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A HOLOMORPHIC REPRESENTATION OF THE SEMIDIRECT SUM OF SYMPLECTIC AND HEISENBERG LIE ALGEBRAS

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Abstract. A representation of the Jacobi algebra by first order differential operators with polynomial coefficients on a Kähler manifold which as set is the product of the complex multidimensional plane times the Siegel ball is presented.

1. Introduction

In this paper we construct a holomorphic polynomial first order differential representation of the Lie algebra which is the semidirect sum $\mathfrak{h}_n \rtimes \mathfrak{sp}(n, \mathbb{R})$, on the manifold $\mathbb{C}^n \times \mathcal{D}_n$, different from the extended metaplectic representation [6]. The case $n = 1$ corresponding to the Lie algebra $\mathfrak{h}_1 \rtimes \mathfrak{su}(1,1)$ was considered in [3]. The natural framework of such an approach is furnished by the so called coherent state (CS)-groups, and the semi-direct product of the Heisenberg-Weyl group with the symplectic group is an important example of a mixed group of this type [11]. We use Perelomov's coherent state aproach [12]. Previous results concern the hermitian symmetric spaces [2] and semisimple Lie groups which admit CS-orbits [4]. The case of the symplectic group was previously investigated in [1], [6], [5],[10], [12]. Due to lack of space we do not give here the proofs, but in general the technique is the same as in [3], where also more references are given. More details and the connection of the present results with the squeezed states [13] will be discussed elsewhere.

2. The differential action of the Jacobi algebra

The Heisenberg-Weyl (HW) group is the nilpotent group with the $2n+1$-dimensional real Lie algebra $\mathfrak{h}_n = < \mathrm{i}s1 + \sum_{i=1}^{n}(x_i a_i^+ - \bar{x}_i a_i) >_{s\in\mathbb{R}, x_i\in\mathbb{C}}$, where a_i^+ (a_i) are the boson creation (respectively, annihilation) operators.

Table 1: *The generators of the symplectic group: operators, matrices, and bifermion operators*

$\boldsymbol{K}^+_{ij}$	$K^+_{ij}=\frac{\mathrm{i}}{2}\begin{pmatrix}0 & e_{ij}+e_{ji}\\ 0 & 0\end{pmatrix}$	$\frac{1}{2}a^+_ia^+_j$
$\boldsymbol{K}^-_{ij}$	$K^-_{ij}=\frac{\mathrm{i}}{2}\begin{pmatrix}0 & 0\\ e_{ij}+e_{ji} & 0\end{pmatrix}$	$\frac{1}{2}a_ia_j$
$\boldsymbol{K}^0_{ij}$	$K^0_{ij}=\frac{1}{2}\begin{pmatrix}e_{ij} & 0\\ 0 & -e_{ji}\end{pmatrix}$	$\frac{1}{4}(a^+_ia_j+a_ja^+_i)$

We consider the realization of the Lie algebra of the group $\mathrm{Sp}(n,\mathbb{R})$ [1], [6]:

$$\mathfrak{sp}(n,\mathbb{R}) =< \sum_{i,j=1}^{n}(2a_{ij}K^0_{ij}+b_{ij}K^+_{ij}-\bar{b}_{ij}K^-_{ij}) >, a^*=-a,\ b^t=b. \quad (1)$$

With the notation: $\boldsymbol{X}:=d\pi(X)$, we have the correspondence: $X\in\ \mathfrak{sp}(n,\mathbb{R})\rightarrow \boldsymbol{X}$, where the real symplectic Lie algebra $\mathfrak{sp}(n,\mathbb{R})$ is realized as $\mathfrak{sp}(n,\mathbb{C})\cap\mathfrak{u}(n,n)$

$$X=\begin{pmatrix}a & b\\ \bar{b} & \bar{a}\end{pmatrix}\leftrightarrow \quad \boldsymbol{X}=\sum_{i,j=1}^{n}(2a_{ij}\boldsymbol{K}^0_{ij}+z_{ij}\boldsymbol{K}^+_{ij}-\bar{z}_{ij}\boldsymbol{K}^-_{ij}),\ b=\mathrm{i}z. \quad (2)$$

The Jacobi algebra is the the semi-direct sum $\mathfrak{g}^J:=\mathfrak{h}_n\rtimes\mathfrak{sp}(n,\mathbb{R})$, where $\mathfrak{h}_n$ is an ideal in $\mathfrak{g}^J$, i.e. $[\mathfrak{h}_n,\mathfrak{g}^J]=\mathfrak{h}_n$, determined by the commutation relations:

$$[a_i,a^+_j] = \delta_{ij};\ [a_i,a_j]=[a^+_i,a^+_j]=0 \quad (3a)$$
$$[K^-_{ij},K^-_{kl}] = [K^+_{ij},K^+_{kl}]=0;\ 2[K^0_{ji},K^0_{kl}] = K^0_{jl}\delta_{ki}-K^0_{ki}\delta_{lj} \quad (3b)$$
$$2[K^-_{ij},K^+_{kl}] = K^0_{kj}\delta_{li}+K^0_{lj}\delta_{ki}+K^0_{ki}\delta_{lj}+K^0_{li}\delta_{kj} \quad (3c)$$
$$2[K^-_{ij},K^0_{kl}] = K^-_{il}\delta_{kj}+K^-_{jl}\delta_{ki};\ 2[K^+_{ij},K^0_{kl}] = -K^+_{ik}\delta_{jl}-K^+_{jk}\delta_{li} \quad (3d)$$
$$2[a_i,K^+_{kj}] = \delta_{ik}a^+_j+\delta_{ij}a^+_k\ :\ 2[K^-_{kj},a^+_i] = \delta_{ik}a_j+\delta_{ij}a_k \quad (3e)$$
$$2[K^0_{ij},a^+_k] = \delta_{jk}a^+_i; 2[a_k,K^0_{ij}] = \delta_{ik}a_j; [a^+_k,K^+_{ij}] = [a_k,K^-_{ij}]=0. \quad (3f)$$

Perelomov's coherent state vectors associated to the group G^J with Lie algebra the Jacobi algebra, based on the complex N-dimensional manifold , $N=\frac{n(n+3)}{2}$,

$$M := \mathrm{HW}/\mathbb{R}\times\mathrm{Sp}(n,\mathbb{R})/\mathrm{U}(n);\ M = \mathcal{D}:=\mathbb{C}^n\times\mathcal{D}_n \quad (4)$$

are defined as

$$e_{z,W} = \exp(\boldsymbol{X})e_0,\ \boldsymbol{X} := \sum_i z_i a_i^+ + \sum_{ij} w_{ij}\boldsymbol{K}_{ij}^+,\ z\in\mathbb{C}^n; W\in\mathcal{D}_n. \quad (5)$$

The non-compact hermitian symmetric space $X_n = \mathrm{Sp}(n,\mathbb{R})/\mathrm{U}(n)$ admits a realization as a bounded homogeneous domain, precisely the Siegel ball [7],[8]:

$$\mathcal{D}_n := \left\{W\in M(n,\mathbb{C}); W = W^t, 1 - W\bar{W} > 0\right\}. \quad (6)$$

The extremal weight vector e_0 verify the equations

$$a_i e_o = 0, i = 1,\cdots,n \quad (7a)$$

$$\boldsymbol{K}_{ij}^+ e_0 \neq 0;\ \boldsymbol{K}_{ij}^- e_0 = 0;\ \boldsymbol{K}_{ij}^0 e_0 = \frac{k}{4}\delta_{ij}e_0. \quad (7b)$$

Proposition 1 *The differential action of the generators of the Jacobi algebra is:*

$$\boldsymbol{a} = \frac{\partial}{\partial z};\ \boldsymbol{a}^+ = z + W\frac{\partial}{\partial z} \quad (8a)$$

$$\mathbb{K}^- = \frac{\partial}{\partial W};\ \mathbb{K}^0 = \frac{k}{4}1 + \frac{1}{2}\frac{\partial}{\partial z}\otimes z + \frac{\partial}{\partial W}W \quad (8b)$$

$$\mathbb{K}^+ = \frac{k}{2}W + \frac{1}{2}z\otimes z + \frac{1}{2}(W\frac{\partial}{\partial z}\otimes z + z\otimes\frac{\partial}{\partial z}W) + W\frac{\partial}{\partial W}W. \quad (8c)$$

Proof. The calculation is an application of the formula $\mathrm{Ad}\,(\exp X) = \exp(\mathrm{ad}\,_X)$. We have used the convention: $\left[\left(\frac{\partial}{\partial W}W\right)f(W)\right]_{kl} := \frac{\partial f(W)}{\partial w_{ki}}w_{il},\ W = (w_{ij})$.

3. The group action

The displacement operator, i.e. $D(\alpha) := \exp(\alpha a^+ - \bar{\alpha}a)$, has the addition property

$$D(\alpha_2)D(\alpha_1) = \mathrm{e}^{\mathrm{i}\theta_h(\alpha_2,\alpha_1)}D(\alpha_2+\alpha_1),\ \theta_h(\alpha_2,\alpha_1) := \mathrm{Im}\,(\alpha_2\bar{\alpha}_1).$$

Concerning the real symplectic group, we extract from [1], [6]

Remark 2 *To every* $g\in\mathrm{Sp}(n,\mathbb{R})$*,* $g\rightarrow g_c\in\mathrm{Sp}(n,\mathbb{C})\cap\mathrm{U}(n,n)$*, or denoted just* g*,* $g = \begin{pmatrix} a & b\\ \bar{b} & \bar{a}\end{pmatrix}$*, where* $aa^* - bb^* = 1;\ ab^t = ba^t;\ a^*a - b^t\bar{b} = 1;\ a^t\bar{b} = b^*a.$

We consider a particular case of the positive discrete series representation [9] of $\mathrm{Sp}(n,\mathbb{R})$ and let us denote $\underline{S}(Z) = S(W)$. The vacuum is chosen such that equation (7b) is satisfied. Here:

$$\begin{aligned}
\underline{S}(Z) &= \exp(\sum z_{ij}\boldsymbol{K}^+_{ij} - \bar{z}_{ij}\boldsymbol{K}^-_{ij}),\ Z=(z_{ij}) && \text{(9a)}\\
S(W) &= \exp(W\boldsymbol{K}^+)\exp(\eta\boldsymbol{K}^0)\exp(-\bar{W}K^-) && \text{(9b)}\\
W &= Z\tanh\frac{\sqrt{Z^*Z}}{\sqrt{Z^*Z}} && \text{(9c)}\\
Z &= \frac{\text{arctanh}\sqrt{WW^*}}{\sqrt{WW^*}}W = \frac{1}{2}\frac{1}{\sqrt{WW^*}}\log\frac{1+\sqrt{WW^*}}{1-\sqrt{WW^*}} && \text{(9d)}\\
\eta &= \log(1-WW^*) = -2\log\cosh\sqrt{ZZ^*}. && \text{(9e)}
\end{aligned}$$

Perelomov's un-normalized CS-vectors for $\mathrm{Sp}(n,\mathbb{R})$ are:

$$e_Z := \exp(\sum z_{ij}\boldsymbol{K}^+_{ij})e_0 = \pi\begin{pmatrix}1 & \mathrm{i}Z\\ 0 & 1\end{pmatrix}e_0,\ Z=(z_{ij});\ Z=Z^t. \tag{10}$$

Remark 3 *For $g\in\mathrm{Sp}(n,\mathbb{R})$, the following relations between the normalized and un-normalized Perelomov's CS-vectors hold:*

$$\underline{S}(Z)e_0 = \det(1-WW^*)^{k/4}e_W \tag{11}$$

$$e_g := \pi(g)e_0 = (\det\bar{a})^{-k/2}e_Z = \left(\frac{\det a}{\det\bar{a}}\right)^{\frac{k}{4}}\underline{S}(Z)e_0, Z=\frac{1}{\mathrm{i}}b\bar{a}^{-1} \tag{12}$$

$$S(g)e_{W/\mathrm{i}} = \det(Wb^*+a^*)^{-k/2}e_{Y/\mathrm{i}} \tag{13}$$

where $W\in\mathcal{D}_n$, and $Z\in\mathbb{C}^n$ in (12) *are related by equations* (9c), (9d)*, and the linear-fractional action of the group* $\mathrm{Sp}(n,\mathbb{R})$ *on the unit ball $\mathcal{D}_n$ in* (13) *is*

$$Y := g\cdot W = (a\,W+b)(\bar{b}\,W+\bar{a})^{-1} = (Wb^*+a^*)^{-1}(b^t+Wa^t). \tag{14}$$

Let us introduce the notation $\tilde{A} := \begin{pmatrix}A\\ \bar{A}\end{pmatrix}$ and

$$\mathcal{D}(Z) = \mathrm{e}^X = \begin{pmatrix}\cosh\sqrt{Z\bar{Z}} & \frac{\sinh\sqrt{Z\bar{Z}}}{\sqrt{Z\bar{Z}}}Z\\ \frac{\sinh\sqrt{\bar{Z}Z}}{\sqrt{\bar{Z}Z}}\bar{Z}Z & \cosh\sqrt{\bar{Z}Z}\end{pmatrix},\ X := \begin{pmatrix}0 & Z\\ \bar{Z} & 0\end{pmatrix}. \tag{15}$$

Remark 4 *The following (Holstein-Primakoff-Bogoliubov) equation is true:* $\underline{S}^{-1}(Z)\tilde{a}\underline{S}(Z) = \mathcal{D}(Z)\tilde{a}$.

Remark 5 *If D is the displacement operator and $\underline{S}(Z)$ is defined by* (9a)*, then*

$$D(\alpha)\underline{S}(Z)=\underline{S}(Z)D(\beta),\ \tilde{\beta}=\mathcal{D}(-Z)\tilde{\alpha};\ \tilde{\alpha}=\mathcal{D}(Z)\tilde{\beta}. \tag{16}$$

Let us introduce the notation

$$S(g)=\underline{S}(Z,A):=\exp(\sum 2a_{ij}\boldsymbol{K}^0_{ij}+z_{ij}\boldsymbol{K}^+_{ij}-\bar{z}_{ij}\boldsymbol{K}^-_{ij}). \tag{17}$$

Remark 6 *If S denotes the representation of* $\mathrm{Sp}(n,\mathbb{R})$*, in the matrix realization of Table 1, we have $S^{-1}(g)\,\tilde{a}\,S(g)=g\cdot\tilde{a}$, and*

$$S(g)D(\alpha)S^{-1}(g)=D(\alpha_g),\ \alpha_g=a\,\alpha+b\,\bar{\alpha}. \tag{18}$$

Lemma 7 *The normalized and un-normalized Perelomov's coherent state vectors*

$$\Psi_{\alpha,W}:=D(\alpha)S(W)e_0;\ e_{z,W'}:=\exp(za^++W'\boldsymbol{K}^+)e_0$$

are related by the relation

$$\Psi_{\alpha,W}=\det(1-W\bar{W})^{k/4}\exp(-\frac{\bar{\alpha}}{2}z)e_{z,W},\ z=\alpha-W\bar{\alpha}. \tag{19}$$

Comment 8 *Starting from* (19)*, we obtain the expression of the reproducing kernel $K=K(\bar{x},\bar{V};y,W)$*

$$\begin{aligned}(e_{x,V},e_{y,W}) &= \det(U)^{k/2}\exp\frac{1}{2}[2<x,Uy> \\ &+ <V\bar{y},Uy>+<x,UW\bar{x}>],\ U=(1-W\bar{V})^{-1}.\end{aligned} \tag{20}$$

From the following proposition we can see the holomorphic action of the Jacobi group $G^J:=\mathrm{HW}\rtimes\mathrm{Sp}(n,\mathbb{R})$ on the manifold (4):

Proposition 9 *Let us consider the action $S(g)D(\alpha)e_{z,W}$, where $g\in\mathrm{Sp}(n,\mathbb{R})$, and the coherent state vector is defined in* (5)*. Then we have:*

$$S(g)D(\alpha)e_{z,W}=\lambda e_{z_1,W_1},\ \lambda=\lambda(g,\alpha;z,W) \tag{21}$$

$$z_1=(Wb^*+a^*)^{-1}(z+\alpha-W\bar{\alpha}) \tag{22}$$

$$W_1=g\cdot W=(aW+b)(\bar{b}W+\bar{a})^{-1}=(Wb^*+a^*)^{-1}(b^t+Wa^t) \tag{23}$$

$$\lambda=\det(Wb^*+a^*)^{-k/2}\exp(\frac{\bar{x}}{2}z-\frac{\bar{y}}{2}z_1)\exp\mathrm{i}\theta_h(\alpha,x) \tag{24}$$

$$x=(1-W\bar{W})^{-1}(z+W\bar{z});\ y=a(\alpha+x)+b(\bar{\alpha}+\bar{x}). \tag{25}$$

Corollary 10 *The action of the Jacobi group G^J on the manifold* (4) *is given by* (21), (22)*. The composition law in G^J is*

$$(g_1,\alpha_1,t_1)\circ(g_2,\alpha_2,t_2)=(g_1\circ g_2,g_2^{-1}\cdot\alpha_1+\alpha_2,t_1+t_2+\mathrm{Im}\,(g_2^{-1}\cdot\alpha_1\bar{\alpha}_2)). \tag{26}$$

The proof of Proposition 9 is based on the previous assertions of this section.

4. The scalar product

Following the general prescription for CS-groups [4], we calculate the Kähler potential f as the logarithm of the reproducing kernel K, and the Kähler two-form:

$$\begin{aligned} f &= -\frac{k}{2}\log\det(1-W\bar{W})+\bar{z}_i(1-W\bar{W})^{-1}_{ij}z_j + \\ & \frac{1}{2}[z_i[\bar{W}(1-W\bar{W})^{-1}]_{ij}z_j+\bar{z}_i[(1-W\bar{W})^{-1}W]_{ij}\bar{z}_j] \end{aligned} \tag{27}$$

$$\begin{aligned} -\mathrm{i}\omega &= \frac{k}{2}\mathrm{Tr}[(1-W\bar{W})^{-1}\mathrm{d}W\wedge(1-\bar{W}W)^{-1}\mathrm{d}\bar{W}] \\ & + \mathrm{Tr}[\mathrm{d}z^t\wedge(1-\bar{W}W)^{-1}\mathrm{d}\bar{z}] \\ & - \mathrm{Tr}[\mathrm{d}\bar{z}^t(1-W\bar{W})^{-1}\wedge\mathrm{d}W\bar{x}]+cc \\ & + \mathrm{Tr}[\bar{x}^t\mathrm{d}W(1-\bar{W}W)^{-1}\wedge\mathrm{d}\bar{W}x]. \end{aligned} \tag{28}$$

Applying the technique of Ch. IV in [8] and a property extracted from the first reference [5] p. 398, we find out for the density of the volume form:

$$Q=\det(1-W\bar{W})^{-(n+2)}. \tag{29}$$

Now we determine the scalar product. If $f_\psi(z):=(e_{\bar{z}},\psi)$, then

$$(\phi,\psi)=\Lambda\int_{z\in\mathbb{C}^n;1-W\bar{W}>0;W=W^t}\bar{f}_\phi(z,W)f_\psi(z,W)QK^{-1}\mathrm{d}z\mathrm{d}W \tag{30}$$

$$\mathrm{d}z=\prod_{i=1}^n\mathrm{dRe}\,z_i\mathrm{dIm}\,z_i;\ \ \mathrm{d}W=\prod_{1\le i\le j\le n}\mathrm{dRe}\,w_{ij}\mathrm{dIm}\,w_{ij}. \tag{31}$$

We take in (30) $\phi,\psi=1$, we change the variable $z=(1-W\bar{W})^{1/2}x$, we apply equations (A1), (A2) in Bargmann [1] and Theorem 2.3.1 p. 46 in [8]

$$\int_{1-W\bar{W}>0,W=W^t}\det(1-W\bar{W})^\lambda\mathrm{d}W=J_n(\lambda)$$

and we find for Λ in (30) (below $p:=(k-3)/2-n>-1$):

$$\Lambda=\pi^{-n}J_n^{-1}(p),J_n(p)=2^n\pi^{\frac{n(n+1)}{2}}\prod_{i=1}^n\frac{\Gamma(2p+2i)}{\Gamma(2p+n+i+1)}. \tag{32}$$

Proposition 11 *Let us consider the Jacobi group G^J with the composition rule* (26)*, acting on the coherent state manifold* (4) *via* (22)–(25)*. The manifold M has the Kähler potential* (27) *and the G^J-invariant Kähler two-form ω given by* (28)*. The Hilbert space of holomorphic functions $\mathcal{F}_K$ associated to the holomorphic kernel $K : M \times \bar{M} \rightarrow \mathbb{C}$ given by* (20) *is endowed with the scalar product* (30)*, where the normalization constant Λ is given by* (32) *and the density of volume given by* (29)*.*

Proposition 12 *Let $h = (g, \alpha) \in G^J$, and we consider the representation $\pi(h) = S(g)D(\alpha)$, $g \in \mathrm{Sp}(n, \mathbb{R})$, $\alpha \in \mathbb{C}^n$, and let the notation $x = (z, W) \in \mathcal{D}$. Then the continuous unitary representation $(\pi_K, \mathcal{H}_K)$ attached to the positive definite holomorphic kernel K defined by* (20) *is $(\pi_K(h).f)(x) = J(h^{-1}, x)^{-1} f(h^{-1}.x)$, where the cocycle $J(h^{-1}, x)^{-1} = \lambda(h^{-1}, x)$ with λ defined by equations* (21)-(25) *and the function f belongs to the Hilbert space of holomorphic functions $\mathcal{H}_K \equiv \mathcal{F}_K$ endowed with the scalar product* (30)*.*

Comment 13 *The value of Λ given by* (32) *corresponds to the one given in* (7.16) *in* [3]*, taking above $n = 1$, $k \rightarrow 4k$. Note that p defying the normalization constant Λ in* (32) *for the Jacobi group is related with $q = \frac{k}{2} - n - 1$ in*

$$(\phi, \psi)_{\mathcal{F}_{\mathcal{H}}} = \Lambda_1 \int_{1 - W\bar{W} > 0; W = W^t} \bar{f}_\phi(W) f_\psi(W) \det(1 - W\bar{W})^q \mathrm{d}W \qquad (33)$$

defining the normalization constant $\Lambda_1 = J_n^{-1}(q)$ for the group $\mathrm{Sp}(n, \mathbb{R})$ by the relation $p = q - \frac{1}{2}$. It is well known [5], [8] *that the* **admissible set** *for k for the space of functions $\mathcal{F}_{\mathcal{H}}$ endowed with the scalar product* (33) *is the set $\Sigma = \{0, 1, \cdots, n-1\} \cup ((n-1), \infty)$. The integral* (33) *deals with a non-negative scalar product if $k \geq n - 1$, in which the domain of convergence $k \geq 2n$ is included, and the separate points $k = 0, 1, ..., n - 1$.*

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References

[1] Bargmann V., *Group Representations on Hilbert Spaces of Analytic Functions*, In: Analytic Methods in Mathematical Physics, R. Gilbert and R. Newton (Eds), Gordon and Breach, New York, London, Paris, 1970, pp 27–63.

[2] Berceanu S. and Gheorghe A., *On Equations of Motion on Hermitian Symmetric Spaces*, J. Math. Phys. **33** (1992) 998–1007; Berceanu S. and Boutet de Monvel L., *Linear Dynamical Systems, Coherent State Manifolds, Flows and Matrix Riccati Equation*, J. Math. Phys. **34** (1993) 2353–2371.

[3] Berceanu S., *A Holomorphic Representation of the Jacobi Algebra*, arXiv: math.DG/0408219, v2 14 Mar2006; to appear in Rev. Math. Phys.

[4] Berceanu S., *Realization of Coherent State Algebras by Differential Operators*, In: Advances in Operator Algebras and Mathematical Physics (Sinaia, 2003), F. Boca, O. Bratteli, R. Longo, H. Siedentop (Eds.), The Theta Foundation, Bucharest 2005, pp 1-24; arXiv: math.DG/0504053

[5] Berezin F., *Quantization in Complex Symmetric Spaces*, Izv. Akad. Nauk SSSR Ser. Mat. **39** (1975), 363–402, 472; Berezin F., *Models of Gross-Neveu Type are Quantization of a Classical Mechanics with a Nonlinear Phase Space*, Commun. Math. Phys. **63** (1978), 131–153.

[6] Folland G., Harmonic Analysis in Phase Space, Princeton University Press, Princeton, New Jersey, 1989.

[7] Helgason S., *Differential Geometry, Lie Groups and Symmetric Spaces*, Academic, New York, 1978.

[8] Hua L. K., *Harmonic Analysis of Functions of Several Complex Variables in the Classical Domains*. AMS, Providence, R.I., 1963.

[9] Knapp A., *Representations Theory of Semisimple Lie groups- An overview Based on Examples*, Princeton University, Princeton, NJ, 1986.

[10] Monastyrsky M. and Perelomov A., *Coherent States and Bounded Homogeneous Domains*, Reports Math. Phys. **6** (1974) 1– 14.

[11] Neeb K.–H., *Holomorphy and Convexity in Lie Theory*, de Gruyter Expositions in Mathematics 28, Walter de Gruyter, Berlin-New York, 2000.

[12] Perelomov A., *Generalized Coherent States and their Applications*, Springer, Berlin, 1986.

[13] Stoler P., *Equivalence Classes of Minimum Uncertainty Packets*, Phys. Rev. D **1** (1970) 3217-3219; —, II, Phys. Rev D **4** (1971) 1925–1926.

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