, the corresponding power vector is computed using IWF. Note that power is still assigned in a distributed way. The payoff of each BS is the number of supported CRs within the cell. According to the power vector, the payoff of BSs in each assignment can be computed. The calculated values can be exchanged by the BSs over a dedicated communication channel for the negotiation. Note that there are some ongoing research studies to assign a worldwide, harmonized, cognitive-supporting pilot channel with a bandwidth less than 50 kHz [12] to allow such negotiations.

Then the disagreement point $d$ is adjusted using Equation (19). Finally, NBS is computed using Equation (17). The pseudo-code for finding the NBS is given by Algorithm 2.

**Algorithm 2** Nash Bargaining Solution

1: find all possible channel assignments
2: find power vector of each channel assignment using IWF
3: find the payoff of BSs in each assignment
4: calculate disagreement point:
$$d = \min_{s_{-i} \in S_{-i}} \max_{s_i \in S_i} u_i(s_i, s_{-i})$$
5: find NBS:
$$s = \arg\max_{s \in S \cup \{d\}} \Pi_{n=1}^{N} (U_n(s) - U_n(d))$$

We implement NBS in the network of Fig. 1. All parameters are set to the values mentioned in Section VI-A. Our simulation results show that there exists a unique solution to the bargaining problem. The outcome of NBS is shown in Fig. 8. Comparing Fig. 8 and Fig. 6, we note that the NBS and the optimal NE of the non-cooperative game coincides. For example, consider the case when the number of PUs is 3. The number of supported CRs at NE points varies between $1$ and $4$ (see Fig. 6). So, out of all possible NE of the non-cooperative games, some of them are optimal and some are not. Whereas in Fig. 8, we observe that when number of PUs is 3, the number of supported CRs as a result of Nash Bargaining process is $4$, which is equal to optimal NE of non-cooperative game.

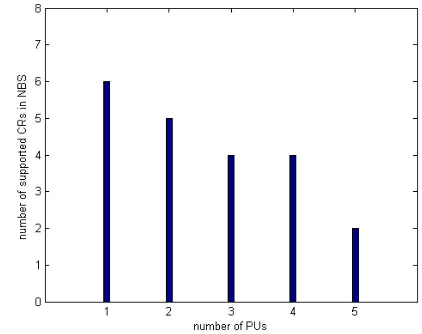



Fig. 8. The number of supported CRs versus the number of PUs, when the Nash bargaining solution is used.

## VIII. Conclusion

In this paper, we have studied the problem of channel/power allocation in IEEE 802.22 cognitive networks. Using a game-theoretic framework, we have analyzed the strategic behaviors of the BSs in these networks. We have derived the optimum strategies and corresponding parameters for a simple 2-Cell IEEE 802.22 network. Then, we have shown that a pure non-cooperative power/channel allocation in IEEE 802.22 network cannot maximize the number of supported CPEs. We have proposed a cooperative solution, based on Nash bargaining, to increase the efficiency of power/channel allocation. The simulation results show that the bargaining solution avoid the non-optimal channel/power allocations.

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## APPENDIX

### A. Proof of Lemma 5.1

*Proof:* We prove the concavity of $u_i$ by showing that its second derivative is always negative in the feasible region. We express $u_1$ as:

$$u_1 = \frac{r_1}{D/\sqrt{2}} - \overline{\gamma} N_0 d p_1^{-\alpha} \zeta^{-1} \cdot \frac{X}{Y}$$

where

$$X = r_2^{-\alpha} + \overline{\gamma}(D - r_1)^{-\alpha}$$

and

$$Y = r_1^{-\alpha} r_2^{-\alpha} - \overline{\gamma}^2 (D - r_1)^{-\alpha} (D - r_2)^{-\alpha}$$

The first derivative of $u_1$ is as follows:

$$\frac{\partial u_1}{\partial r_1} = \frac{1}{D/\sqrt{2}} - \overline{\gamma} N_0 d p_1^{-\alpha} \cdot \frac{\frac{\partial X}{\partial r_1} Y - \frac{\partial Y}{\partial r_1} X}{Y^2}$$

The second derivative is:

$$\frac{\partial^2 u_1}{\partial r_1^2} =$$

$$-\overline{\gamma} N_0 d p_1^{-\alpha} \cdot \frac{(\frac{\partial^2 X}{\partial r_1^2} Y - \frac{\partial^2 Y}{\partial r_1^2} X) Y^2 - 2 \frac{\partial Y}{\partial r_1} Y (\frac{\partial X}{\partial r_1} Y - \frac{\partial Y}{\partial r_1} X)}{Y^4}$$

The first derivative of X is:

$$\frac{\partial X}{\partial r_1} = -\overline{\gamma} \alpha (D - r_1)^{-\alpha - 1}$$

Its second derivative is:

$$\frac{\partial^2 X}{\partial r_1^2} = \overline{\gamma} \alpha (\alpha + 1)(D - r_1)^{-\alpha - 2}$$

The first derivative of Y is:

$$\frac{\partial Y}{\partial r_1} = -\alpha r_1^{-\alpha - 1} r_2^{-\alpha} - \alpha \overline{\gamma}^2 (D - r_1)^{-\alpha - 1} (D - r_2)^{-\alpha}$$

Its second derivative is:

$$\frac{\partial^2 Y}{\partial r_1^2} = \alpha(\alpha + 1) r_1^{-\alpha - 2} r_2^{-\alpha}$$

$$-\alpha(\alpha + 1) \overline{\gamma}^2 (D - r_1)^{-\alpha - 2} (D - r_2)^{-\alpha}$$

Putting all together and considering the feasible region (13), we conclude that:

$$\frac{\partial^2 u_1}{\partial r_1^2} < 0$$

The same reasoning is true for $u_2$. We conclude that $u_i$ is a concave function of $r_i$. ■

### B. Iterative Water Filling Algorithm

The following algorithm shows the iterative water filling algorithm to find the transmit power vector of BSs in a distributed way.

**Algorithm 3** Iterative Water Filling

1: IWF gets $m$ BSs transmitting towards $m$ CRs and returns $P = (P_1, P_2, \cdots, P_m)$
2: $P \leftarrow (0, 0, \cdots, 0)$
3: **for** $i = 1 : m$ **do**
4: update $P_i$ based on all $P_j, j \neq i$
5: **end for**
6: **for** $i = 1 : m$ **do**
7: **if** $\gamma(CR_i) > \overline{\gamma}$ **then**
8: reduce $\gamma$
9: **end if**
10: **if** $\gamma(CR_i) < \overline{\gamma}$ **then**
11: reduce $\gamma$
12: **end if**
13: **end for**
14: **if** all CRs reach $\overline{\gamma}$ **then**
15: goto 19
16: **else**
17: goto 3
18: **end if**
19: check the constraint of protecting PUs
20: **if** not satisfied **then**
21: $P \leftarrow (0, 0, \cdots, 0)$
22: **end if**
23: return $P$

### *C. Non-Cooperative Game*

Here, we propose an algorithm to be played among BSs to reach the NE of the game. The proposed algorithm starts with a channel assignment which is chosen uniformly at random out of all possible channel assignments. Then BSs find the corresponding transmit power by using IWF. If this channel/power assignment is a NE of the game, the algorithm finishes. Otherwise it continues in this way: the BS serving the first CR, assigns the next channel to it. If this assignment is a NE, the algorithm finishes. Otherwise, the BS serving the second CR assigns the next channel to it. If this assignment is a NE, the algorithm finishes. Otherwise, it continues with the third CR and so on. $counter$ saves the number of iterations of the algorithm and is increased by one in each step. The algorithm finishes whenever either it reaches a NE or $counter$ becomes a predefined maximum value. The pseudo-codes for this algorithm are given in Fig. 4. Note that by implementing this algorithm, BSs may converge to any NE from any arbitrary initial assignment.

**Algorithm 4** Non-Cooperative Game of Channel/Power Allocation in IEEE 802.22

```
counter ← 0
choose one channel assignment uniformly at random
find the corresponding power vector
if this channel/power assignment is a NE then
    return this NE
    goto 19
end if
while counter < max_counter do
    for i = 1 : N do
        counter ← counter + 1
        assign the next channel to CR_i
        find the corresponding power vector
        if this channel/power assignment is a NE then
            return this NE
            goto 19
        end if
    end for
end while
return counter
```

In Fig. 9 and 10, we look at the performance of the proposed non-cooperative game in Section C. We run the algorithm 100 times. Fig. 9 shows the percentage of the times the game converges versus the number of PUs.

As we see in some cases the proposed algorithm converges with high probability. But in other cases the probability of convergence is very small. In Fig. 10 we just consider the times the game converges and we calculate the average of convergence time. As we see in this figure, the game converges almost quickly in all cases. We conclude that non-cooperative algorithm may result in non-convergence or undesirable NE point.

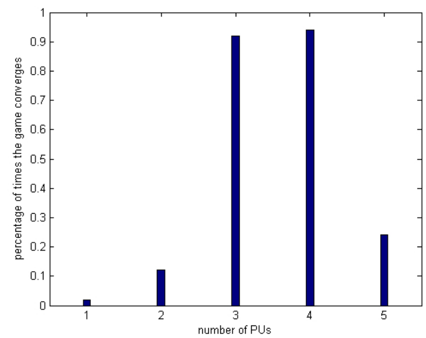


Fig. 9. percentage of the times the game converges versus number of PUs

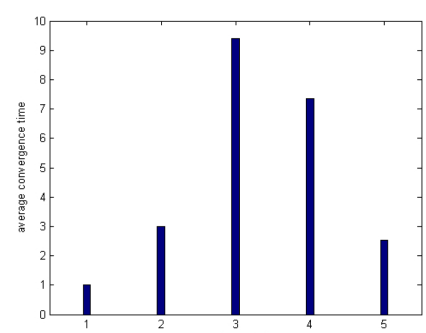


Fig. 10. Average convergence time of the game versus number of PUs