Yüce, Salim; Kuruoğlu, Nuri

Holditch-type theorems under the closed planar homothetic motions. (English summary)


In the paper under review, Holditch-type theorems are presented for closed homothetic motions. For these motions, the linked frame that is attached to the selected moving point in-between $A$ and $B$ distorts according to a homothetic scale $h(t)$ (i.e. a sort of continuous “zooming in and out” motion). Therefore, two moving frames in the Euclidean planes are defined, one for the fixed reference frame $E' = \{O'; e'_1, e'_2\}$ and one for the moving referential $E = \{O; e_1, e_2\}$. For a closed homothetic motion, the moving frame is related to the fixed one through

$$e_1 = \cos(\varphi(t))e'_1 + \sin(\varphi(t))e'_2,$$

$$e_2 = -\sin(\varphi(t))e'_1 + \cos(\varphi(t))e'_2,$$

and the point $x' = (x'_1, x'_2) \in E'$ is related to $x = (x_1, x_2) \in E$ through

$$x' = h(t)x - \left(\begin{array}{c} u_1(t) \\ u_2(t) \end{array}\right),$$

where $h(t)$ is the homothetic periodic scale, $\varphi(t)$ is a periodic rotation and $u_1(t)$ and $u_2(t)$ are periodic translation functions, that is, there exists $T > 0$ such that $h(t+T) = h(t)$, $\varphi(t+T) = \varphi(t) + 2\varphi \nu$, $u_1(t+T) = u_1(t)$ and $u_2(t+T) = u_2(t)$. The integer $\nu \in \mathbb{Z}$ is called the rotation number of the closed planar homothetic motion.

The first theorem states that if the points $A = (0, 0)$ and $B = (a, 0) \in E$ draw closed-trajectory curves $k_A$ and $k_B$ with the orbit areas $F_A$ and $F_B$, respectively, then the point $C = (a, c) \in E$ describes a closed-trajectory curve $k_C$ with area

$$F_C = [aF_B + bF_A]/(a + b) + (c^2 - ab)h^2(t_0)\pi\nu - 2h^2(t_0)\nu c L_{AB}/(\oint h d\varphi),$$

where $L_{AB}$ is the length of the rod (segment $AB$) and $t_0$ is a certain time instant during the homothetic motion.

The main theorem of the paper states that if three points $A = (0, 0)$, $B = (b, 0)$ and $C = (c, d)$
$\in E$ describe closed-trajectory curves of areas $F_A$, $F_B$ and $F_C$ respectively, then a point $X = (x, y) \in E$ describes the area

$$F_X = \left(1 - \frac{x}{b} + \frac{c - b}{bd} y\right) F_A + \left(\frac{x}{b} - \frac{cy}{bd}\right) F_B + \frac{y}{d} F_C$$

$$+ \left(x^2 + y^2 - bx - \frac{c^2 + d^2}{d} y + \frac{bc}{d} y\right) h^2(t_0) \pi \nu.$$

Reviewed by Philippe A. Müllhaupt

© Copyright American Mathematical Society 2008