LRP 662/00 February 2000

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H. Müller, A.A. Howling, Ch. Hollenstein

submitted for publication in
J. Physics D: Appl. Phys.

ISSN 0458-5895

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H. Müller, A.A. Howling, Ch. Hollenstein

Centre de Recherches en Physique des Plasmas, Ecole Polytechnique Fédérale de Lausanne,

PPB-Ecublens, CH-1015 Lausanne, Switzerland

Abstract

In the high voltage, capacitive sheath of a gas discharge driven by a rf current, the ions are periodically screened by the electrons. For a square wave discharge current, an equation describing the time-averaged charge screening is derived: $(n_i - \bar{n}_e)/n_i = \left(\bar{E}/\bar{E}|_{el}\right)^{1/2}$.

n_i is the ion density, $\bar{n}_e$ is the time-averaged electron density; $\bar{E}$ is the time-averaged electric field; the index 'el' designates the electrode position.

By combining the new charge screening equation with established rf sheath equations, the spatial dependences of the time-averaged electric potential, electric field, net charge carrier density, and of the ion density, are determined. It is found that these parameters are related to the distance from the plasma / sheath edge by power laws.

For collisionless ion motion through the rf sheath, the respective exponents are

3/2, 1/2, –1/2, and –3/4 (collisionless dc sheath : 4/3, 1/3, –2/3, –2/3).

For highly collisional ion motion through the rf sheath, the respective exponents are

2, 1, 0, and –1/2 (collisional dc sheath : 5/3, 2/3, –1/3, –1/3).

The time-averaged rf sheath is found to be little dependent on the waveform of the discharge current: for sinusoidal current, the above power laws are still approximately valid.

I. Introduction

Capacitive rf gas discharges are widely used for material surface processing (sputtering, etching, film deposition) eg in the semiconductor industry. Gas discharge models, ie models for the bulk plasma and for the sheaths between the bulk plasma and the electrodes, are therefore of practical interest.

Lieberman analyzed the rf sheath model used in this report, see section **II**, for a sinusoidal rf discharge current, and calculated complicated formulas for the spatial dependences of the time-averaged electric potential $\bar{\Phi}$, time-averaged electric field $\bar{E}$, time-averaged net charge carrier density $\bar{n} = n_i - \bar{n}_e$, and of the ion density n_i in the sheath [**1, 2**]. In this report, we clarify the character of the spatial dependences of these parameters, by assuming a square wave, instead of sinusoidal, rf discharge current.

In section **III,** we derive a new equation describing the ion screening by the electrons in the rf sheath. In section **IV**, we choose equations of ion motion, according to the ion collisionality in the sheath. In section **V**, we combine the charge screening equation and the equation of ion motion, to derive power laws for the spatial dependences of $\bar{\Phi}$, $\bar{E}$, $\bar{n}$ and n_i. In section **VI,** the validity of the rf sheath results is discussed.

II. Model

The high voltage capacitive rf sheath is shown schematically in fig. **1**. The total current per unit area J_t is positive when flowing to the left. The electron boundary at the distance s from the plasma / sheath edge then moves to the right. In our model, the ion density diverges at the plasma / sheath edge : a finite ion current per unit area $J_i = en_i v_i$ enters the sheath with velocity $v_i = 0$ (**A4**). We let the electron boundary, at the rf phase angle $\omega t = 0$, connect with the plasma / sheath edge, $s = 0$, and at $\omega t = \pm\pi$, connect with the electrode, $s = d$.

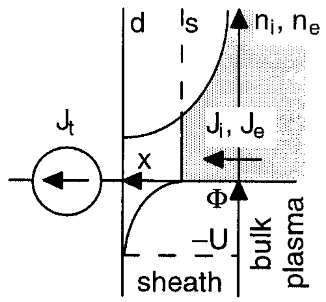

figure 1:

Schematic representation of the high voltage, capacitive rf sheath.

The assumptions of the rf sheath model are :

A1 The ion motion in the sheath is determined by the time-averaged electric field.

A2 The electrons are inertialess, ie they respond immediately to the electric field.

A3 The electric field at the plasma / sheath edge is negligible : $\bar{E}(x = 0) \rightarrow 0$.

A4 The ions enter the sheath with negligible velocity : $v_i(x = 0) = 0$.

A5 The Debye length is much shorter than the sheath width : $\lambda_D/d \rightarrow 0$; the electron density in the sheath is discontinuous : $n_e = n_i$ for $x < s$, $n_e = 0$ for $x > s$ (see fig. 1).

A6 The ion current is much smaller than the total current : $J_i << J_t$.

A7 Ionization in the sheath is negligible, ie the ion current is conserved.

We note :

- **A1, A2** are associated to 'capacitive rf sheath' : ion plasma angular frequency ω_{pi} < discharge fundamental angular frequency ω < electron plasma angular frequency ω_{pe}.
- **A3, A4, A5** are associated to 'high voltage rf sheath' : electron temperature T_e << sheath voltage U.
- For $x < s$ the total current J_t is composed of electron current J_e and ion current J_i (see fig. 1). **A6** allows to relate the electron motion in the sheath to the total current (ie the discharge current) alone : $J_e \cong J_t$.

III. Charge Screening in the rf Sheath

The net charge carrier density $\overline{n}$ and the electric field $\overline{E}$ are related by Maxwell's equation

$$\overline{n} = \frac{\varepsilon_0}{e}\frac{d\overline{E}}{dx} \qquad (1)$$

The total current per unit area $J_t(t)$ drives the electrons through the ion distribution in the sheath, and thereby determines the movement of the electron boundary at the distance $s(t)$ from the plasma / sheath edge :

$$en_i(s)\frac{ds}{dt} = -J_t \qquad (2)$$

The ratio, time-averaged charge carrier density $\overline{n}$ to ion density n_i is equal to the time per rf cycle when the ions are not screened by the electrons. In the phase angle interval $-\pi < \omega t < \pi$,

$$\left(\frac{\overline{n}}{n_{\mathrm{i}}}\right)(s)=\frac{|\omega t|}{\pi} \quad (3)$$

Differentiating (**3**) with respect to s, and using (**2**), we obtain

$$\frac{d}{ds}\left(\frac{\overline{n}}{n_{\mathrm{i}}}\right)=-\mathrm{sign}(t)\frac{\omega}{\pi}\frac{en_{\mathrm{i}}(s)}{J_{\mathrm{t}}} \quad (4)$$

From (**1**) evaluated at $x = s$, (**3**), and (**4**) follows, for $-\pi < \omega t < \pi$,

$$\frac{d}{ds}\left(\frac{\overline{n}}{n_{\mathrm{i}}}\right)^2=-\mathrm{sign}(t)\frac{2\omega}{\pi}\frac{\varepsilon_0}{J_{\mathrm{t}}}\frac{d\overline{E}}{ds} \quad (5)$$

For a general, periodic time-dependence of the total current J_{t}, we cannot integrate (**5**).

We assume now a square wave total current; for $-\pi < \omega t < \pi$, we set

$$J_{\mathrm{t}}=-|J_{\mathrm{t}}|\,\mathrm{sign}(t) \quad (6)$$

with $|J_{\mathrm{t}}|$ a constant.

We insert (**6**) into (**5**); $s(t)$ alone remains time-dependent, and can be replaced by x :

$$\frac{d}{dx}\left(\frac{\overline{n}}{n_{\mathrm{i}}}\right)^2=\frac{2\omega}{\pi}\frac{\varepsilon_0}{|J_{\mathrm{t}}|}\frac{d\overline{E}}{dx} \quad (7)$$

We integrate (**7**) using the boundary conditions $(\overline{n}/n_{\mathrm{i}})(0)=0$ (from (**3**) evaluated at $\omega t = 0$) and $\overline{E}(0) = 0$ (**A3**) :

$$\left(\frac{\overline{n}}{n_{\mathrm{i}}}\right)^2 = \varepsilon_0 \frac{\omega}{\frac{\pi}{2}|J_{\mathrm{t}}|}\overline{E} \qquad (8)$$

From (**8**) evaluated at $x = d$, and $(\overline{n}/n_{\mathrm{i}})(d)=1$ (from (**3**) evaluated at $\omega t = \pm\pi$) follows

$$\overline{E}(d)=\frac{1}{\varepsilon_0}\frac{\frac{\pi}{2}|J_{\mathrm{t}}|}{\omega} \qquad (9)$$

(**9**) can be easily understood: $\frac{\pi}{2}|J_{\mathrm{t}}|/\omega$ is the time-averaged sheath charge per unit area; the outflowing electric field is directed away from the bulk plasma, towards the electrode.

Inserting (**9**) into (**8**), and using (**1**), we obtain an equation describing the screening of the ions by the electrons :

$$\frac{\overline{n}}{n_{\mathrm{i}}}=\sqrt{\frac{\overline{E}}{\overline{E}(d)}} \qquad (10)$$

In the abstract we have written (**10**) using $n_{\mathrm{i}} - \overline{n_{\mathrm{e}}}$ instead of $\overline{n}$, and $\overline{E}|_{\mathrm{el}}$ instead of $\overline{E}(d)$.

We note that in this section we have not used assumption **A7** : the charge screening equation (**10**) is valid regardless of ionization in the sheath.

IV. Ion Motion through the Sheath

The ion motion depends on the type and number of ion collisions in the sheath. There are equations of ion motion which cover the whole range of collisionality in the sheath, from collisionless to highly collisional motion [3]. The results in section **V** are simple only for the two equations of ion motion for the aforesaid limiting cases.

Collisionless ion motion is determined by ion energy conservation (**A1, A4, A7**) :

$$\frac{m_i}{e}\frac{v_i^2}{2} = -\overline{\Phi} \quad (11a)$$

From the ion mobility law for fast, collisional ion motion [**4a**] follows (**A1**) :

$$\frac{m_i}{e}\frac{v_i^2}{\frac{2}{\pi}\lambda_i} = \overline{E} \quad (11b)$$

To eliminate the ion velocity, we insert (**11**) into the definition of the ion current

$$J_i = en_i v_i \quad (12)$$

and obtain what we will call the ion current equation :

collisionless rf sheath

$$J_i = en_i\sqrt{\frac{2e\left(-\overline{\Phi}\right)}{m_i}} \quad (13a)$$

collisional rf sheath

$$J_i = en_i\sqrt{\tfrac{2}{\pi}\lambda_i\frac{e\overline{E}}{m_i}} \quad (13b)$$

We note that in this section we have used assumption **A7** only for the derivation of (**13a**). The ion current equation (**13b**) for the highly collisional rf sheath is valid regardless of ionization in the sheath.

V. The Time-Averaged rf Sheath

In this section, we combine Maxwell's equation (**1**), the sheath charge equation (**9**), the charge screening equation (**10**), the ion current equation (**13**), and the definition of the electrostatic potential

$$\overline{E} = \frac{d(-\overline{\Phi})}{dx} \quad (14)$$

to obtain the solution of the time-averaged rf sheath, ie $\overline{\Phi}(x)$, $\overline{E}(x)$, $\overline{n}(x)$, $n_i(x)$, for the square wave discharge current.

For $\overline{\Phi}(x)$ we make the power-law ansatz

$$(-\overline{\Phi}) = \overline{U}\left(\frac{x}{d}\right)^{\alpha} \quad (15)$$

Inserting (**15**) into the definition of the electrostatic potential (**14**), we obtain $\overline{E}(x)$:

$$\overline{E} = \alpha\frac{\overline{U}}{d}\left(\frac{x}{d}\right)^{\alpha-1} \quad (16)$$

Inserting (**16**) into Maxwell's equation (**1**), we obtain $\overline{n}(x)$:

$$\overline{n} = \alpha(\alpha-1)\frac{\varepsilon_0}{e}\frac{\overline{U}}{d^2}\left(\frac{x}{d}\right)^{\alpha-2} \quad (17)$$

Inserting (**16**), (**17**) into the charge screening equation (**10**), we obtain $n_{\mathrm{i}}(x)$:

$$n_{\mathrm{i}} = \alpha(\alpha-1)\frac{\varepsilon_0}{e}\frac{\overline{U}}{d^2}\left(\frac{x}{d}\right)^{\frac{\alpha-3}{2}} \qquad (18)$$

Inserting (**16**) into the sheath charge equation (**9**), we obtain the amplitude of the square wave total current, $|J_{\mathrm{t}}|$, to which we keep associated the factor $\pi/2$:

$$\tfrac{\pi}{2}|J_{\mathrm{t}}| = \alpha\varepsilon_0\omega\frac{\overline{U}}{d} \qquad (19)$$

Inserting (**15**), (**16**), (**18**) into the ion current equation (**13**), we obtain the ion current at the electrode :

collisionless rf sheath

$$J_{\mathrm{i}}(d) = \alpha(\alpha-1)\varepsilon_0\sqrt{\frac{2e}{m_{\mathrm{i}}}}\frac{\overline{U}^{3/2}}{d^2} \qquad (20.1\mathrm{a})$$

collisional rf sheath

$$J_{\mathrm{i}}(d) = \alpha(\alpha-1)\sqrt{\frac{\alpha}{2}}\varepsilon_0\sqrt{\frac{2e}{m_{\mathrm{i}}}\tfrac{2}{\pi}\lambda_{\mathrm{i}}}\frac{\overline{U}^{3/2}}{d^{5/2}} \qquad (20.1\mathrm{b})$$

and the spatial dependence of the ion current :

collisionless rf sheath

$$\frac{J_{\mathrm{i}}(x)}{J_{\mathrm{i}}(d)} = \left(\frac{x}{d}\right)^{\alpha-3/2} \qquad (20.2\mathrm{a})$$

collisional rf sheath

$$\frac{J_{\mathrm{i}}(x)}{J_{\mathrm{i}}(d)} = \left(\frac{x}{d}\right)^{\alpha-2} \qquad (20.2\mathrm{b})$$

To close the problem, we must decide on the spatial dependence of the ion current in the rf sheath. In the case of the collisionless rf sheath, we were already forced in section IV to make up our minds; for simplicity we chose ion current conservation **A7**. We apply **A7** now also to the collisional rf sheath, and obtain the value of the power α:

collisionless rf sheath		collisional rf sheath	
$\alpha = 3/2$	(21a)	$\alpha = 2$	(21b)

Equations (**15**) – (**21**) are the full solution of the time-averaged, high voltage, capacitive rf sheath driven by a square wave current.

(**15**) – (**18**), (**21**) relate $\bar{\Phi}(x)$, $\bar{E}(x)$, $\overline{n}(x)$, $n_i(x)$ to $\bar{U}$, d.

(**19**), (**20**), (**21**) relate $\bar{U}$, d to J_i, $|J_t|$.

In practice the only free rf discharge parameter is the total current amplitude $|J_t|$. The ion current J_i follows from the physics of the entire discharge (sheaths and bulk plasma) [**4b**].

VI. Discussion

In this section, all rf sheath results for the square wave discharge current are taken from the previous sections in this report, and all rf sheath results for the sinusoidal discharge current are taken from refs. [**1,2**]; there the same rf sheath model is used.

VI.1 Validity of the Charge Screening Equation for Sinusoidal rf Discharge Current

The charge screening equation (**10**) was derived for the square wave discharge current (**6**). To test the validity of (**10**) for the sinusoidal discharge current

$$J_{\mathrm{t}} = -\tilde{J}_{\mathrm{t}} \sin(\omega t) \qquad (22)$$

we define the number density $n_{\mathrm{i}}^* = \bar{n} \times (\bar{E}(d)/\bar{E})^{1/2}$. (**10**) says, $n_{\mathrm{i}} = n_{\mathrm{i}}^*$ for the square wave discharge current. In fig. **2**, we compare n_{i}^* with n_{i} and $\bar{n}$ for the sinusoidal discharge current; we find, $n_{\mathrm{i}}^* \cong n_{\mathrm{i}}$, ie charge screening is still approximately described by (**10**).

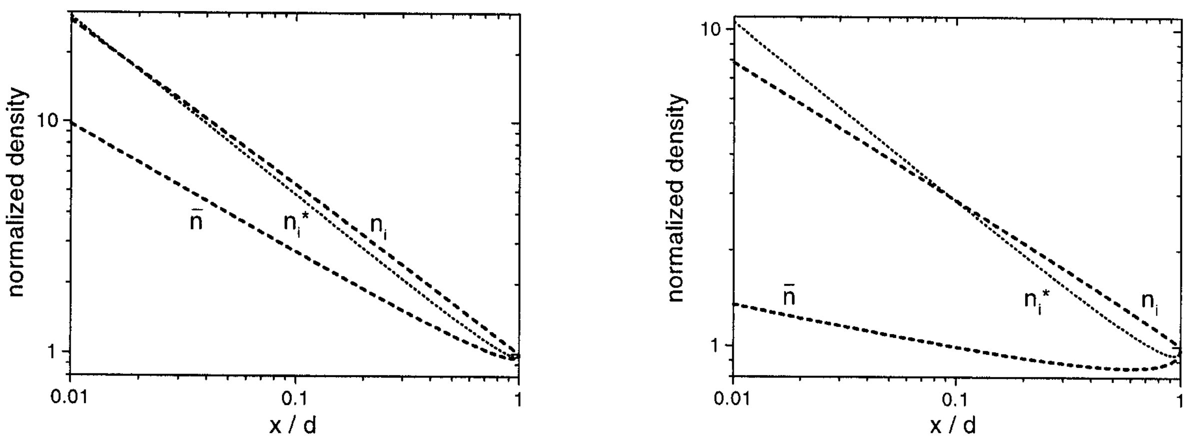

figure 2a : collisionless rf sheath *figure 2b : collisional rf sheath*

Time-averaged net charge carrier density $\bar{n}$, ion density n_i (both dashed), and $n_{\mathrm{i}}^* = \bar{n} \times (\bar{E}(d)/\bar{E})^{1/2}$ (short dashed) versus distance x from the plasma / sheath edge, for sinusoidal rf discharge current. $\bar{n}$, n_i, n_i^* are normalized to their values at the electrode, x is normalized to the sheath width d.

VI.2 Comparison of rf Sheath Results for Square Wave and Sinusoidal rf Discharge Current

Equations (**1**), (**9**), (**10**), (**13**), (**14**) determine the time-averaged rf sheath driven by the square wave current (**6**). In fact, (**1**), (**13**), (**14**) are valid regardless of the current waveform, and (**9**) is valid for the sinusoidal current (**21**) when $\frac{\pi}{2}|J_t|$ is replaced by $\tilde{J}_t$ (because $\frac{\pi}{2}|J_t|/\omega$ for square wave current, and $\tilde{J}_t/\omega$ for sinusoidal current, are both the time-averaged sheath charge). The charge screening equation (**10**) is the only rf sheath equation that is valid only for a square wave discharge current. However, we just showed that (**10**) is at least approximately valid for a sinusoidal discharge current. We expect therefore that the time-averaged rf sheath for square wave and for sinusoidal discharge current are similar, for equal ion current and equal sheath charge. To test this assumption, we compare the spatial dependences of n_i and $\bar{n}$, and the rf Child laws, for the two current waveforms.

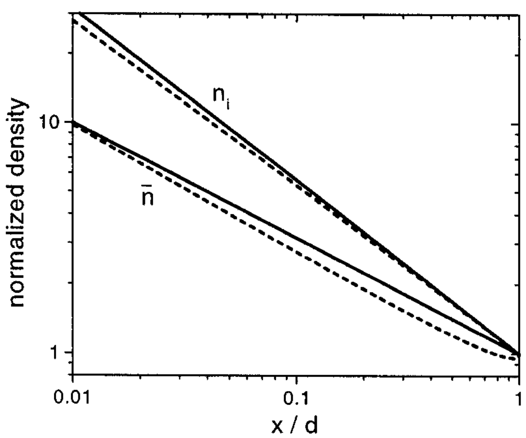

figure 3a : collisionless rf sheath

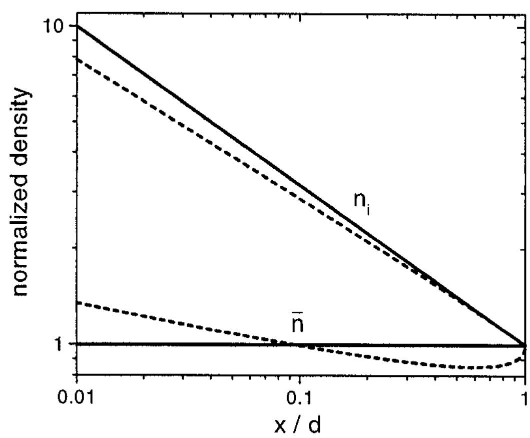

figure 3b : collisional rf sheath

Time-averaged net charge carrier density $\bar{n}$ and ion density n_i, versus distance x from the plasma / sheath edge, for square wave (solid) and sinusoidal (dashed) rf discharge current. $\bar{n}$, n_i are normalized to their values at the electrode, x is normalized to the sheath width d.

In fig. **3** we find indeed that the spatial dependence of $\bar{n}$, n_i is similar for square wave and sinusoidal discharge current.

The rf Child law (**23**), too, is almost identical for square wave and sinusoidal total current; only the numerical constant is slightly different :

collisionless rf sheath

$$J_i = \left.\begin{matrix}(\text{square:})\ 3/4 \\ (\text{sinus:})\ \ 0.82\end{matrix}\right\}\varepsilon_0\sqrt{\frac{2e}{m_i}}\frac{\overline{U}^{3/2}}{d^2} \quad (23a)$$

collisional rf sheath

$$J_i = \left.\begin{matrix}(\text{square:})\ 2 \\ (\text{sinus:})\ \ 2.10\end{matrix}\right\}\varepsilon_0\sqrt{\frac{2\lambda_i}{\pi}\frac{2e}{m_i}\frac{\overline{U}^{3/2}}{d^{5/2}}} \quad (23b)$$

In [**5**] the rf sheath solutions for square wave and sinusoidal rf discharge current are compared in greater detail.

VI.3 Comparison of the Time-Averaged rf Sheath (Square Wave Current) and the dc Sheath

The dc Child law contains simultaneously the relation between the sheath voltage and width, $U(d)$ and, by substituting $U \rightarrow -\Phi$, $d \rightarrow x$, the spatial dependence of the potential, $\Phi(x)$ [**4c,6**]

collisionless dc sheath

$$J_i = \frac{4}{9}\varepsilon_0\sqrt{\frac{2e}{m_i}}\left\{\begin{matrix}U^{3/2}/d^2 \\ (-\Phi)^{3/2}/x^2\end{matrix}\right. \quad (24a)$$

collisional dc sheath

$$J_i = 1.43\varepsilon_0\sqrt{\frac{2\lambda_i}{\pi}\frac{2e}{m_i}}\left\{\begin{matrix}U^{3/2}/d^{5/2} \\ (-\Phi)^{3/2}/x^{5/2}\end{matrix}\right. \quad (24b)$$

In table 1 we compare the power laws for $U(d)$ (for constant ion current) and $\Phi(x)$ for the dc sheath, from (**24**), and for $\overline{U}(d)$ (for constant ion current) and $\overline{\Phi}(x)$ for the time-averaged rf sheath driven by a square wave current, from (**15**), (**20**), (**21**).

dc sheath	rf sheath
$U \propto d^{4/3}$	$\overline{U} \propto d^{4/3}$
$-\Phi \propto x^{4/3}$	$-\overline{\Phi} \propto x^{3/2}$

table 1a : collisionless sheath

dc sheath	rf sheath
$U \propto d^{5/3}$	$\overline{U} \propto d^{5/3}$
$-\Phi \propto x^{5/3}$	$-\overline{\Phi} \propto x^{2}$

table 1b : collisional sheath

Power law relations between the (time-averaged) sheath voltage U, $\overline{U}$ and sheath width d, and between the (time-averaged) sheath potential Φ, $\overline{\Phi}$ and the distance from the plasma / sheath edge x, for the dc sheath, and the rf sheath driven by a square wave current.

$U(d)$ and $\Phi(x)$ for the dc sheath, and $\overline{U}(d)$ across the rf sheath, follow the same power law; the exponent is 4/3 for the collisionless sheath, and 5/3 for the collisional sheath.

$\overline{\Phi}(x)$ in the rf sheath follows a different power law; the exponent is 3/2 for the collisionless sheath, and 2 for the collisional sheath.

The rf power laws for $\overline{U}(d)$ and $\overline{\Phi}(x)$ are different because of the partial charge screening in the time-averaged rf sheath.

VI.4 Validity of the rf Sheath Model

Our rf sheath model somewhat simplifies the ion and electron motion within the sheath, **A1**, **A2**, and the physics at the plasma / sheath edge, **A3**, **A4**, and at the electron boundary, **A5**, **A6**. Until here we are on relatively safe grounds. The dubious assumption is **A7** : no ionization in the sheath. The model would be improved if ionization (of neutral gas atoms by bulk electrons, or secondary electrons from the electrode) could be accounted for, by means of the (time-averaged) equation of ion continuity [**4d**]

$$\frac{dJ_{\mathrm{i}}}{dx} = \left(\frac{\partial n_{\mathrm{i}}}{\partial t}\right)_{\mathrm{iz}} \qquad (25)$$

Out of the complete set of rf sheath equations, (**1**), (**9**), (**10**), (**13**), (**14**), only the equation of collisionless ion motion (**13a**) is *not* valid when there is ionization in the sheath. (**13a**) applies to the collision*less* rf sheath, which we exclude for the moment from our discussion. For the collision*al* rf sheath and the power law ansatz (**15**) follow again (**16**) – (**19**) and (**20b**). Combining (**20b**) and (**25**), we obtain

$$(\alpha - 2)\frac{J_{\mathrm{i}}(d)}{d}\left(\frac{x}{d}\right)^{\alpha-1} = \left(\frac{\partial n_{\mathrm{i}}}{\partial t}\right)_{\mathrm{iz}} \qquad (26b)$$

From (**26b**) and $(dn_{\mathrm{i}}/dt)_{\mathrm{iz}} > 0$ follows $\alpha > 2$: By inserting $\alpha > 2$ into (**20.2b**) we obtain for the ion current entering the sheath, $J_{\mathrm{i}}(0) = 0$, which is unphysical. We conclude that when there is ionization in the sheath, the time-averaged, collision*al*, high voltage, capacitive rf sheath driven by a square wave current can no longer be accurately described by the general power laws (**15**) – (**18**). For the collision*less* rf sheath, matters are too complicated for a brief discussion, but as for the collisional sheath, we expect deviation from the power laws. The ionization problem should be kept in mind when one tries to measure the power laws experimentally, or to reproduce them by numerical (PIC, Monte-Carlo) simulations.

VII. Conclusion

For the high voltage, capacitive rf sheath driven by a square wave current, an equation describing the screening of the ions by the electrons is derived. The charge screening equation in combination with other, well-established rf sheath equations leads to power laws for the space dependences of electrical parameters (electric potential and field, net charge carrier density and ion density) in the time-averaged rf sheath.

The charge screening equation is found to be approximately applicable to a rf sheath driven by the more common sinusoidal current. Accordingly, the time-averaged rf sheath is little dependent on the waveform of the driving current, ie it retains its power law character for the sinusoidal current.

Acknowledgements :

The authors thank Dr. Laurent Sansonnens for critically reading the manuscript.

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