Intermittent motion and sediment transport: experimental and theoretical insights

Christophe Ancey & Anthony Davison

École Polytechnique Fédérale de Lausanne

Collaborators: Tobias Böhm, Philippe Frey

Cemagref (Grenoble, France)

Magali Jodeau

Cemagref (Lyon, France)

DRI Workshop, 6 November 2007

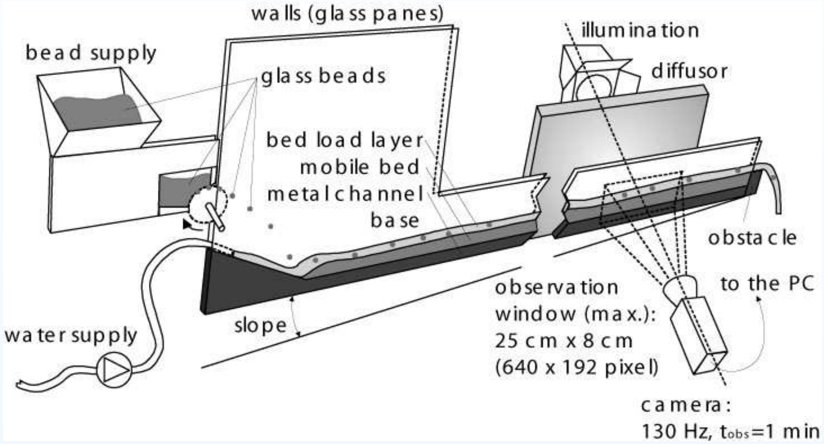
`christophe.ancey@epfl.ch`

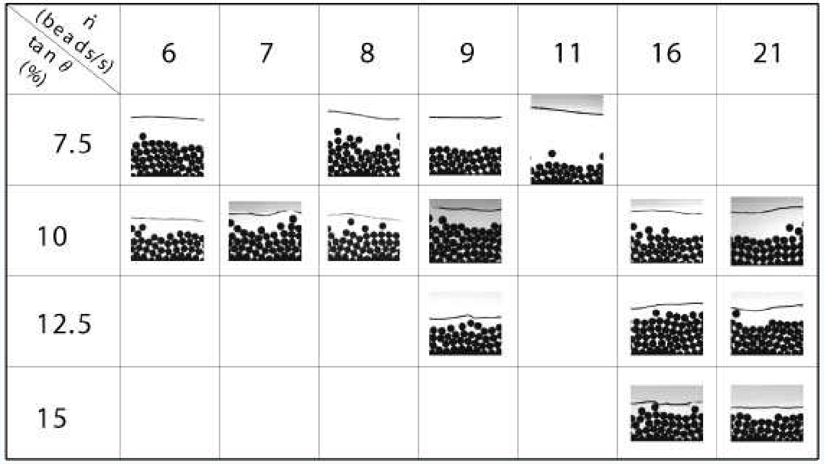
Content

- Experimental facility: 2D flume
- Experimental observations: origin of fluctuations?
- Model: Birth-death, emigration-immigration Markov model
- Comparison with data

[Ancey *et al.*, *Phys. Rev. E* **66** (2002) 036306; Ancey *et al.*, *Phys. Rev. E* **67** (2003) 011303; Boehm *et al.*, *Phys. Rev. E* **69** (2004) 061307; Ancey *et al.*, *Phys. Rev. E* **74** (2006) 011302; Ancey *et al.* in press in *J. Fluid Mech.*]

A toy (for big boys)



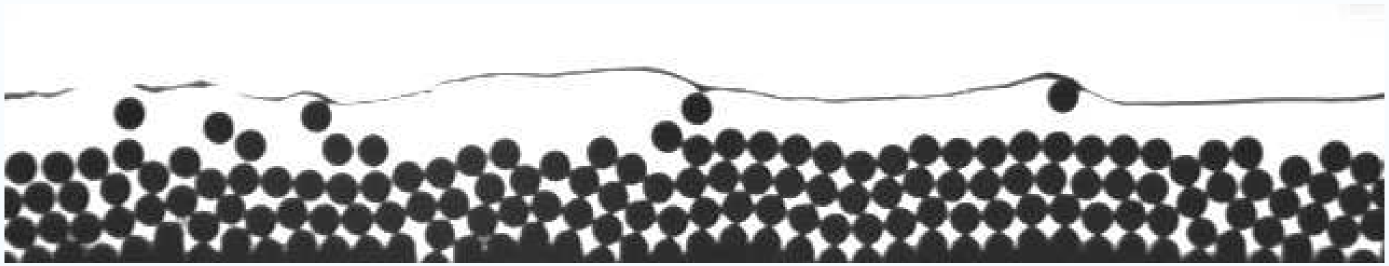


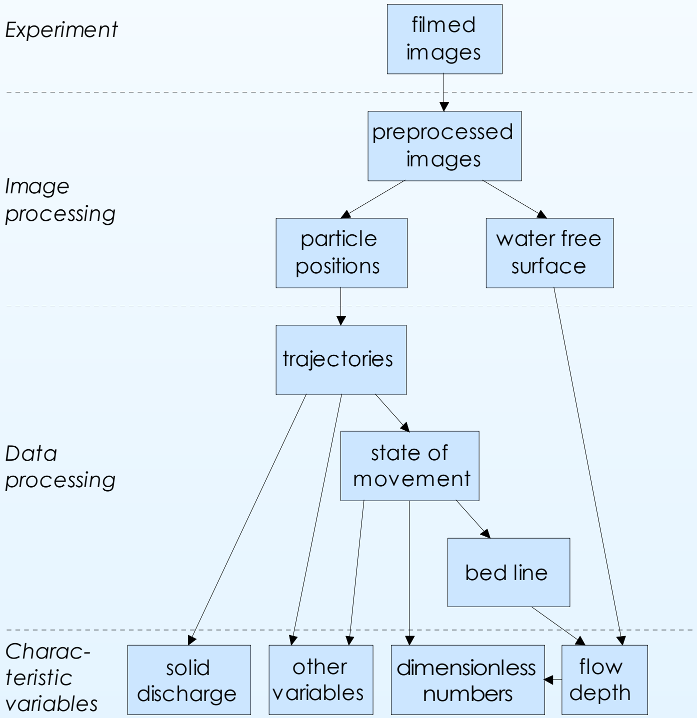
2D Flume

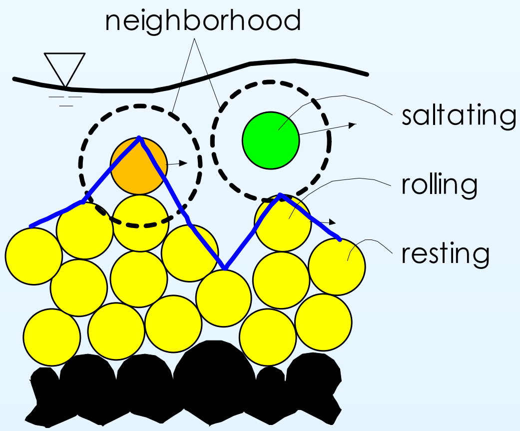
Experiments

- Slope from 7% to 15%
- Flow rate range: 6–21 beads/s
- bed equilibrium
- 8000 images at 140 Hz (approx. 60 s.)

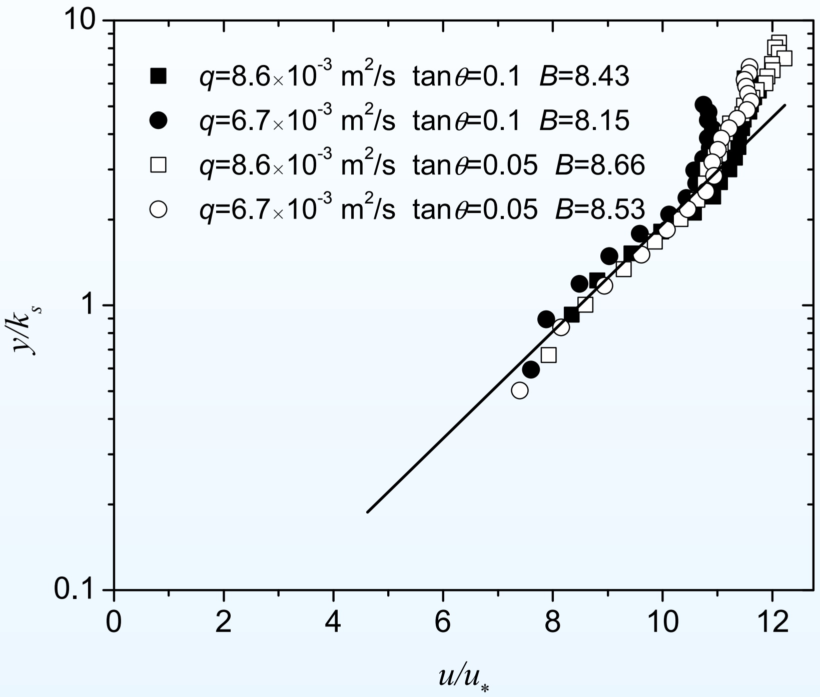
Image processing



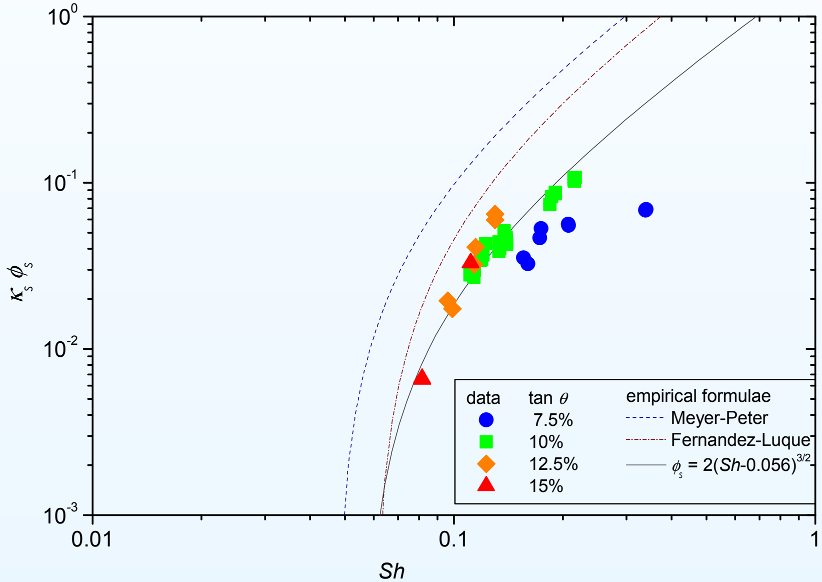




Representative of flow conditions usually met?



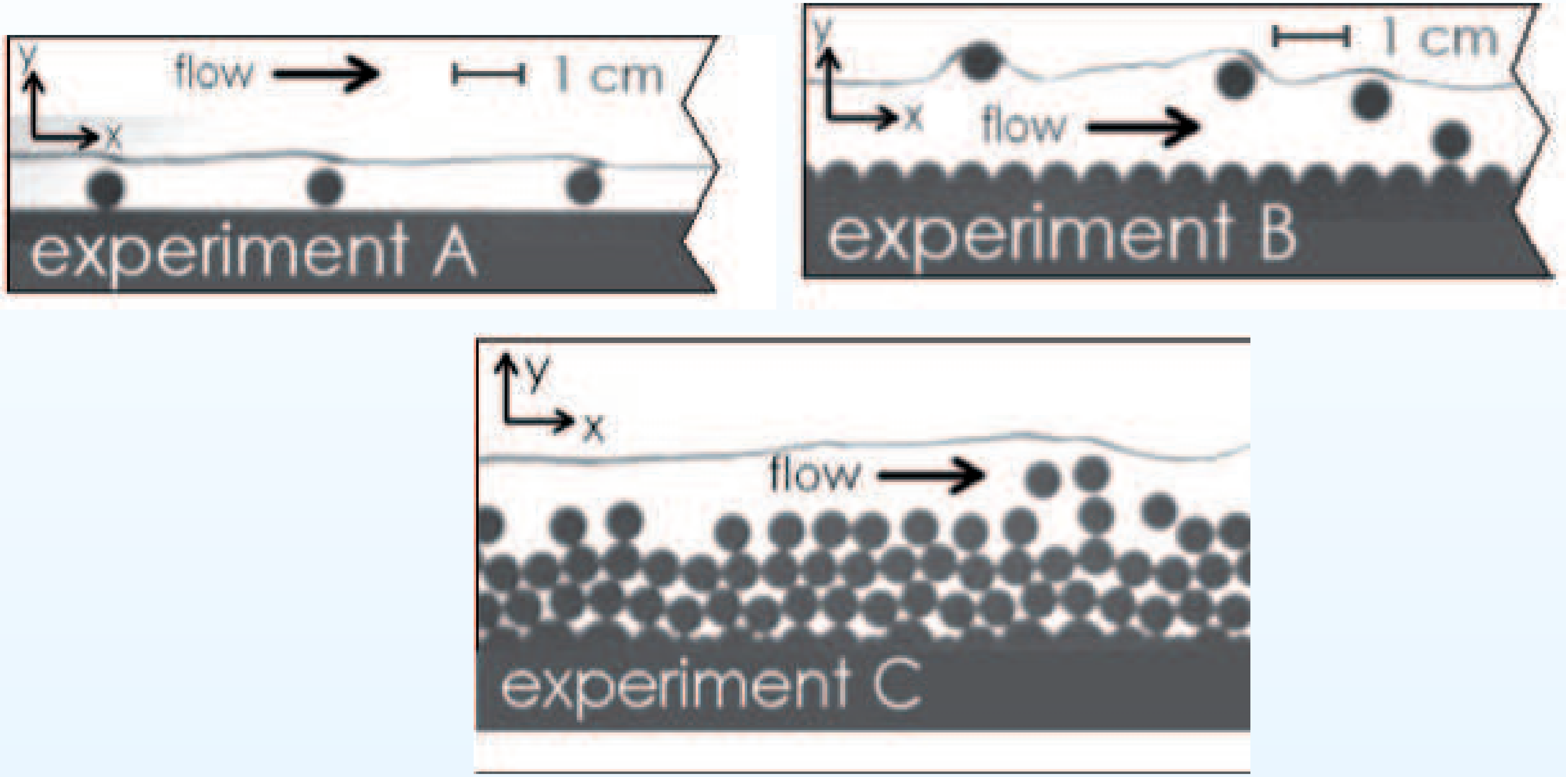
Velocity profile



Saltating contribution

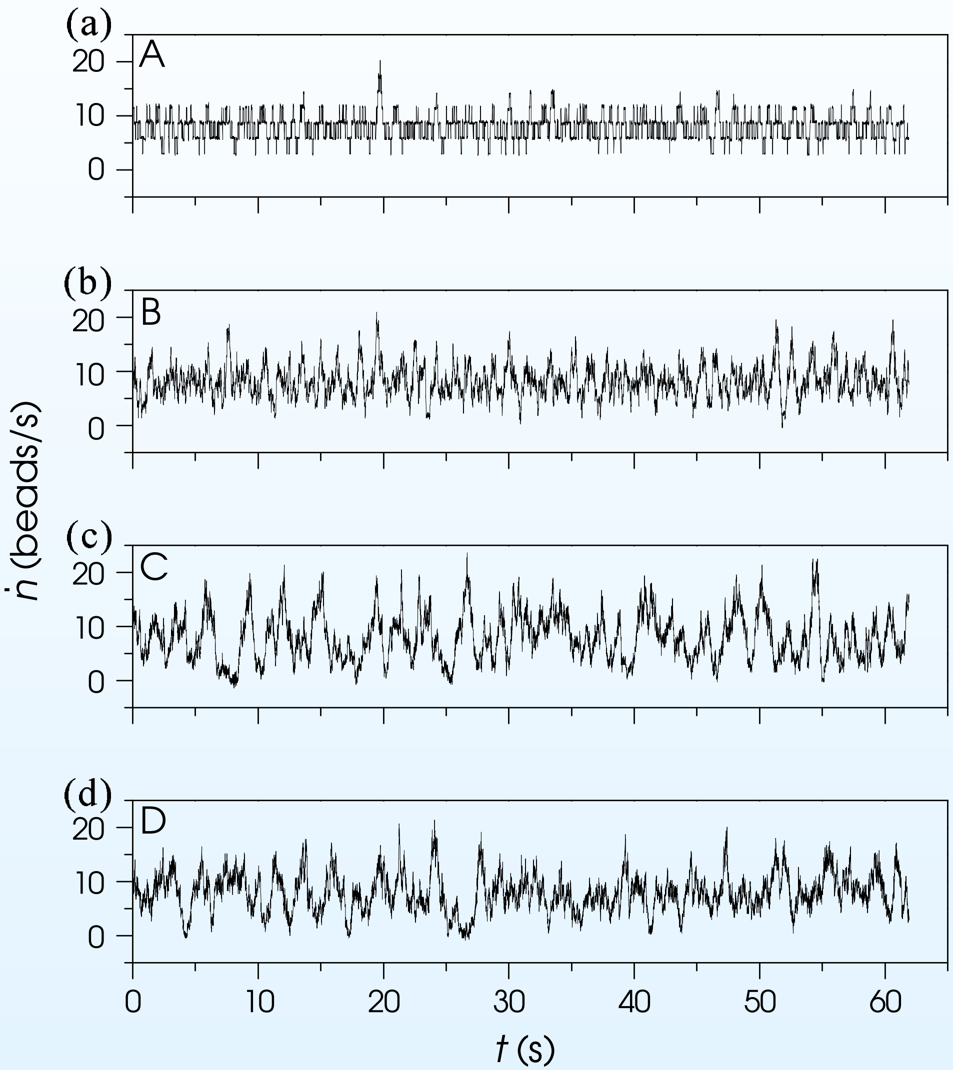
- Hydraulic conditions: flow controlled by bottom for shallow flow ($h/(2a) = O(1)$), otherwise by sidewalls
- When the bed-load discharge is computed with the saltating contribution alone, we retrieve empirical laws.

Mobile and fixed bed



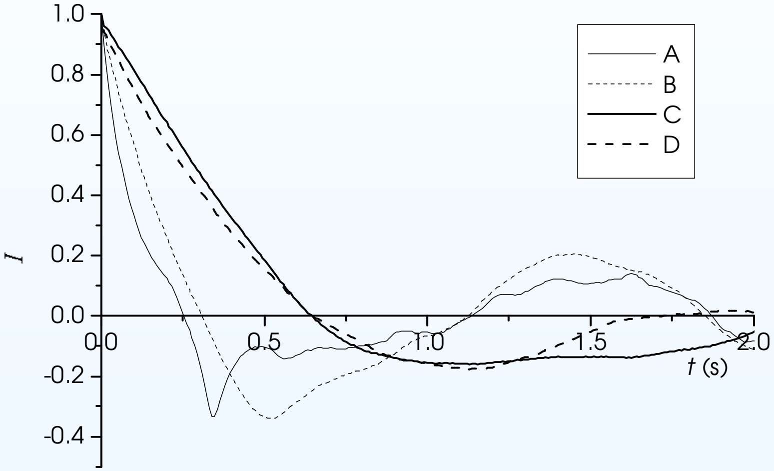
Experiment A: flat bed. Experiment B: corrugated bottom.
Experiment D: disordered mobile bed.

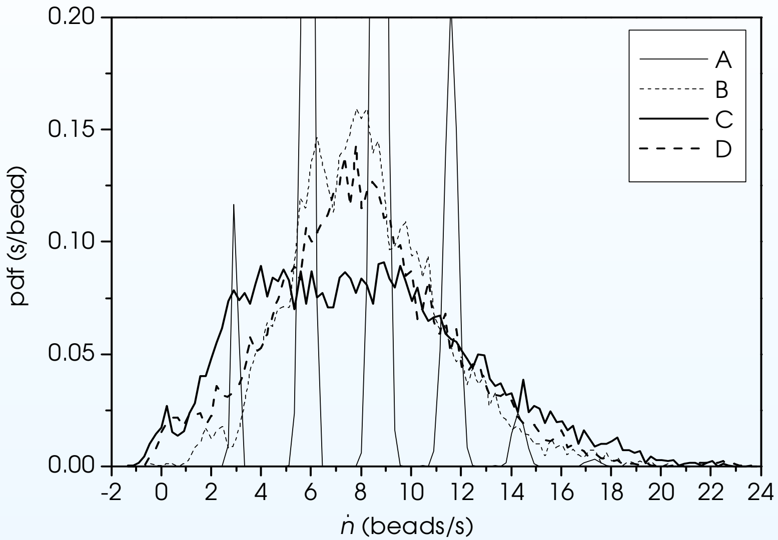
Effect of the bed on the solid discharge



Experiment A: flat bed. Experiment B: corrugated bottom. Experiments C and D: mobile bed.

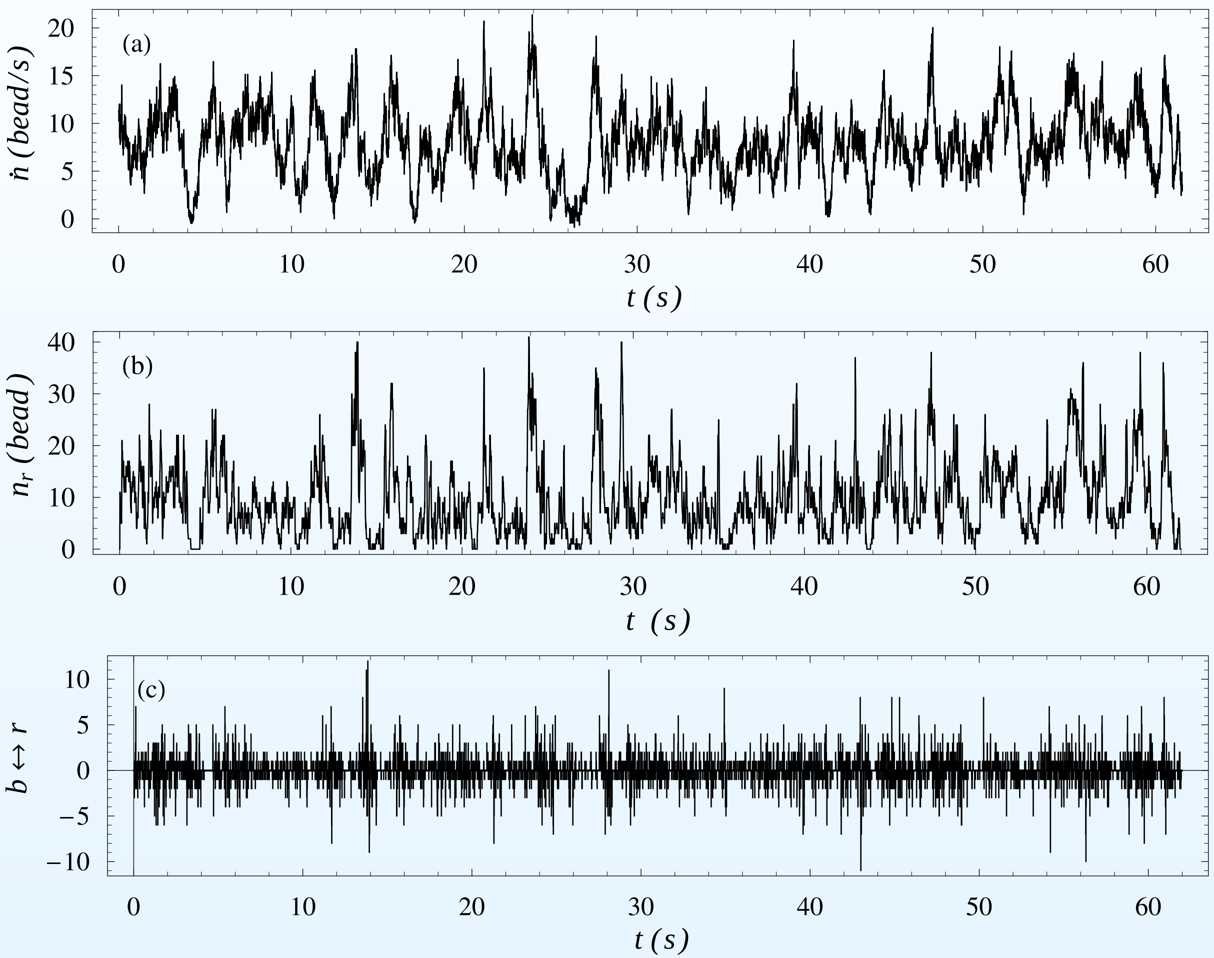
Effect of the bed on the solid discharge (2)



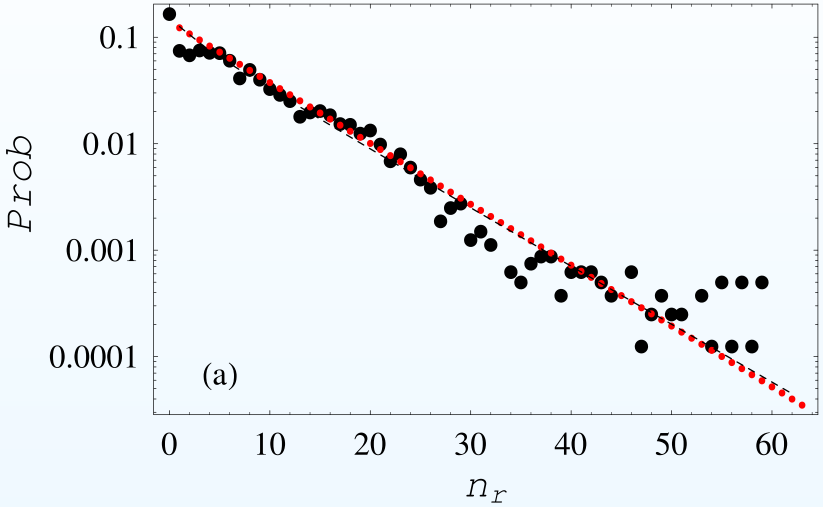


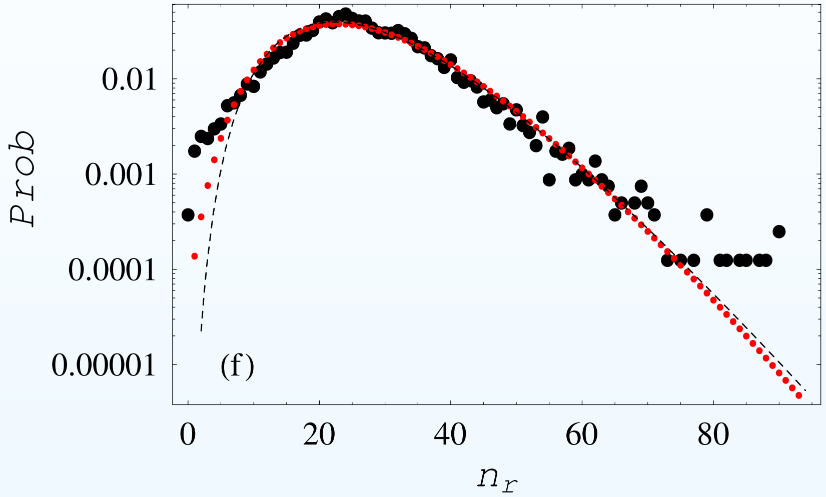
Autocorrelation function and pdf of the solid discharge Experiment A: flat bed. Experiment B: corrugated bottom. Experiments C and D: mobile bed. Exchanges with the bed cause longer autocorrelation time and wider fluctuations.

Times series (experiment C: mobile bed)



Some key observations



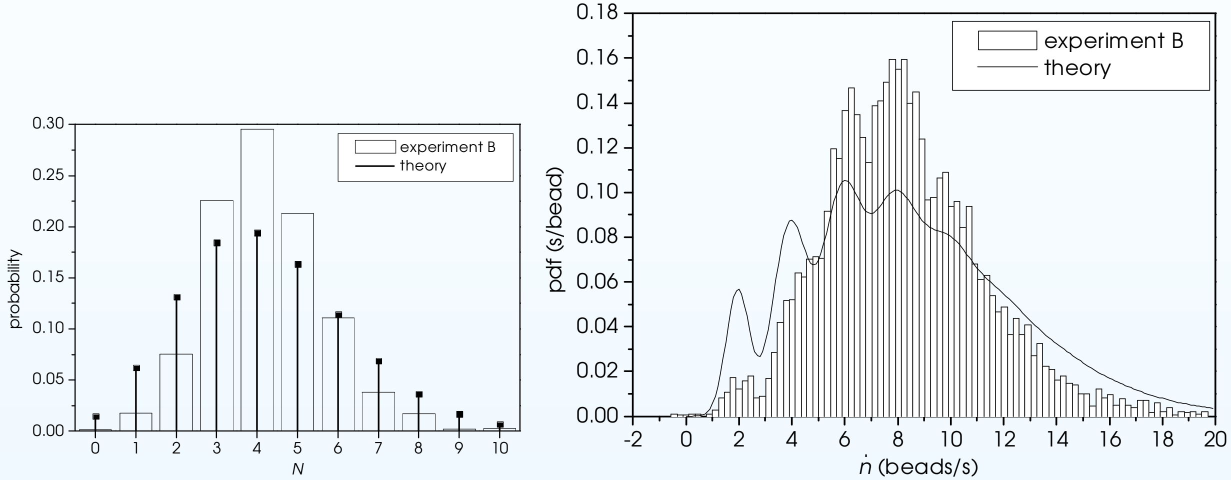


PDF of the solid discharge (low and large water discharges) $\rightsquigarrow$ Negative binomial distribution rather than binomial distribution

- variance $>$ mean $\rightsquigarrow$ wide fluctuations
- long correlation time

$\Rrightarrow$ Existence of long range fluctuations?

Model for stationary beds

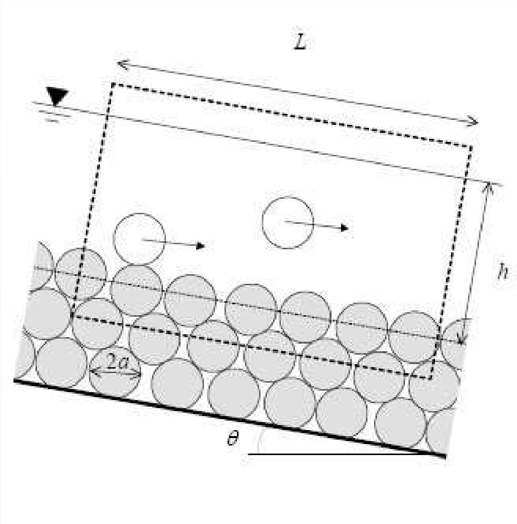


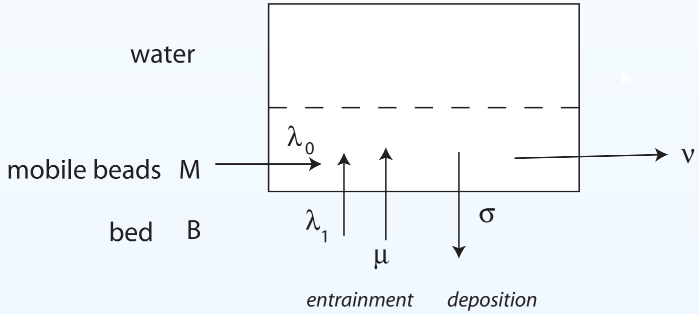
Assumptions: (1) the number of particles included in the observation window is distributed according to a Poisson distribution, (2) the streamwise components of particle velocity is Maxwellian, (3) the particle velocity distribution is independent of the particle number (because flow is dilute).

$$P_{\dot{n}}(\dot{n}|L) = \sum_{k=1}^{\infty} \frac{e^{-\mu}\mu^k}{k!\sqrt{2\pi k}} \frac{L}{\sigma_u} \exp\left[-\frac{(L\dot{n} - k\bar{u})^2}{2k\sigma_u^2}\right]$$

where σ_u^2 is the particle velocity variance (streamwise component), $\bar{u}$ its average, and μ is the average number of particles moving in the observation window.

Model for mobile beds





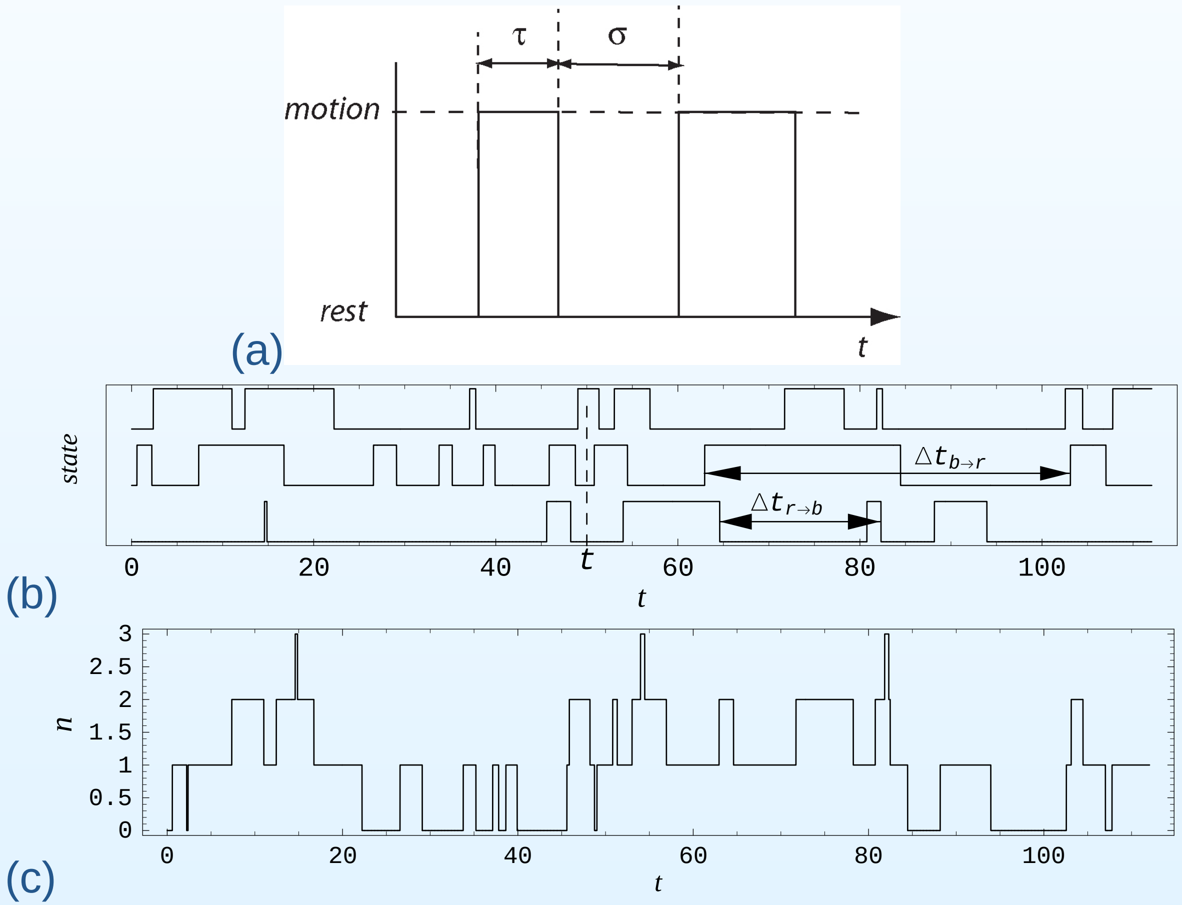
$$q_s = \int_{\mathcal{S}} \int_{\mathbb{R}^2} P[\mathbf{u}_p \mid \mathbf{x},\, t]\, \mathbf{u}_p \cdot \mathbf{k}\, |d\mathbf{x}| d\mathbf{u}_p. \Rightarrow \dot{n} = \frac{q_s}{v_p} = \frac{1}{L} \sum_{i=1}^{N} u_i.$$

q_s: solid discharge per unit width [m^2/s], $\dot{n}$: solid discharge in terms of beads/s, L: window length, v_p: particle volume, N: number of particles within the window, u_i: particle velocity

Model for mobile beds (2): Einstein's theory revisited

Let us consider a closed system with N particles.
(a) Succession of resting ($u_i = 0$) and moving ($u_i > 0$) phases for a single particle.
(b) Superimposition of N telegrapher's processes.
(c) The number of moving particles is computed as the sum of the state variables; the waiting time for a single particle is defined as the time period elapsed between two events of the same type.
Issue: the sum of N Bernoulli variables follows a binomial distribution: no chance to generate thick tails...

$$\dot{n} = \frac{q_s}{v_p} = \frac{1}{L}\sum_{i=1}^{N} u_i$$



Model (3)

To compute N, we consider the following exchanges:

- A moving bead enters the window at rate $\lambda_0 > 0$ from the left (immigration). The corresponding probability of arrival of a particle in the time interval $[t, t + \Delta t)$ is independent of t and N, and we can write

$$P(n \to n + 1; \Delta t) = \lambda_0 \Delta t + o(\Delta t).$$

- Moving beads leave the window independently at rate $\nu > 0$ (emigration). The transition probability is

$$P(n \to n - 1; \Delta t) = n\nu \Delta t + o(\Delta t).$$

Model (4)

- Two processes enable entrainment of particles from the bed (birth): a particle can be dislodged from the bed by the water stream at rate $\lambda_1 > 0$; or a moving bead can destabilize a stationary one and set it moving. This occurs at rate μ for any moving bead within the observation window. The corresponding transition probabilities are respectively

$$P(n \to n+1; \Delta t) = \lambda_1 \Delta t + o(\Delta t), \quad P(n \to n+1; \Delta t) = \mu n \Delta t + o(\Delta t).$$

- A moving bead can settle (i.e., come to rest) within the window, independently at rate σ for each moving bead (death). The transition probability is thus

$$P(n \to n-1; \Delta t) = n\sigma \Delta t + o(\Delta t).$$

Model (5): master equation

We arrive at

$$P(n;t+\Delta t) = \alpha(n+1)\Delta t P(n+1;t) + P(n-1;t)\{\lambda + (n-1)\mu\}\Delta t +$$
$$P(n;t)\{1 - \Delta t(\lambda + n\alpha + n\mu)\} + o(\Delta t),$$

for $n = 1, 2 \ldots$, and

$$P(0;t+\Delta t) = \alpha P(1;t)\Delta t + P(0;t)(1 - \lambda\Delta t) + o(\Delta t),$$

for $n = 0$, with $\alpha = \sigma + \nu$ and $\lambda = \lambda_1 + \lambda_0$. On rearranging the terms and letting $\Delta t \to 0$, we obtain

$$\frac{\partial}{\partial t}P(n;t) = (n+1)\alpha P(n+1;t) + (\lambda + (n-1)\mu)\, P(n-1;t) - (\lambda + n(\alpha + \mu))\, P(n;t),$$

$$\frac{\partial P(0,t)}{\partial t} = \alpha P(1;t) - \lambda P(0;t).$$

Model (6): General solution

We introduce the probability generating function

$$G(z,t) = \sum_{n=0}^{\infty} z^n P(n;t),$$

and find

$$G(z,t) = \left(\frac{\alpha - \mu}{(K\mu - \mu)z + \alpha - K\mu} \right)^{n+\lambda/\mu} \left(\frac{(K\alpha - \mu)z + \alpha(1-K)}{\alpha - \mu} \right)^{n},$$

where $K = \mathrm{e}^{-t(\alpha-\mu)}$ corresponds to the autocorrelation function for flows at equilibrium.

Steady state solution

For steady flow conditions, the number of particles within the observation window forms a stationary random process whose probability distribution

$$P_s(n) = \mathrm{NegBin}(n; r, p) = \frac{\Gamma(r+n)}{\Gamma(r)n!} p^r (1-p)^n, n = 0, 1, \ldots,$$

with $r = \lambda/\mu$ and $p = 1 - \mu/\alpha$, Γ the gamma function. The mean is $\lambda/(\alpha - \mu)$ and the variance is

$$\mathrm{var} N = \frac{\lambda\alpha}{(\alpha-\mu)^2}.$$

For $\mu = 0$, we obtain $G_s(z) = \mathrm{e}^{-\lambda(z-1)/\alpha}$, corresponding to the Poisson distribution of rate $r' = \lambda/\alpha$,

$$P_s(n) = \frac{(r')^n}{n!}\mathrm{e}^{-r'}, \quad n = 0, 1, \ldots.$$

Steady state solution (2)

Under stationary conditions, the autocorrelation function of the number of particles in motion within the window may be written as

$$\operatorname{corr}\{N(0), N(\tau)\} = \rho(\tau) = \frac{\mathbb{E}[N(\tau)N(0)] - \mathbb{E}[N(0)]^2}{\operatorname{Var}(N_0)}, \quad \tau > 0.$$

The mean of $N(\tau)N(0)$ can be expressed as $\mathbb{E}[N(0)\mathbb{E}\{N(\tau)\} \mid N(0)]$, in which the conditional mean of $N(\tau)$ given $N(0)$ appears; this can be obtained from G

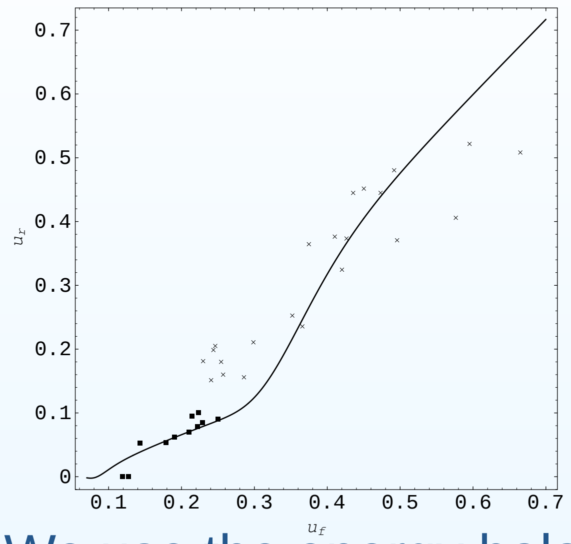
$$\mathbb{E}\{N(\tau) \mid N(0) = n\} = \left(\frac{\partial G}{\partial z}\right)_{z=1} = nK + \lambda \frac{1-K}{\alpha - \mu}.$$

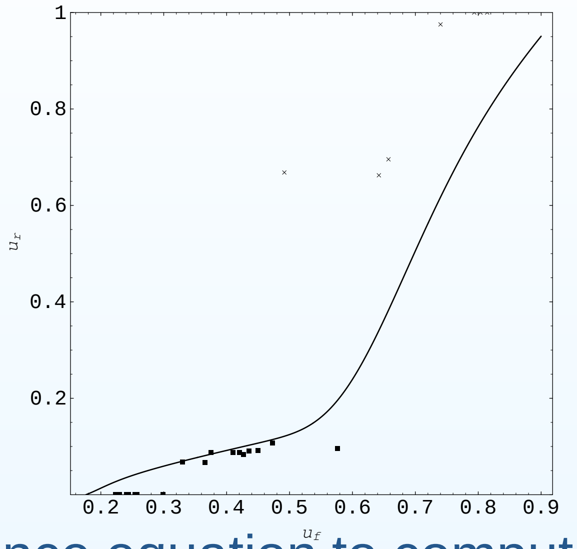
We find that

$$\rho(\tau) = \mathrm{e}^{-\tau/t_c},$$

where $t_c = \frac{1}{\alpha - \mu} > 0$ is the autocorrelation time.

Velocity of a single particle



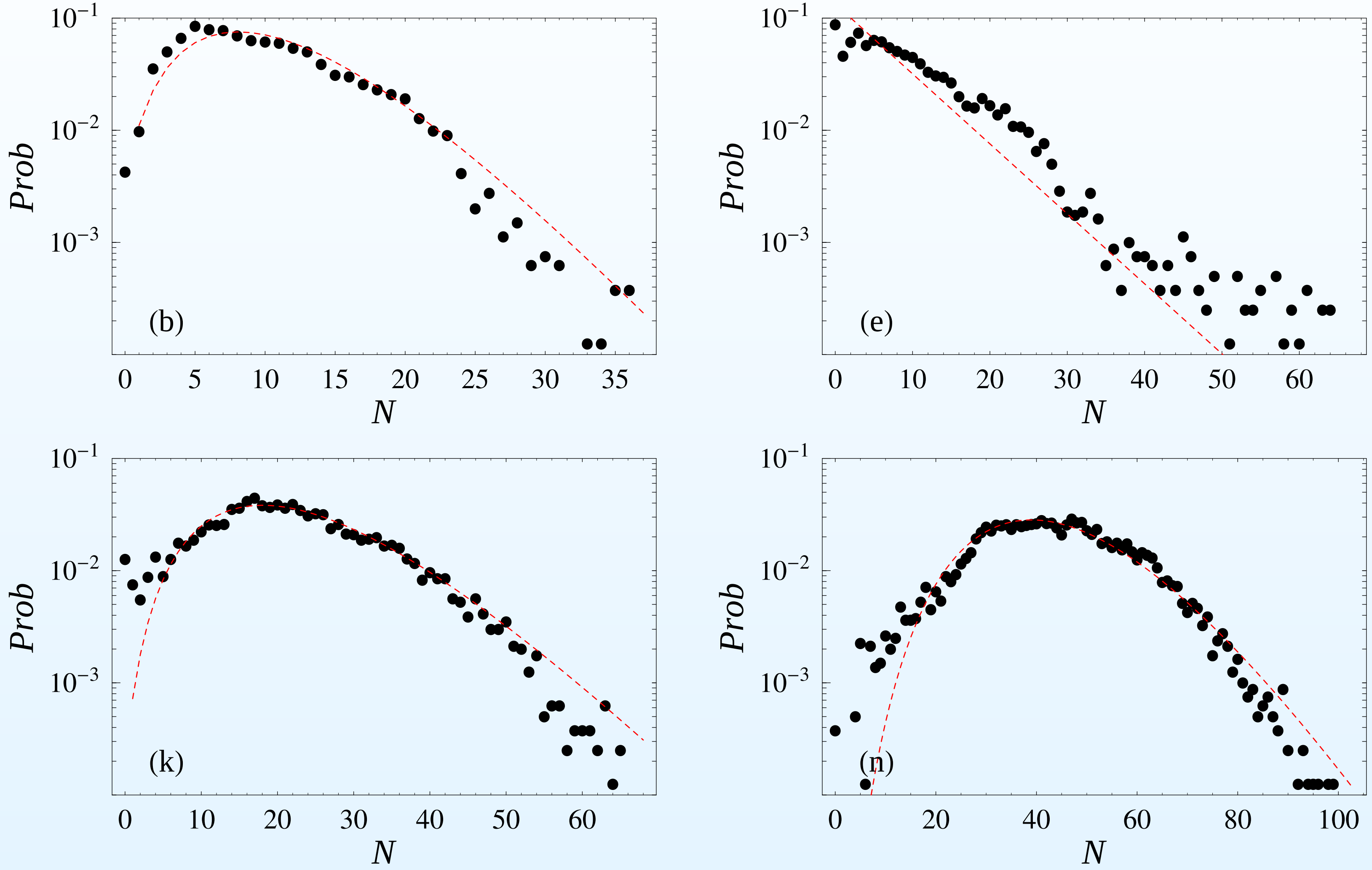


We use the energy balance equation to compute the particle velocity of a single rolling particle,

$$m' g \bar{u}_p \sin\theta + \bar{P}_d = \bar{P}_c,$$

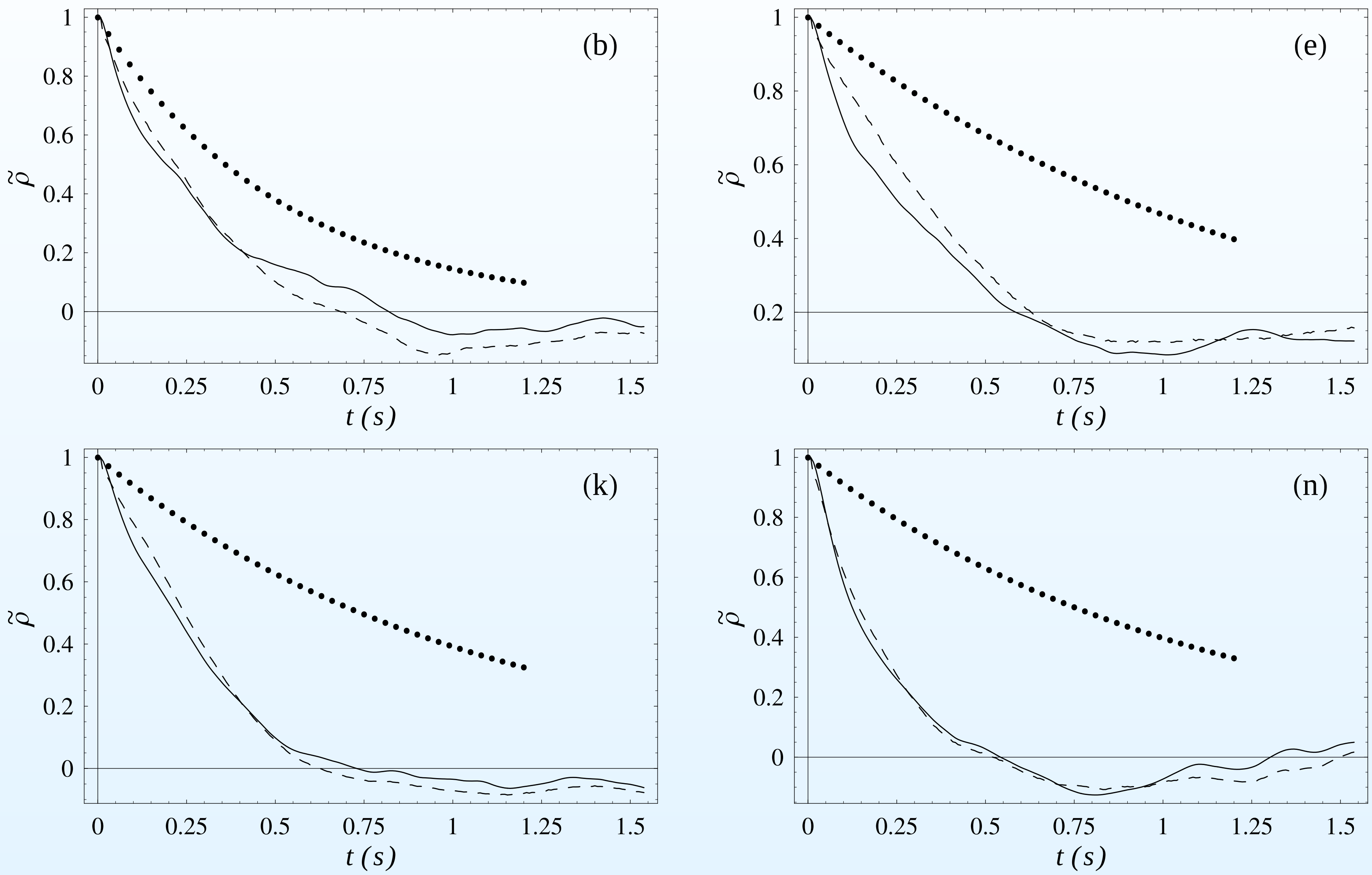
where $m' = m - 4\pi\rho_f a^3/3$ is the buoyant mass, $\bar{P}_d = \bar{\mathbf{F}}_d \cdot \bar{\mathbf{u}}_p$ is the power of drag forces supplied to the particle, where $\bar{\mathbf{F}}_d = C_d \pi a^2 |\mathbf{u}_f - \mathbf{u}_p|(\mathbf{u}_f - \mathbf{u}_p)$ is the drag force, with C_d the drag coefficient.

Comparison with experiment (1)



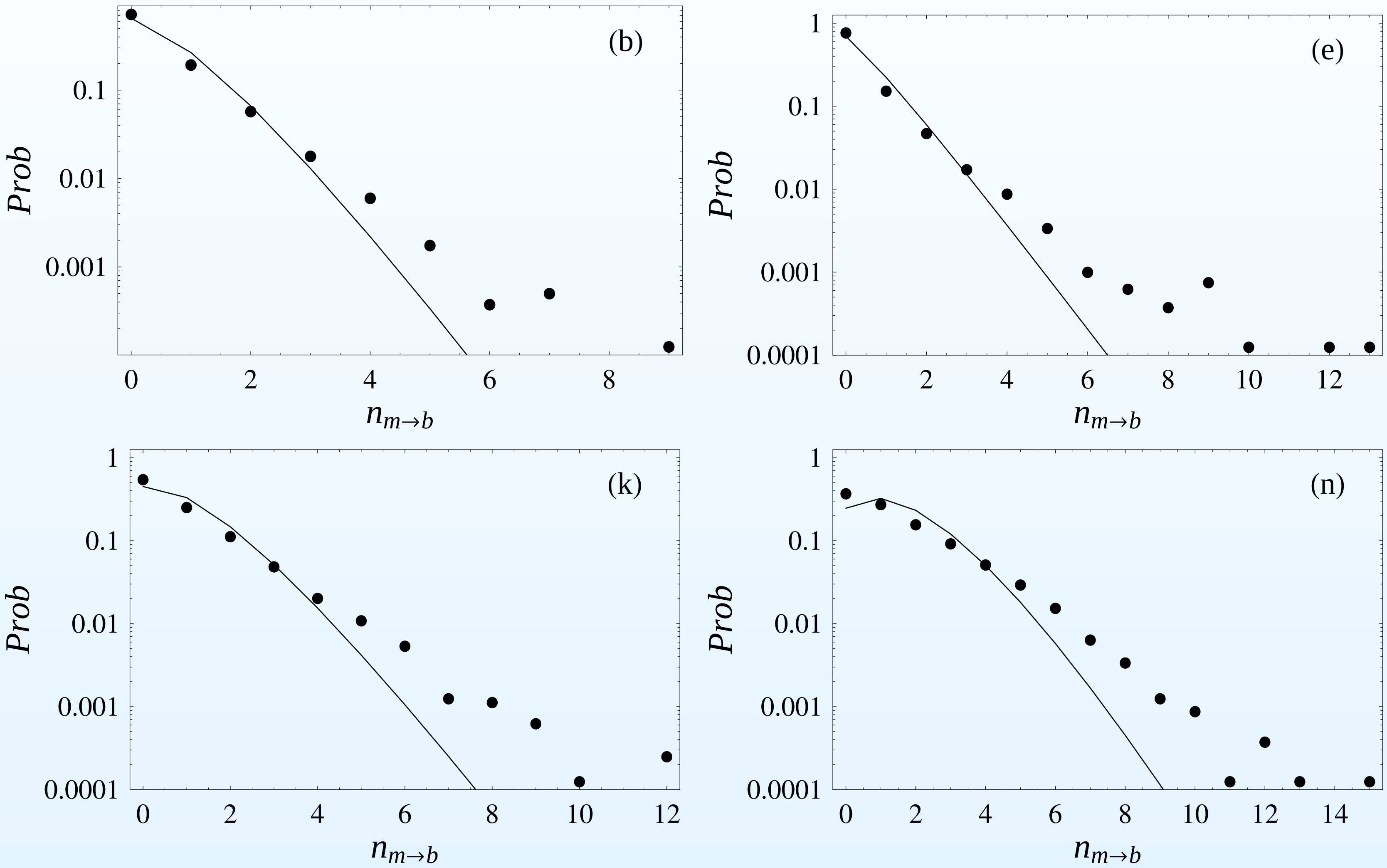
Probability of observing N moving beads in the window

Comparison with experiment (2)



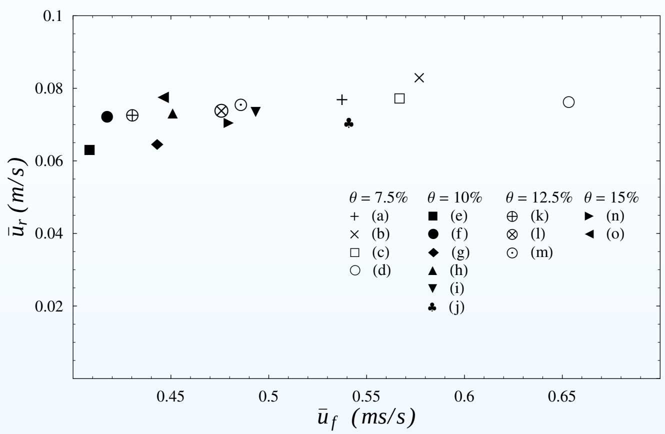
autocorrelation time

Comparison with experiment (3)

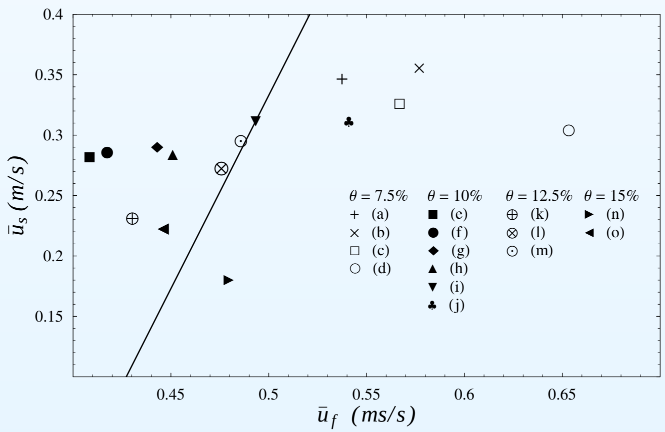


Probability distribution of the number of particles that come to a halt during a time interval δt

Comparison with experiment (4)



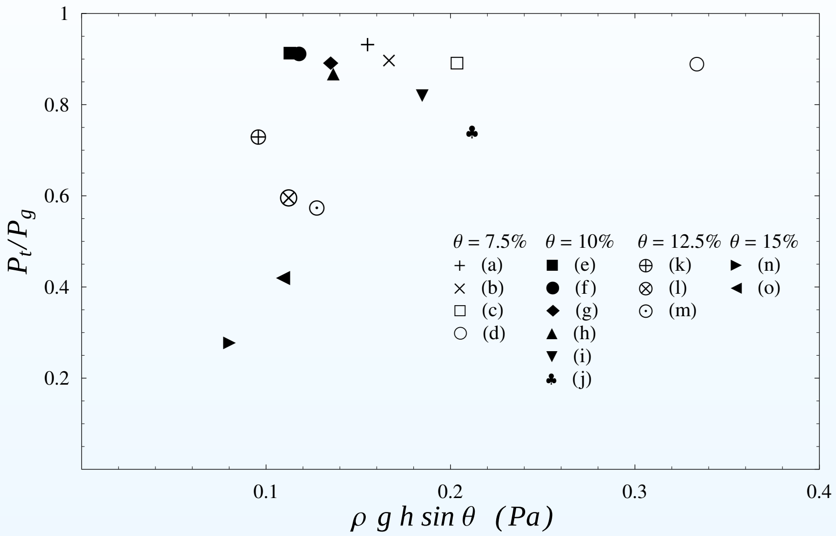
(a)



(b)

Variation of the rolling velocity with the mean fluid velocity. (b) Variation of the saltating velocity with the mean fluid velocity.

Comparison with experiment (4)



power supplied by gravity to the control volume

$$P_g = \int_{\mathcal{V}} \rho \mathbf{g} \cdot \mathbf{u} \mathrm{d}\mathcal{V} = \rho g q_w L \sin\theta,$$

the turbulent power

$$P_t = \rho g q_w L \sin\theta - \bar{N}\bar{F}_D \bar{u}_p.$$

A few words of conclusion

- Idealized hydraulic conditions, but not too unrealistic ;
- Wide fluctuations $\rightsquigarrow$ cooperative aspects between particle, non trivial fluid-solid coupling ;
- Solid discharge: strong dependence on N, weak dependence on u_p ;
- Main shortcoming: autocorrelation time shorter in experiments than in theory.