

Shear strength in one- and two-way slabs according to the Critical Shear Crack Theory

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ABSTRACT: Currently, there is no generally-accepted theory giving a physical explanation of the shear strength in one- and two-way slabs. Furthermore, for members without transverse reinforcement, shear strength is estimated in most codes of practice following empirical or semi-empirical approaches. In this paper, the fundamentals of the Critical Shear Crack Theory (CSCT) are introduced. This theory, based on a mechanical model, is shown to provide a unified approach for one- and two-way shear in slabs, leading to simple design expressions for estimating the strength and deformation capacity of such members. The paper also details a code-like formulation based on this theory and developed for the Swiss code for structural concrete. Comparisons of the theory to a wide range of test data are finally presented.

1 INTRODUCTION

Shear has been investigated in structural concrete from its early developments (Ritter 1899, Morsch 1908) with theoretical and experimental works covering both one-way shear (Fig. 1a) and two-way shear (also known as punching shear, Fig. 1b).

With respect to beams and one-way slabs with transverse reinforcement, several consistent approaches based on the theory of plasticity as strut-and-tie models (Schlaich et al. 1987) or stress fields (Muttoni et al. 1997) have been successfully applied. Also, theories accounting for compatibility conditions as the Modified Compression Field Theory (MCFT, Vecchio and Collins 1986) or the fixed-angle softened-truss model (Pang and Hsu 1996) can also be applied to such members giving accurate estimates of their strength and deformation capacity. On the other hand, scanty rational approaches have been developed in other fields concerning shear:

- With respect to beams and one-way slabs without transverse reinforcement, currently most design approaches are based on empirical models (ACI 2005, Eurocode 2 2004). However, some physically-sound theories like the MCFT have been successfully applied to such members, opening a promising approach for the unified treatment of one-way shear in members with and without transverse reinforcement. Recently, a simplified form of the MCFT has been developed and introduced into the Canadian code for structural concrete CSA (2004).

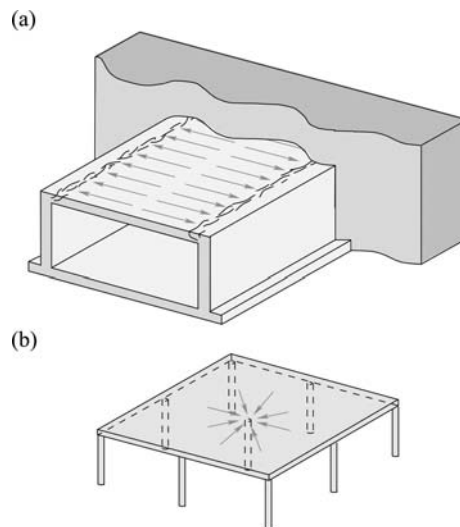


Figure 1. Shear in R/C slabs: (a) one-way shear developing in top slab of a cut-and-cover tunnel; and (b) two-way shear developing in a flat slab supported by columns.

- With respect to punching of slabs without transverse reinforcement, the first rational approach developed with a certain success was that of Kinnunen and Nylander (1960), which inspired the work of other researchers (Hallgren, 1996, Broms, 2006). However, most design approaches are still based

on empirical formulations (ACI 2005, Eurocode 2 2004). Also, none of the aforementioned theories for beams and one-way slabs (Vecchio and Collins 1986, Pang and Hsu 1996) have yet successfully been applied to the problem of punching shear.

- With respect to punching of slabs with transverse reinforcement, most design approaches are based on the empirical formulations adopted for members without transverse reinforcement (reducing the concrete contribution) and adding a contribution of the punching shear reinforcement (ACI 2005, Eurocode 2 2004). The design of such members relies thus mainly on empirical formulations. However, some efforts are currently being devoted to the development of physically-sound models for dimensioning of such members (Birkle, 2004).

In this paper, the theoretical ground of the Critical Shear Crack Theory (CSCT) is presented and discussed. The CSCT is a theory based on a mechanical model that allows treating in a unified manner both shear and punching-shear problems. The application of the CSCT to the design of members without transverse reinforcement (one-way and two-way slabs) is finally presented in this paper, introducing the implementation of this theory into the Swiss code SIA 262 (2003).

The CSCT is also applicable to shear-reinforced members (Fernández Ruiz and Muttoni 2007). However, details on this topic will not be treated within this paper.

2 HISTORICAL DEVELOPMENT OF THE CRITICAL SHEAR CRACK THEORY

The basic principles of the critical shear crack theory were introduced for the first time in the Draft code proposal (Muttoni, 1985) of the Swiss code for structural concrete SIA 162 (1993). An experimental campaign followed (Muttoni and Thürlimann, 1986), where the role of the crack width in the shear cracks developing through the web of beams was investigated. Two works (Muttoni 1989 and Muttoni and Schwartz 1991) summarized the main theoretical findings, and were the basis of the approach for members without transverse reinforcement in the Swiss code for structural concrete SIA 162 in 1993. Further improvements of the theory for shear in one-way slabs and punching shear (Muttoni 2003a, 2003b and 2003c) were recently included in the new version of the Swiss code for structural concrete (SIA 262, 2003), which can be considered to be fully based on this theory for the shear design of members without stirrups.

Currently, the theory continues to be improved and extended. New developments account for the shear strength in beams (Vaz Rodrigues 2007) and slabs

(Guandalini 2005) with plastic strains, the position of the control section for beams under distributed loading (Muttoni 2003b, Muttoni and Fernández Ruiz 2007), analytical formulations of load-rotation relationships for axisymmetric slabs (Muttoni 2007) and the strength of slabs with shear reinforcement (Fernández Ruiz and Muttoni 2007). Also, several experimental campaigns have recently been performed (Guandalini and Muttoni 2004, Vaz Rodrigues 2004, Vaz Rodrigues 2006, Tassinari et al. 2007, Guidotti et al. 2007) allowing to verify several phenomena predicted by the CSCT.

3 THEORETICAL APPROACH OF THE CSCT

The critical shear crack theory is based on the assumption that the shear strength in members without transverse reinforcement, traditionally correlated to the square root of the concrete compressive strength after the works of Moody (1954), is governed by the width and by the roughness of a shear crack which develops through the inclined compression strut carrying shear (Muttoni 2007, Muttoni and Fernández Ruiz 2007), see Figure 2.

This dependence is expressed as follows:

$$\frac{V_R}{b_0 \cdot d} = \sqrt{f_c} \cdot f(w, d_g) \quad (1)$$

where V_R is the shear strength, b_0 is a control perimeter (equal to the width of the member (b) in beams and set at $d/2$ of the border of a column for punching shear), d is the effective depth of the member, f_c is the compressive strength of the concrete, w is the width of the shear critical crack and d_g is the maximum size

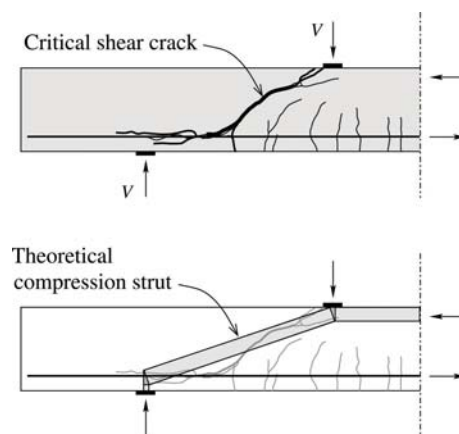


Figure 2. Critical shear crack developing through the shear-carrying compression strut in a simply supported beam.

of the aggregate (accounting for the roughness of the lips of the cracks).

According to the CSCT, the width of the shear critical crack (w) in one way-slabs is assumed to be proportional to the strain in a control depth (ε , at $0.6d$ from the compression face) times the effective depth of the member (see Fig. 3a):

$$w \propto \varepsilon \cdot d \quad (\text{One - way slabs}) \quad (2)$$

In two-way slabs, the width of the critical shear crack is assumed proportional to the slab rotation (ψ) times the effective depth of the member (see Fig. 3b):

$$w \propto \psi \cdot d \quad (\text{Two - way slabs}) \quad (3)$$

Based on these assumptions, the following failure criteria (Muttoni 2003a) have been derived for members without stirrups:

$$\frac{V_R}{b_0 \cdot d \cdot \sqrt{f_c}} = \frac{1/3}{1 + 120 \frac{\varepsilon \cdot d}{d_{g0} + d_g}} \quad (4a)$$

(One - way slabs [SI - units : MPa, mm])

$$\frac{V_R}{b_0 \cdot d \cdot \sqrt{f_c}} = \frac{3/4}{1 + 15 \frac{\psi \cdot d}{d_{g0} + d_g}} \quad (4b)$$

(Two - way slabs [SI - units : MPa, mm])

where d_{g0} is a reference aggregate size equal to 16 mm.

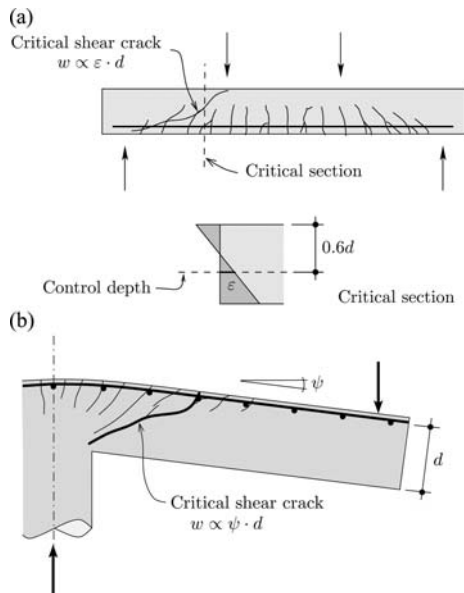


Figure 3. Correlation of the critical shear crack width for: (a) beams and one-way slabs; and (b) slabs.

A comparison of the failure criteria proposed by the CSCT to the test results of 285 beams (Muttoni and Fernández Ruiz 2007) and 99 slabs (Muttoni 2007) are given in Figure 4. The comparison shows an excellent agreement between theory and experiments, with a very small coefficient of variation. Such results are better than those obtained with some codes of practice (ACI 2005, Eurocode 2 2004) as shown in Muttoni and Fernández Ruiz (2007) and Muttoni (2007).

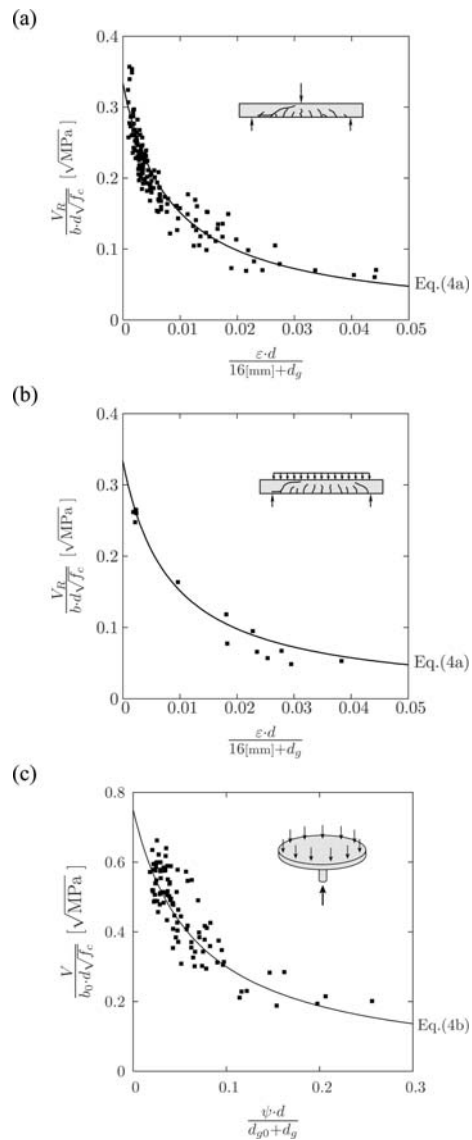


Figure 4. Comparison of the failure criteria (Eq. (4)) to test results of members without transverse reinforcement: (a,b) beams subjected to point load and uniform loading respectively; and (c) slabs subjected to punching shear.

It can be noted that the deformation capacity at ultimate can also be obtained from the failure criteria (expressed by the rotation of the slab for punching and by the strains in the critical section for beams).

4 CODE-LIKE FORMULATION

Adopting some reasonable assumptions (Muttoni and Fernández Ruiz 2007, Muttoni 2007), the previous failure criteria can be used to derive design formulae for members without transverse reinforcement. For beams in shear (Muttoni and Fernández Ruiz 2007), the following design formula results:

$$\frac{V_{Rd}}{b_0 \cdot d \cdot \sqrt{f_{ck}}} = \frac{0.2}{1 + 0.0022 \cdot d \cdot \frac{m_{Ed}}{m_{Rd}}} \quad (5)$$

[SI - units : MPa, mm]

where m_{Ed} is the design bending moment applied at the control section and m_{Rd} is the design flexural strength of the section. Since m_{Ed} depends on the applied shear, the shear strength (V_{Rd}) can be obtained explicitly for instance in members subjected to point loads by solving the resulting quadratic expression. A comparison (Muttoni and Fernández Ruiz 2007) of Eq. (5) to the test results shown in Figures 4a,b leads to an average value of the ratio $V_{test}/V_{CSCT} = 1.09$ with a coefficient of variation of 0.11.

For punching shear, the strength of a slab can be checked using the following design formula:

$$\frac{V_{Rd}}{b_0 \cdot d \cdot \sqrt{f_{ck}}} = \frac{0.2}{0.45 + 0.135 \cdot L \cdot \left(\frac{m_{0d}}{m_{Rd}}\right)^{3/2}} \quad (6)$$

[SI - units : MPa, mm]

where L is the span of the slab and m_{0d} is a reference moment depending on the type and boundary conditions of the column (inner column $m_{0d} = V_d/8$; border column $m_{0d} = V_d/4$ or edge column $m_{0d} = V_d/2$). A comparison (Muttoni 2007) of Eq. (6) to the test results shown in Figure 4c leads to an average value of the ratio $V_{test}/V_{CSCT} = 1.07$ with a coefficient of variation of 0.09.

More details on the implementation of the CSCT to the Swiss code for structural concrete SIA 262 (2003) can be found in references (Muttoni 2003a,b,c).

5 CONCLUSIONS

The main conclusions of this paper are:

1. The Critical Crack Width Theory (CSCT) provides a consistent approach to study the strength of

members subjected to one-way shear or two-way shear (punching).

2. The CSCT can be applied to both members with and without shear reinforcement.
3. Accurate results are obtained with the CSCT when compared to test results with a very small coefficient of variation.
4. Design formulae of the failure criteria, together with some reasonable assumptions for the load-deformation relationship of the members, allow to implement the theory in a code-friendly manner.
5. The current Swiss code for structural concrete SIA 262 (2003) for members without transverse reinforcement is fully based on the design formulations of the CSCT.

NOTATION

The following symbols are used in the paper:

| | |
|---------------|---|
| E_s | = modulus of elasticity of reinforcement |
| L | = main span of a slab system |
| V | = shear force |
| V_d | = factored shear force |
| V_R | = nominal punching shear strength |
| V_{Rd} | = design punching shear strength |
| V_{CSCT} | = shear strength according to the CSCT |
| V_{test} | = measured shear strength |
| b_0 | = perimeter of the critical section for punching shear |
| b | = width of a beam |
| d | = distance from extreme compression fibre to the centroid of the longitudinal tensile reinforcement |
| d_g | = maximum diameter of the aggregate |
| d_{g0} | = reference aggregate size (16 mm (0.63 in)) |
| f_c | = average compressive strength of concrete (cylinder) |
| f_{ck} | = design value of the compressive strength of concrete (cylinder) |
| f_{yd} | = factored value of the yield strength of reinforcement |
| m_{Ed} | = applied moment capacity at section of control |
| m_{0d} | = reference moment depending on the type of the column |
| m_{Rd} | = design moment capacity per unit width |
| w | = width of the critical shear crack |
| ε | = reference strain in beams taken at $0.6 \cdot d$ from the compression face |
| ψ | = rotation of slab outside the column region |

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