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Analysis of Phase Measurements in Combined Interferometer and Polarimeter Systems

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Abstract. Combined interferometer and polarimeter systems, using a single detecting element per line of sight, are susceptible to perturbation of the interferometric phase [1] when modulation of the polarization vector is applied. This issue has been investigated extensively [2] for the case of a rotating elliptically polarized probing beam, demonstrating that here a perturbation is inevitable. In this article the anology between this analysis and earlier work is pointed out, and the underlying physics discussed. It will be demonstrated that schemes have been proposed in which the perturbation has been avoided or kept well within acceptable limits.

1. Introduction

There is increasing interest in combined interferometer and polarimeter systems for simultaneous measurement of electron density and poloidal magnetic field in fusion plasma devices [3]. In recent years sophisticated schemes have been proposed and built [4, 5, 6] which require only a single detecting element per line of sight, a goal frequently accomplished through active modulation of the polarization of the probing wave. It has however been demonstrated [1, 2] that these schemes are susceptible to a perturbation of the interferometric measurement. In [2] a general expression is given for a rotating polarization ellipse with arbitrary ellipticity, leading to the conclusion that with this type of scheme the perturbation of the interferometric phase is inevitable and can only be suppressed by electronic filtering at the expense of bandwidth.

In section 2 of this article a link is established between the analysis presented in [2] and previously published work [6, 7], while in section 3 a physical interpretation of this mathematical analysis is presented. To conclude, section 4 treats two presently used experimental techniques which are able to avoid the perturbation of the interferometric phase by applying a double probing beam.

2. Mathematical description

A description of the polarization state and phase evolution in a combined interferometer and polarimeter system can be obtained either via the Stokes [2, 8] or the Jones

[7, 9] formalism. Both formalisms can be applied to analyse the phase perturbation in combined interferometer and polarimeter systems employing polarization modulation. While in a recent paper by Segre [2] this type of analysis was given on the basis of the Stokes formalism, an equivalent analysis based on Jones matrix calculus was presented in [6] and, in more detail, in [7].

It is relatively straightforward, but illustrative, to demonstrate the equivalence of the results. In [2] the phase perturbation term φ_m is given as the difference of two arctangents (see Eq. 21)

$$\varphi_m = \varphi_p - \varphi_o = \arctan(S/C) - \arctan(S_o/C_o) \tag{1}$$

where the notation is modified to stress that φ_p is equal to φ_o in absence of plasma, i.e. when $\delta = 0$ and $\Delta = 0$. In [6, 7] the above expression has been rearranged (see App.) to retain only a single arctangent, yielding

$$\varphi_m = \arctan\left[\frac{S\,C_o - C\,S_o}{C\,C_o + S\,S_o}\right] \tag{2}$$

In this Eq. the Eqs. 23 and 24 of [2] can now be substituted, and after some tedious algebra one obtains

$$\varphi_m = \arctan \left(\frac{-\tan\delta\,\left[2\tan\chi_o\tan^{-1}\chi_2 + \frac{1}{2}\left[1+\tan^2\chi_o + (1-\tan^2\chi_o)\cos(2\psi_o)\right]\left[\tan^{-2}\chi_2 - 1\right]\right]}{\left[\cos^2\psi_o + \sin^2\psi_o\,\tan^2\chi_o\right]\left[1+\tan^{-2}\chi_2\right] + \tan\delta\,\sin(2\psi_o)\tan^{-1}\chi_2\,\left[1-\tan^2\chi_o\right]} \right) \tag{3}$$

a result which is equivalent to Eq. 8 of [6],

$$\varphi_m = -\arctan \left(\frac{\tan(\mathrm{d}\psi/2)\,\left[2\epsilon\alpha + \frac{1}{2}\left[1+\epsilon^2 - (1-\epsilon^2)\cos(2\xi)\right]\left[\alpha^2 - 1\right]\right]}{\left[\sin^2\xi + \epsilon^2\cos^2\xi\right]\left[1+\alpha^2\right] + \tan(\mathrm{d}\psi/2)\,\sin(2\xi)\,\alpha\left[1-\epsilon^2\right]} \right) \tag{4}$$

when applying the coordinate transform

$$\begin{array}{lllllll} \xi & \longleftrightarrow & \psi_o - \frac{\pi}{2} & \qquad & \mathrm{d}\Phi & \longleftrightarrow & -\varphi_I \\ \alpha & \longleftrightarrow & \tan\chi_1 = -1/\tan\chi_2 & & \mathrm{d}\phi & \longleftrightarrow & \Delta \\ \epsilon & \longleftrightarrow & -\tan\chi_o & & \mathrm{d}\psi & \longleftrightarrow & 2\delta \end{array} \tag{5}$$

Here the symbols on the left-hand side refer to the notation used in [6, 7] and those on the right-hand side to the notation in [2]. The factor $\frac{\pi}{2}$ in the first of the above equations arises because ψ_o in [2] is taken with respect to the coordinate axis parallel to the magnetic field, whereas ξ in [6, 7] is taken with respect to the perpendicular axis.

3. Physical interpretation

The complex mathematical expression given above makes an interpretation of the result somewhat obscure. However, it can be made more intuitive by separating the phase evolution of the probing and reference beam with respect to a non-rotating (fictive) reference.

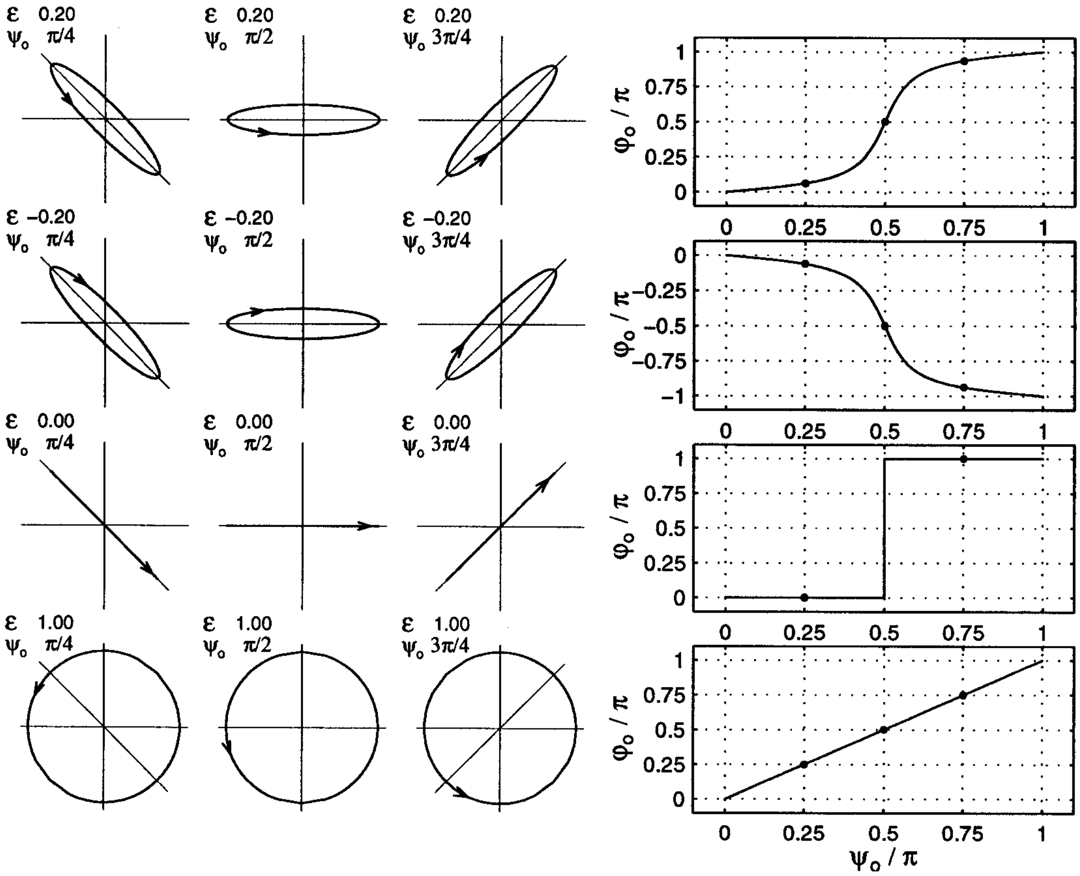

Figure 1. Behaviour of the phase of the y-component for positive, negative, and zero ellipticity, as well as for a left-handed circularly polarized wave. In all cases a phase transition of magnitude π occurs per half a revolution of the polarization vector. The phase of the signal component along the minor axis of the ellipse determines whether this transition is positive or negative. In the case where this component is absent (ϵ=0) the choice is arbitrary since the signal amplitude goes to zero. In the case of circular polarization, both components have equal amplitude and the transition degenerates.

In all existing polarimetric schemes both the probing and reference waves are passed through an analyzing polarizer before detection. In general this polarizer is oriented to select the polarization component parallel to the external magnetic field (henceforth chosen to be the y-direction), and the phase of this component is determined. If a wave with rotating linear polarization is detected in this manner, its phase with respect to the fictive reference would be constant up to the moment when the polarization vector is oriented exactly perpendicular to the detection–axis ($\psi = \frac{\pi}{2}$). At this point the level of the detected signal will be zero and its phase undefined. After passage, the phase will have changed by π radians as a result of the change of sign of the detected component (see Fig. 1).

In the case where the wave ellipticity is not zero the phase transition will become more gradual as a result of the existence of a signal component along the minor axis of the ellipse. The sign of the ellipticity, i.e. whether the phase of this smaller component is $\frac{\pi}{2}$ ahead or behind the phase of the main component, determines whether this transition

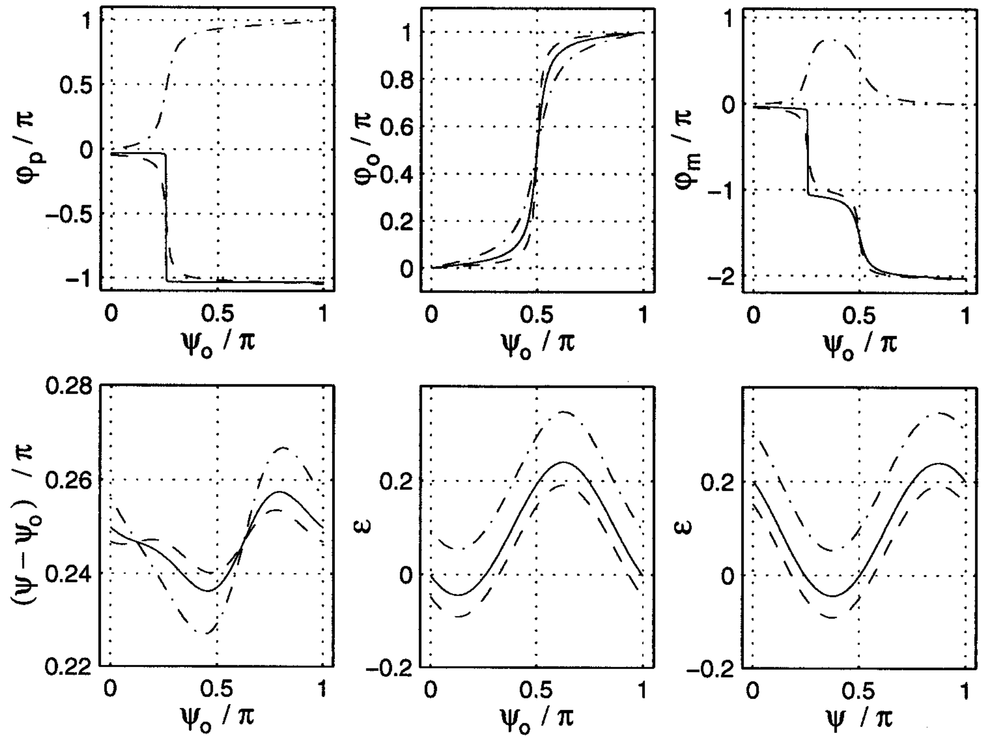

Figure 2. The individual wave phases φ_p and φ_o for probing and reference wave, resulting perturbative phase φ_m, change of azimuth of the probing wave $\psi - \psi_o$ and output ellipticity of the probing wave, all as a function of input azimuth ψ_o. The last plot shows the output ellipticity as a function of output azimuth ψ. The parameters are chosen so as to allow comparison with Fig. 1 of [2], i.e. $\sin(2\chi_2) = -0.98$, $\tan\delta = 1$ (hence $\tan\chi_o^* = 0.098$) and $\tan\chi_o = 0.05$ (dashed), 0.098 (solid) and 0.20 (dash-dotted).

is positive or negative. Half a revolution later, at $\psi = \frac{3\pi}{2}$, a second phase transition will occur with the same sign. After one full revolution the wave will therefore have acquired a phase difference of 2π with respect to the stationary reference, as is to be expected. In the limiting case where the wave polarization approaches circular, the slope of the transition will go to unity and the phase difference will grow linear in time (see Fig. 1).

In polarimetric schemes which use waves with a rotating polarization ellipse the phase transitions will affect both the probing and the reference wave. Therefore in general two transitions will be present when the phase difference between a probing and a rotating (actual) reference wave is considered. In the absence of a plasma the azimuthal angle ψ and ellipticity ϵ of both waves (and thus the exact position and slope of the transitions) will be identical. The two transitions will cancel and the phase difference will be stationary. However, a minor mismatch in the orientation of the analyzer in probe and reference arms will already compromise this situation. In the presence of a plasma the azimuth of the polarization of the probing wave is changed as a result of Faraday rotation, causing a shift of the position of its transition with respect to that of the reference. The two phase transitions will now no longer coincide, and the interferometric phase will be perturbed by a 'spike', having a width determined by the amount of rotation.

In addition, the Cotton-Mouton effect will cause a change in ellipticity of the probing wave. For given plasma parameters, represented by δ and χ_2, a value for the initial ellipticity of the probing wave exists for which the ellipticity after passage through the plasma will be zero at the moment when the main axis is oriented perpendicular to the axis of detection. This threshold value is given by the variable χ_o^* as introduced in [2]. A wave with ellipticity between $\tan(\chi_o^*)$ and zero (irrespective of the sign of χ_o^*) will, after passage through the plasma, have its ellipticity reversed with respect to that of the reference wave at the time of their respective passages through the x–axis ($\psi, \psi_o = \pm\frac{\pi}{2}$). The two signals will undergo phase transitions with opposite signs, which do not cancel but instead accumulate after half a revolution of the polarization vector. If the ellipticity of the incident wave lies outside the interval mentioned above, the sign of the ellipticity at the time of passage through the x–axis will be identical for probing and reference waves, and the phase difference will follow the evolution described in the previous paragraph. The difference in ellipticity of the two waves will nevertheless cause the slopes of the transitions to differ and consequently the 'spike' becomes asymmetrical.

This effect is illustrated in Fig. 2, where three of the curves of Fig. 1 of [2] are reproduced. Additional plots describe the change in polarization of the probing wave after passage through the plasma slice. For the case where χ_o equals χ_o^*, the ellipticity of the emerging wave is shown to pass through zero at the moment when its azimuth is parallel to the x–axis ($\psi = \frac{\pi}{2}$). Contrary to the presentation given in [2], in this plot the associated phase transition has been preserved. However, since in this particular case the sign of the slope is undetermined, one may alternatively assume an upward transition, causing the curve to qualitatively follow the upper branch.

4. Review of improved schemes

From the analysis presented above and in [2] the impression may be obtained that a perturbation of the interferometric phase is inevitable in combined interferometer and polarimeter systems. However, one has to realize that up until now only schemes implying a single probing beam have been considered. In the following, two schemes are discussed in which the problem of phase perturbation has been avoided through the application of two probing beams.

4.1. The RTP scheme

In spite of some apparent similarities with a scheme proposed in [10], the method implemented at the RTP tokamak [6, 7] should not be regarded as one based on a single probing beam with rotating linear polarization. It would be better to treat it as the combination of two independent interferometers, one of which employs a left-handed and the other a right-handed circularly polarized probing beam (i.e. $\chi_o = \pm 1$).

The phase change of both interferometers is determined independently by mixing each with a linearly polarized LO beam. Each of the probing beams will, upon passage through the plasma, undergo a phase change $\Delta + \varphi_m$, where Δ is the conventional interferometric phase change and φ_m a perturbation thereon. As discussed previously, φ_m depends on the polarization of the probing wave and on plasma parameters. Starting from Eq. 4 it can be demonstrated [6, 7] that for circular waves φ_m equals the probing wave ellipticity ϵ (i.e. +1 or −1) multiplied by the conventional Faraday rotation angle α_F. The phase difference between the two probing waves then equals twice this angle and its average value yields the interferometric phase.

In practice the two independent interferometers can be combined on the same optical setup by using frequency separation. In this case the probing waves will also mix after passage through the analyzer, and the phase of this mixing product gives an alternative means of determining the phase difference between the probing waves. Although advantageous, this feature is not essential to the method.

One important distinction between the RTP method and the one by Dodel-Kunz is that it does not require equal beam power in the two probing beams, i.e. the polarization of the composite wave is not relevant. Also separation of carrier and envelope signal is not necessary after detection. Instead, individual phase measurements are performed on distinct signals, well separated in the frequency domain. This completely different method of detection avoids all artifacts described in section 3.

4.2. The TCV scheme

In the case of the combined interferometer/polarimeter presently being installed at the TCV tokamak [5, 11], two linearly polarized waves are used to probe the plasma. The polarization of one, with amplitude A, is rotating with angular frequency ω_r, while the polarization of the other, with amplitude B, is stationary and directed parallel to the external magnetic field. Upon detection the component in this direction is selected with an analyzer. The composite incident wave can be described by

$$E = \begin{pmatrix} A\cos(\omega t)\sin(\omega_r t) \\ -A\cos(\omega t)\cos(\omega_r t) + B\cos(\omega t - \phi_o) \end{pmatrix} \tag{6}$$

where the rotating polarization vector is taken to be aligned parallel to the external magnetic field at $t = 0$. The parameter ϕ_o represents a phase difference between the two probing waves which in the TCV scheme will be adjusted to a multiple of π. In this case the composite incident wave will again be linearly polarized with time-varying amplitude and azimuth. Upon passage through the plasma both the non-rotating and the rotating component will undergo a polarization rotation as a result of the Faraday effect. Consequently, the rotating part and hence the amplitude modulation will show a phase lag with respect to the reference, from which the polarimetric phase can be retrieved.

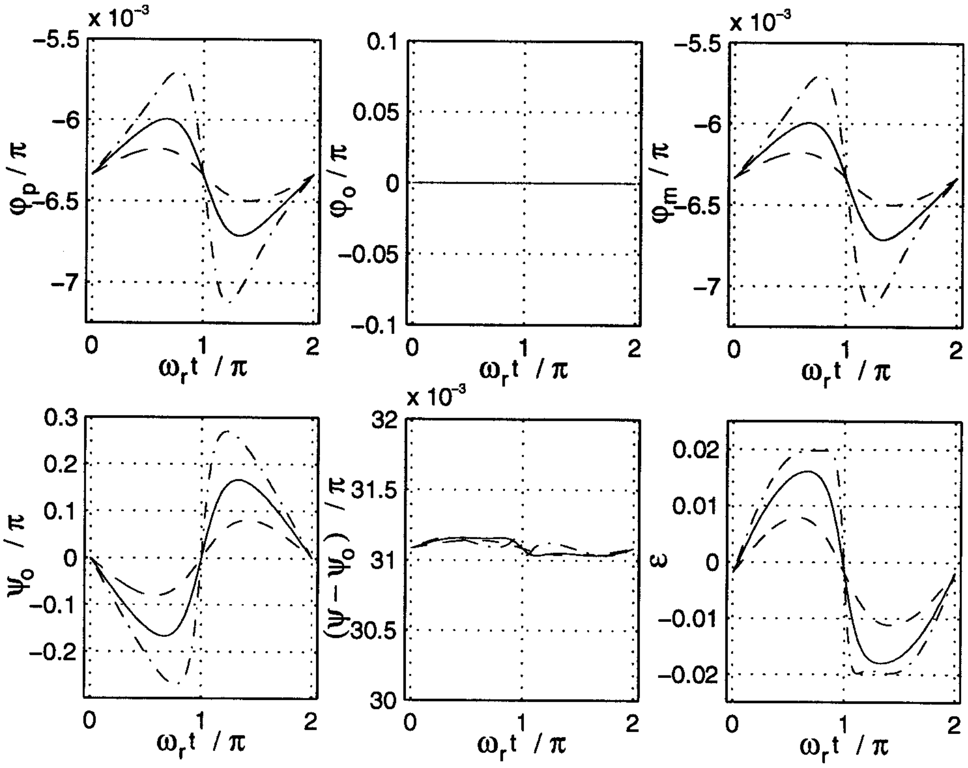

Figure 3. The individual wave phases φ_p and φ_o for probing and reference wave, resulting perturbative phase φ_m, incident azimuth of the probing wave ψ, change of azimuth $\psi - \psi_o$ and output ellipticity of the probing wave, all as a function of modulation angle $\omega_r t$. To facilitate comparison with Fig. 2, plasma parameters were again chosen to be $\sin(2\chi_2) = -0.98$ and $\tan\delta = 1$. The ratio of rotating to non-rotating wave amplitudes A/B was taken to be 0.25 (dashed), 0.50 (solid) and 0.75 (dash-dotted).

The azimuth ψ_o of the incident wave is given by

$$\psi_o = \arctan\left(\frac{-A\,\sin(\omega_r t)}{-A\,\cos(\omega_r t) + B}\right) \tag{7}$$

where ψ_o is again defined with respect to the y-axis. This Eq. reduces to $\psi_o = \omega_r t$ if the component with stationary polarization is absent ($B = 0$). In reality the ratio of the probing wave amplitudes A/B is chosen to be sufficiently small. This is done to avoid phase transitions of the type depicted in Fig. 1 and to keep the modulation depth limited, which is beneficial to an accurate determination of the interferometer phase.

Eq. 7 can now be used to express the rotation of the A–component in terms of the azimuth ψ_o so that the formalism of [2] can be applied. The results are shown in Fig. 3 and indicate a negligible perturbation of the interferometric phase for this modulation scheme and the chosen plasma parameters (note that $\tan\delta$=1 corresponds to a significant amount of birefringence). At each moment during the modulation cycle the polarizations of both probing and reference waves remain close to linear and sufficiently far removed from the phase transition points at $\psi, \psi_o = \pm\frac{\pi}{2}$.

5. Conclusions

It has been shown that identical results for the phase perturbation term in a combined interferometer and polarimeter system can be obtained on the basis of different formalisms (Stokes/Jones matrices). The results given in [2] can indeed also be derived from equations previously presented in [6, 7].

A physical interpretation of the mathematical equations has been developed by considering the behaviour of probing and reference waves individually, thus revealing the meaning of the parameter χ_o^* as introduced in [2]. It has been shown that a phase transition occurs also in the case where the incident wave has an ellipticity $\tan\chi_o^*$, but that then the slope of the transition is undefined because the detected signal vanishes. The bifurcative behaviour of the curves shown in [2] is explained as being the result of a change in sign of the probing wave ellipticity.

In the latter part two single-detector polarimetric schemes were discussed neither of which suffers from a perturbation of the interferometric phase. The RTP method basically involves two separate phase measurements using orthogonally polarized waves, the polarization of each of which remains stationary. The interferometric phase and Faraday angle are obtained by combining these two measurements. Since modulation is absent, so are the modulation artifacts. The scheme proposed for the TCV tokamak uses a composite wave which only scans a limited azimuthal interval (between $\pm\arctan(A/B)$ in the case of the reference beam). It avoids ψ values close to the positions of the phase transition, and thus keeps the perturbation of the interferometric phase small.

Appendix

Realizing that any phase can be considered as the ratio of imaginary to real component of a complex signal, one may write

$$\exp[-j\varphi_p] = \exp[-j\arctan(S/C)] = \frac{C + jS}{\sqrt{C^2+S^2}} \tag{8}$$

and an analogous expression for φ_o relating it to C_o and S_o. The phase difference between these two terms, φ_m, can then be obtained by determining the phase of the quotient of the two,

$$\begin{aligned}\exp[-j\varphi_m] &= \exp[-j\varphi_p] \;/\; \exp[-j\varphi_o] \\ &= \sqrt{\frac{C_o^2+S_o^2}{C^2+S^2}}\;\frac{CC_o+SS_o-j(SC_o-CS_o)}{C_o^2+S_o^2}\end{aligned} \tag{9}$$

which leads to Eq. 2 of the main text.

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