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Velocity Ratio Measurement using the Frequency of Gyro Backward Wave

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ABSTRACT

The operating diagram of a low quality factor, 8GHz TE^{o}_{01} gyrotron exhibits oscillations between 6.8 and 7.3GHz. These oscillations are identified as the backward wave component of the TE^{o}_{21} traveling mode. As the resonance condition of this mode depends on the average parallel velocity $<v_{\|}>$ of the beam electrons ($\omega_{BW} \cong \Omega_c/\gamma - k_{\|}<v_{\|}>$), the measurement of ω_{BW} for given Ω_c and γ, is used as a diagnostic for the beam electrons velocity ratio $\alpha=<v_{\perp}>/<v_{\|}>$. The values of α, deduced from ω_{BW} through the linear dispersion relation for the electron cyclotron instability in an infinite waveguide, are unrealistic. A non-linear simulation code gives α values which are in very good agreement with the ones predicted by a particle trajectory code (+10% to +20%). We find numerically that the particles' velocity dispersion in $v_{\perp}$ and $v_{\|}$ increases ω_{BW}. This effect explains part of the discrepancy between the values of α inferred from ω_{BW} without velocity dispersion and the expected values.

The velocity ratio $\alpha=\langle v_\perp\rangle/\langle v_\parallel\rangle$ of the beam electrons is a critical parameter in high power, high efficiency gyrotrons. The values of α given by the gun design code are usually used for output power calculation. Nonlinear theory[1] shows that the maximum perpendicular efficiency $\eta_{\perp\max}$ for a first-harmonic gyromonotron is $\eta_{\perp\max}\cong 0.7$. The maximum total efficiency $\eta_{\max}$ is directly proportional to the square of the perpendicular electron velocity $v^2_{\perp 0}$ since $\eta_{\max} = \gamma\beta^2_{\perp 0}/(2(\gamma-1))\eta_{\perp\max}$, where $\beta_{\perp 0}=v_{\perp 0}/c$, c is the speed of light, and γ is the relativistic factor. The gyrotron efficiency not only strongly depends on α, but also on the perpendicular velocity spread[2] $\Delta v_\perp/\langle v_\perp\rangle$. Diagnostics on electron beam of microwave sources have been proposed using capacitive probes[3], measurement of electron cyclotron spontaneous emission[4] (ECE) and Thomson scattering on the electrons.[5] Thomson scattering and ECE measurements provide information on the velocity spread. Capacitive probe measurements in a gyrotron[6] have already shown good agreement between the measured α and the expected values, at low beam currents ($I_b\leq 20$ A). They have also shown saturation of α at high beam current and at high magnetic compression. This letter describes another method which determines α from the frequency of the backward wave in a gyrotron.

Our low-diffractive-Q ($Q_{\mathrm{diff}}=160$) TE^{o}_{011} 8GHz gyrotron can be operated as a TE^{o}_{21} Gyrotron Backward Wave Oscillator (Gyro BWO) by lowering the magnetic field below the value of the TE^{o}_{011} cyclotron resonance [Figs. 1 and 2]. In principle, the Gyro BWO instability is absolute since the negative group velocity of the waveguide mode provides an internal feedback to the forward traveling cyclotron wave generated in the electron beam. However, for a finite interaction length L_i of the electron beam with the wave, the beam current must exceed a threshold value before the instability is observed. The oscillation frequency of the backward TE^{o}_{21} mode depends on the average parallel velocity $\langle v_\parallel\rangle$ of the beam electrons ($\omega_{BW}\cong\Omega_c/\gamma - k_\parallel\langle v_\parallel\rangle$, where $k_\parallel$ is the component of the wave number parallel to the electron beam) and, thus, on α for a given electron cyclotron frequency $\Omega_c=eB_0/m$ (where e and m are the electron charge and mass) and a given γ.

The linear dispersion relation for the interaction between a cold annular electron beam, consisting of electrons spiraling around the lines of a uniform guiding magnetic field B_0, and a TE_{mn} wave guide mode has been derived by Chu.[7] Solving this polynomial equation for ω_{BW} using the experimental values of, beam accelerating voltage V_c, beam current I_b, beam radius r_b, cavity radius r_c, cavity magnetic field B_0, and α, yields the TE^{o}_{21} oscillation frequency. A fully-relativistic self-consistent gyrotron simulation code[8], based on slow time-scale equations for the rf electric field F, which is assumed to vary as $e^{-i\omega t}$, and for the particles' momentum and phase, has been modified to model the Gyro BWO. The boundary conditions for the rf electric field have been adapted to our gyrotron cavity. On the gun side, the backward wave impedance has to match the cavity impedance:

$$\frac{dF}{dz}\Big|_{z=0} = -i\, k_{||}\, F.$$

The backward power is reflected by the cut-off section of the cavity and travels without interaction towards the load. On the collector side, the end of the Gyro BWO interaction region ($z=L_i$) is defined by the fast change of $k_{||}$ in the output uptaper which detunes the resonance condition,

$$\omega_{BW} \cong \Omega_c/\gamma - k_{||} \langle v_{||} \rangle$$

and we impose

$$\frac{dF}{dz}\Big|_{z = L_i} = +i\, k_{||}\, F.$$

The numerical simulations show that this condition is sufficient to insure that the electric field is very small for $z>L_i$. The straight part of the cavity is 20.5 centimeters long and the interaction length is found to be around 24 centimeters ($L_i \cong 5.5$ - $6\ \lambda_{vacuum}$). The value of α can be deduced from the experimental ω_{BW} through both the linear and non-linear calculations of ω_{BW} versus α [Fig. 3].

The tube was designed to operate in the TE^{o}_{011} mode at 8GHz with a magnetic field around 3.13kG. Its parameters are identical to the ones given in Ref. 8, except for the diffractive quality factor which has been lowered to 160. The experimental mode map [Fig. 2] shows that, for the measurement of α, the tube can be operated as a Gyro BWO at low magnetic field. The output window, originally matched for the TE^{o}_{01} mode at 8GHz, has been adapted to the TE^{o}_{21} mode by adding a second Al_2O_3 disk. Such a two windows system has a reflection coefficient between 0% and 15% in the frequency range from 6.8 to 7.3GHz. The oscillation frequency was insensitive to this matching. However, the frequency band structure which was observed with a single window when varying α (or any other physical parameter), was almost completely suppressed. In addition the Gyro BWO output power is reduced by a factor of about two with this window system. The electron gun has a standard MIG diode structure. For a fixed cavity field, the α was varied by changing the magnetic field in the cathode region. The domain of α experimentally accessible was limited by beam interception in the gun region. The oscillation frequency was precisely measured (±1MHz) with a spectrum analyzer during a 15msec microwave pulse. Measurements were carried out for beam currents between 2 and 8A and magnetic fields of 2.970kG, 3.033kG and 3.095kG [see Fig. 2]. Figure 4 shows a plot of the Gyro BWO frequency for a fixed magnetic field profile, i.e. at constant α, versus I_b. The beam current has no observable effect on α, for currents between 2 and 8A. However, experimental frequencies imply α values 10-15% higher than those calculated with the non-linear simulation. The EGUN code[9] which includes magnet coil locations and currents, space charge and relativistic effects, was used to predict α values (α_{EGUN}) from the experimental parameters. Figures 5 (a) and 5 (b) are plots of α derived from the experimental Gyro BWO frequencies, using both the linear dispersion relation and the non-linear simulation, versus the expected values α_{EGUN}. The linear dispersion relation overestimates α by more than 50% suggesting that the cavity is too short ($L_i \cong 5.5 - 6\ \lambda_{vacuum}$) for the interaction to be described by the linear dispersion relation for an infinite medium. Although the output power is quite low, non-linear effects may still play an important role. The values of α found through the non-linear simulations for a cold electron beam and with a realistic geometry are in very good agreement with α_{EGUN} (+10% to 20%). Typical electron beam parameters as given by the EGUN code are shown in Table I.

α	$<\gamma>$	$\Delta\gamma/<\gamma>$	$\Delta v_{\parallel}/<v_{\parallel}>$	$\Delta v_{\perp}/<v_{\perp}>$
1.23	1.1460	0.0001	0.027	0.018
1.32	1.1458	0.0001	0.031	0.016
1.65	1.1456	0.0002	0.036	0.013
2.37	1.1451	0.0002	0.072	0.013
2.95	1.1448	0.0002	0.114	0.014

Table I: Electron beam parameters at the cavity entrance from the particle trajectory code[9] for V_c=75kV, I_b=5A and B_0=3.033kG. Δ means the standard deviation.

The parallel velocity spread strongly increases with α whereas the perpendicular velocity spread stays constant and the energy spread is negligible at our typical beam currents. Figure 6 shows that velocity spreads tend to increase ω_{BW} and thus lead to overestimates of the inferred α. Table I gives minimum values for the spreads, since effects such as the physics of the emission at the cathode surface or possible azimuthal beam asymmetries are not taken into account in the EGUN code. Estimations of the velocity spreads at the cathode due to temperature $(\Delta v_\perp/<v_\perp>)_T$ and due to cathode roughness $(\Delta v_\perp/<v_\perp>)_r$ were given by Tsimring.[10] The perpendicular velocity spread in the cavity can be evaluated from the spread at the cathode since, in low space charge beams, this quantity is approximately conserved during the adiabatic compression. For our expe-riment typical values are $(\Delta v_\perp/<v_\perp>)_T \approx 6 \cdot 10^{-6}$ and $(\Delta v_\perp/<v_\perp>)_r \approx 3.5 \cdot 10^{-2}$, where we consider hemispherical bumps of radius $r=10^{-6}$m as representative of the surface roughness. The latter value has to be added as a statistically independent variance to the ones given in Table I, with its corresponding parallel component given by $\Delta v_{||}/<v_{||}>=\alpha^2 \Delta v_\perp/<v_\perp>$ since $\Delta\gamma \approx 0$. The measured Gyro BWO output power was considerably lower than expected from the calculations with zero velocity spreads. This indicates that these spreads are non-negligible in our case and perhaps larger than the computed values [see Table I].

The beam electrons velocity ratio was obtained by measuring the frequency of the TE^{o}_{21} Gyro BWO mode. The linear dispersion relation yields unrealistic values of α. A non-linear particle simulation code was used to infer α from ω_{BW}. These results are in very good agreement (+10% to +20%) with the α values expected from the EGUN code. The beam current has no observable effect on α, for currents between 2 and 8A. We plan to increase the beam current up to 50A. Calculations show that velocity spreads tend to increase ω_{BW} and thus to overestimate α. Presence of velocity spread is confirmed by the low Gyro BWO output power.

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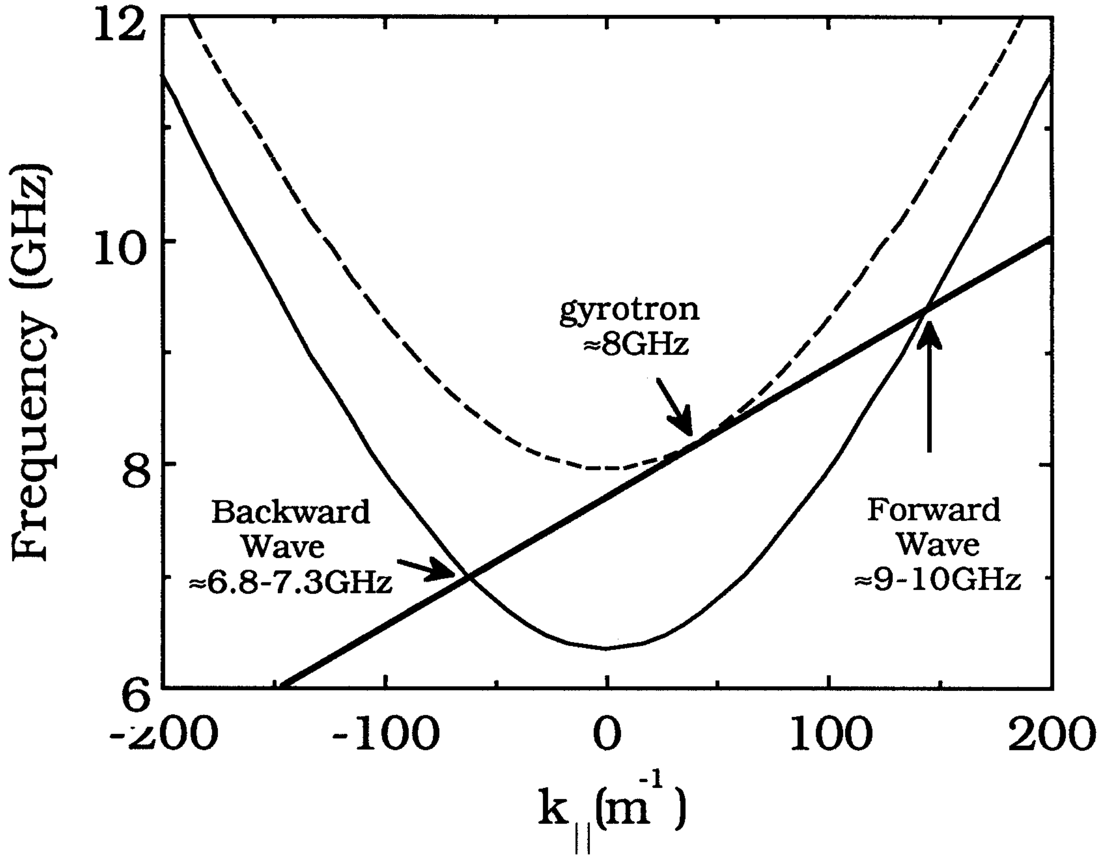

Fig. 1: Dispersion curves for the TE°_{01} (dashed line) and TE°_{21} (thin line) waveguide modes and the fast-beam cyclotron mode (thick line).

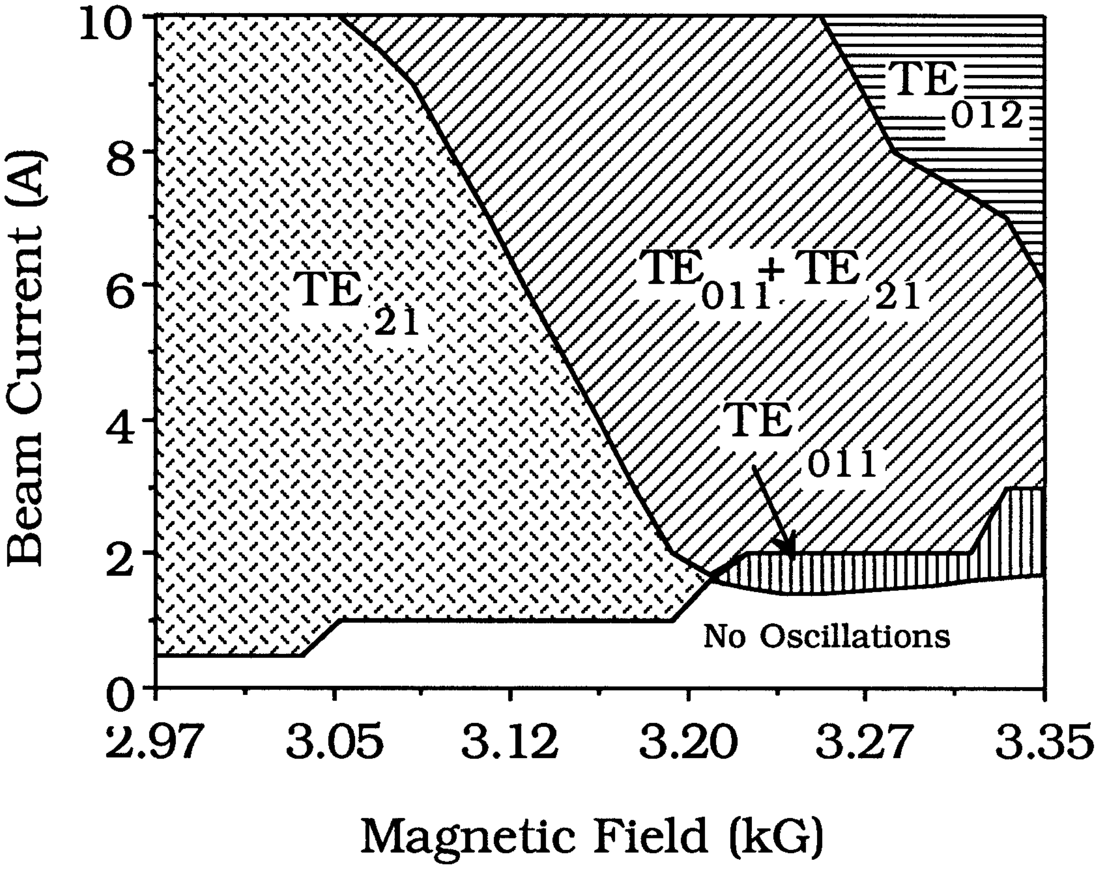

Fig. 2: Experimental mode chart for V_c=75kV and α=1.7.

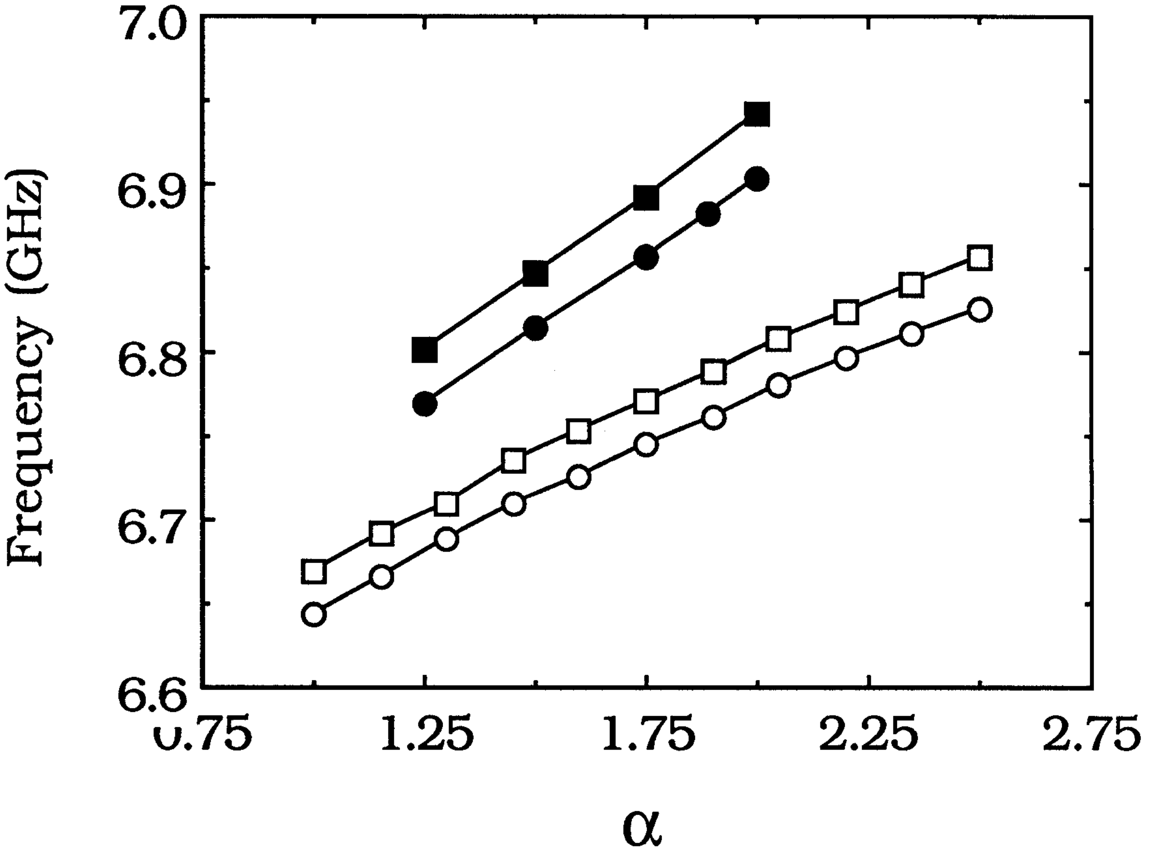

Fig. 3: Dependency of the Gyro BWO frequency on α as computed with the linear dispersion relation (open symbols) and with the non-linear simulation for a cold beam (filled symbols) for V_c=75kV, B_0=2.970kG, I_b=4A (circles) and I_b=8A (squares).

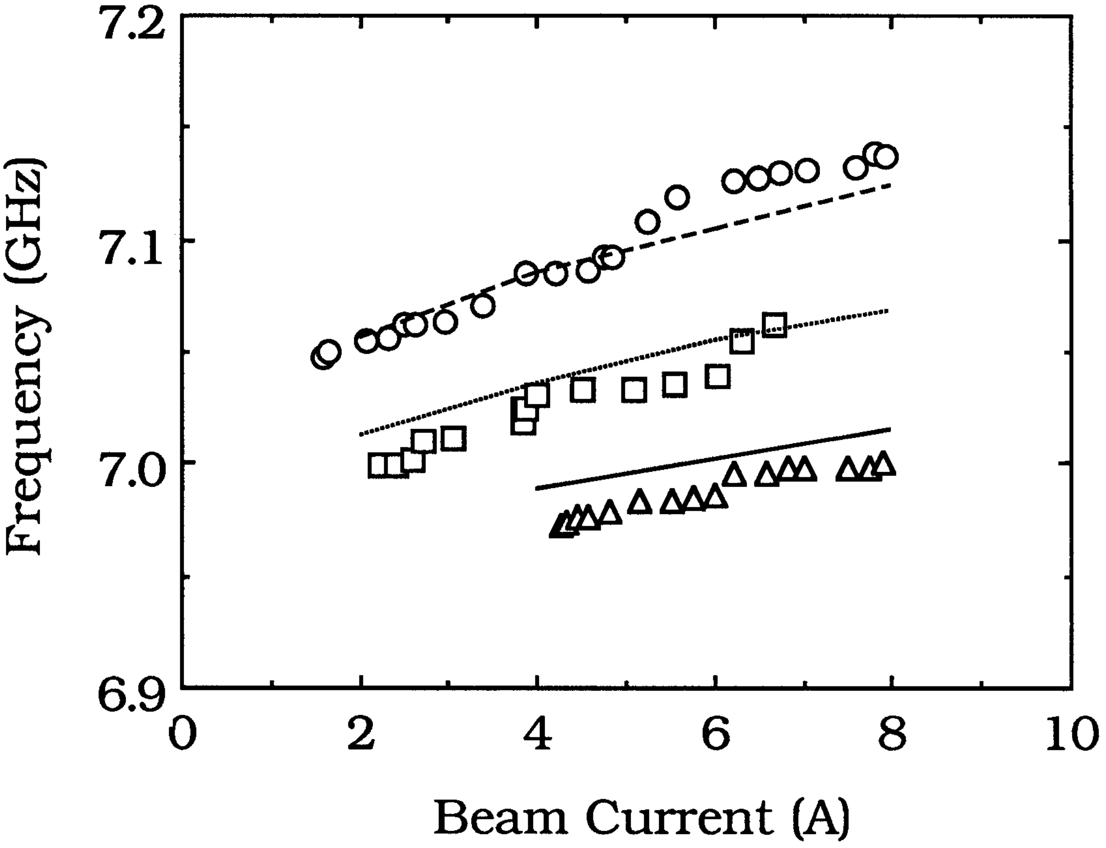

Fig.4: Gyro BWO frequency as a function of the beam current for V_c=75kV, B_0=3.095kG, computed with the non-linear simulation for α=1.5 (full line), α=1.75 (dotted line) and α=2.0 (dashed line); and measured values for α=1.29 (triangles), α=1.52 (squares), and α=1.81 (circles).

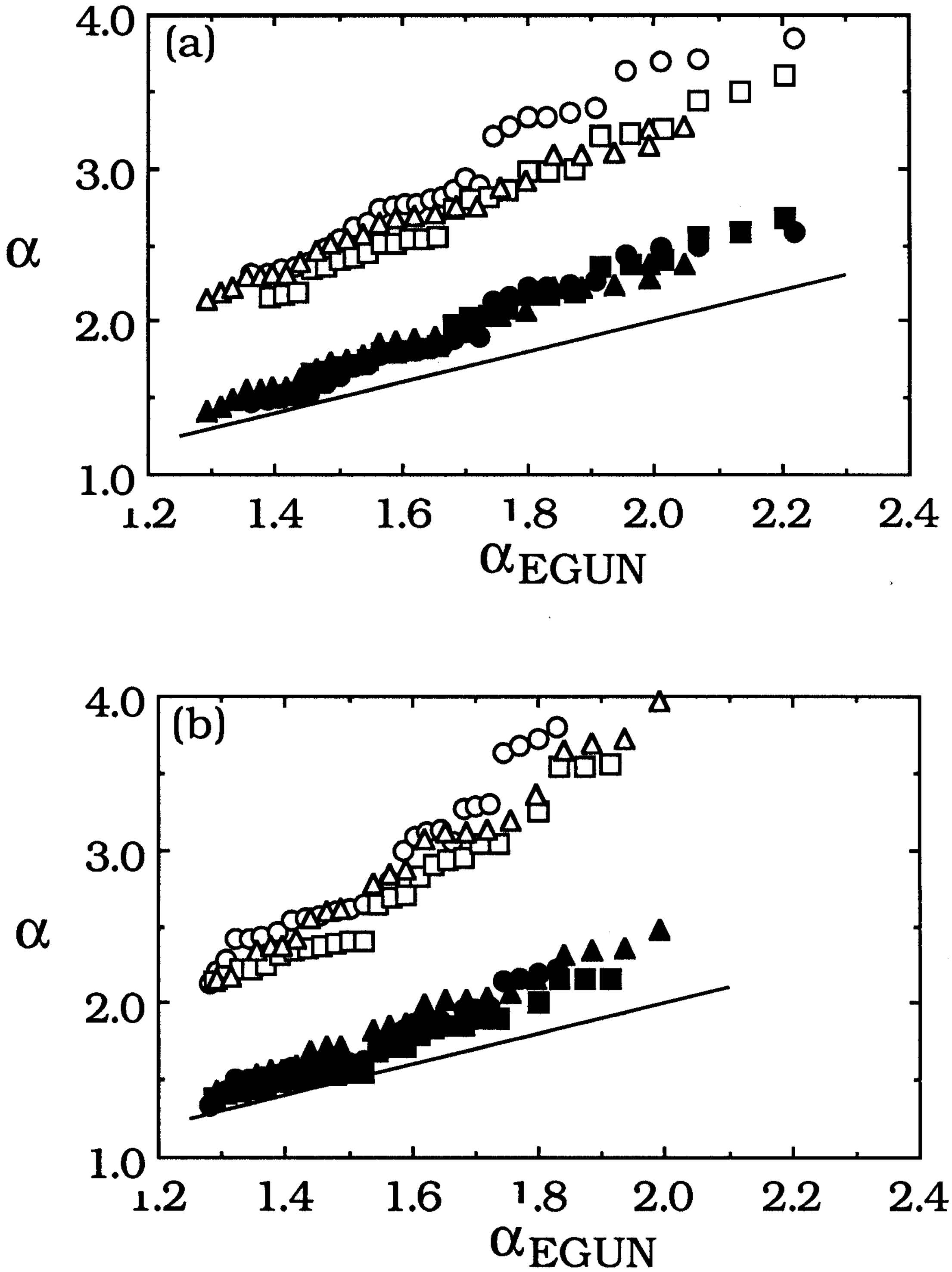

Fig. 5: Comparison of α inferred from ω_{BW} to the α given by the EGUN code (using the experimental magnetic field profiles). The α inferred from ω_{BW} is calculated using the linear dispersion relation (open symbols) and the non-linear simulation (filled symbols). Data are plotted for: B_0=2.970kG (circles),3.033kG (squares),3.095kG (triangles), V_c=75kV and (a) I_b=4A, (b) I_b=8A. Also plotted the line $\alpha=\alpha_{EGUN}$.

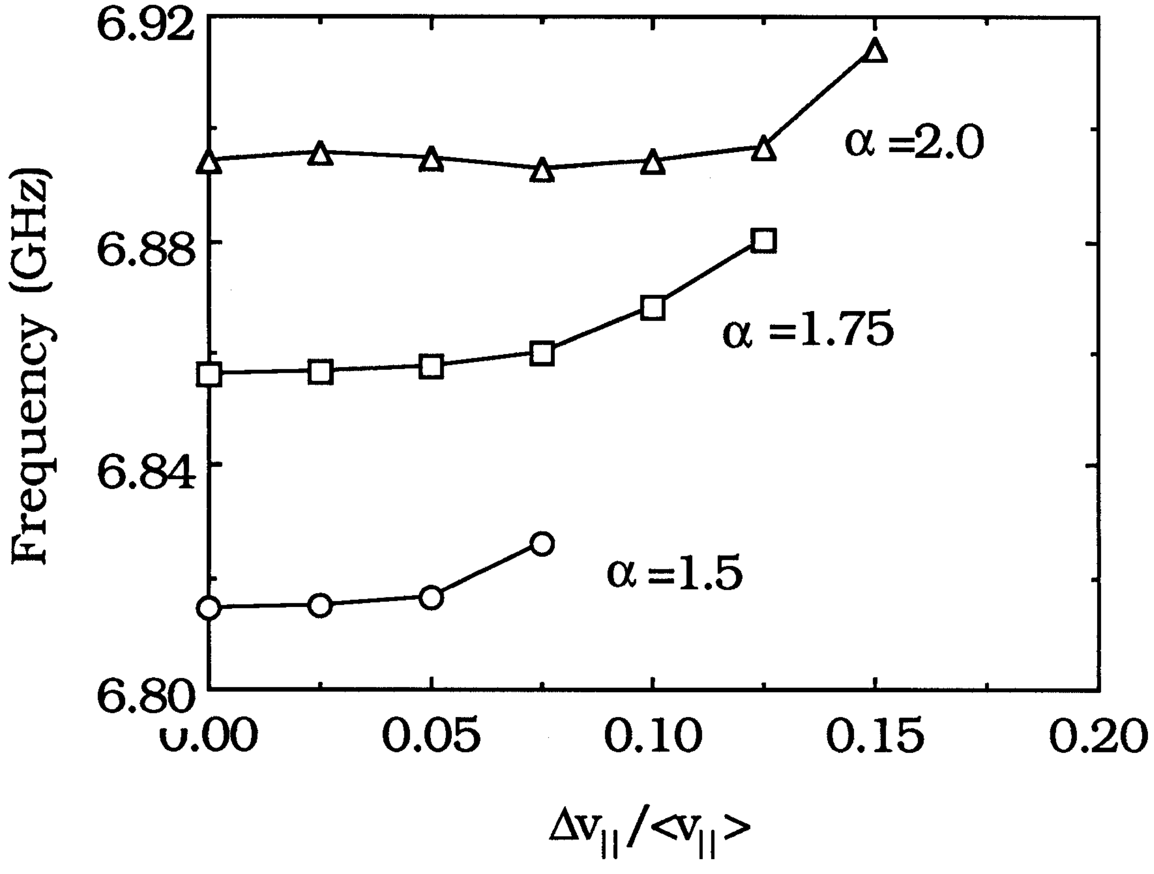

Fig. 6: Effect of velocity spread on Gyro BWO frequency. For these calculations V_c=75kV, I_b=4A and B_0=2.970kG, $\Delta\gamma=0$ is assumed and thus $\Delta v_\perp/<v_\perp>=(1/\alpha^2)\Delta v_{||}/<v_{||}>$.