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A NUMERICAL STUDY

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**Precision of Ion Temperature Measurements in a  
Tokamak Plasma Using Collective Thomson Scattering:  
A numerical study**

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**ABSTRACT**

A numerical simulation has been performed to investigate the precision of a  $T_i$ -measurement of a tokamak plasma via collective Thomson scattering at 385  $\mu\text{m}$ . The study is based on a parameter and data set referring to an experiment that has been carried out at the TCA tokamak using a pulsed  $\text{D}_2\text{O}$  laser and a heterodyne receiver system. Apart from limitations due to the signal-to-noise ratio of the detection system, we investigate the influence of uncertainties in various plasma parameters required for the  $T_i$ -evaluation from a scattered spectrum. In the experiment typical relative errors of  $\pm 25\%$  have already been obtained, but careful optimization of the system, taking into account the results of this study, and further technical improvements should allow to reduce this limit to about  $\pm 10\%$  for the plasma conditions considered.

## 1. INTRODUCTION

Recently, experiments at the TCA tokamak have succeeded in measuring the ion temperature via collective Thomson scattering from thermal density fluctuations. Using a high power pulsed D<sub>2</sub>O laser emitting at 385 $\mu$ m and a heterodyne receiver comprising a Schottky diode mixer, relative uncertainties of a single shot measurement of less than 25% have been obtained<sup>1</sup>.

It is the aim of this study to investigate the limitations in precision of such measurements and possibilities to achieve further improvements. For this purpose numerical simulations based on the parameters of the particular scattering experiment at TCA have been carried out. It will be shown that, apart from limitations in the signal-to-noise ratio determined by the parameters of the experimental setup, uncertainties in the other plasma parameters required to evaluate the ion temperature from measured spectra play an important role. This study is therefore complementary to earlier publications by Sharp et al.<sup>2</sup> and Watterson et al.<sup>3</sup>, who investigated the feasibility of the method in general.

## 2. PARAMETER SET

For this study we have chosen a parameter set which corresponds to our scattering experiments carried out on the TCA tokamak<sup>1</sup>. The plasma parameters refer to ohmically heated discharges. A summary of the standard data set, which applies to all cases unless stated otherwise, is given in table 1.

## 3. SPECTRAL SHAPE

In the collective regime ( $\alpha > 1$ ) the shape of the spectrum of scattered radiation depends on a number of plasma parameters in addition to  $T_i$ . Several cases are of particular interest :

- the influence of the ratio  $T_e / T_i$
- the contributions from impurity ions
- the influence of the magnetic field

Starting from our standard data set as given in table 1, several examples are shown in Figs 1 to 3 to illustrate the characteristic changes in the shape of the spectra in these cases. We assume that there is no significant drift of the plasma parallel to the scattering wave vector  $k_d = k_s - k_i$  ( $k_s$  and  $k_i$  are the scattered and incident wave vector respectively). This implies that the spectrum of scattered light is symmetrical (in frequency space), so we only need to consider one half-spectrum. It is obvious that the result of a fitting procedure which evaluates  $T_i$  from a measured spectrum will depend, sometimes rather critically, on the correct value of other plasma parameters (such as  $T_e$ ,  $Z_{eff}$  etc) required as input data. Also, certain plasma conditions may be less favorable than others for a precise  $T_i$  measurement.

In particular, the cases presented in Fig. 2 show that high-Z impurity ions mainly affect the central part of the spectrum<sup>4</sup>. As long as there is not enough information about local impurity concentrations in the plasma, it is preferable to exclude this central part from the analysis. Our simulation is based on the assumption that a range of spectral channels of 80MHz width are available, centered at 0, 80, 160, 240, ... MHz. The first four channels and hence signals below 280 MHz are excluded.

The effect of the angle between the magnetic field and the difference wave vector  $k_d = k_s - k_i$  is shown in Fig. 3a for  $\theta_B$  close to  $90^\circ$ . The enhancement of the scattered intensity in the centre of the spectrum is due to the magnetized electrons<sup>5</sup>. Averaging over the solid angle of observation reduces the effect slightly but for a cone with half angle of  $2^\circ$  centered at  $90^\circ$  it is still noticeable (see solid curve in Fig. 3a). This distortion can be avoided if the direction of observation is tilted out of the poloidal plane. Assuming that the scattering volume is near the plasma centre (as is the case in our experiment) a tilt angle of  $4^\circ$  is sufficient to reduce the influence of the B-field to a negligible level (Fig. 3b).  $\theta_B$  can also be varied by scattering from a volume located away from the magnetic axis of the plasma, where the poloidal field component contributes to the total magnetic field.

Following these qualitative statements, we study the influence of various plasma parameters and of their uncertainties in more detail. In section 4 we investigate the sensitivity of the  $T_i$  evaluation to externally supplied plasma parameters. In section 5 the influence of uncertainties in these parameters as well as noise in the measured signals is studied. For this purpose the "real" conditions of an

experimental measurement are simulated numerically using a Monte-Carlo method.

#### 4. INFLUENCE OF OTHER PLASMA PARAMETERS ON THE EVALUATION OF $T_i$

If density fluctuations at the thermal level are assumed, the spectrum of the scattered radiation for a given set of plasma parameter values can be calculated<sup>6</sup>. The measurement of  $T_i$  is based on a fitting procedure which compares calculated spectra with the data recorded in a number of spectral channels. By taking a calculated spectrum instead of measured data as the input to the fitting routine we reproduce the original  $T_{i0}$  value (in Figs 4 to 8 this point is identified by the intersection of the dashed horizontal and vertical lines). If, however, one of the plasma parameters supplied to the fitting routine is deliberately changed (to simulate an error in the measured value of this parameter) a different result for  $T_i$  is obtained. The amount by which this value differs from the correct one is a measure of the sensitivity of the  $T_i$  measurement to uncertainties in this particular parameter. This procedure has been applied in the following to investigate the influence of several plasma parameters on the evaluation of  $T_i$ .

##### 4.1 EFFECT OF UNCERTAINTY IN $N_E$

The total scattered intensity is proportional to the number of scattering centers within the volume of observation and hence proportional to the electron density  $n_e$ . Therefore, in principle the spectral intensity could provide useful information about the plasma state. However, for a scattering experiment at submillimetre wavelengths, measurements of the absolute intensity are subject to large uncertainties. Calibration methods like Rayleigh or Raman scattering, which are employed in the visible range of the spectrum, are no longer feasible in the far infrared. Hence all parameters which influence the signal level, such as optical losses in the scattered beam path, receiver sensitivity, conversion efficiency and amplifier gain, would have to be known to a high precision to determine the absolute sensitivity of the complete system. To avoid this difficulty no attempt is made to convert the signal levels into absolute intensities. Instead, a two-parameter fitting procedure is used, one parameter being the ion temperature and the other one the scaling factor for the intensity.

The density also influences the spectra in a more subtle way, via the Debye length and the scattering parameter  $\alpha$ . It is this influence which leads to the results presented in Fig. 4 (curve 1) showing the deviation in the evaluated  $T_i$  as a function of the supplied  $n_e$  value. From curve 1 we find that the ion temperature is overestimated by only 25% if the assumed electron density is wrong by a factor of 2. An inaccuracy of  $\pm 10\%$  in  $n_e$  translates to a 2.25% error in fitted  $T_i$ .

The situation is much worse if fits to intensity-calibrated spectra are considered (Fig. 4, curve 2). An error in the supplied value of  $n_e$  is regarded by the fitting procedure as an incorrect value of scattered intensity. The fitting procedure attempts to produce a good fit by varying the only free variable,  $T_i$ , resulting in a very erroneous value of  $T_i$ , as shown in Fig. 4, curve 2. Incorrect ion temperatures are obtained especially when  $n_e$  is underestimated. An underestimate of only 10% in the density results in 40% error in  $T_i$ . This is the main reason why in practically all our investigations the absolute spectral intensity is ignored.

#### 4.2 EFFECT OF UNCERTAINTY IN $T_e$

It has already been stated above that the ratio  $T_e/T_i$  has an important effect on the spectral shape and hence also on the precision of the measurement. Cases for various ion temperatures (300 eV, 400 eV and 500 eV) have been studied and the results are presented in Fig. 5. We find that the larger the ratio  $T_e/T_i$ , the smaller are the errors in  $T_i$  for incorrect  $T_e$  - values. For  $T_{i0} = 400\text{eV}$  a 10% error in  $T_e$  results in over 15% error in  $T_i$ . For this reason it is essential to have an accurate independent  $T_e$  diagnostic available. Underestimating  $T_e$  leads to smaller errors in  $T_i$  than overestimating it.

The spectrum of a deuterium plasma, with the standard parameter values given in table 1, does not show a pronounced ion-acoustic feature. This is quite different for a helium plasma with similar parameters where the ion acoustic resonance is well developed due to the higher mass and charge of the ions. We show in the same Fig. 5 the sensitivity of a  $T_i$  measurement to variations in  $T_e$  for a He-plasma with otherwise identical parameters to our standard D-plasma. It is clearly far less sensitive to uncertainties in  $T_e$ .

#### 4.3 EFFECT OF UNCERTAINTY IN B

As has been mentioned earlier the electron gyration enforced by the magnetic field leads to changes in the spectra also in the case of collective scattering. Since this effect depends strongly on the orientation of the difference wave vector  $k_d$  to the total magnetic field  $B$ , we expect that errors in the value of the angle  $\theta_B$  supplied to the fitting routine will lead to significant errors in the evaluation of  $T_i$ . Fig. 6 shows this dependence for two cases :  $\theta_B = 88^\circ$ , which corresponds to the observation of a scattering volume near the plasma centre along a direction in the poloidal plane; and for  $\theta_B = 86^\circ$ , which refers to the conditions of the latest experiments on TCA<sup>1</sup>.

Since the influence of the magnetic field is only important for  $\theta_B$  near  $90^\circ$ , the errors do not further increase for angles supplied to the fitting routine below  $70^\circ$ . For the case of  $\theta_B = 86^\circ$  an error of not more than 20% is incurred if the influence of the magnetic field is neglected completely, whereas at  $\theta_B=88^\circ$  the temperature would be overestimated by 75%.

For a given angle the field magnitude is far less critical. This is shown in Fig. 7 for our standard conditions. A 10% error in the field value results in only 4.4% uncertainty in  $T_i$ .

#### 4.4 EFFECT OF UNCERTAINTY IN $Z_{EFF}$

The sensitivity of the  $T_i$ -measurement to uncertainties in  $Z_{eff}$  is shown in Fig. 8. For the three cases presented here different species of impurity ions have been included (curve 1 : mixture of C, O, and Fe; curve 2 : C and O; curve 3 : Fe only). The concentrations have been adjusted to obtain the same true  $Z_{eff}$  of 2.53 in all cases. When  $Z_{eff}$  is changed to study its influence on the  $T_i$ -evaluation, the concentrations of all impurity species are varied in proportion. In accordance with the standard parameter set the central part of the spectrum ( $\nu < 280$  MHz) has been excluded. This explains why in the case with a heavy impurity only (curve 3) the influence of  $Z_{eff}$  is rather small. However, the presence of light impurities can lead to large errors in a  $T_i$  measurement especially if their concentrations are underestimated.

## 5. NUMERICAL SIMULATION OF A SCATTERING EXPERIMENT

So far we have studied the influence of uncertainties or errors in various plasma parameters by deliberately changing the data fed into the fitting routine. Instead of experimental data, calculated spectra were used which did not contain any statistical fluctuations. In the following the real conditions of a scattering experiment are simulated numerically<sup>3</sup>. Using a Monte-Carlo technique random noise is added to the calculated spectra. The noise level is expressed in terms of an NEP (noise equivalent power referred to the receiver input) and comprises the influence of fluctuations in the background radiation and the contributions from detector and amplifier noise. In the case of a heterodyne receiver with bandwidth of each spectral channel  $dv$  and an integration time  $\tau$  the signal-to-noise ratio is expressed by equ. (1) :

$$(1) \quad \frac{S}{N} = \frac{P_s}{(P_s + P_n)} * \sqrt{1 + dv \cdot \tau},$$

where  $P_s$  is the power of the scattered radiation collected by the receiver and  $P_n$  represents the power equivalent to the noise. A sample spectrum  $P_s(\omega_i)$  is obtained from  $i = 1, \dots, N$  theoretical values  $P_t(\omega_i)$  by :

$$(2) \quad P_s(\omega_i) = \chi_m^2 [P_t(\omega_i) + P_n(\omega_i)] - P_n(\omega_i)$$

with a  $\chi^2$ -distribution of  $m=2(dv \tau + 1)$  degrees of freedom<sup>3</sup>. Each simulation consists of a series of sample spectra (in our case 31). From a fitting procedure we obtain a set of  $T_i$  values which represent possible results of a measurement for the given conditions.

The numerical simulation also permits us to investigate by statistical means the influence of uncertainties associated with measured plasma parameters that have to be supplied to the fitting routine. For each sample spectrum the value of each important plasma parameter is taken at random from a Gaussian distribution which is centered at the measured value of the particular parameter. The width of this distribution (usually expressed as fractional variation in the parameter) is related to the uncertainty of the measurement from which this parameter has been obtained. The statistical analysis of the set of  $T_i$  values allows determination of the accuracy of the  $T_i$  evaluation and reveals systematic deviations.



## 5.1 THE QUALITY OF THE FIT

For the fitting procedure we define as merit function  $F(T_i)$  the RMS value, summed over all spectral channels, of the differences between the artificially created noisy spectrum and the theoretical spectrum which corresponds to the particular  $T_i$  found by the fitting routine. It is in fact this quantity that is minimized during the fitting procedure. Therefore, the smaller  $F(T_i)$  the better the fit. Also, in order to obtain meaningful results,  $F(T_i)$  should have a pronounced minimum. If the minimum is rather shallow we cannot expect to resolve small changes in  $T_i$ . Such a situation can occur when either the spectrum does not change significantly with ion temperature or the signal-to-noise ratio is too small.

Figure 9a shows  $F(T_i)$  versus  $T_i$  for an ideal, noiseless case where all the parameters are exactly and correctly known. It is evident that the curve has a very pronounced minimum (0 to within numerical precision in this case) at exactly 400 eV (the true  $T_i$  value). In Figure 9b the five curves plotted differ from Fig. 9a (and each other) due to the presence of noise in the "experimental" spectrum. The curves, corresponding to five separate "shots", all show a clear minimum, albeit much less pronounced than in the noiseless case. The estimated  $T_i$  values are also no longer exactly 400 eV.

If we assume uncertainties in  $n_e$ ,  $T_e$  and  $B$  at the same time we obtain the results presented in Fig. 9c. Some of the shots show  $F(T_i)$  with either a broad, poorly defined minimum in the vicinity of 1000eV (shot 5), or else decreasing to some minimum value well beyond the plotted range (shot 1).

The curve  $F(T_i)$  is clearly asymmetric. This reflects the fact that variations in the spectral shape are much more pronounced for small  $T_i$  values. Hence it is less likely that the evaluated  $T_i$  will be too low, whereas a spectrum with a large  $T_i$  might still produce a reasonably good fit.

## 5.2 EFFECTS OF UNCERTAINTIES IN OTHER PLASMA PARAMETERS

In addition to the statistical fluctuations caused by noise we can also assume uncertainties (expressed as a fractional variation) in one or several of the externally supplied plasma parameters. In Fig. 10 we show the result of simulation runs for specified fractional variations in  $T_e$ . In Fig. 10a the standard deviation of

$T_i$ -fits (S) is expressed as percentage value of the true ion temperature and Fig. 10b shows the difference between the mean of the fits and the true  $T_i$ -value (E), also expressed as percentage of the true value. For the data set of table I, and with all plasma parameters precisely known, the mean systematic error in  $T_i$  is  $\pm 3\%$  with a shot to shot scatter of  $17 \pm 3\%$ . As the fractional variation in  $T_e$  is increased from zero to 0.2 or more, the mean systematic error rises to about 9% of  $T_{i0}$ , and the shot-to-shot scatter to over 35%. If, however,  $T_e$  can be reliably measured to within 10% of the true value, the values of E and S remain small. Figure 11 shows the relationship between  $T_e$  supplied to the fitting routine and  $T_i$  estimates. It confirms the behaviour observed with noiseless spectra (Fig. 5, curve labeled 400 eV).

For the investigation of the influence of uncertainties in the impurity contents a single variable,  $\Delta F_i$ , determines the statistical scatter in the supplied abundances of all three impurity ions. In Fig. 12 E and S are plotted against  $\Delta F_i$ . For each shot the abundances of the three impurity species are independent of each other. Since they vary randomly and independently, we plot in Fig. 13 estimated  $T_i$  against the fractional abundance of the deuterium ion. To account for uncertainties in the other plasma parameters we assume  $\Delta n_e = 0.05$ ,  $\Delta T_e = 0.10$  and  $\Delta B = 0.03$ . Figures 12 and 13 show only a slight dependence of estimated  $T_i$  on impurity abundance. The mean systematic error in  $T_i$  remains constant at about  $5 \pm 5\%$  for  $\Delta F_i$  up to 0.8, and the shot-to-shot scatter rises from about 22% to 30% as  $\Delta F_i$  ranges from 0 to 0.8.

According to the results presented in Fig. 8, the  $T_i$ -values obtained are too large if the impurity concentration is underestimated (fractional abundance of deuterium ions  $> 0.968$ ) and vice versa. This is also observed in Fig. 13. The results of Figures 12 and 13 represent a case where the concentrations of all impurities have been measured independently (with limited precision). In reality often  $Z_{eff}$  only is measured (sometimes only estimated). Under these circumstances systematic effects like the peak around  $Z_{eff}=1.5$  in Fig. 8 are more important.

The measurement precision depends strongly on the shape of the spectrum. In the case of  $T_e / T_i \gg 1$  the spectrum (Fig. 1) shows a pronounced ion-acoustic resonance. Its width and position are very sensitive to a change in  $T_i$  at fixed  $T_e$ . Therefore a more precise  $T_i$  measurement should be obtained at low  $T_i$  where the ratio of  $T_e / T_i$  is large. This is demonstrated with simulation results in Fig. 14. As function of the ion temperature, for fixed electron temperature of 700eV, we show

the standard deviation of 31  $T_i$  values with respect to a single measurement expressed as percentage of the true value. Two sets of simulation runs were done for each  $T_i$  value assuming that all other plasma parameters are known exactly. The shot-to-shot scatter of 5% for small  $T_i$  increases to 20% for plasmas with  $T_e=T_i$ .

### 5.3 EFFECT OF THE RECEIVER SENSITIVITY AND POSITION OF THE SPECTRAL CHANNELS

Up till now we have discussed the influence of imprecisely known plasma parameters on the precision of an ion temperature measurement with collective Thomson scattering. If these parameters were all accurately known, the measurement precision would be limited by the signal-to-noise ratio. According to equation 1 the S/N ratio achievable is finally limited by the square-root term since  $P_s/(P_s+P_n)$  can never exceed one. The maximum value possible for the system discussed (80MHz channels, 1.38 $\mu$ s integration time) is 10.5.

In Fig. 15 we show the spectrum for our standard conditions (solid line), the signals in each channel obtained from a particular simulation run (circles) and the standard deviations  $\sigma$  of 31 simulations, represented as error bars of length  $2\sigma$ , centered on the means. A similar graph would be obtained from a real scattering experiment. The S/N ratios range from approx. 7 to 10 and hence are very close to the theoretical limit. Experimentally signal to noise ratios of only half of these values have been achieved so far<sup>1</sup>. S/N ratios of 8 and 4 translate into pre-integration signal-to-noise ratios of 3.2 and 0.6 respectively. The precision or shot-to-shot scatter achievable in the case of exact plasma parameters is degraded by approximately 50% in going from the standard NEP of  $5 \cdot 10^{-19}$  W/Hz to  $2.5 \cdot 10^{-18}$  W/Hz (Fig. 16). Hence further efforts to either increase the detection sensitivity by up to a factor of 5 (possible with state-of-the-art Schottky diode mixers) or the power delivered by the laser (difficult to achieve) would be worthwhile. Also, losses in the optical system should be further reduced.

Optimization of the instrumental parameters should also deal with the position of the receiver channels with respect to the spectrum to be observed. In the following we assume that the number of channels and hence the total covered bandwidth are fixed but their position relative to the spectrum can be changed. We have already mentioned that the inclusion of the line center is not advisable since parasitic stray light, scattering from impurities and contributions from non-thermal fluctuations all influence the signal close to line center. On the

other hand, if the spectral band is too far out in the wings of the thermal feature, poor signal-to-noise ratio precludes an accurate fit. Obviously there must be an optimum between these extremes.

For the following simulation runs (Fig. 17) it has been assumed that  $\Delta n_e=0.05$ ,  $\Delta T_e=0.10$  and  $\Delta B=0.03$ . It is also assumed that the total spectral band covered is 960MHz, consistent with the 12 channels of 80MHz bandwidth each as given in table I. The precision is of the order of 25% for cutoff frequencies up to 360 MHz. It then drops to below 10% where it remains up to a cutoff frequency as high as 1080 MHz. Thereafter it rises rapidly. From these results we conclude that it is advantageous to include receiver channels near the wings of the spectrum, even if the scattered intensity is low. Such an arrangement reacts in a more sensitive way to changes in  $T_i$ . The signals in the central channels are higher, permitting better signal-to-noise ratios, but their relative variation with  $T_i$  is smaller.

## 6. CONCLUSIONS

Using numerical simulations we have investigated the precision of an ion temperature measurement of a tokamak plasma by collective Thomson scattering from thermal density fluctuations. The parameters used during this study refer to experiments carried out at the TCA tokamak. They are typical for a scattering experiment using a pulsed D<sub>2</sub>O laser at 385  $\mu\text{m}$  and a heterodyne receiver comprising a Schottky barrier diode mixer. Therefore the case studied here may serve as an example which points out some important aspects that have to be taken into account when designing a collective scattering experiment to measure  $T_i$ . In particular, we have shown that it is essential to know other plasma parameters (especially  $T_e$ ) with sufficient precision. Care should be taken when selecting the distribution of the spectral channels of the receiver system in order to obtain optimum sensitivity to variations in  $T_i$ .

The quoted precision of a  $T_i$ -measurement depends on the parameters of the tokamak plasma and the specifications of the experimental set-up. Although they can be regarded as typical, a careful reevaluation will be necessary for other cases taking into account the specific conditions which may be different from those considered in this study. As far as our experiment on TCA is concerned, further improvements still seem to be possible (e.g. improving the receiver sensitivity, rearranging the spectral channels). In this way the relative

uncertainty of a  $T_i$  measurement could be reduced to about 10% under optimum conditions.

In the future, other coherent radiation sources (e.g. gyrotrons or FELs) allowing longer pulses and integration times may become available. In this case, either a substantial improvement in the signal-to-noise ratio can be obtained or measurements at lower plasma densities become feasible.

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## FIGURE CAPTIONS

- Fig. 1: Spectra for the standard conditions defined in table I and variable  $T_i$ . For fixed  $T_e=700\text{eV}$  the ratio  $T_i/T_e$  is 0.1, 0.2, 0.5, 1, 2, 5, 10 for the curves 1 to 7.
- Fig. 2: Spectra for standard conditions and no impurities ( curve 1), light impurities (C, O, curve 2), the heavy impurity Fe (curve 3) and the standard impurity composition according to table I (curve 4). The impurity concentrations have been adjusted in proportion to give  $Z_{\text{eff}}=2.53$  (curves 2-4).
- Fig. 3: Spectra for different angles  $\theta_B$ . The curves labeled "averaged" have been calculated by integration over the solid angle assuming a Gaussian intensity profile for a cone of half angle  $2^\circ$ , centered at  $\theta_B=90^\circ$  (a) and  $\theta_B=86^\circ$  (b). The "averaged" shape is well approximated by an angle of  $89^\circ$  in (a), whereas for case (b) an integration is superfluous.
- Fig. 4: Ion temperatures obtained by fitting "standard" (see table I) noiseless spectra with spectra calculated for incorrect density values.  
Curve 1: freely adjustable intensity scale;  
Curve 2: calibrated intensity scale.
- Fig. 5: Ion temperatures obtained by fitting "standard" (see table I) noiseless spectra with spectra calculated for incorrect  $T_e$  values.  
Curves are shown for 3 ion temperatures in D and for  $T_i = 400 \text{ eV}$  in He.
- Fig. 6: Ion temperatures obtained by fitting "standard" (see table I) noiseless spectra with spectra calculated for incorrect  $\theta_B$  values. The corresponding curve for a "true"  $\theta_B$  value of  $86^\circ$  is also shown.
- Fig. 7: Ion temperatures obtained by fitting "standard" (see table I) noiseless spectra with spectra calculated for incorrect magnetic field values.
- Fig. 8: Ion temperatures obtained by fitting "standard" (see table I) noiseless spectra with spectra calculated for incorrect impurity concentrations (curve 1), light impurities C and O only (curve 2) and heavy impurity Fe

only (curve 3). The "correct"  $Z_{\text{eff}}$  is always 2.53 and the relative impurity concentrations are kept fixed.

Fig. 9 Log of the least-squares-fitting merit function  $F$  plotted against  $T_i$ . In all cases the true  $T_i$  value is 400 eV.

(a) Noiseless "experimental" spectrum, plasma parameters  $n_e$ ,  $T_e$  and  $B$  exactly and correctly known.

(b) Merit function for five simulated shots where "experimental" spectrum is noisy, but  $n_e$ ,  $T_e$ , and  $B$  are exactly and correctly known. The fitted  $T_i$ -values are: 345, 406, 411, 446, 472 eV.

(c) Merit functions for five simulated shots where "experimental" spectrum is noisy, and  $n_e$ ,  $T_e$  and  $B$  are only approximately known. The fitted  $T_i$ -values are: 146, 354, 357, 1166, 1541 eV.

Fig. 10 (a) Percentage shot-to-shot standard deviation,

(b) mean systematic percentage error in fitted  $T_i$ , both as function of fractional variation in given  $T_e$ .

Each point was obtained from 31 simulated shots. Values of all other parameters are as given in table 1.

Fig. 11 Fitted  $T_i$  as a function of the supplied value of  $T_e$  for about 300 simulated shots. The correct values of  $T_e$  and  $T_i$  are 700 eV and 400 eV respectively. Values of all other parameters are as given in table 1.

Fig. 12 (a) Percentage shot-to-shot standard deviation,

(b) mean systematic percentage error in fitted  $T_i$ , both as function of fractional variation in given impurity abundance.

Each point was obtained from 31 simulated shots. Values of all other parameters are as given in table 1.

Fig. 13 Fitted  $T_i$  as a function of the supplied fractional abundance of deuterium ions, for about 210 shots. The true values of  $D$  and  $T_i$  are 0.968 and 400 eV respectively.

Values of all other parameters are as given in table 1.

Fig. 14 Percentage shot-to shot standard deviation in fitted  $T_i$  as a function of "true  $T_i$ ". Each point was obtained from 31 simulated shots. Values of all other parameters are as given in table 1.

Fig. 15 Scattered spectra. The circles represent the signal recorded in each channel for a typical single simulated shot. The vertical bars give  $\pm 1$  standard deviation obtained from 31 such shots, and the solid line is the noiseless spectrum calculated from standard conditions given in table 1.

Fig. 16 (a) Percentage shot-to-shot standard deviation,  
(b) mean systematic percentage error in fitted  $T_i$ , both as function of receiver NEP.  
Each point was obtained from 31 simulated shots. Values of all other parameters are as given in table 1.

Fig. 17 (a) Percentage shot-to-shot standard deviation,  
(b) mean systematic percentage error in fitted  $T_i$ , both as function of lower cutoff frequency of the first receiver channel.  
Each point was obtained from 31 simulated shots. Values of all other parameters are as given in table 1.



Table I. Standard input data set for a D-plasma.

<i>Subject</i>	<i>Parameter</i>	<i>Symbol</i>	<i>Value</i>	
<b>Plasma</b>	electron density	$n_e$	$8.10^{13} \text{ cm}^{-3}$	
	electron temperature	$T_e$	700 eV	
	magnetic field	B	1.5 Tesla	
	true ion temperature	$T_{i0}$	400 eV	
	fractional abundance of the ion species :			
		Deuterium		0.968
		Carbon		0.02
		Oxygen		0.01
		Iron		0.002
	mass (in amu):			
		Deuterium		2.0
		Carbon		12.0
		Oxygen		16.0
		Iron		55.8
	charge (in electron charge units) :			
		Deuterium		1
		Carbon		6
		Oxygen		8
		Iron		19
		temperature : (for all impurity species)	$T_{imp}$	400 eV
uncertainties expressed in standard deviations (SD) :				
	SD for $n_e$	$\Delta n_e$	0 (0.05)*	
	SD for $T_e$	$\Delta T_e$	0 (0.10)*	
	SD for B	$\Delta B$	0 (0.03)*	
	SD for impurity abundance	$\Delta F_i$	0 (0.4)*	
	SD for impurity ion temp.	$\Delta T_{imp}$	0	
<b>Scattering experiment</b>	scattering angle	$\theta$	90°	
	angle to magnetic field**	$\theta_B$	88°	
	f number (focusing and detection optics)	f	13.5	
<b>Laser</b>	wavelength	$\lambda$	385 $\mu\text{m}$	
	power	P	1.0 MW	
	pulse duration	$\tau$	1.38 $\mu\text{s}$	
<b>Receiver</b>	noise equivalent power	NEP	$5 \cdot 10^{-19} \text{ W/Hz}$	
	minimum IF frequency	$\nu_0$	280 MHz ***	
	channel width	$\Delta\nu$	80 MHz	
	number of channels	N	12	

Derived <sup>+</sup>	diameter of scattering volume	$D_V$	6.6 mm
	scattering parameter	$\alpha$	1.97
	Debye length	$\lambda_D$	22 $\mu\text{m}$
	effective charge	$Z_{\text{eff}}$	2.53

\* The values in brackets are estimated for typical TCA tokamak conditions.

\*\*  $\theta_B$  is the angle between the scattering k-vector and the magnetic field B.

\*\*\* The zero of the IF-scale corresponds to the D<sub>2</sub>O laser line and hence the centre of the spectrum. The first IF-channel covers the band from 280 to 360 MHz.

+ These values are calculated from the input parameters.

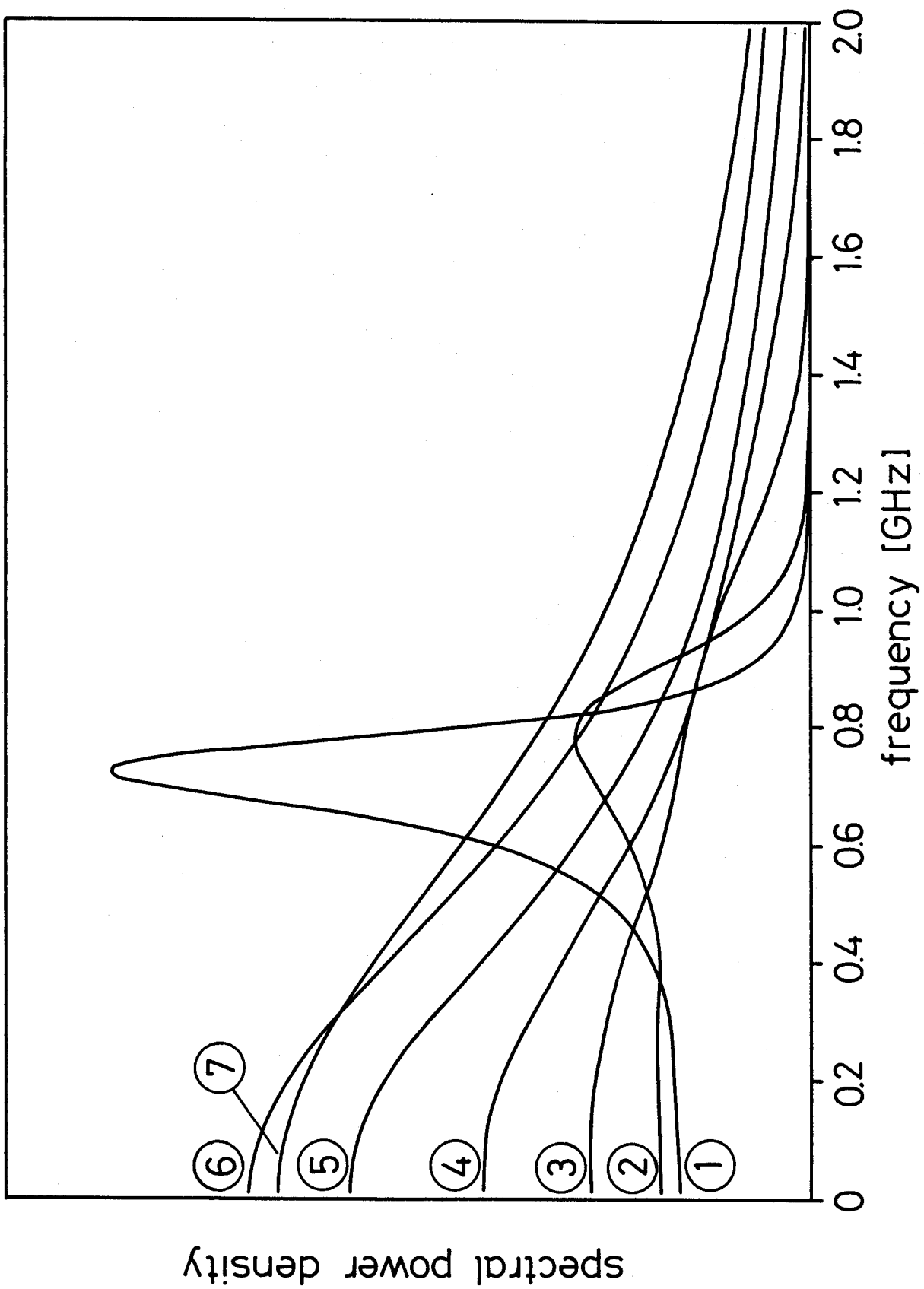


FIG. 1

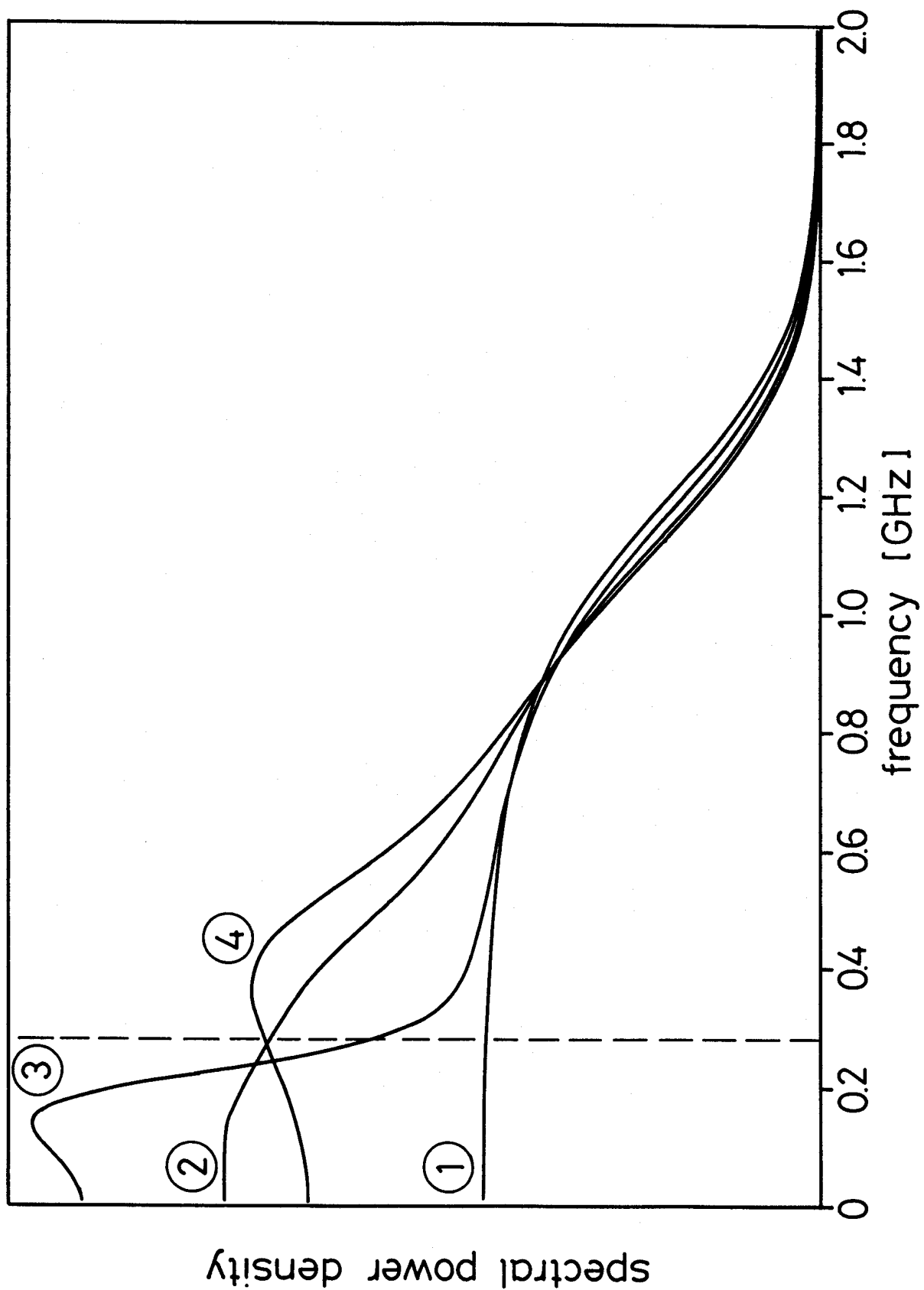


FIG. 2

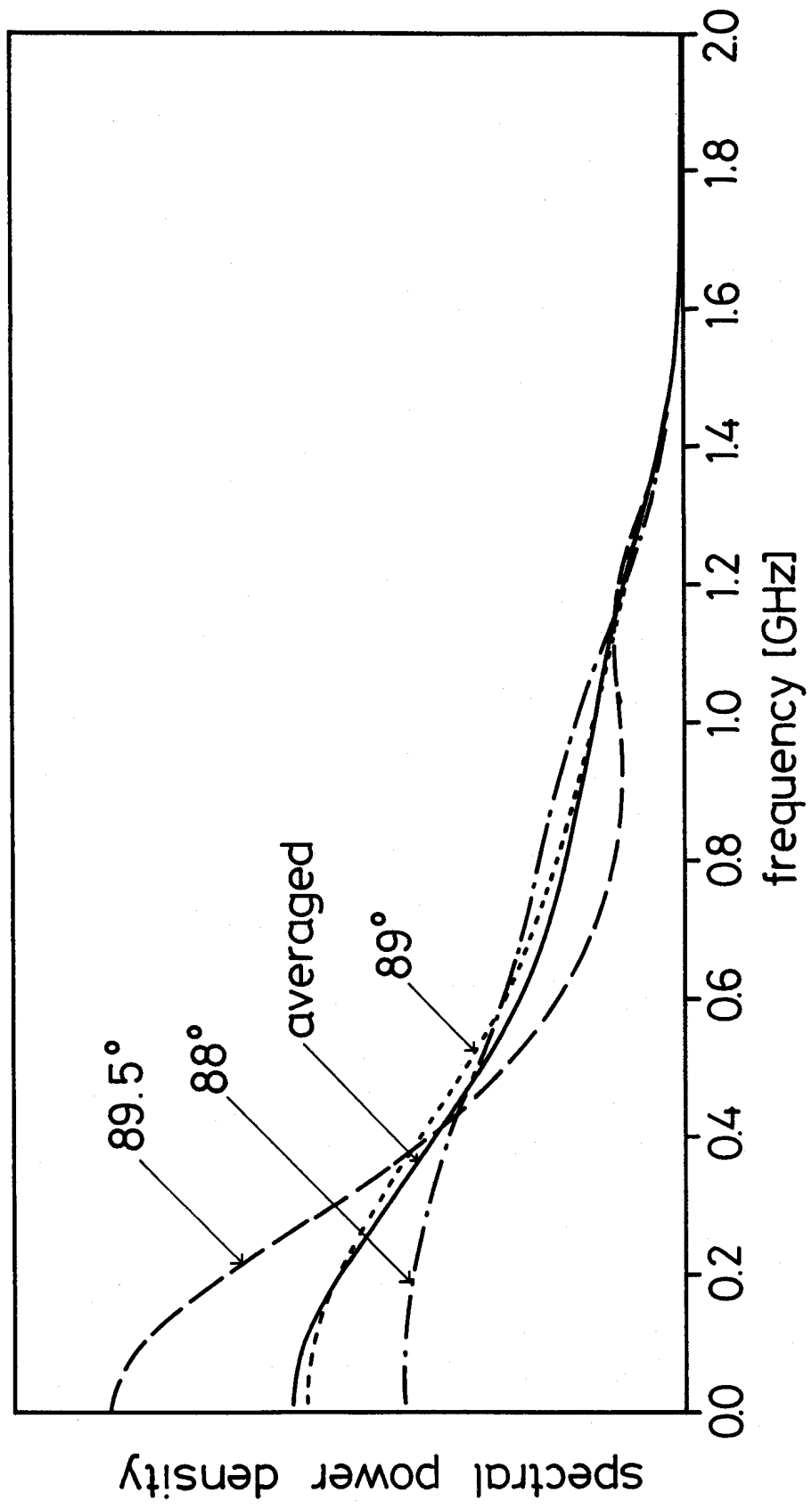


FIG. 3 A

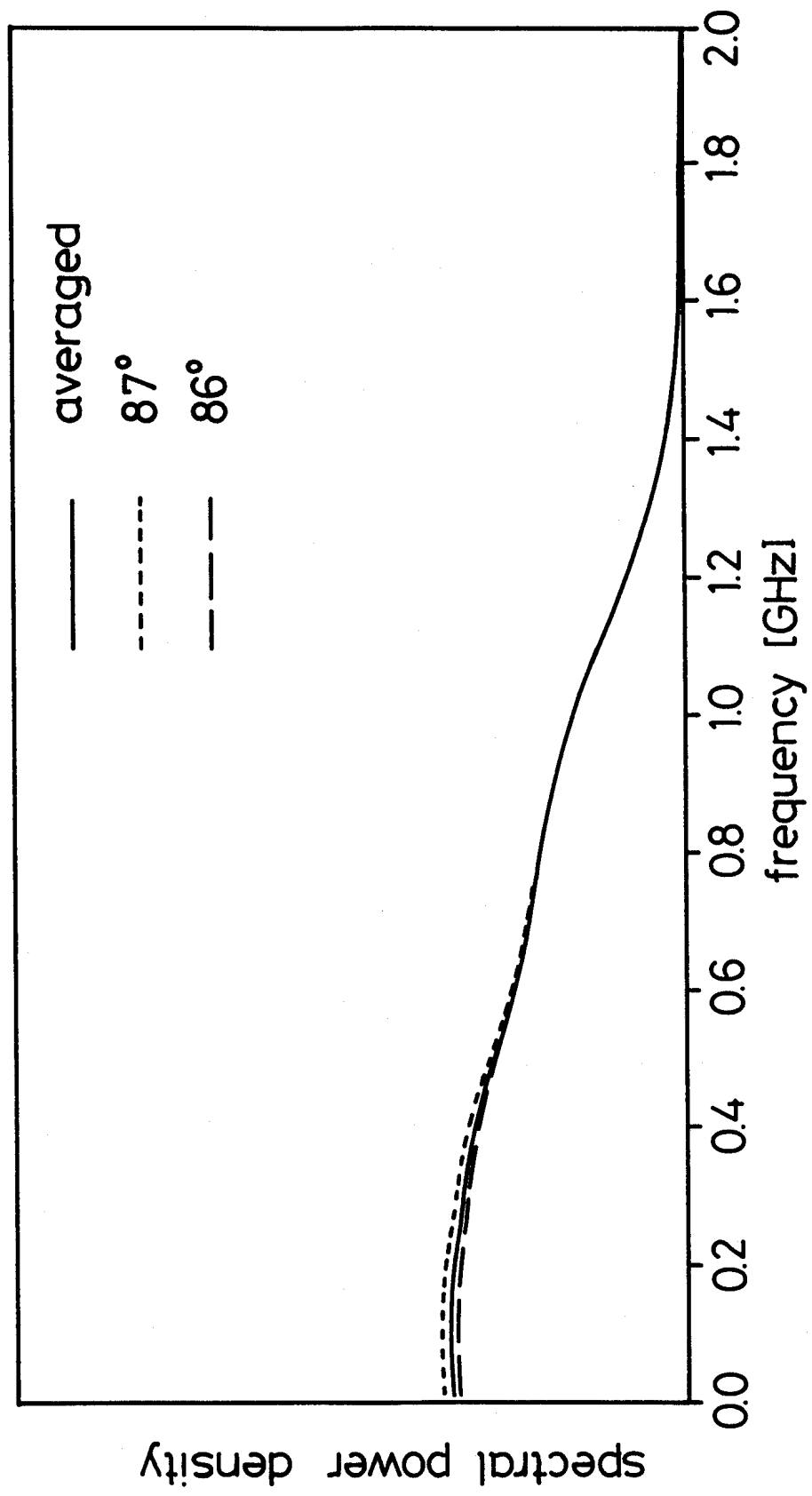


FIG. 3B

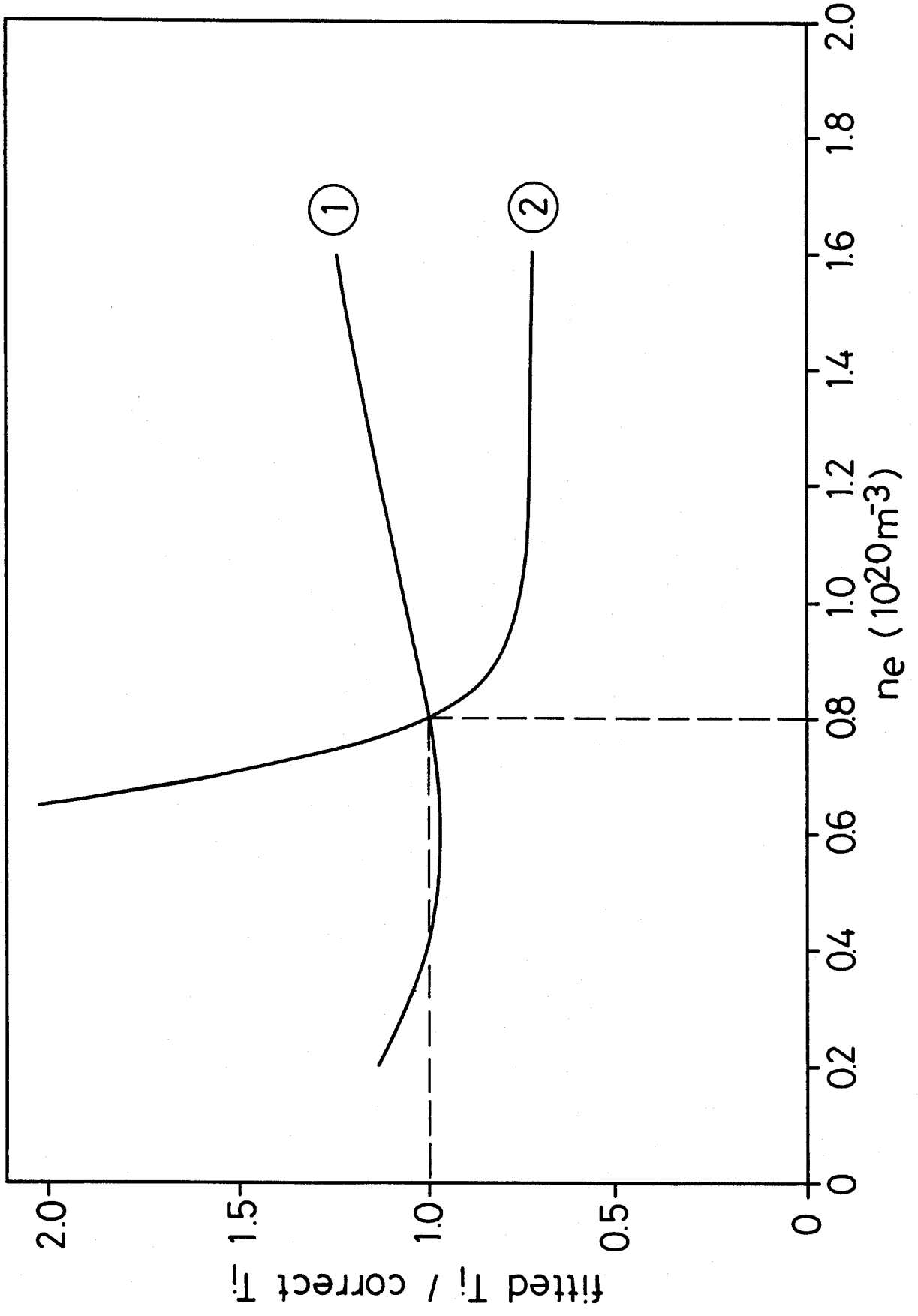


FIG. 4

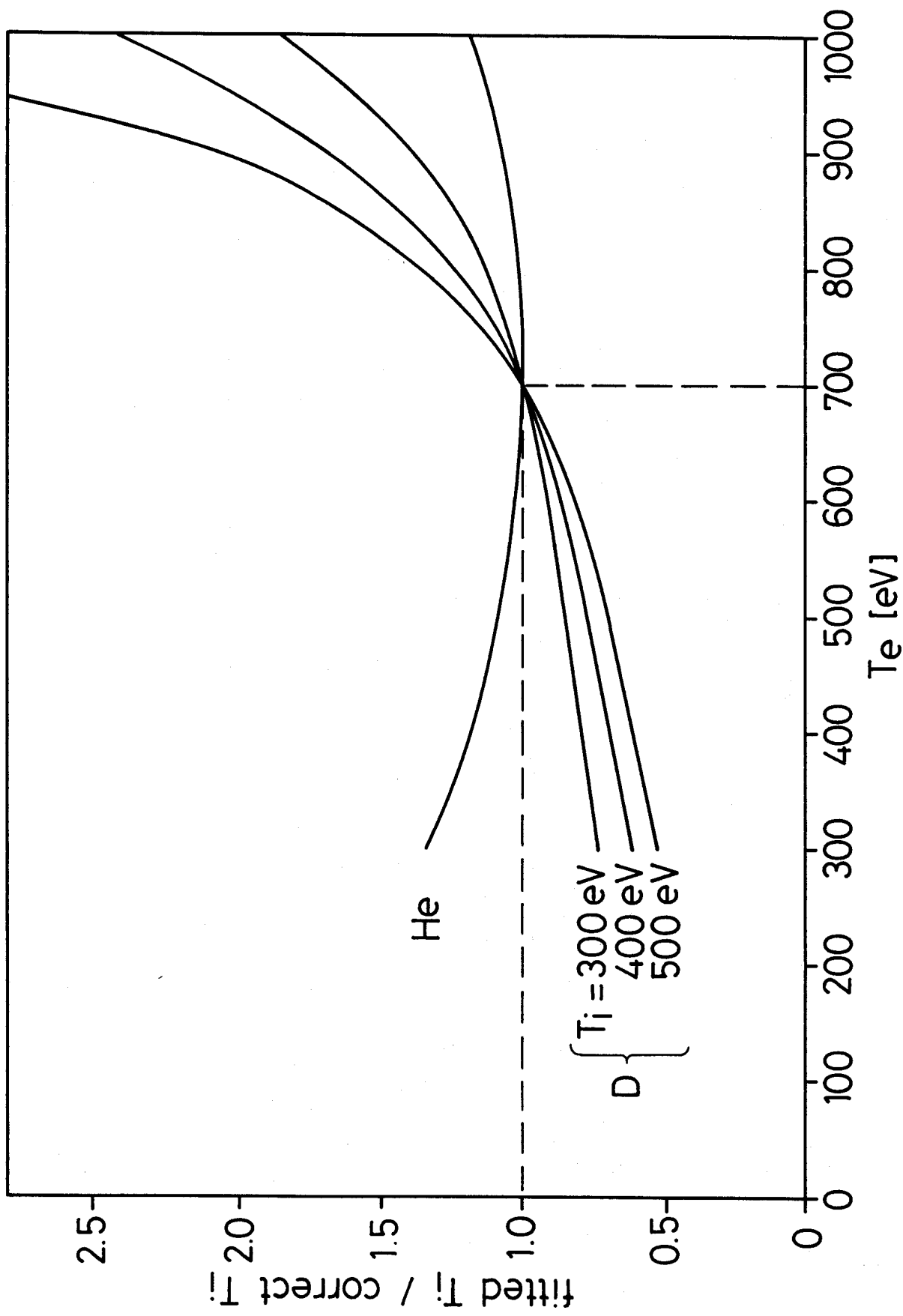


FIG. 5



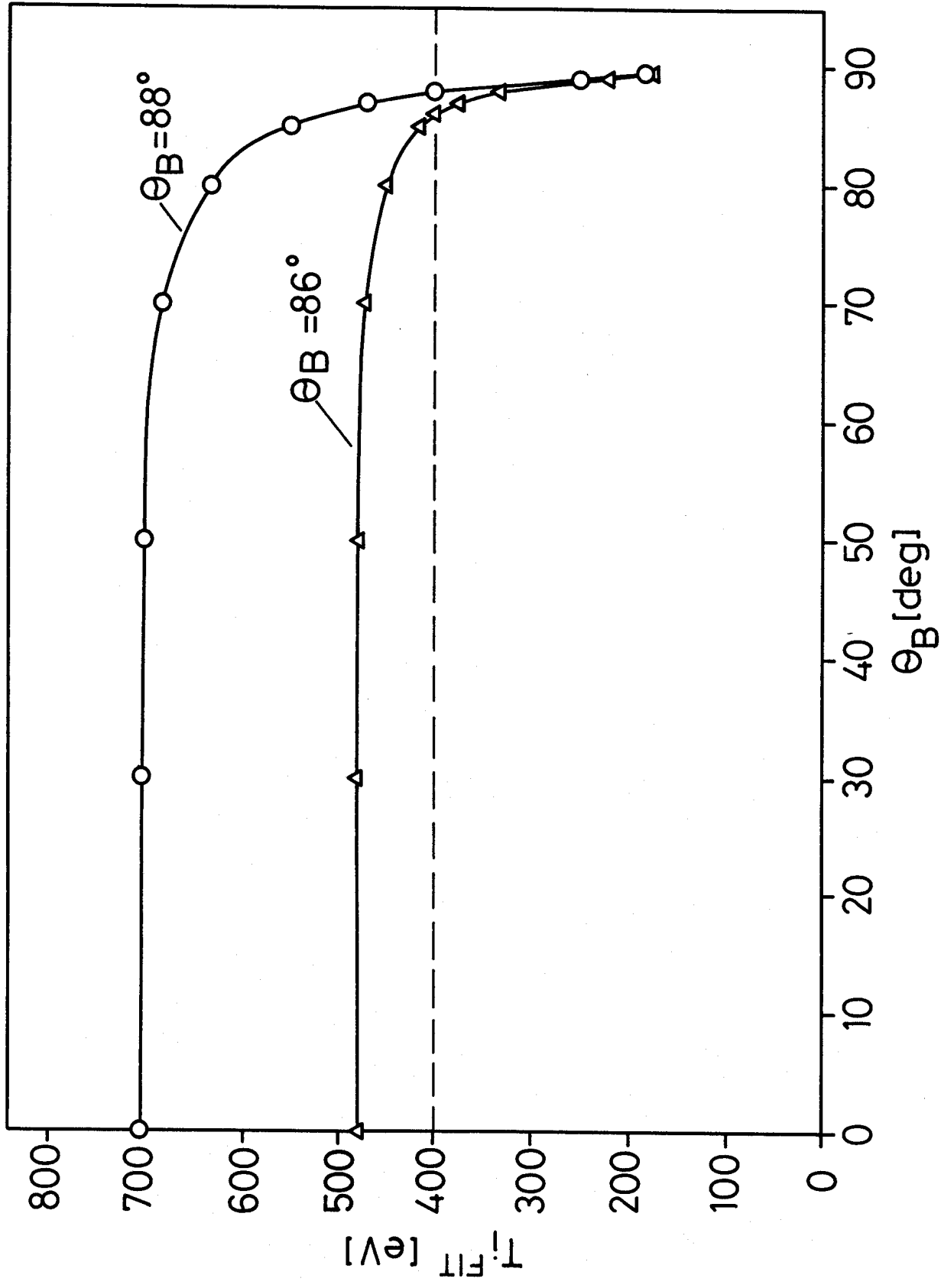


FIG. 6

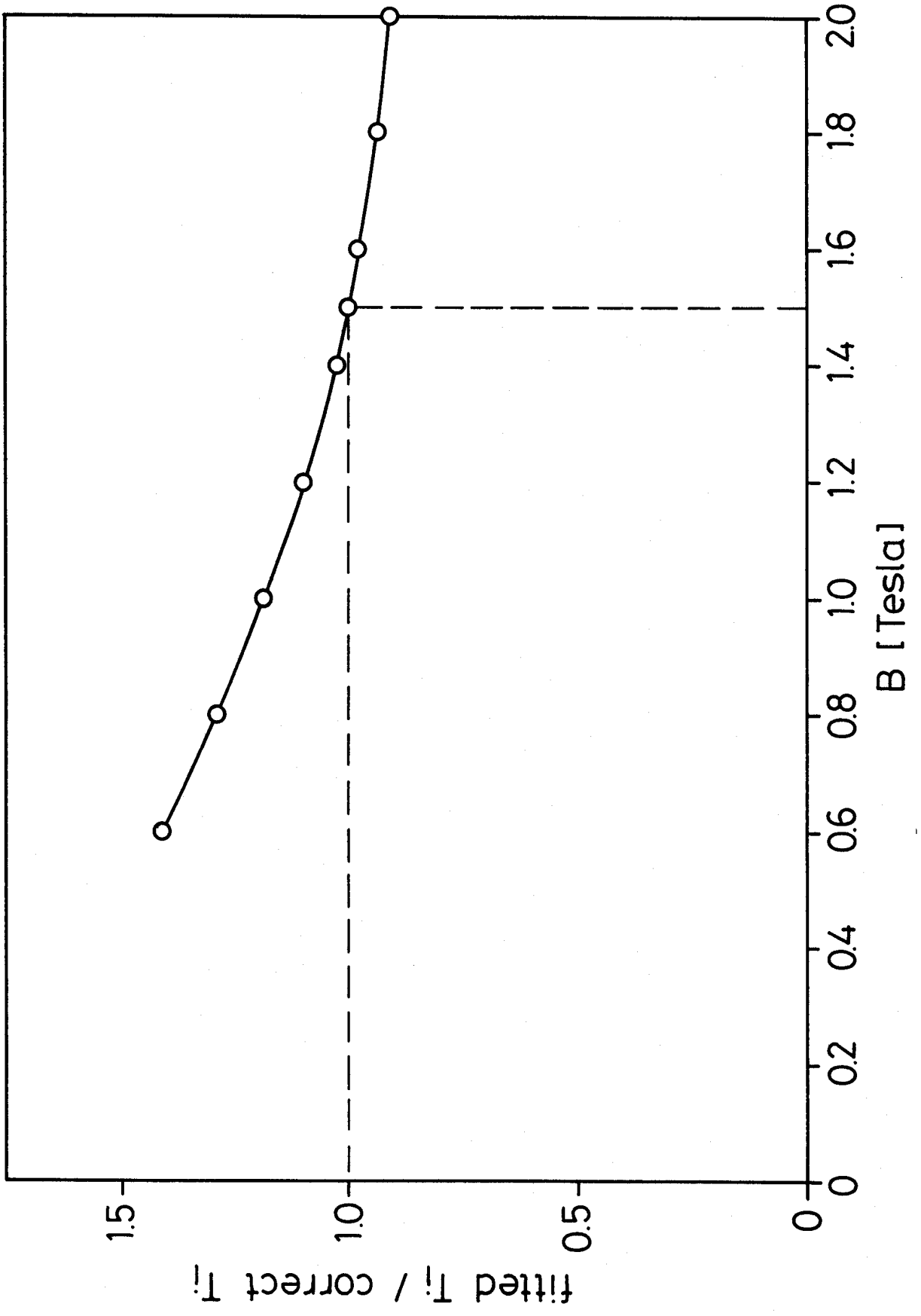


FIG. 7

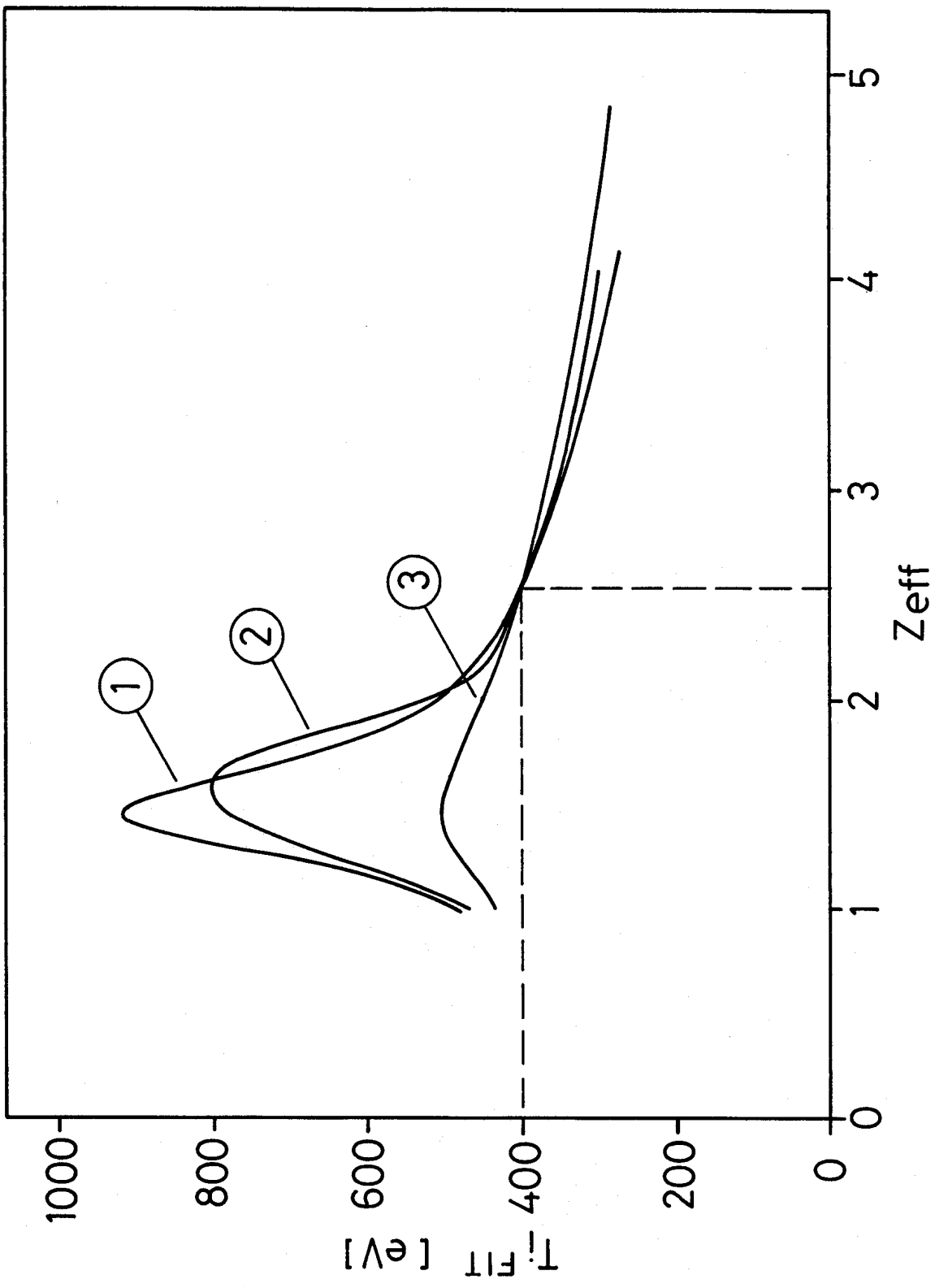


FIG. 8

FIG. 9

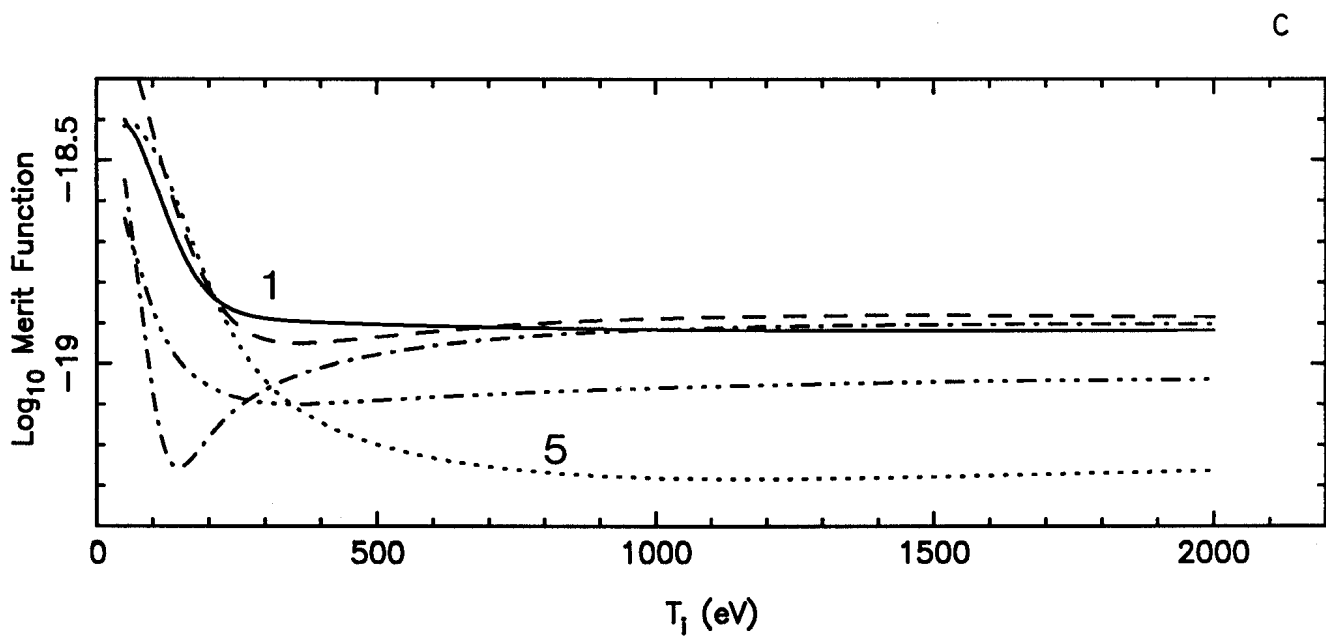
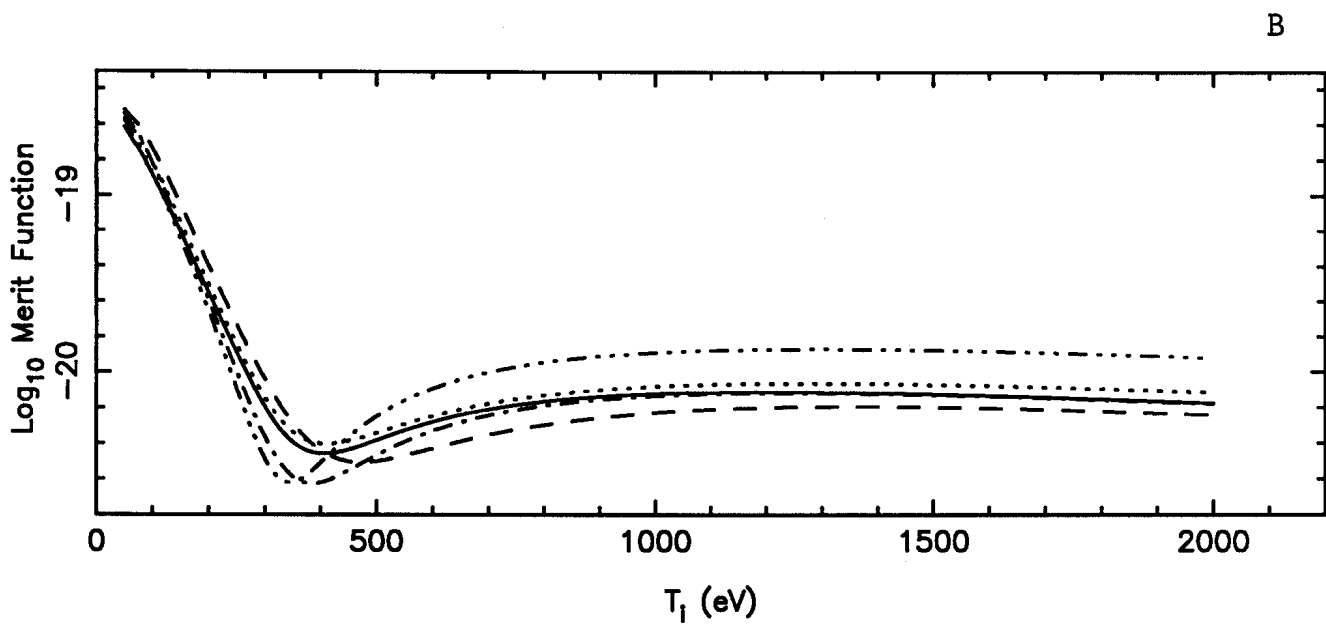
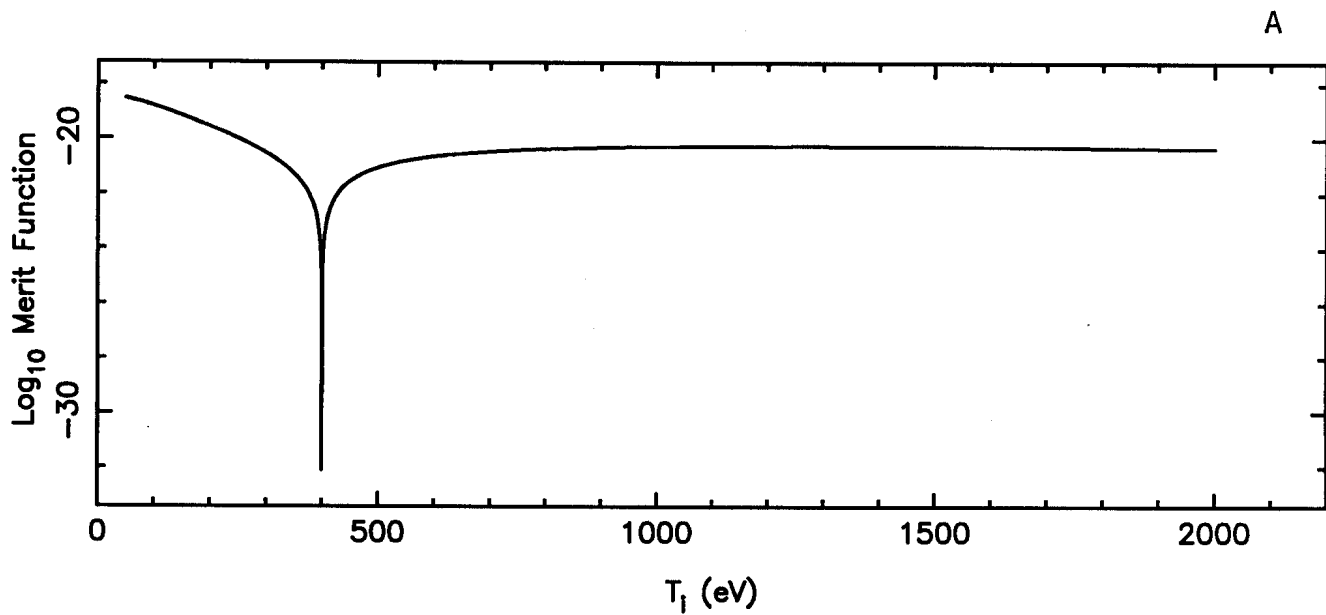


FIG. 10

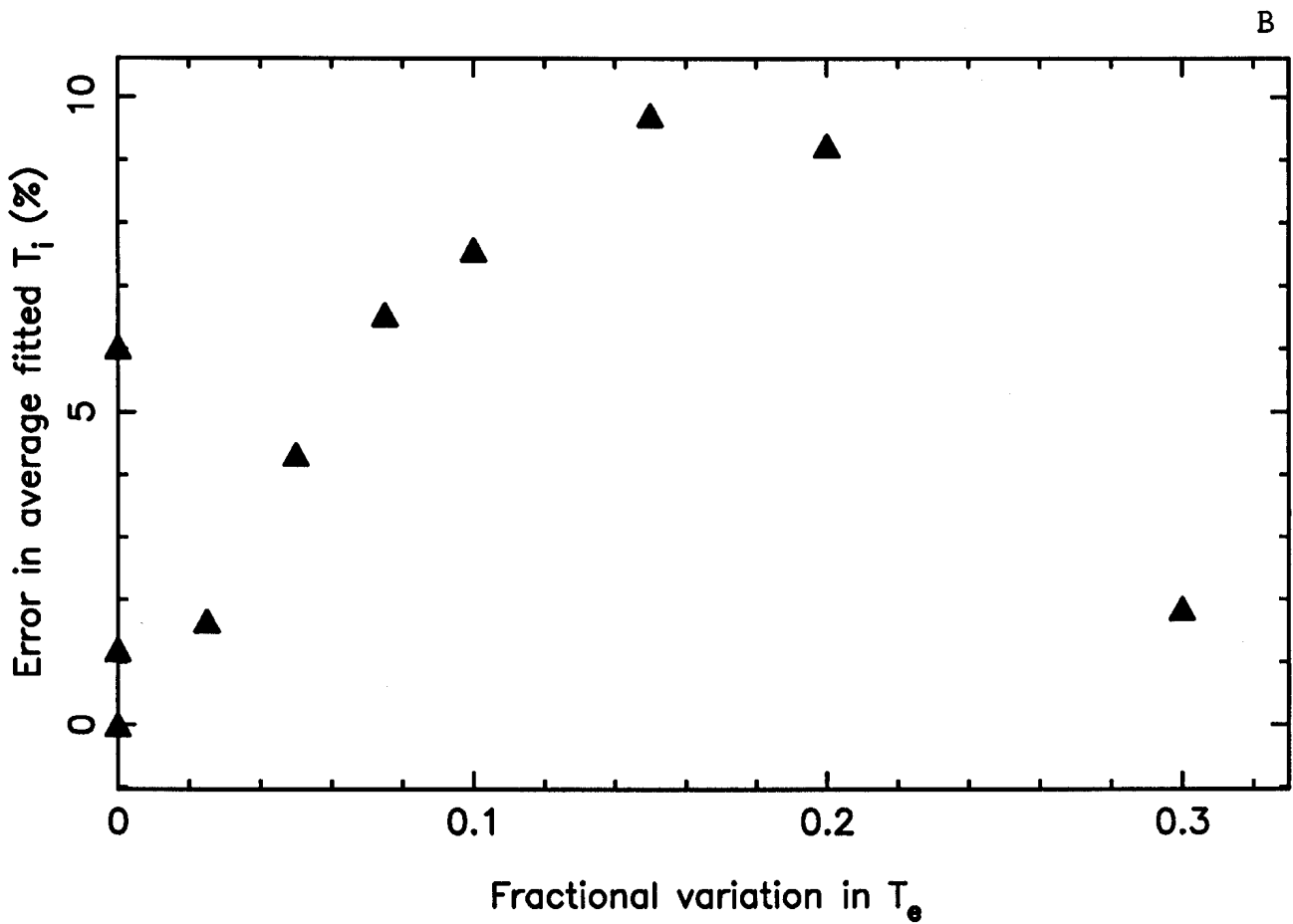
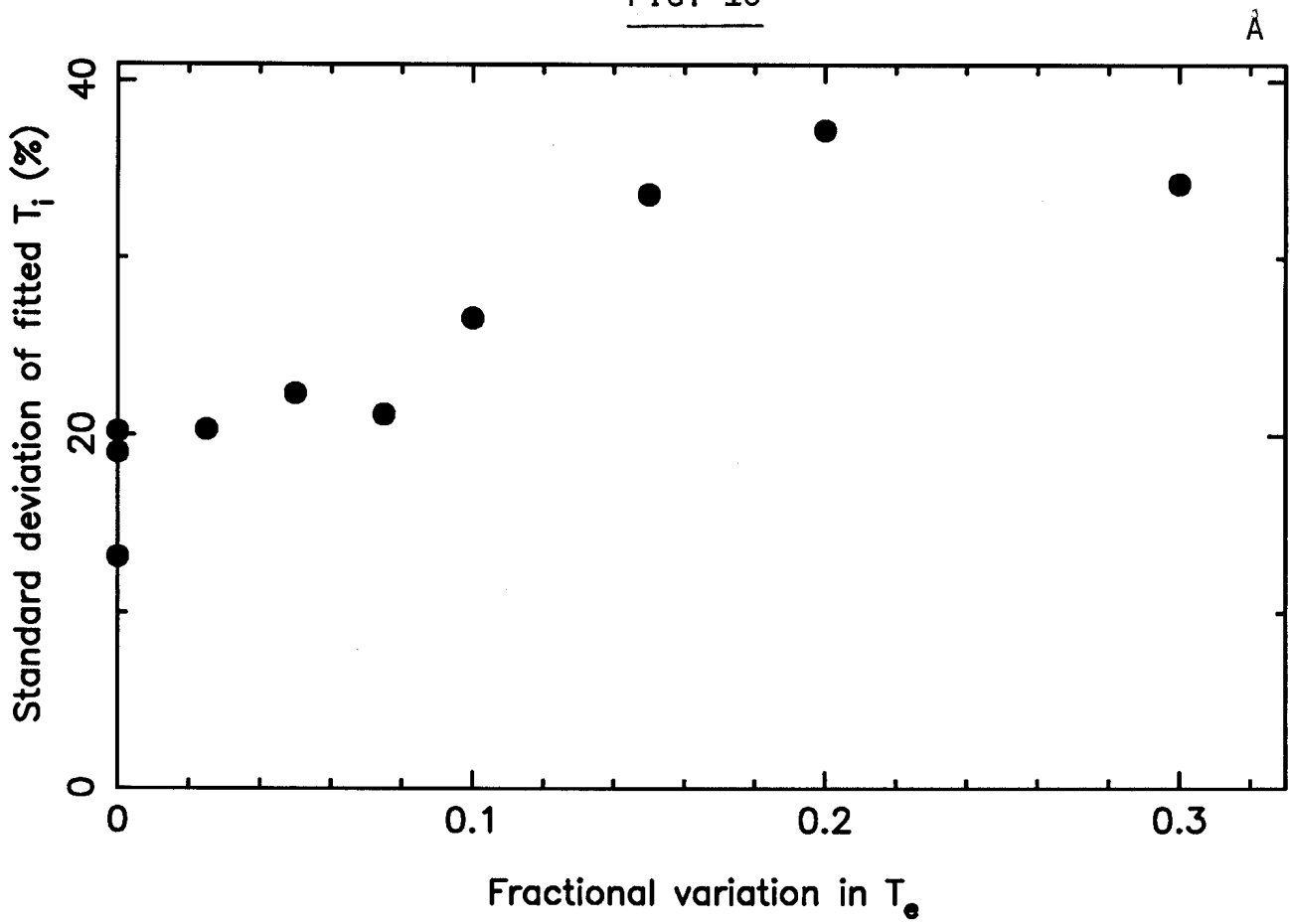


FIG. 11

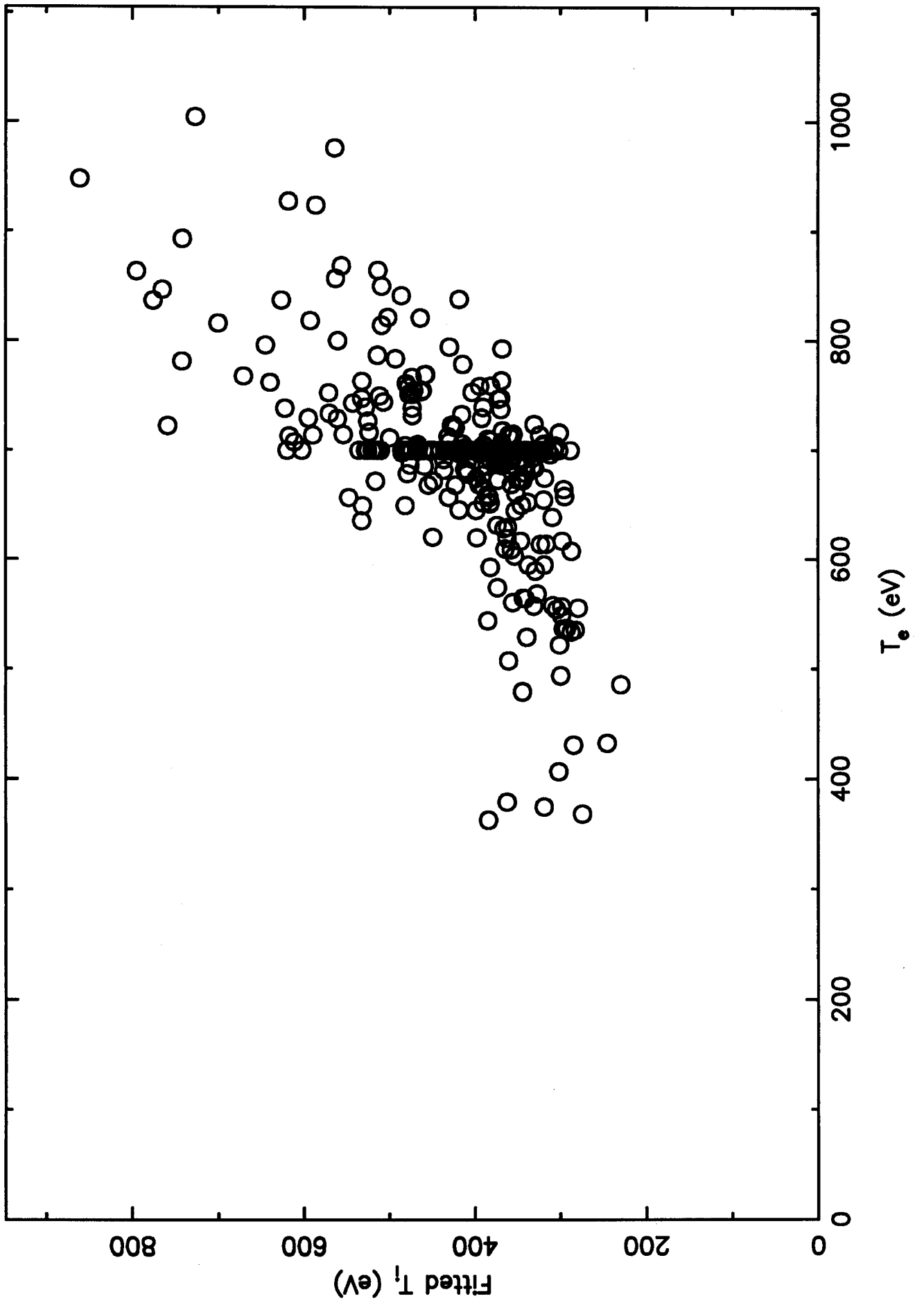


FIG. 12

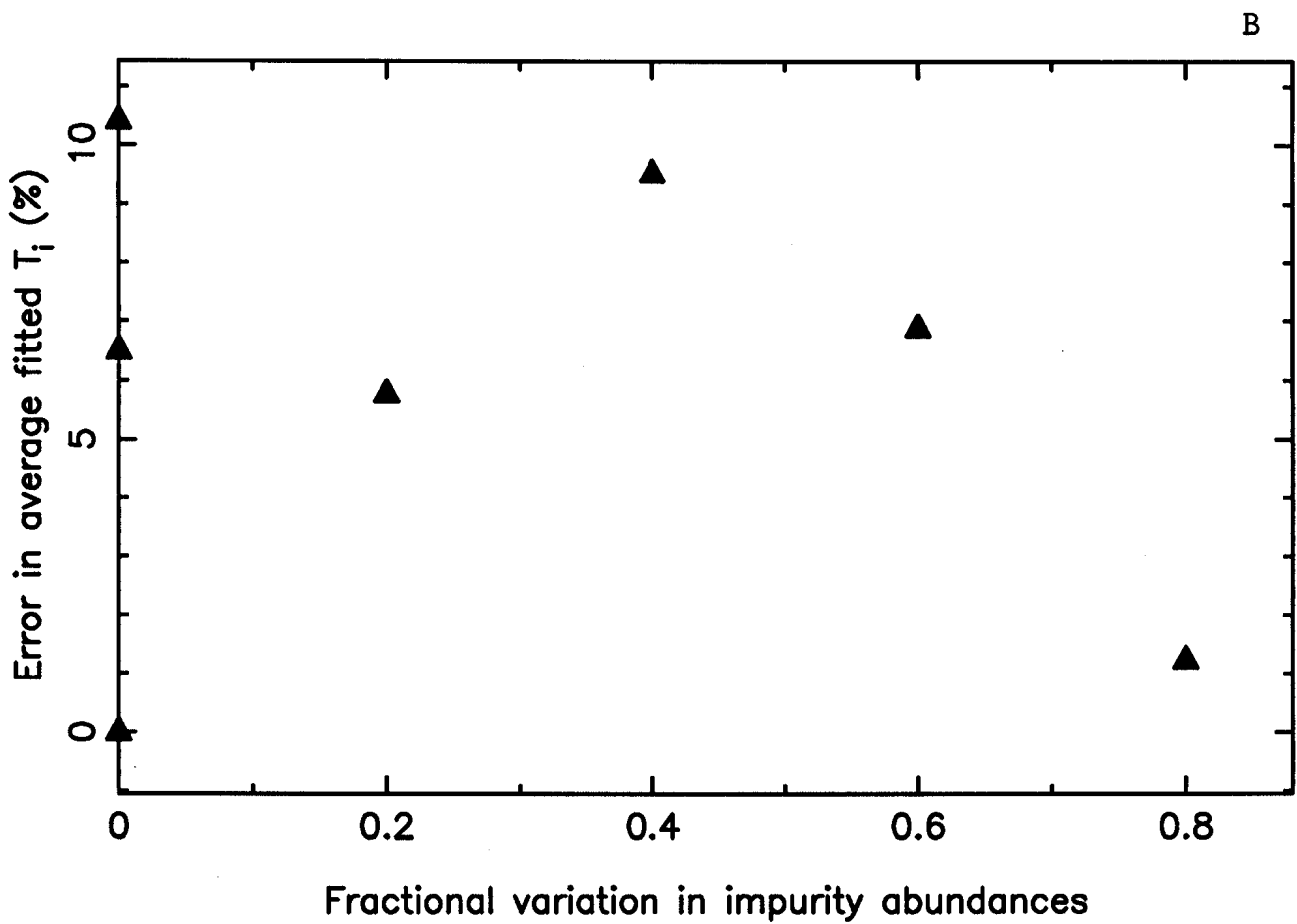
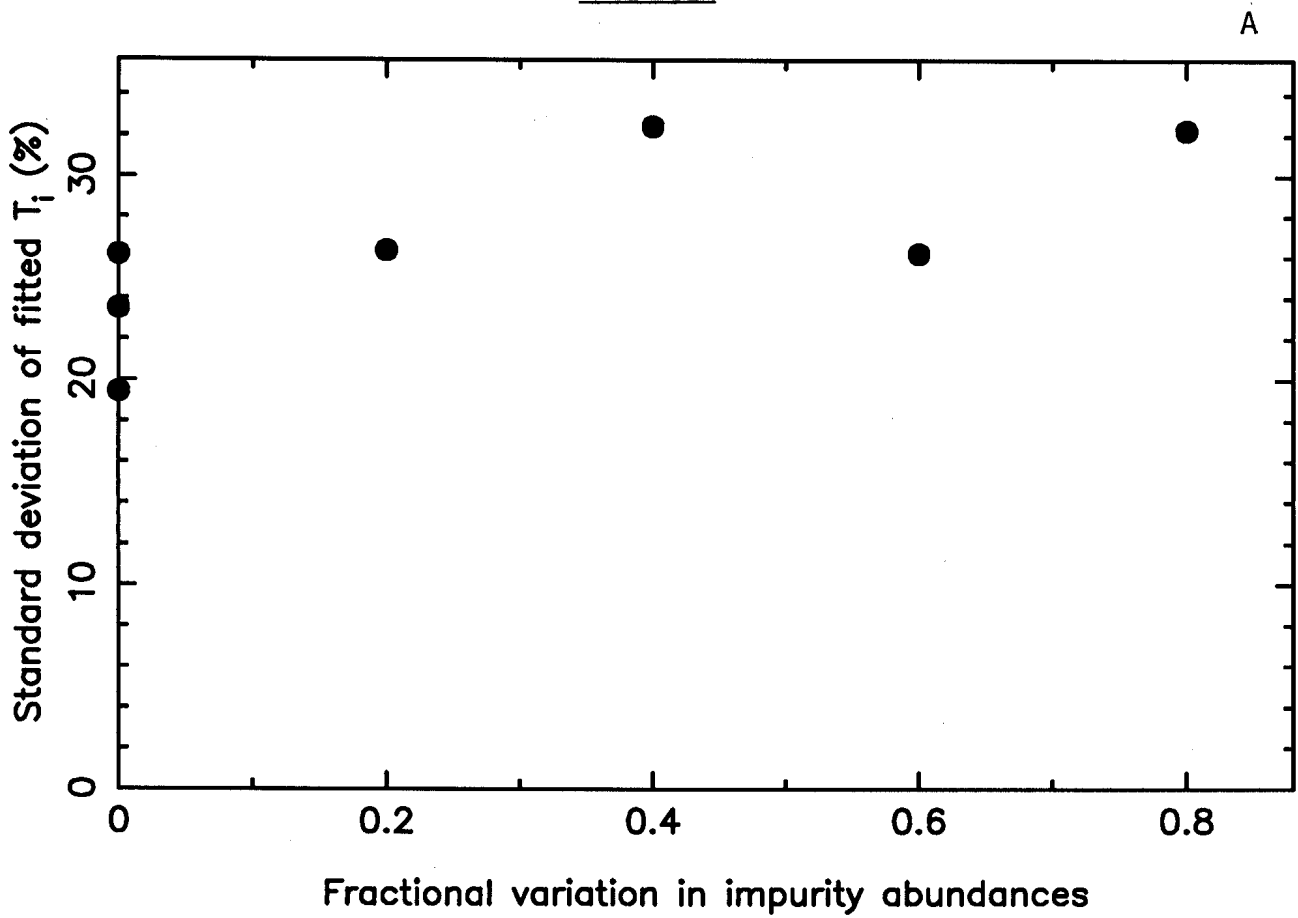
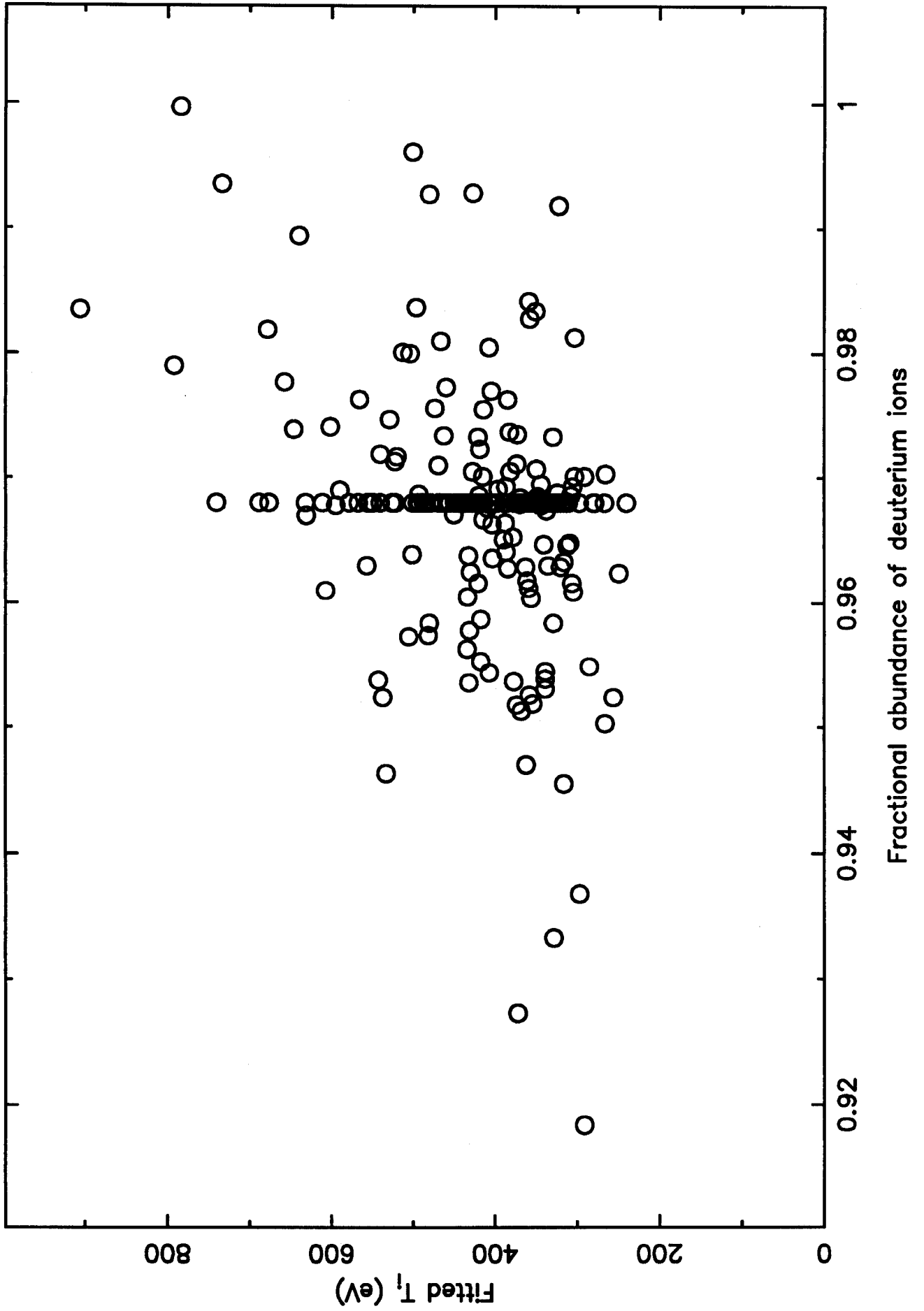


FIG. 13





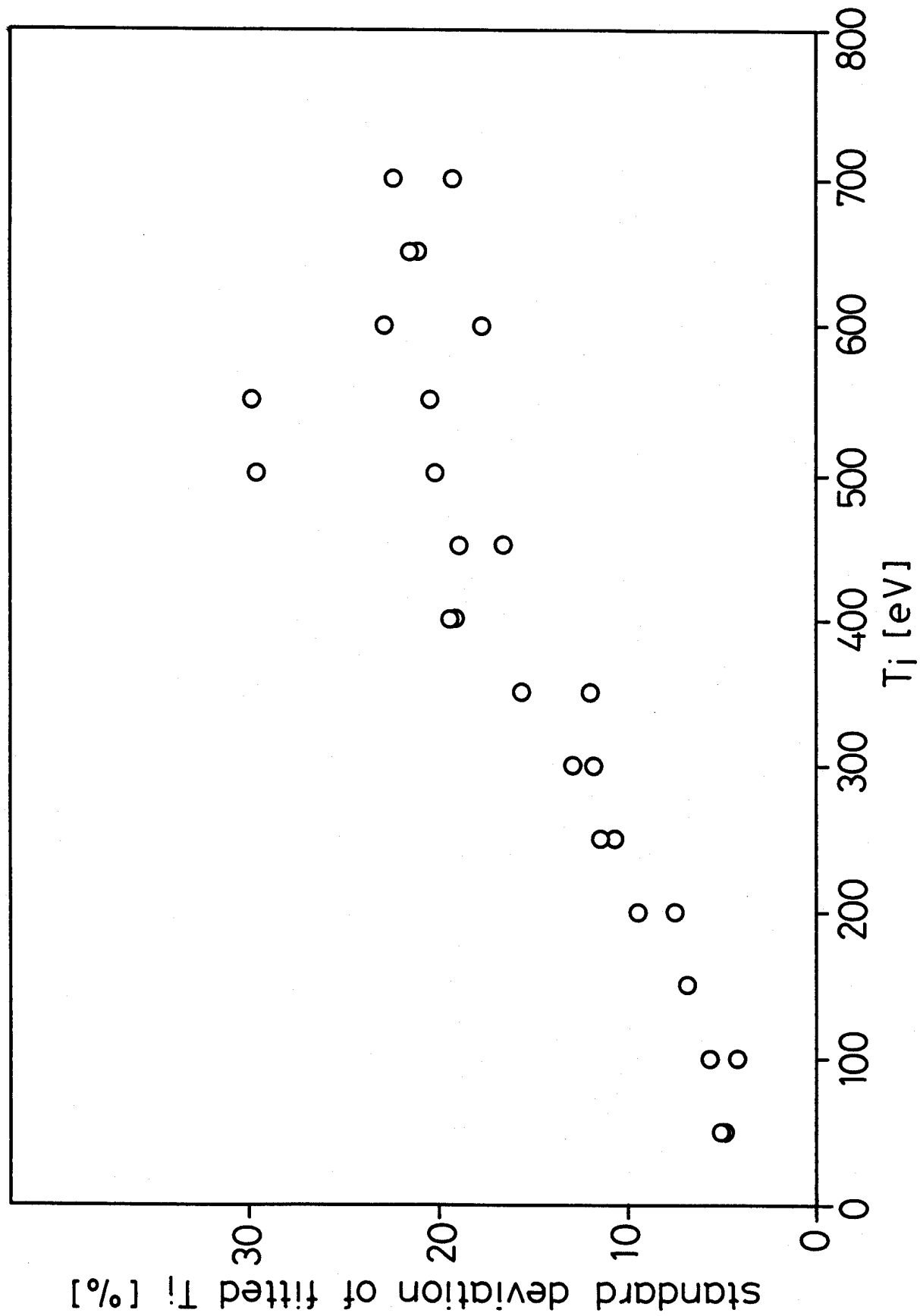


FIG. 14

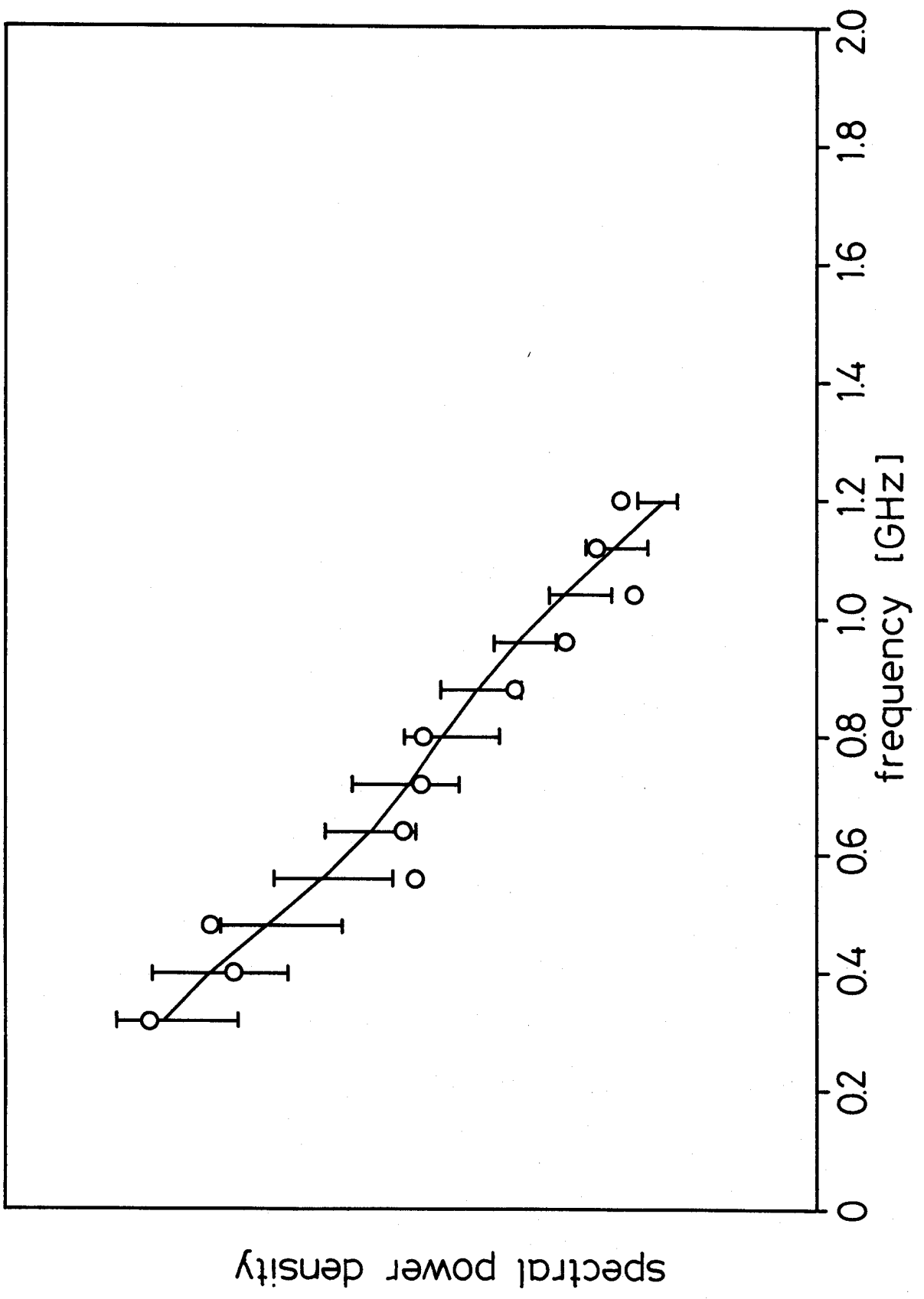


FIG. 15

FIG. 16

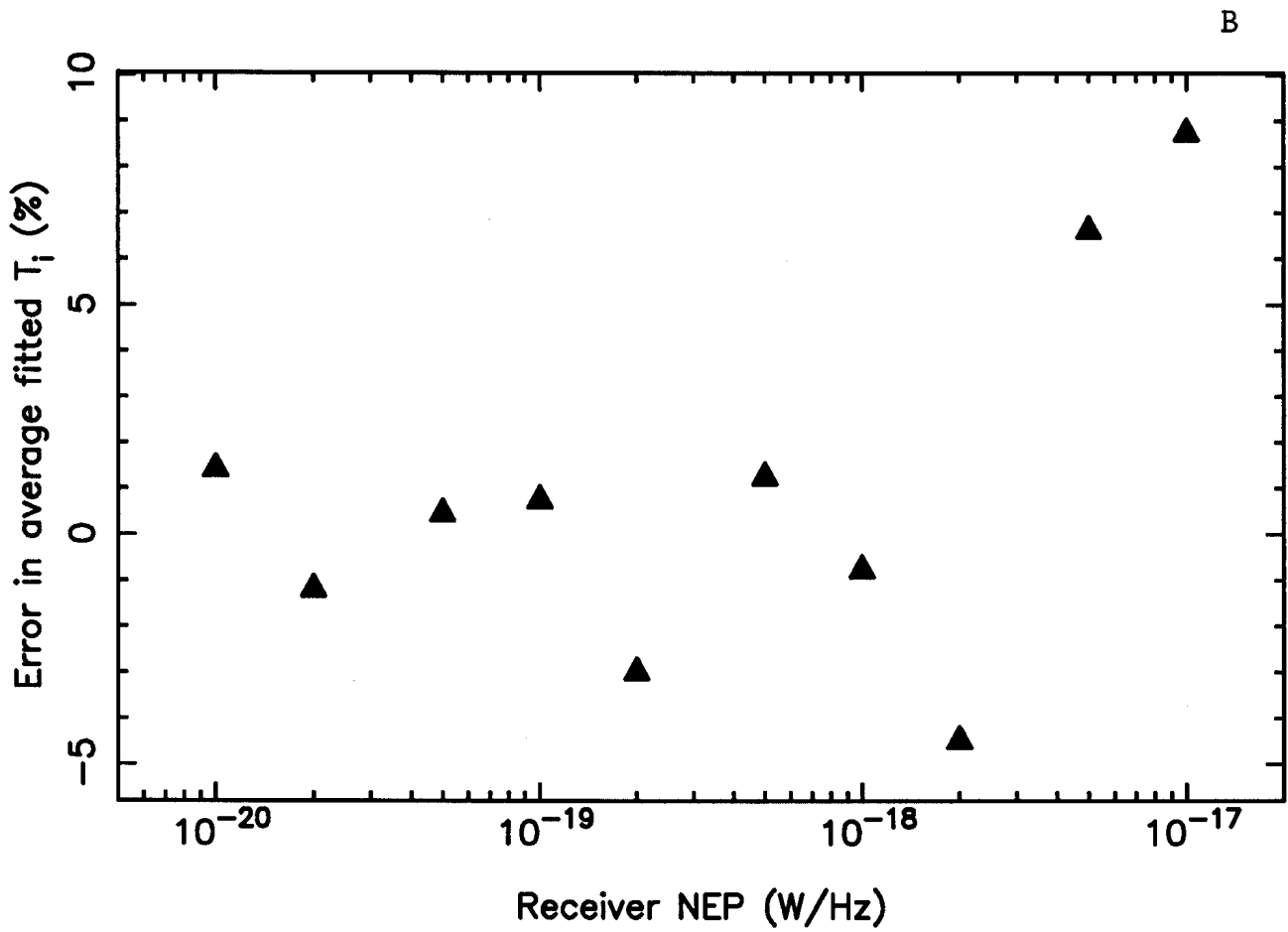
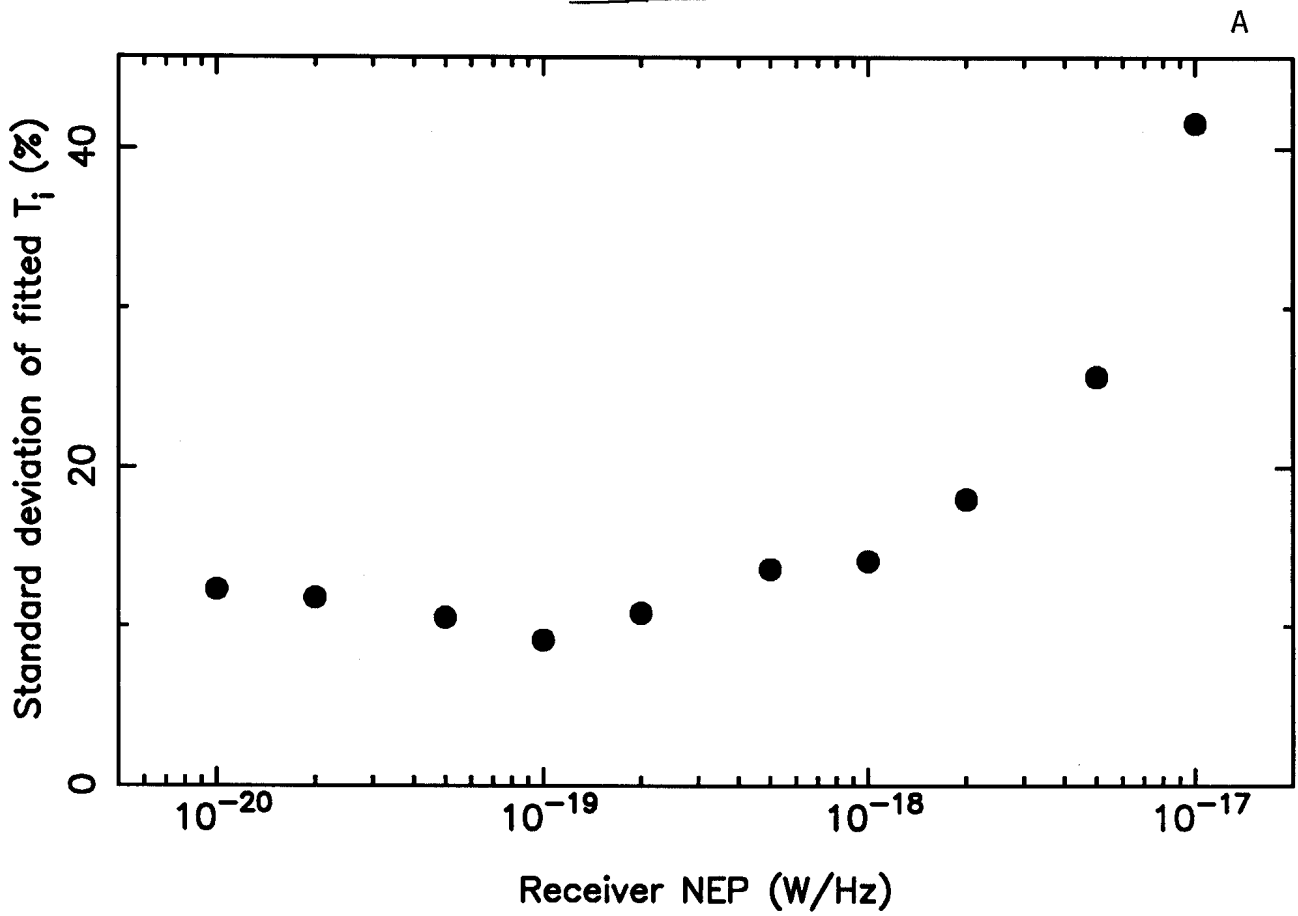


FIG. 17

